

CSE2403-Discrete Mathematics

Problem Sheet-2

Topic: Mathematical Logic

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1. Establish the validity of the following arguments

- (a) Some monkeys eat bananas. All monkeys are primates. Therefore some primates eat bananas.
- (b) All computer scientists are clever or wealthy. No computer scientist is wealthy. Therefore all computer scientists are clever or witty.
- (c) Everyone shouts or cries. Not everyone cries. So some people shout and don't cry.
- (d) Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised. Therefore Babies can't manage crocodiles.
- (e) One student in this class know how to write a programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore some in this class can get a high-pay job.
- (f) All integers are rational numbers. Some integers are power of 2. Therefore some rational numbers are power of 2.
- 2. Define the following:

F(x): x is greater than five

E(x): x is an even number

N(x): x is negative

Consider the following universe of discourse:

- integers
- real numbers
- negative numbers

Determine the truth value of each of the following propositions in each universe of discourse.

- (a) $\exists x F(x)$
- (b) $\forall x N(x)$
- (c) $\forall x(F(x) \land E(x))$
- 3. Express the negations of the following statements using quantifiers and in English.

- (a) If the teacher is absent, then some students do not keep quiet
- (b) All the students are keep quiet and the teacher is present
- (c) Some of the students do not keep quiet or the teacher is absent.
- 4. Prove the following by using direct method

(a)
$$P \land Q, (P \leftrightarrow Q) \rightarrow (R \lor S) \Rightarrow (R \lor S)$$

(b)
$$(P \land Q) \rightarrow R$$
, $\neg R \lor S$, $\neg S \Rightarrow (\neg P \lor \neg Q)$

(c)
$$P \rightarrow Q, (\neg Q \lor R) \land \neg R, \neg (\neg P \land S) \Rightarrow \neg S$$

(d)
$$(P \rightarrow Q) \rightarrow R$$
, $P \land S$, $Q \land T \Rightarrow R$

- 5. Prove the following by using indirect method
 - $(a) \ P {\rightarrow} Q, Q {\rightarrow} R, \neg (P {\wedge} R), (P {\vee} R) {\Rightarrow} R.$

(b)
$$P \rightarrow Q$$
, $\neg Q$, $P \lor R \Rightarrow R$

(c)
$$S \rightarrow \neg Q$$
, $S \lor R$, $\neg R$, $\neg R \leftrightarrow Q \Rightarrow \neg P$

6. Prove the following by using the CP rule

(a)
$$(P \lor Q) \to R \Rightarrow (P \land Q) \to R$$

(b)
$$\neg P \lor Q$$
, $\neg Q \lor R$, $R \rightarrow S \Rightarrow (P \rightarrow S)$

(c)
$$P \rightarrow (Q \rightarrow S), \neg R \lor P, Q \Rightarrow R \rightarrow S$$

7. Prove that each of the following sets of premises is inconsistent.

(a)
$$P \rightarrow Q$$
, $P \rightarrow R$, $Q \rightarrow \neg R$, P

(b)
$$P \rightarrow Q, (Q \lor R) \rightarrow S, S \rightarrow \neg P, P \land \neg R$$

(c)
$$P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S), P \land Q \land \neg S$$