Minimization of DFA by Using MyHill-Nerode Theorem

Steps:

1) Draw a table for all pairs of states (P,Q)
2) Mark all pairs where $P \in F$ and $Q \notin F$
3) If there are any Unmarked pairs (P,Q) such that $[\delta(P,x), \delta(Q,x)]$ is marked, then mark [P,Q] where 'x' is an input symbol

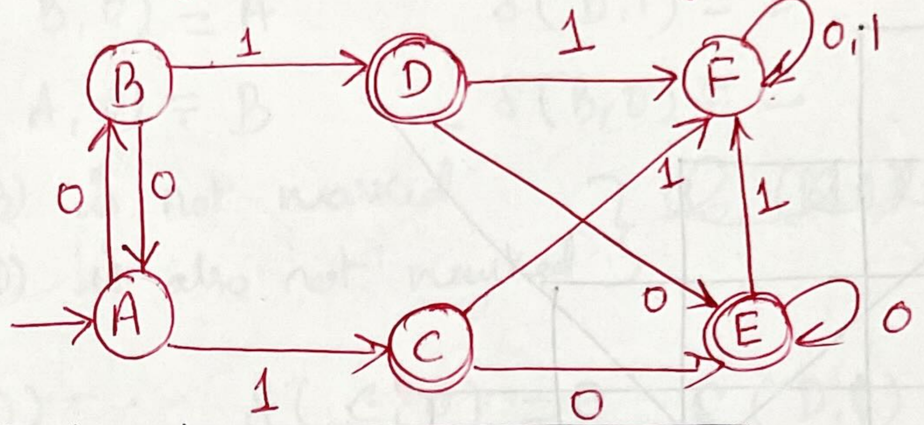
   REPEAT THIS UNTIL NO MORE MARKINGS CAN BE MADE
4) Combine all the Unmarked Pairs and make them a single state in the

Minimization of DFA

Table filling Method

My Hill Nerode Theorem

Q. Minimize the following DFA



Solⁿ:

$F - C, D, E$
$NF - A, B, F$

	0	1
→ A	B	C
B	A	D
*C	E	F
*D	E	F
*E	E	F
F	F	F

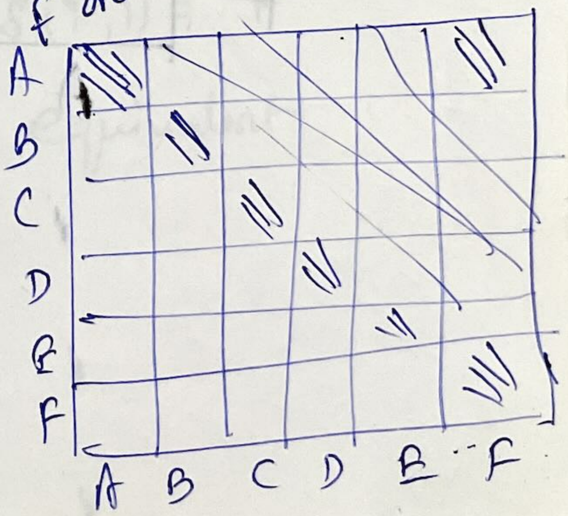
First we have to make two pair of set final and Non final state -

F = { C, D, E } NF = { A, B, F }
Final Nonfinal

A, 0 = B A, 0 = C
B, 0 = A B, 0 = D

	A	B	C	D	E
B					
C	X	X			
D	X	X			
E	X	X			
F			X	X	X

F and NF



$\delta(A,0) =$ $\delta(A,1) =$

$\delta(B,0) =$ $\delta(B,1) =$

B						
C						
D						
E						
F						
G						
	A	B	C	D	E	F

B					
C	X	X			
D	X	X			
E	X	X			
F	X	X	X	X	X
	A	B	C	D	E

Consider the pair (B, A)

$\delta(B, 0) = A$ $\quad$ $\delta(B, 1) = D$

$\delta(A, 0) = B$ $\quad$ $\delta(B, 0) = C$

(A, B) is not marked
(C, D) is also not marked } So, (B, A) not equivalent

(C, D) $\quad$ $\delta(C, 0) = E$ $\quad$ $\delta(D, 1) = F$

$\delta(D, 0) = E$ $\quad$ $\delta(D, 1) = F$

(C, E) $\quad$ $\delta(C, 0) = E$ $\quad$ $\delta(C, 1) = F$

$\delta(E, 0) = E$ $\quad$ $\delta(E, 1) = F$

(D, E) $\quad$ $\delta(D, 0) = E$ $\quad$ $\delta(D, 1) = F$

$\delta(E, 0) = E$ $\quad$ $\delta(E, 1) = F$

(A, F) $\quad$ $\delta(A, 0) = B$ $\quad$ $\delta(A, 1) = C$

$\delta(F, 0) = F$ $\quad$ $\delta(F, 1) = F$

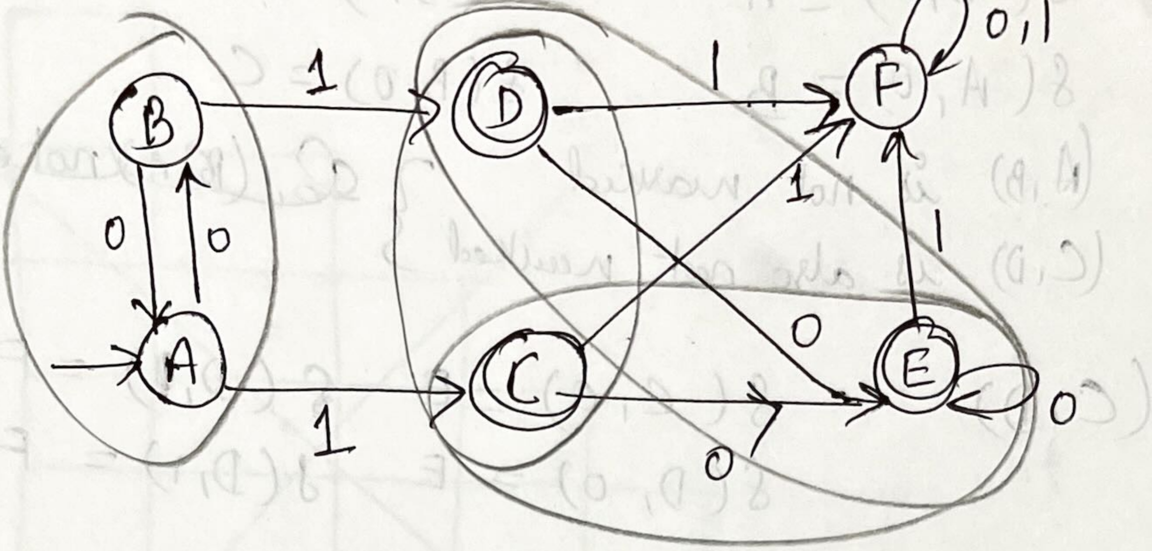
So (A, F) is not equivalent.

(B, F) $\quad$ $\delta(B, 0) = A$ $\quad$ $\delta(B, 1) = D$

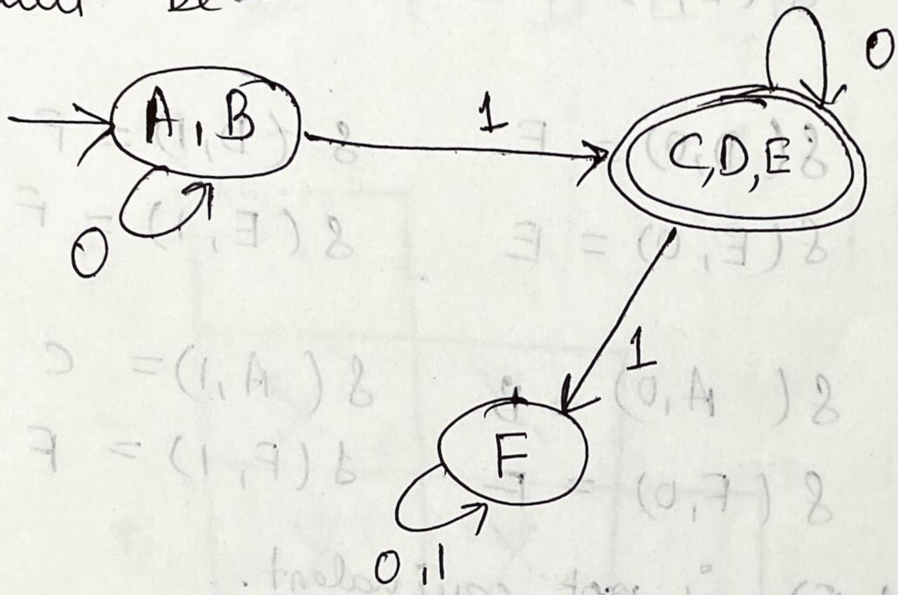
$\delta(F, 0) = F$ $\quad$ $\delta(F, 1) = F$

So, (B, F) is also not equivalent

(A, B) (D, C) (E, C) (E, D)



So, the states for the minimized DFA will be—



Minimized DFA

Minimize the following Automata

	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_4	q_3
q_2	q_4	q_3
$*q_3$	q_5	q_6
$*q_4$	q_7	q_6
q_5	q_3	q_6
q_6	q_6	q_6
q_7	q_4	q_6

Solⁿ:- Step 1:- Two set of final and Non final states:-

$P = \{q_3\ q_4\}$ $\quad Q = \{q_0\ q_1\ q_2\ q_5\ q_6\ q_7\}$

Mark all the pairs of final and Non final states -

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_1							
q_2							
q_3	X	X	X				
q_4	X	X	X				
q_5				X	X		
q_6				X	X		
q_7				X	X		

Step 2:- Consider the pair (q_0, q_1)

$\delta(q_0, a) = q_1 \qquad \delta(q_0, b) = q_2$

$\delta(q_1, a) = q_4 \qquad \delta(q_1, b) = q_3$

$\Rightarrow (q_0\ q_1)$ is non equivalent.

pair (q_0, q_2)

$\delta(q_0, a) = q_1 \qquad \delta(q_0, b) = q_2$

$\delta(q_2, a) = q_4 \qquad \delta(q_2, b) = q_3$

$\Rightarrow (q_0\ q_2)$ is non equivalent.

(q_1, q_2)

$\delta(q_1, a) = q_4 \qquad \delta(q_1, b) = q_3$

$\delta(q_2, a) = q_4 \qquad \delta(q_2, b) = q_3$

$\Rightarrow q_1 q_2$ is equivalent.

(q_3, q_4)

$\delta(q_3, a) = q_5 \qquad \delta(q_3, b) = q_6$

$\delta(q_4, a) = q_7 \qquad \delta(q_4, b) = q_6$

$\Rightarrow q_3 q_4$ is equivalent.

(q_0, q_5)

$\delta(q_0, a) = q_1 \qquad \delta(q_0, b) = q_2$

$\delta(q_5, a) = q_3 \qquad \delta(q_5, b) = q_6$

$\Rightarrow (q_0, q_5)$ is non equivalent

(q_1, q_5)

$\delta(q_1, a) = q_4 \qquad \delta(q_1, b) = q_3$

$\delta(q_5, a) = q_3 \qquad \delta(q_5, b) = q_6$

$\Rightarrow (q_1 q_5)$ is non equivalent

(q_2, q_5)

$\delta(q_2, a) = q_4 \qquad \delta(q_2, b) = q_3$

$\delta(q_5, a) = q_3 \qquad \delta(q_5, b) = q_6$ $\Rightarrow (q_2 q_5)$ is non equivalent

(q_0, q_6)

$\delta(q_0, a) = q_1 \qquad \delta(q_0, b) = q_2$

$\delta(q_6, a) = q_6 \qquad \delta(q_6, b) = q_6$ $\Rightarrow (q_0, q_6)$ is equivalent

(q_1, q_6)

$\delta(q_1, a) = q_4 \qquad \delta(q_1, b) = q_3$

$\delta(q_6, a) = q_6 \qquad \delta(q_6, b) = q_6$ $\Rightarrow (q_1, q_6)$ is non equivalent.

(q_2, q_6)

$\delta(q_2, a) = q_4 \qquad \delta(q_2, b) = q_3$

$\delta(q_6, a) = q_6 \qquad \delta(q_6, b) = q_6$ $\Rightarrow (q_2, q_6)$ is non equivalent

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_1	X						
q_2	X						
q_3	X	X	X				
q_4	X	X	X				
q_5	X	X	X	X	X		
q_6		X	X	X	X	X	
q_7	X	X	X	X	X		X

(q_5, q_6)

$\left.\begin{array}{ll} \delta(q_5, a) = q_3 & \delta(q_5, b) = q_6 \\ \delta(q_6, a) = q_6 & \delta(q_6, b) = q_6 \end{array}\right\} \Rightarrow (q_5, q_6)$ non equivalent

(q_0, q_7)

$\delta(q_0, a) = q_1 \quad \delta(q_0, b) = q_2 \Rightarrow (q_0 q_7)$ non equivalent

$\delta(q_7, a) = q_4 \quad \delta(q_7, b) = q_6$

(q_1, q_7)

$\delta(q_1, a) = q_4 \quad \delta(q_1, b) = q_3 \Rightarrow (q_1, q_7)$ non equivalent

$\delta(q_7, a) = q_4 \quad \delta(q_7, b) = q_6$

(q_2, q_7)

$\delta(q_2, a) = q_4 \quad \delta(q_2, b) = q_3 \Rightarrow (q_2, q_7)$ non equivalent

$\delta(q_7, a) = q_4 \quad \delta(q_7, b) = q_6$

(q_5, q_7)

$\delta(q_5, a) = q_3 \quad \delta(q_5, b) = q_6 \Rightarrow (q_5 q_7)$ equivalent

$\delta(q_7, a) = q_4 \quad \delta(q_7, b) = q_6$

(q_6, q_7)

$\left.\begin{array}{ll} \delta(q_6, a) = q_6 & \delta(q_6, b) = q_6 \\ \delta(q_7, a) = q_4 & \delta(q_7, b) = q_6 \end{array}\right\} \Rightarrow (q_6, q_7)$ non equivalent

Step 3:- Remaining Unmarked pairs

(q_1, q_2) $\quad \delta(q_1, a) = q_4 \quad \delta(q_1, b) = q_3$

$\quad \delta(q_2, a) = q_4 \quad \delta(q_2, b) = q_3$

$(q_1 q_2)$ equivalent

(q_3, q_4) $\quad \delta(q_3, a) = q_5 \quad \delta(q_3, b) = q_6$

$\quad \delta(q_4, a) = q_7 \quad \delta(q_4, b) = q_6$

$(q_3 q_4)$ equivalent

(q_0, q_6) $\quad \delta(q_0, a) = q_1 \quad \delta(q_0, b) = q_2$

$\quad \delta(q_6, a) = q_6 \quad \delta(q_6, b) = q_6$

$\Rightarrow q_0 q_6$ non equivalent

(q_5, q_7) $\quad \delta(q_5, a) = q_3 \quad \delta(q_5, b) = q_6$

$\quad \delta(q_7, a) = q_4 \quad \delta(q_7, b) = q_6$

(q_5, q_7) equivalent

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_1	X						
q_2	X						
q_3	X	X	X				
q_4	X	X	X				
q_5	X	X	X	X	X		
q_6	X	X	X	X	X	X	
q_7	X	X	X	X	X		X

Step 4:- Remaining Pairs

$(q_1 q_2) \Rightarrow$ equivalent

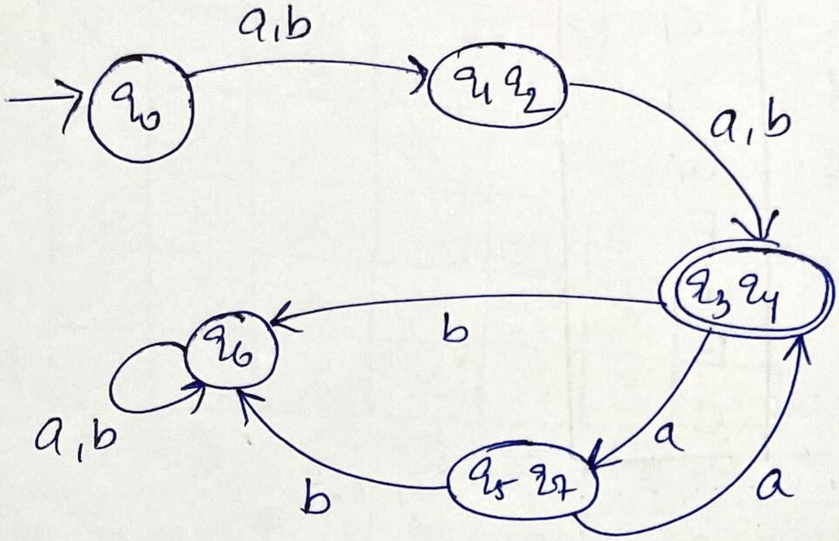
$(q_3 q_4) \Rightarrow$ equivalent

$(q_5 q_7) \Rightarrow$ equivalent

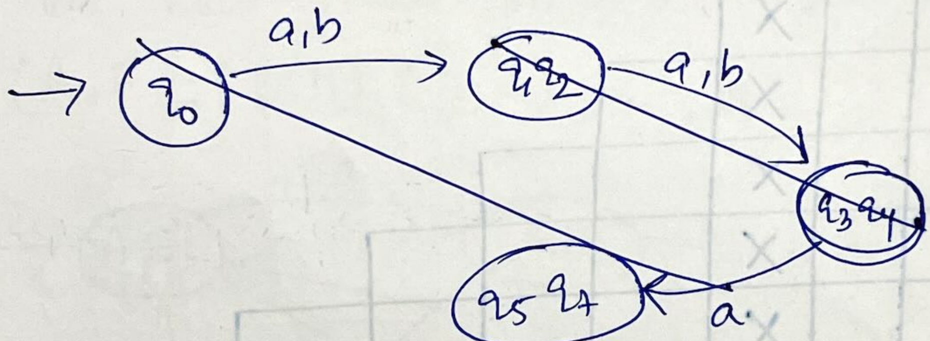
No mark will be added in the existing table.

Step 5: New states for the minimized DFA are:-

$\{q_0\}$ $\{q_1\,q_2\}$ $\{q_3\,q_4\}$ $\{q_5\,q_7\}$ $\{q_6\}$



~~q_6 will get eliminated~~



Q:- Minimize the following DFA :-

States	a	b
→ A	B	F
B	G	C
* C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

Solⁿ:-

Step 1:-

	A	B	C	D	E	F	G
B							
C	X	X					
D			X				
E			X				
F			X				
G			X				
H			X				

Step 2:-

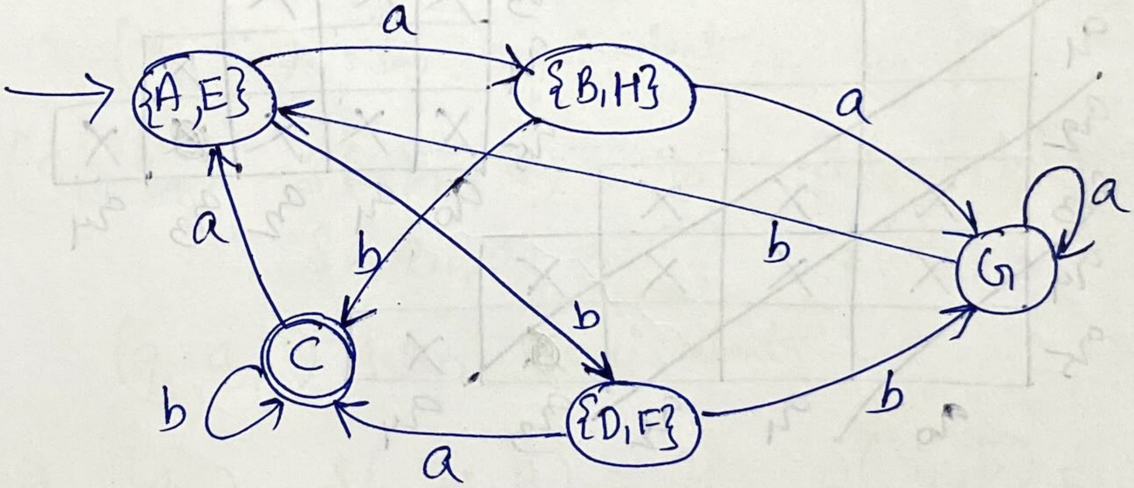
	A	B	C	D	E	F	G
B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G	X	X	X	X	X	X	
H	X		X	X	X	X	X

B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G	X	X	X	X	X	X	
H	X		X	X	X	X	X
	A	B	C	D	E	F	G

Non marked states can't be marked as they are equivalent.

So, the new states in minimized DFA,

(A, E), (B, H), (D, F), C, G



Minimized DFA

Q: Minimize the following DFA,

	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_4
$*q_3$	q_5	q_5
q_4	q_3	q_3
$*q_5$	q_5	q_5

Soln: $P = \{q_3, q_5\}$ $Q = \{q_0, q_1, q_2, q_4\}$

Step 1: Marking all the pair of final and nonfinal States.

	q_0	q_1	q_2	q_3	q_4
q_1					
q_2					
q_3	X	X	X		
q_4	X	X	X	X	
q_5				X	X

	q_0	q_1	q_2	q_3	q_4
q_1					
q_2					
q_3	X	X	X		
q_4				X	
q_5	X	X	X	X	X

Step 2: Unmarked Pairs are –

(q_0, q_1) $\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_3$

$\delta(q_1, a) = q_0$ $\delta(q_1, b) = q_3$

$q_0\, q_1$ – equivalent

(q_0, q_2) $\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_3$

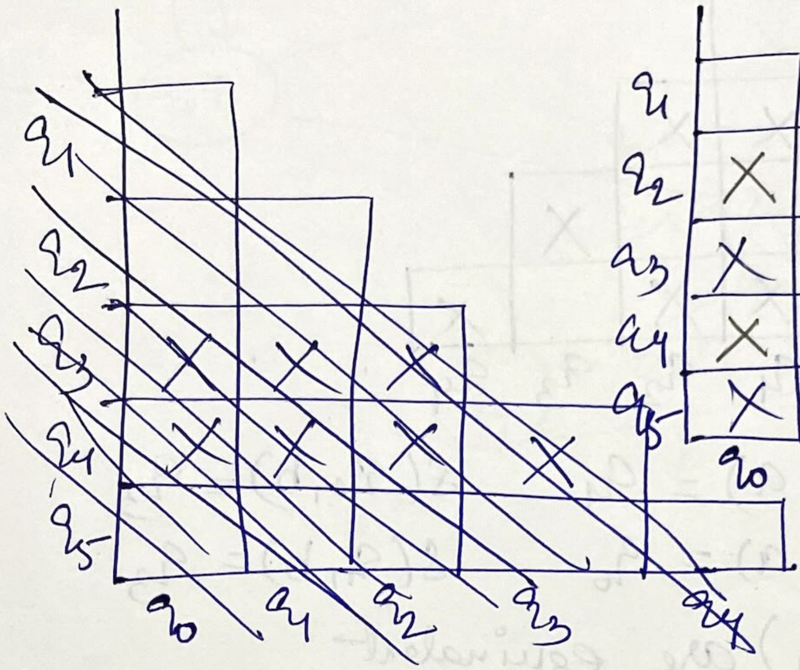
$\delta(q_2, a) = q_1$ $\delta(q_2, b) = q_4$

(q_0, q_2) Non equivalent

(q_1, q_2) $\delta(q_1, a) = q_0$ $\delta(q_1, b) = q_3$

$\delta(q_2, a) = q_1$ $\delta(q_2, b) = q_4$

(q_1, q_2) Non equivalent



q_1					
q_2	X	X			
q_3	X	X	X		
q_4	X	X	X	X	
q_5	X	X	X		X
	q_0	q_1	q_2	q_3	q_4

(q_0, q_4) $\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_3$

$\delta(q_4, a) = q_3$ $\delta(q_4, b) = q_3$

$(q_0\, q_4)$ Non equivalent

(q_1, q_4) $\delta(q_1, a) = q_0$ $\delta(q_1, b) = q_3$

$\delta(q_4, a) = q_3$ $\delta(q_4, b) = q_3$

(q_1, q_4) Non equivalent

(q_2, q_4) $\delta(q_2, a) = q_1$ $\delta(q_2, b) = q_4$

$\delta(q_4, a) = q_3$ $\delta(q_4, b) = q_3$

(q_2, q_4) Non equivalent

(q_3, q_5)

$\delta(q_3, a) = q_5$ $\delta(q_3, b) = q_5$

$\delta(q_5, a) = q_5$ $\delta(q_5, b) = q_5$

(q_3, q_5) equivalent

Step 3: Check for the remaining unmarked pairs (q_0, q_1) and (q_3, q_5)

q_1					
q_2	X	X			
q_3	X	X	X		
q_4	X	X	X	X	
q_5	X	X	X		X
	q_0	q_1	q_2	q_3	q_4

(q_0, q_1) $\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_3$

$\delta(q_1, a) = q_0$ $\delta(q_1, b) = q_3$.

(q_0, q_1) are equivalent

(q_3, q_5) $\delta(q_3, a) = q_5$ $\delta(q_3, b) = q_5$

$\delta(q_5, a) = q_5$ $\delta(q_5, b) = q_5$

(q_3, q_5) are equivalent.

Step 4: All the unmarked pairs are :-

$\{q_0, q_1\}$ $\{q_3, q_5\}$

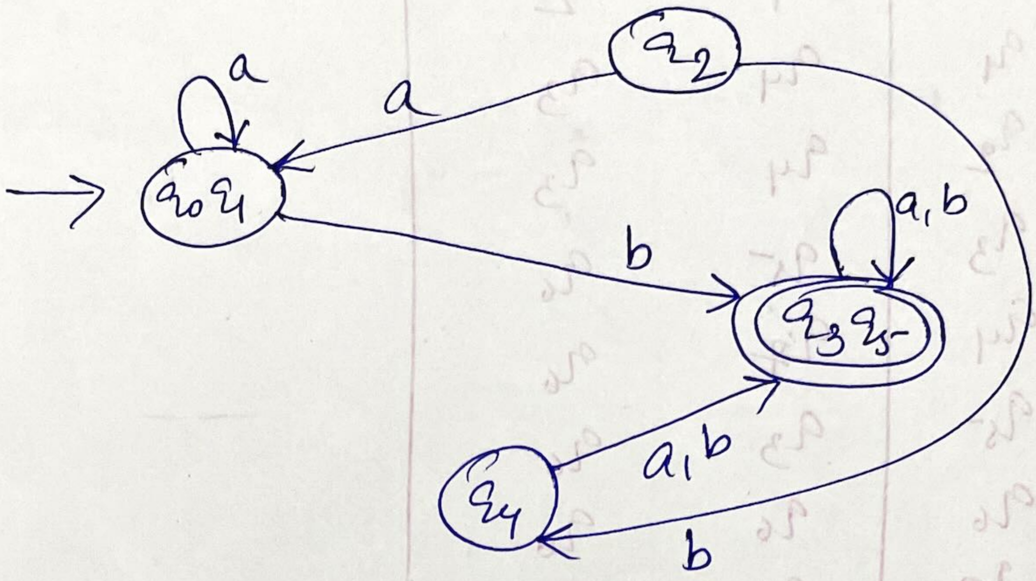
So, the states of the minimized DFA are -

$\{q_0, q_1\}$ q_2 $\{q_3, q_5\}$ q_4.

Transition Table

	a	b
$\rightarrow \{q_0 q_1\}$	$q_0 q_1$	q_3, q_5
q_2	$q_0 q_1$	q_4
$* \{q_3 q_5\}$	$q_3 q_5$	$q_3 q_5$
q_4	$q_3 q_5$	$q_3 q_5$

Transition Diagram of Minimized DFA



Minimization of DFA

Q:- Minimize the states of the automata given in the following table.

State	I/P a	I/P b
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
$*q_6$	q_1	q_3

Solⁿ:- We have to prepare the chart for the pairs of the states :-

Step 1:-

	q_0	q_1	q_2	q_3	q_4	q_5
q_1						
q_2						
q_3						
q_4						
q_5						
$*q_6$	X	X	X	X	X	X

Initially, we will mark 'x' to all pairs of final and nonfinal states

eg:- $(q_0\, q_6)$ $(q_1\, q_6)$ $(q_2\, q_6)$ $(q_3\, q_6)$ $(q_4\, q_6)$ $(q_5\, q_6)$

Step 2:- Now, we will consider the remaining pairs of the table. The pairs which are marked by 'x' are not equivalent.

Consider, the pair (q_4, q_5)

$\delta(q_4, a) = q_0$ $\quad\quad$ $\delta(q_4, b) = q_6$

$\delta(q_5, a) = q_1$ $\quad\quad$ $\delta(q_5, b) = q_4$

The pair (q_4, q_5) is not equivalent bcoz $(q_4\ q_5)$ is not equivalent on b.

(q_3, q_5) $\quad$ $\left.\begin{array}{ll} \delta(q_3, a) = q_0 & \delta(q_3, b) = q_5 \\ \delta(q_5, a) = q_1 & \delta(q_5, b) = q_4 \end{array}\right\}$

So, $(q_3\ q_5)$ is not equivalent

q_1	X					
q_2	X	X				
q_3	X	X	X			
q_4	X	X	X	X		
q_5	X	X	X	X	X	
q_6	X	X	X	X	X	X
	q_0	q_1	q_2	q_3	q_4	q_5

As, we find no states equivalent, we cannot remove any of them. Hence, the given transition table itself is the minimum state automaton.

The pairs (A, E), (B, H) and (D, F) are not marked and hence considered indistinguishable. and the States G and C are Called distinguishable.

So, the different groups can be represented as the states of minimized DFA:

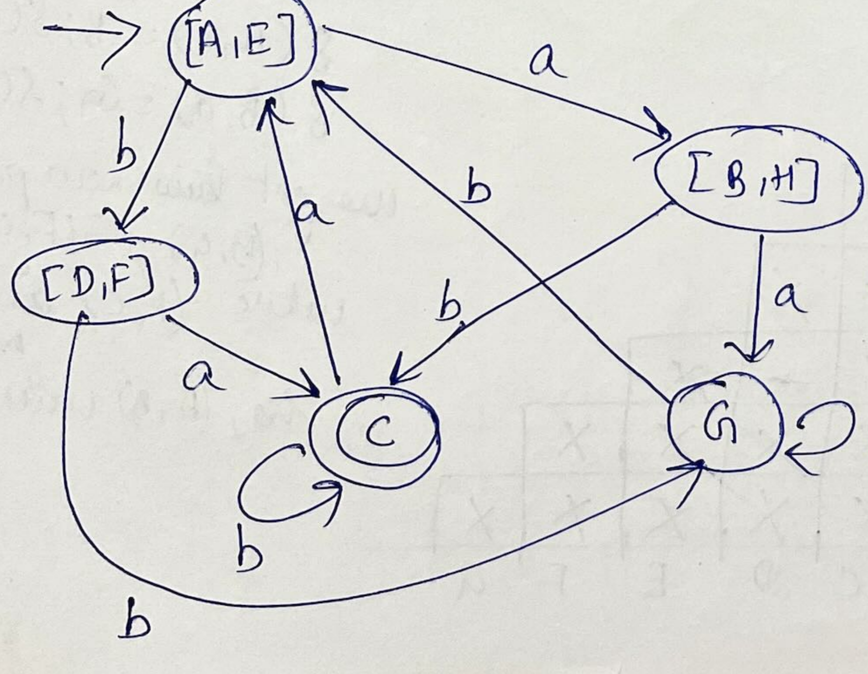
(A, E) (B, H), C, (D, F), G.

Transition table for minimized DFA:-

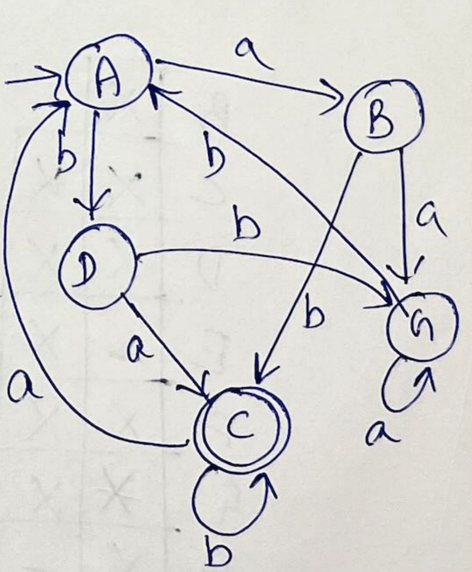
States	I/P Symbol a	b
→(A, E)	(B, H)	(D, F)
(B, H)	G	C
*C	(A, E)	C
(D, F)	C	G
G	G	(A, E)

A = E
B = H
*C
D = F
G

Transition diagram for the minimized DFA:-



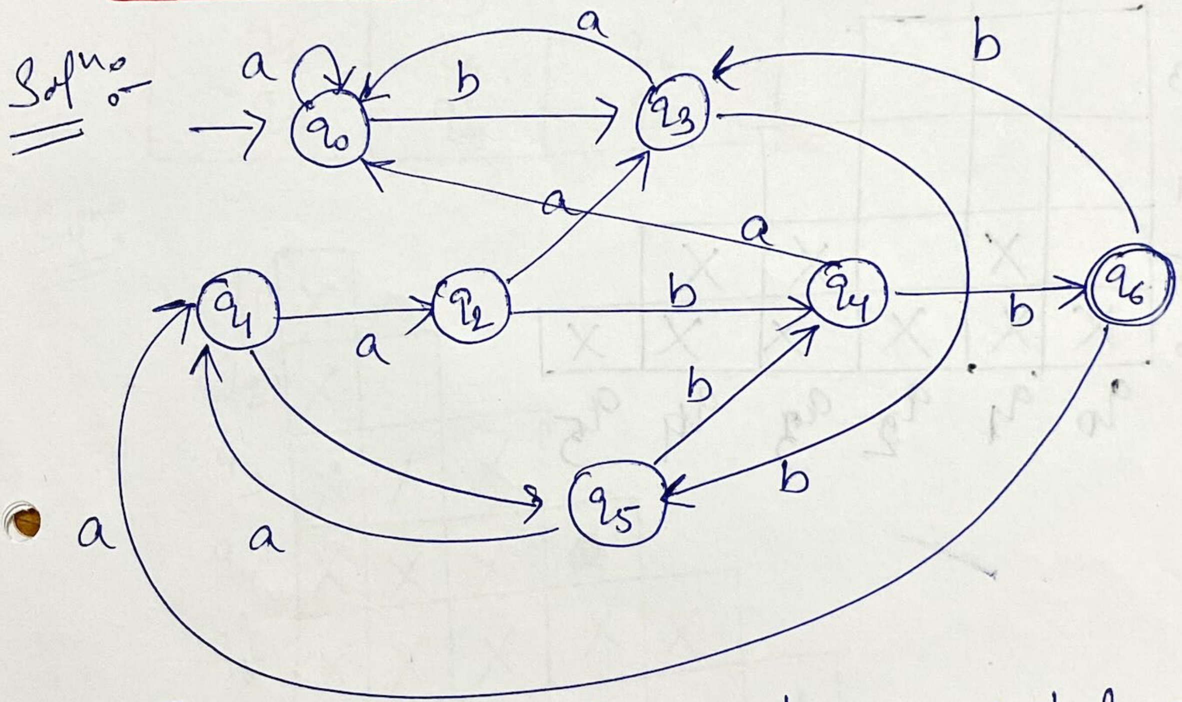
or



Q:- Minimize the states of the automata given in the following table:-

State	Input a	Input b
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
$*q_6$	q_1	q_3

Sol^n:-



The pair of states can be represented as:-

	q_0	q_1	q_2	q_3	q_4	q_5
q_1	X					
q_2	X	X				
q_3	X	X	X			
q_4	X	X	X	X		
q_5	X	X	X	X	X	
q_6	X	X	X	X	X	X

We can't find any states equivalent. So, we can't remove any of them. It's already minimized DFA.

i)

q_1						
q_2						
q_3						
q_4						
q_5						
q_6	X	X	X	X	X	X
	q_0	q_1	q_2	q_3	q_4	q_5

ii)

q_1						
q_2						
q_3						
q_4						
q_5		X		X	X	
q_6	X	X	X	X	X	X
	q_0	q_1	q_2	q_3	q_4	q_5

Q:- Construct a minimum state automata equivalent to a DFA whose transition table is defined as follows:-

State/Σ	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_4	q_3
q_2	q_4	q_3
$*q_3$	q_5	q_6
$*q_4$	q_7	q_6
q_5	q_3	q_6
q_6	q_6	q_6
q_7	q_4	q_6

Soln:-

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_1	X						
q_2	X						
q_3	X	X	X				
q_4	X	X	X	X			
q_5	X	X	X	X	X		
q_6		X	X	X	X	X	
q_7	X	X	X	X	X	X	X

Minimize the following automata.

	0	1
→ A	B	E
B	C	F
* C	D	H
D	E	H
E	F	I
* F	G	B
G	H	B
H	I	C
* I	A	E

Solⁿ:

	A	B	C	D	E	F	G	H
B	X							
C	X	X						
D		X	X					
E	X	–	X	X				
F	X	X	–	X	X			
G		X	X	=	X	X		
H	X	–	X	X	–	X	X	
I	X	X	–	X	X	–	X	X

So, from the table we will have,

A = G = D
B = H = E
C = F = I

	0	1
→ A	B	B
B	C	C
* C	A	B

Find minimum state automaton equivalent to the following automata:-

	0	1
→ a	b	a
b	a	c
c	d	b
* d	d	a
e	d	f
f	g	e
g	f	g
h	g	d

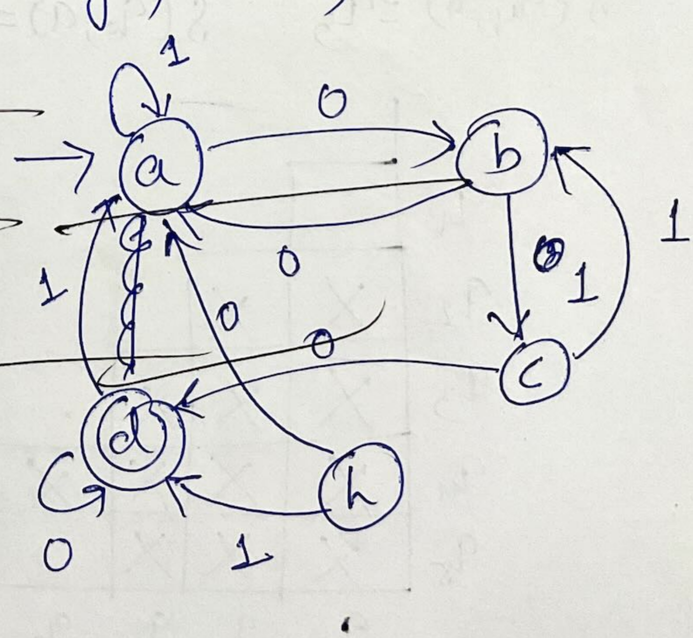
Soln:-

	a	b	c	d	e	f	g
b	X						
c	X	X					
d	X	X	X				
e	X	X		X			
f	X		X	X	X		
g		X	X	X	X	X	
h	X	X	X	X	X	X	X

~~⇒ No minimization~~

~~So, here we have a = g ; b = f ; c = e~~

	0	1
→ a	b	a
b	a	c
c	d	b
* d	d	a
h	a	d



Q:- Minimize the following DFA where q_0 is start state and q_3 and q_5 are final state.

	a	b
→ q_0	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_4
* q_3	q_5	q_5
q_4	q_3	q_3
* q_5	q_5	q_5

Solⁿ:-

q_1					
q_2					
* q_3	X	X	X		
q_4				X	
* q_5	X	X	X		X
	q_0	q_1	q_2	q_3	q_4

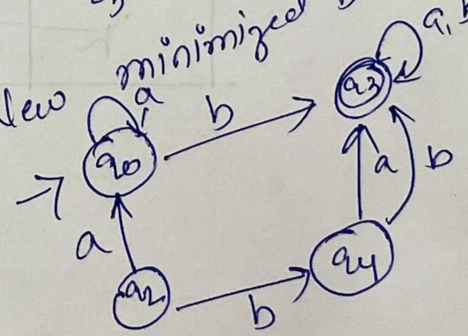
$\delta(q_0, a) = q_1 \quad \delta(q_0, b) = q_3$

$\delta(q_4, a) = q_3 \quad \delta(q_0, a) = q_1$

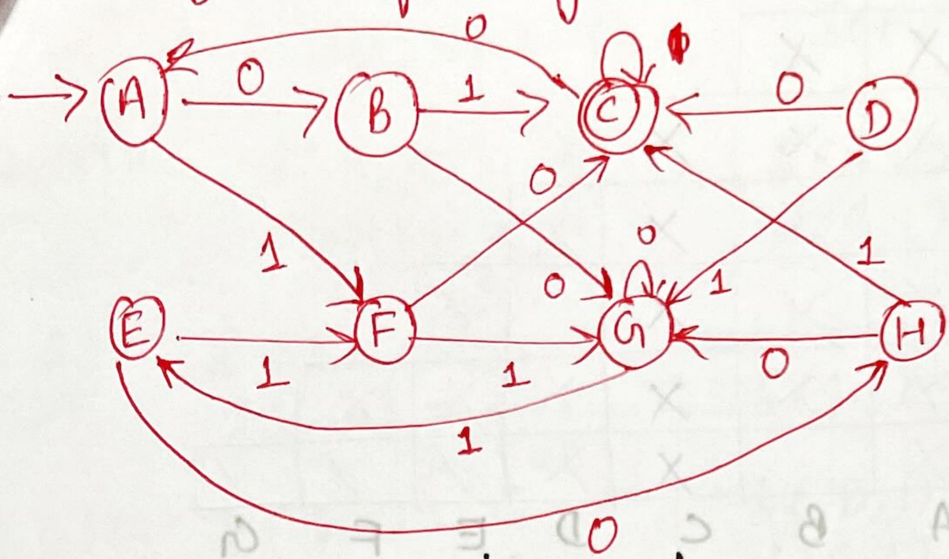
q_1					
q_2	X	X			
q_3	X	X	X		
q_4	X	X	X	X	
q_5	X	X	X		X
	q_0	q_1	q_2	q_3	q_4

$\therefore q_0 = q_1$
$q_3 = q_5$

New minimized DFA



Minimize the following DFA:-



Solⁿ:

	0	1
→ A	B	F
B	G	C
* C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

F = C

NF = A B D E F G H

Step 1:

	A	B	C	D	E	F	G
B							
C	X	X					
D			X				
E			X				
F			X				
G			X				
H			X				

Step 2:

	A	B	C	D	E	F	G
B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G	X	X	X	X	X	X	
H	X		X	X	X	X	X

Consider the pair (A, B) :-

$\delta(A, 0) = B$ $\qquad \delta(A, 1) = F$

$\delta(B, 0) = G$ $\qquad \delta(B, 1) = C$

(A, D) :-
$\delta(A, 0) = B$ $\qquad \delta(A, 1) = F$
$\delta(D, 0) = C$ $\qquad \delta(D, 1) = G$

(B, D) :-
$\delta(B, 0) = G$ $\qquad \delta(B, 1) = C$
$\delta(D, 0) = C$ $\qquad \delta(D, 1) = G$

(A, E) :-
$\delta(A, 0) = B$ $\qquad \delta(A, 1) = F$
$\delta(E, 0) = H$ $\qquad \delta(E, 1) = F$

(B, E) :-
$\delta(B, 0) = G$ $\qquad \delta(B, 1) = C$
$\delta(E, 0) = H$ $\qquad \delta(E, 1) = F$

(D, E) :-
$\delta(D, 0) = C$ $\qquad \delta(D, 1) = G$
$\delta(E, 0) = H$ $\qquad \delta(E, 1) = F$

(A, F) :-
$\delta(A, 0) = B$ $\qquad \delta(A, 1) = F$
$\delta(F, 0) = C$ $\qquad \delta(F, 1) = G$

(B, F) :-
$\delta(B, 0) = G$ $\qquad \delta(B, 1) = C$
$\delta(F, 0) = C$ $\qquad \delta(F, 1) = G$

(D, F) :-
$\delta(D, 0) = C$ $\qquad \delta(D, 1) = G$
$\delta(F, 0) = C$ $\qquad \delta(F, 1) = G$

(E, F) :- $\delta(E,0) = H$ $\delta(E,1) = F$
$\delta(F,0) = C$ $\delta(F,1) = G$

(A, G) :- $\delta(A,0) = B$, $\delta(A,1) = F$
$\delta(G,0) = G$ $\delta(G,1) = E$

(B, G) :- $\delta(B,0) = G$ $\delta(B,1) = C$
$\delta(G,0) = G$ $\delta(G,1) = E$

(D, G) :- $\delta(D,0) = C$ $\delta(D,1) = G$
$\delta(G,0) = G$ $\delta(G,1) = E$

(E, G) :- $\delta(E,0) = H$ $\delta(E,1) = F$
$\delta(G,0) = G$ $\delta(G,1) = E$

(F, G) :- $\delta(F,0) = C$ $\delta(F,1) = G$
$\delta(G,0) = G$ $\delta(G,1) = E$

(A, H) :- $\delta(A,0) = B$ $\delta(A,1) = F$
$\delta(H,0) = G$ $\delta(H,1) = C$

(B, H) :- $\delta(B,0) = G$ $\delta(B,1) = C$
$\delta(H,0) = G$ $\delta(H,1) = C$

(D, H) :- $\delta(D,0) = C$ $\delta(D,1) = G$
$\delta(H,0) = G$ $\delta(H,1) = C$

(E, H) :- $\delta(E,0) = H$ $\delta(E,1) = F$
$\delta(H,0) = G$ $\delta(H,1) = C$

(F, H) :- $\delta(F,0) = C$ $\delta(F,1) = G$
$\delta(H,0) = G$ $\delta(H,1) = C$

(G, H) :- $\delta(G,0) = G$ $\delta(G,1) = E$
$\delta(H,0) = G$ $\delta(H,1) = C$

Step 3:-

	A	B	C	D	E	F	G
B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G	X	X	X	X	X	X	
H	X		X	X	X	X	X

(A,E):- $\delta(A,0) = B$ $\quad \delta(A,1) = F$
$\delta(E,0) = H$ $\quad \delta(E,1) = F$

(D,F):- $\delta(D,0) = C$ $\quad \delta(D,1) = G$
$\delta(F,0) = C$ $\quad \delta(F,1) = G$

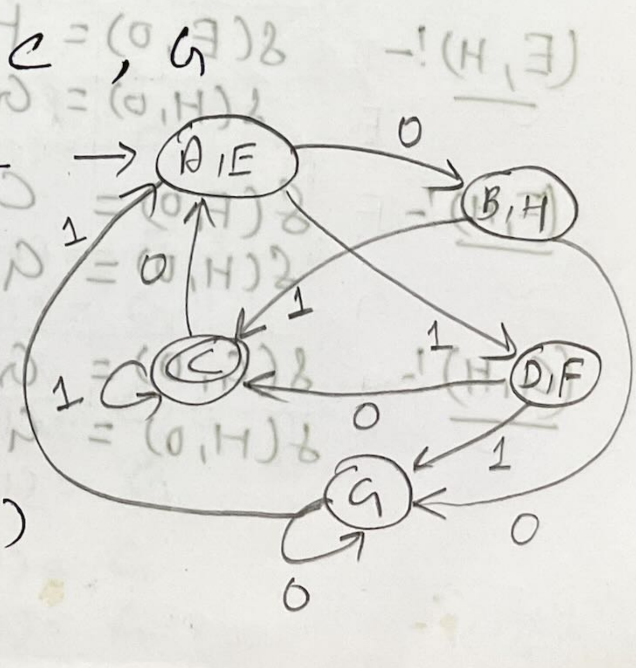
(B,H):- $\delta(B,0) = G$ $\quad \delta(B,1) = C$
$\delta(H,0) = G$ $\quad \delta(H,1) = C$

Same pairs are not getting marked in this Step also.

Step 4:- Unmarked pairs will get Combined into One State.

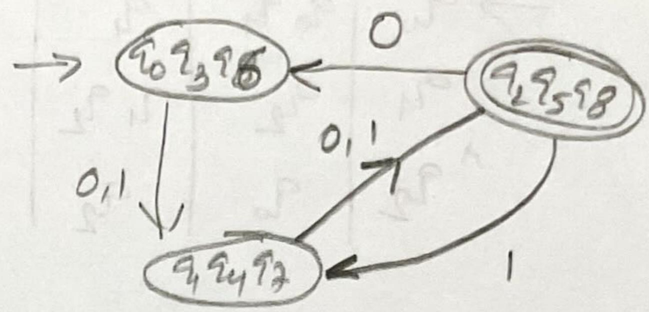
(A,E), (D,F), (B,H), C, G

	0	1
→ (A,E)	(B,H)	F
(B,H)	G	C
* C	(A,E)	C
(D,F)	C	G
G	G	(A,E)



Q: Minimize the DFA.

	0	1
→ q_0	q_1	q_4
q_1	q_2	q_5
* q_2	q_3	q_7
q_3	q_4	q_7
q_4	q_5	q_8
* q_5	q_6	q_1
q_6	q_7	q_1
q_7	q_8	q_2
* q_8	q_0	q_4



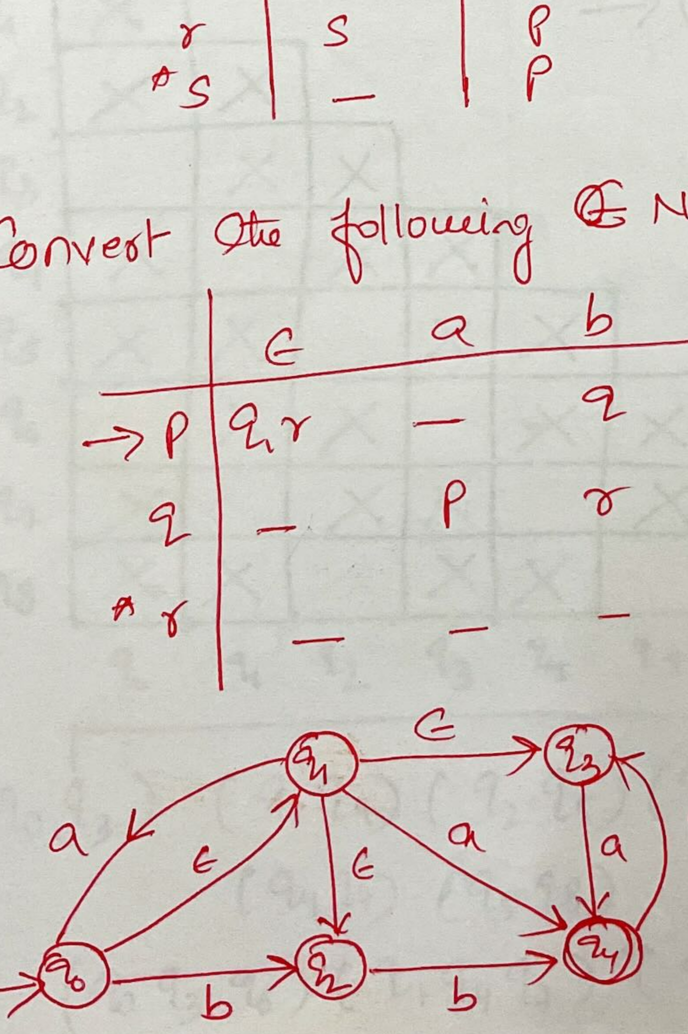
Q: Convert the following NFA to DFA.

	0	1
→ p	q, s	q
* q	r	q, r
r	s	p
* s	—	p

Q: Convert the following ε NFA to DFA.

	ε	a	b	c
→ p	q, r	—	q	r
q	—	p	r	p, q
* r	—	—	—	—

Q:-



Q: Convert the following NFA to an equivalent DFA.

	a	b	ϵ
$\rightarrow q_0$	$q_0 q_1$	q_1	—
q_1	q_2	$q_1 q_2$	—
$*q_2$	q_0	q_2	q_1

Q: Design a DFA to determine whether ternary number (base 3) is divisible by 5.

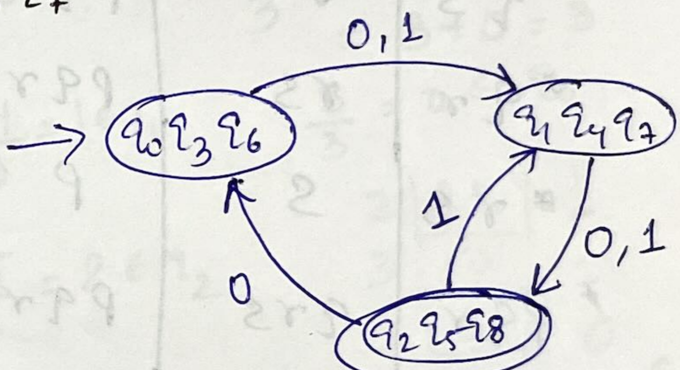
Minimize the DFA.

	0	1
$\rightarrow q_0$	q_1	q_4
q_1	q_2	q_5
$*q_2$	q_3	q_7
q_3	q_4	q_7
q_4	q_5	q_8
$*q_5$	q_6	q_1
q_6	q_7	q_1
q_7	q_8	q_2
$*q_8$	q_0	q_4

Solⁿ :- $F \Rightarrow q_2, q_5, q_8$

$NF \Rightarrow q_0, q_1, q_3, q_4, q_6, q_7$

	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
q_1	X							
q_2	X	X						
q_3		X	X					
q_4	X		X	X				
q_5	X	X		X	X			
q_6		X	X		X	X		
q_7	X		X	X		X	X	
q_8	X	X		X	X		X	X



(q_0, q_3) $(q_1 q_4)$ $(q_2 q_5)$ $(q_0 q_6)$ $(q_3 q_6)$ $(q_1 q_7)$ $(q_2 q_8)$

$(q_4 q_7)$ $(q_5 q_8)$

$(q_0 q_3 q_6)$ $(q_1 q_4 q_7)$ $^*(q_2 q_5 q_8)$

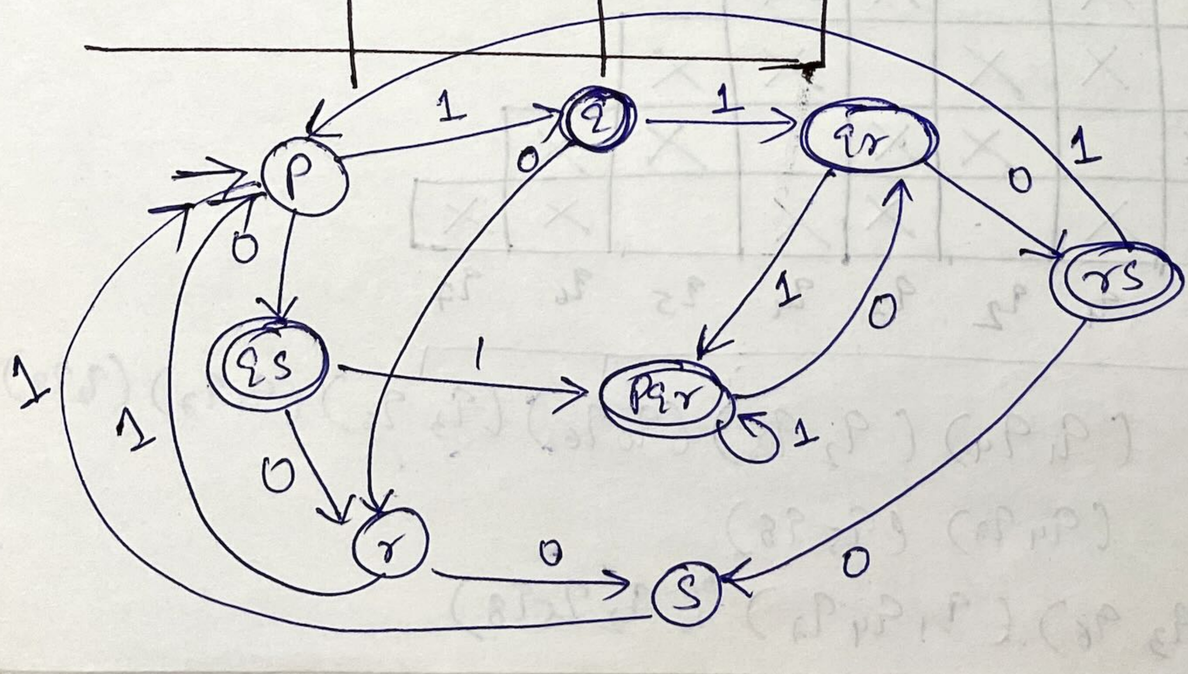
Q: Convert the following NFA to DFA.

	0	1
→ p	q, s	q
* q	r	q, r
r	s	p
* s	–	p

Soln:

	0	1
→ p	qs	q
* q	r	qr
r	s	p
* s	–	p
* qs	r	pqr
* qr	rs	pqr
* rs	s	p
* pqr	qrs	pqr
* qrs	rs	pqr

	0	1
→ p	qs	q
* q	r	qr
r	s	p
* s	–	p
* qs	r	pqr
* qr	rs	pqr
* rs	s	p
* pqr	qr	pqr



Q: DFA for Ternary no. divisible by 3.

Soln: $\Sigma = \{0, 1, 2\}$

Remainders :- $0 - q_0$

$1 - q_1$

$2 - q_2$

	δ	0	1	2
0	$\rightarrow * q_0$	q_0	q_1	q_2
1	q_1	q_0	q_1	q_2
2	q_2	q_0	q_1	q_2

$\frac{00}{3} = 0 \in q_0$

$\frac{01}{3} = 1 \in q_1$

$\frac{02}{3} = 2 \in q_2$

$\frac{10}{3} = 1\times3^1 + 0\times3^0$

$= 3+0 = 3$

$\frac{3}{3} = 0 \in q_0$

$$\frac{11}{3} = \frac{1\times3^1 + 1\times3^0}{3} = \frac{4}{3} = 1 \in q_1$$

$$\frac{12}{3} = \frac{1\times3^1 + 2\times3^0}{3} = \frac{5}{3} = 2 \in q_2$$

$$\frac{20}{3} = \frac{2\times3^1 + 0\times3^0}{3} = 0 \in q_0$$

$$\frac{21}{3} = \frac{2\times3^1 + 1\times3^0}{3} = \frac{7}{3} = 1 \in q_1$$

$$\frac{22}{3} = \frac{2\times3^1 + 2\times3^0}{3} = \frac{8}{3} = 2 \in q_2$$

3	32	2
3	10	1
3	3	0
	1	

1012

$1\times3^3 + 0\times3^2 + 1\times3^1 + 2\times3^0$

$= 27 + 3 + 2$

$= 32$

1002

$1\times3^3 + 2\times3^0$

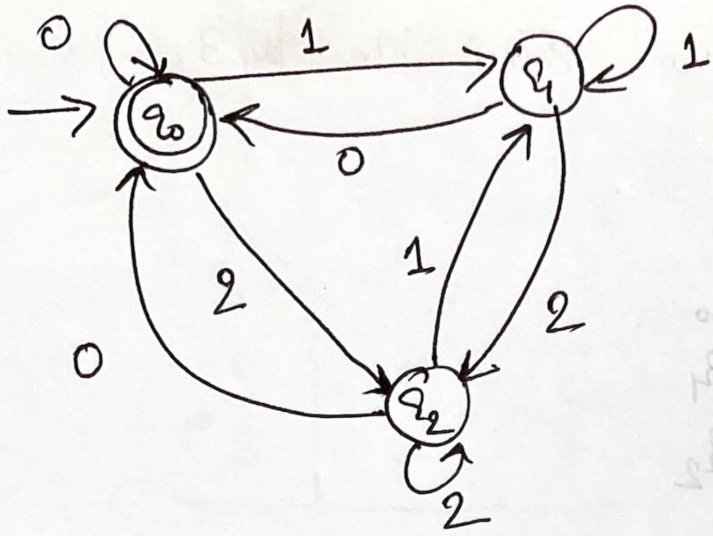
$= 27 + 2$

$= 29$

3	29	2
3	9	0
3	3	0
	1	

1010

27 + 3.



i/p:- $(120)_3 = 1\times3^2 + 2\times3^1 + 0\times3^0$

$= 9 + 6 + 0 = 15$

$15 \bmod 3 = 0$

$(122)_3 = 1\times3^2 + 2\times3^1 + 2\times3^0$

$= 9 + 6 + 2$

$= 17$

$17 \bmod 3 = 2$

$(2122)_3 = 2\times3^3 + 1\times3^2 + 2\times3^1 + 2\times3^0$

$= 54 + 9 + 6 + 2$

$= 71$

$71 \bmod 3 = 2$

$(2102)_3 = 2\times3^3 + 1\times3^2 + 0\times3^1 + 2\times3^0$

$= 54 + 9 + 0 + 2$

$= 65$

$65 \bmod 3 = 2$

Q. DFA for ternary no. divisible by 5.

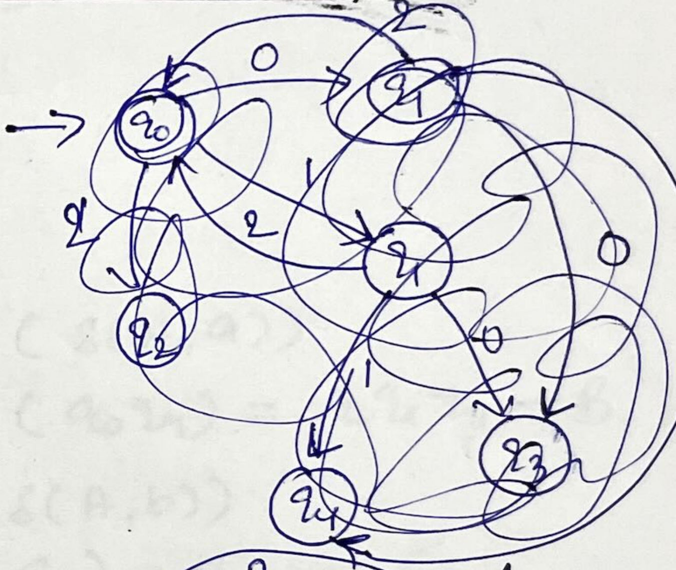
Sol^n:- $\Sigma = \{0, 1, 2\}$

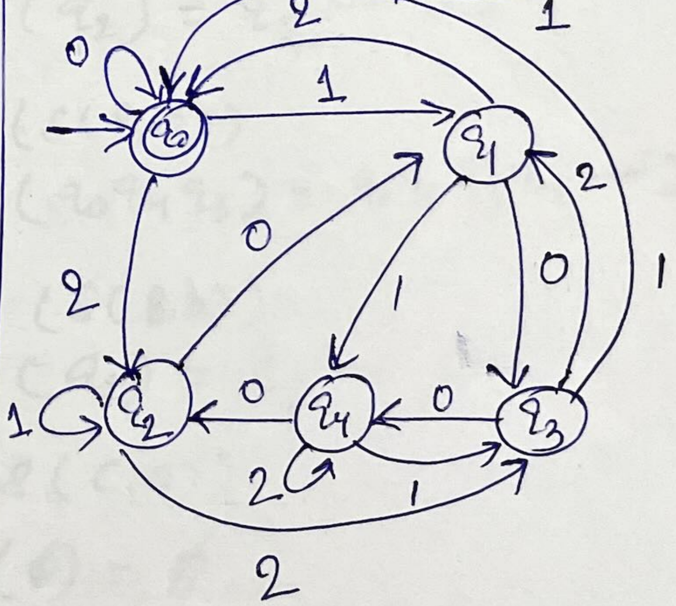
Remainders:- $\{0, 1, 2, 3, 4\}$

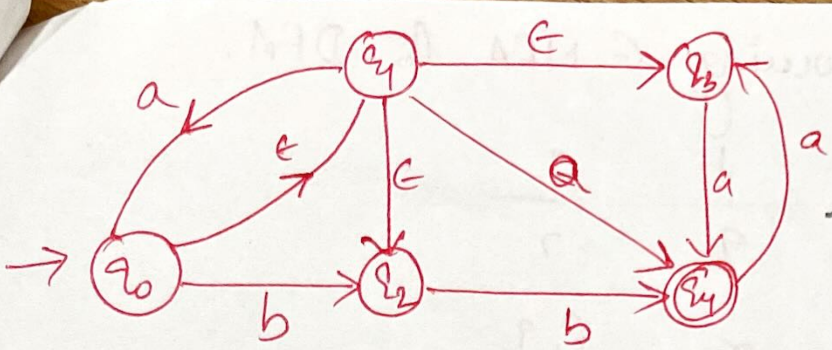
$q_0 - 0$
$q_1 - 1$
$q_2 - 2$
$q_3 - 3$
$q_4 - 4$

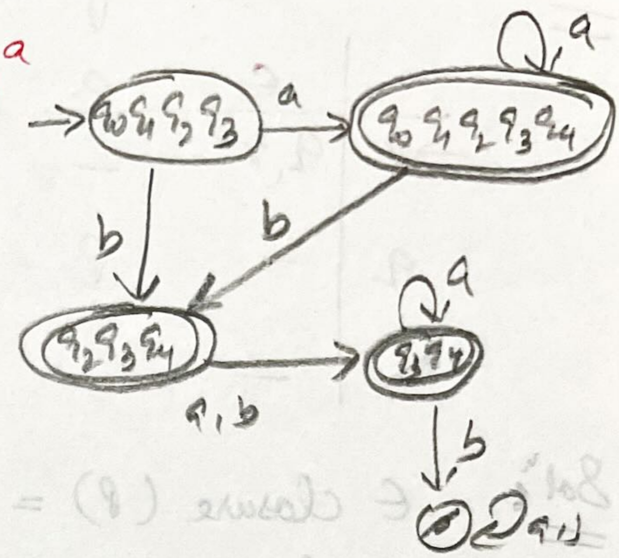
			0	1	2
0	0	$\rightarrow * q_0$	q_0	q_1	q_2
1	1	q_1	q_3	q_4	q_0
2	2	q_2	q_1	q_2	q_3
3	10	q_3	q_4	q_0	q_1
4	11	q_4	q_2	q_3	q_4

Decimal No.	Ternary No.
0	0
1	1
2	2
3	10
4	11
5	12
6	20
7	21
8	22
9	100
10	101
11	102









Sol^n:

	ε	a	b
→ q_0	q_1	–	q_2
q_1	q_2q_3	q_0q_4	–
q_2	–	–	q_4
q_3	–	q_4	–
* q_4	–	q_3	–

ϵ closure $(q_0) = q_0q_1q_2q_3$ — A

ϵ closure $(q_1) = q_1q_2q_3$

ϵ closure $(q_2) = q_2$

ϵ closure $(q_3) = q_3$

ϵ closure $(q_4) = q_4$

$\delta'(A, a) = \epsilon$ closure $(\delta(A, a))$
$= \epsilon$ closure $(q_0q_4) = q_0q_1q_4$ — B

$\delta'(A, b) = \epsilon$ closure $(\delta(A, b))$
$= \epsilon$ closure $(q_2) = q_2$ — C

$\delta'(B, a) = \epsilon$ closure $(\delta(B, a))$
$= \epsilon$ closure $(q_0q_4q_3) = q_0q_1q_3q_4$ — D

$\delta'(B, b) = \epsilon$ closure $(\delta(B, b))$
$= \epsilon$ closure $(q_2) = q_2$ — C

$\delta'(C, a) = \epsilon$ closure $\delta(C, a))$
$= \epsilon$ closure $(\phi) = \phi$

$\delta'(C, b) = \epsilon$ closure$(\delta(C, b)) = q_4$ — E

Q:- Convert the following ∈ NFA to DFA.

	∈	a	b	c
→p	q, r	—	q	r
q	—	p	r	p, q
*r	—	—	—	—

Solⁿ: ∈ closure (p) = {p, q, r} — A

∈ closure (q) = {q}

∈ closure (r) = {r}

$\delta'(A, a) = \in closure(\delta(A, a))$
$= \in closure(p) = (p, q, r)$ — A

$\delta'(A, b) = \in closure(\delta(A, b))$
$= \in closure(q, r) = q, r$ — B

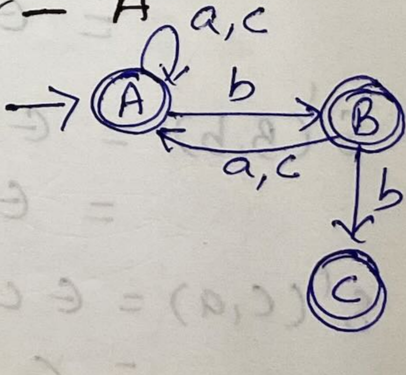
$\delta'(A, c) = \in closure(\delta(A, c))$
$= \in closure(p\,q\,r) = (p\,q\,r)$ — A

$\delta'(B, a) = \in closure(\delta(B, a))$
$= \in closure(p) = p\,q\,r$ — A

$\delta'(B, b) = \in closure(\delta(B, b))$
$= \in closure(r) = r$ — C

$\delta'(B, c) = \in closure(\delta(B, c))$
$= \in closure(p, q) = p\,q\,r$ — A

$\delta'(C, a) = \in closure(\delta(C, a))$
$= \in closure(\phi) = \phi$



$\delta'(C, b) = \in closure(\delta(C, b))$
$= \in closure(\phi) = \phi$

$\delta'(C, c) = \in closure(\delta(C, c))$
$= \in closure(\phi) = \phi$

$\delta'(D, a) = \epsilon\ closure\ (\delta(D, a))$

$= \epsilon\ closure(q_0 q_3 q_4) = q_0 q_1 q_3 q_4 - D$

$\delta'(D, b) = \epsilon\ closure\ (\delta(D, b))$

$= \epsilon\ closure\ (q_2) = q_2 - C$

$\delta'(E, a) = \epsilon\ closure\ (\delta(E, a))$

$= \epsilon\ closure\ (q_3) = q_3 - F$

$\delta'(E, b) = \epsilon\ closure\ (\delta(E, b))$

$= \epsilon\ closure\ (\phi) = \phi$

$\delta'(F, a) = \epsilon\ closure\ (\delta(F, a))$

$= \epsilon\ closure\ (q_4) = q_4\ E$

$\delta'(F, b) = \epsilon\ closure\ (\delta(F, b))$

$= \phi$

	0	1
→ A	B	C
* B	D	C
C	—	E
* D	D	C
* E	F	—
F	E	—