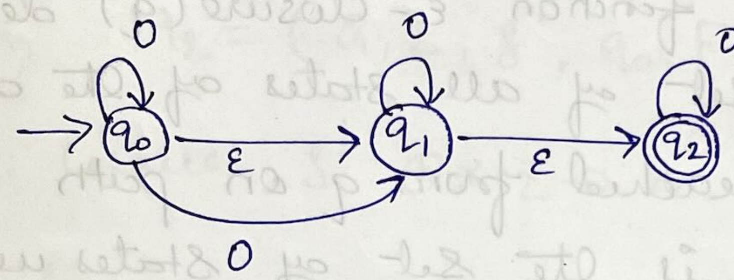
Finite Automata with ε-Transition

— NFA having transitions without input symbols. It can change their state with empty ~~string~~ input string that is without any input or ε as an input.



This is ~~an~~ NFA with ε moves, because it is possible to move from q_0 to q_1 ~~of~~ q_1 to q_2. without accepting any input symbols. Being a finite automata, NFA with ε transitions will also be denoted as five tuples.

$$M = \{Q, \Sigma, \delta, q_0, F\}$$

Q - finite set of states

Σ - i/P set of alphabet

δ - $Q \times (\Sigma \cup \varepsilon)$ to 2^Q

q_0 - initial state

F - set of final state $F \subseteq Q$.

Acceptance of a string by NFA with ε-Moves

A string w in Σ^* will be accepted by NFA with ε-moves if there exists atleast one path corresponding w, which starts in an initial state, and ends in one of the final states. This path may be formed by ε-transition as well as non ε-transitions.

ε-Closure (q) – epsilon closure of q.

ε-closure(q):- The function ε-closure(q) denotes the set of all states of the automata which can be reached from q on path labelled ε i-e, it is the set of states with distance zero from state 'q'.

Algorithm:- Method of finding ε-closure(q)

To find the set of ε-closure(q), say P we do the following step:-

1) add q to P.
2) find all the sets $\delta(q, \varepsilon)$, for each element $q \in P$ and add to P all the elements of these sets that are not ~~allowed~~ already included in P. Stop, when this step does not change P.

Construction of NFA without ϵ-moves from ϵ-moves

$$M = (Q, \Sigma, \delta, q_0, F)$$

where $\delta : Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$.

It can be converted to NFA without ϵ-moves,

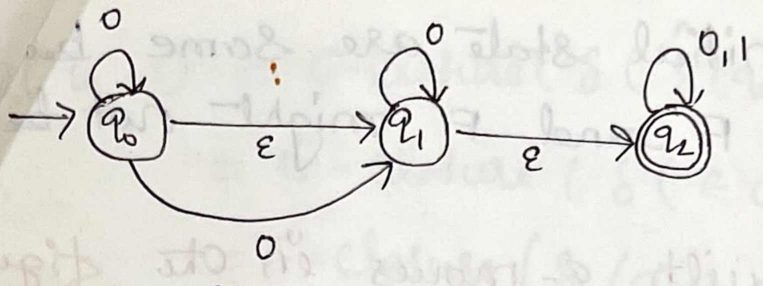
$$M_2 = (Q, \Sigma, \delta', q_0, F')$$

where $\delta' : Q \times \Sigma \rightarrow 2^Q$

$$\delta'(q, a) = \epsilon\text{ closure}(\delta(\hat{\delta}(q, \epsilon), a))$$

where $\hat{\delta}(q, \epsilon) = \epsilon\text{ closure}(q)$

$$\delta'(q, a) = \epsilon\text{ closure}(\delta(\underbrace{\epsilon\text{ closure}(q)}_{\hat{\delta}(q,\epsilon)}; a))$$



ϵ- closure $(q_0) = \{q_0, q_1, q_2\}$
ϵ- Closure $(q_1) = \{q_1 q_2\}$
ϵ- closure $(q_2) = \{q_2\}$

It will be noted that ϵ-closure (q) will never be empty set, because q is always reachable from itself without consuming any input symbol.

Construction of NFA without ϵ- moves from NFA with ϵ - moves

For every NFA with ϵ- moves there exists an equivalent NFA without ϵ- moves accepting the same language. If NFA with ϵ - moves is given by,

$$M_1 = (Q, \Sigma, \delta, q_0, F)$$

where $\delta: Q \times (\Sigma \cup \epsilon) \to 2^Q$.

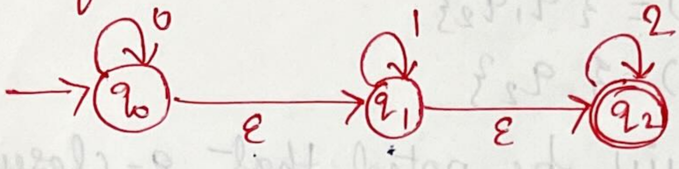
It can be converted to NFA without ϵ-moves

$$M_2 = (Q, \Sigma, \delta', q_0, F')$$

where $\delta': Q \times \Sigma \to 2^Q$

Q, Σ and initial state are same but set of final states F and F' might not be.

Q:- Convert NFA with ε-moves in the figure to equivalent NFA without ε-moves.



$Q, \Sigma, \delta', q_0, F'$

Sol^n:- Here,

ϵ closure $(q_0) = \{q_0 q_1 q_2\}$

ϵ closure $(q_1) = \{q_1 q_2\}$

ϵ closure $(q_2) = \{q_2\}$

As we see that q_2 is in ε-closure(q_0) and ϵ-closure(q_1) that means q_2 is at a distance zero from q_0 & q_1.

Thus, the set of final states of resultant equivalent NFA without ϵ-moves is,

$$F' = \{q_0, q_1, q_2\}$$

Transition function δ'

$$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 0))$$
$$= \epsilon\text{-closure}(\delta(\epsilon\text{closure}(q_0), 0))$$
$$= \epsilon\text{-closure}(\delta(q_0 q_1 q_2), 0)$$
$$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$$
$$= \epsilon\text{-closure}(q_0)$$
$$= \{q_0 q_1 q_2\}$$

$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 1))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{ closure}(q_0), 1))$

$= \epsilon\text{-closure}(\delta(q_0 q_1 q_2, 1))$

$= \epsilon\text{ closure}(q_1)$

$= \{q_1 q_2\}$

$\delta'(q_0, 2) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 2))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{ closure}(q_0), 2))$

$= \epsilon\text{-closure}(\delta(q_0 q_1 q_2, 2))$

$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}$

Similarly,

$\delta'(q_1, 0) = \epsilon\text{ closure}(\delta(\hat{\delta}(q_1, \epsilon), 0))$

$= \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), 0))$

$= \epsilon\text{-closure}(\delta(q_1 q_2, 0))$

$= \epsilon\text{ closure}(\phi) = \phi$

$\delta'(q_1, 1) = \epsilon\text{ closure}(\delta(\hat{\delta}(q_1, \epsilon), 1))$

$= \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), 1))$

$= \epsilon\text{ closure}(\delta(q_1 q_2, 1))$

$= \epsilon\text{ closure}(q_1)$

$= q_1 q_2$

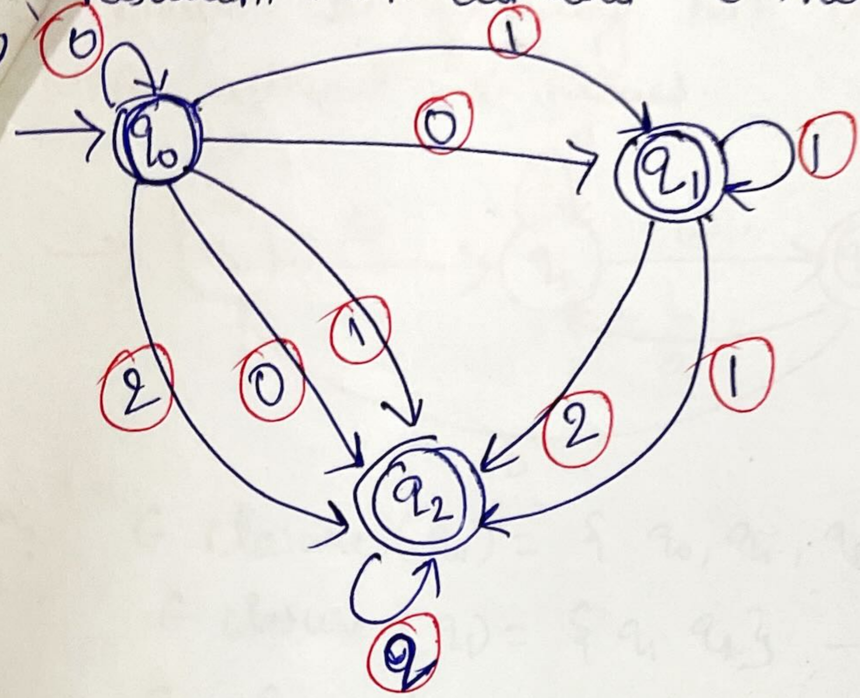
$$\delta'(q_1, 2) = \epsilon\text{ closure}(\delta(\hat{\delta}(q_1, \epsilon), 2))$$
$$= \epsilon\text{ Closure}(\delta(\epsilon\text{ Closure}(q_0), 2))$$
$$= \epsilon\text{ closure}(\delta(q_0 q_1 q_2), 2))$$
$$= \epsilon\text{ Closure}(q_2)$$
$$= \{q_2\}$$

$$\delta'(q_2, 0) = \epsilon\text{ Closure}(\delta(\hat{\delta}(q_2, \epsilon), 0))$$
$$= \epsilon\text{ Closure}(\delta(\epsilon\text{ Closure}(q_2), 0))$$
$$= \epsilon\text{ Closure}(\delta(q_2, 0))$$
$$= \phi$$

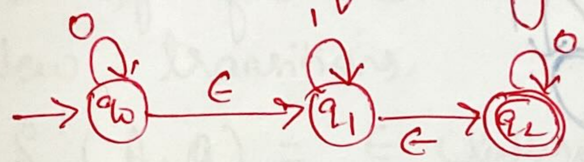
$$\delta'(q_2, 1) = \epsilon\text{ Closure}(\delta(\hat{\delta}(q_2, \epsilon), 1))$$
$$= \epsilon\text{ Closure}(\delta(\epsilon\text{ Closure}(q_2), 1))$$
$$= \epsilon\text{ closure}(\delta(q_2, 1))$$
$$= \phi$$

$$\delta'(q_2, 2) = \epsilon\text{ Closure}(\delta(\hat{\delta}(q_2, \epsilon), 2))$$
$$= \epsilon\text{ Closure}(\delta(\epsilon\text{ Closure}(q_2), 2))$$
$$= \epsilon\text{ Closure}(\delta(q_2, 2))$$
$$= \epsilon\text{ Closure}(q_2)$$
$$= \{q_2\}$$

Resultant NFA without ϵ moves -



Q2 :- Consider the following NFA with ϵ moves.



Construct DFA.

Solⁿ :-

ϵ closure $(q_0) = \{q_0, q_1, q_2\}$

ϵ closure $(q_1) = \{q_1, q_2\}$

ϵ closure $(q_2) = \{q_2\}$

New set of final states $F' = \{q_0, q_1, q_2\}$

New transitions :- δ'

~~$\delta'(q_0, 0) = \delta'($ ϵ closure $(\delta_0(\hat{\delta}, 0), 0)$~~

$$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 0))$$

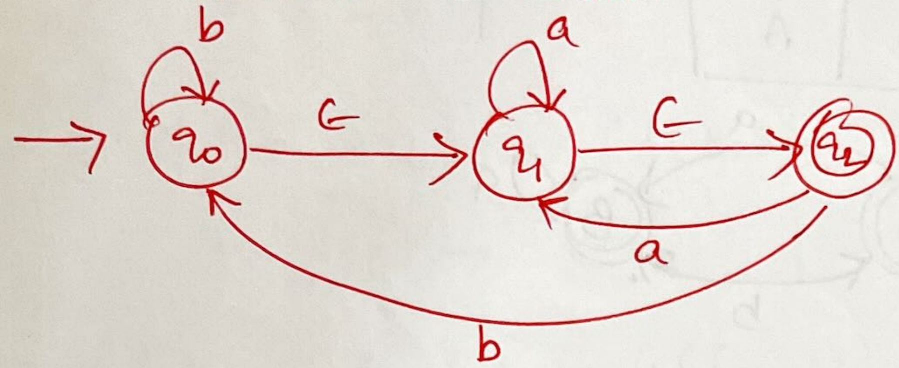
$$= \epsilon\text{-closure}(\delta(\epsilon\text{ closure}(q_0), 0))$$

$$= \epsilon\text{-closure}(\delta(\{q_0 q_1 q_2\}, 0))$$

$$= \epsilon\text{-closure}(q_0 q_2)$$

$$= \{q_0 q_1 q_2\}$$

Convert the following NFA with ϵ moves to NFA without ϵ-moves.



	a	b	ϵ
$\rightarrow q_0$	–	q_0	q_1
q_1	q_1	–	q_2
$* q_2$	q_1	q_0	–

Solⁿ: ϵ Closure $(q_0) = \{q_0, q_1, q_2\}$ — A

ϵ closure $(q_1) = \{q_1\ q_2\}$ — B

ϵ Closure $(q_2) = \{q_2\}$ — C

Set of final states $F' = \{q_0, q_1, q_2\}$

New transitions,

$\delta'(A, a) = \epsilon$ Closure $(\delta(A, a))$

$= \epsilon$ Closure $(q_1) = q_1 q_2$ (B)

$\delta'(A, b) = \epsilon$ Closure $(\delta(A, b))$

$= \epsilon$ Closure $(q_0) = q_0 q_1 q_2$ — (A)

$\delta'(B, a) = \epsilon$ Closure $(\delta(B, a))$

$= \epsilon$ Closure $(q_1) = q_1 q_2$ — B

$\delta'(B, b) = \epsilon$ Closure $(\delta(B, b))$

$= \epsilon$ Closure $(q_0) = q_0 q_1 q_2$ — A

$\delta'(C, a) = \epsilon$ Closure $(\delta(C, a))$

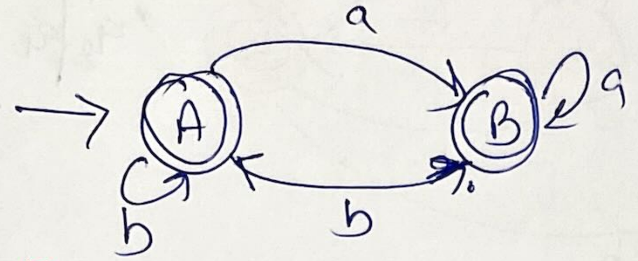
$= q_1 q_2$ — B

$\delta'(C, b) = \epsilon$ Closure $(\delta(C, b))$

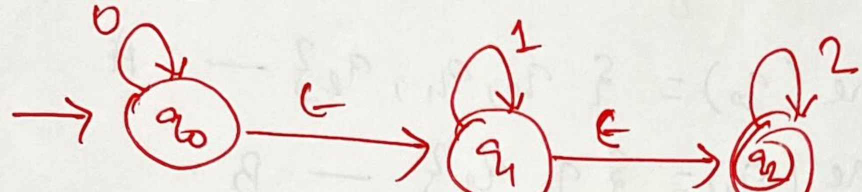
$= q_0 q_1 q_2$ - A

	a	b
→*A	B	C
*B	B	A
*C	B	A

	a	b
→*A	B	A
*B	B	A



Q!-



Solⁿ:-

ϵ closure $(q_0) = \{q_0\ q_1\ q_2\}$ — A

ϵ closure $(q_1) = \{q_1\ q_2\}$ — B

ϵ closure $(q_2) = \{q_2\}$ — C

$\delta'(A,0) = \epsilon$ closure $(\delta(A,0))$

$= \epsilon$ closure (q_0)

$= \{q_0 q_1 q_2\}$ — A

$\delta'(A,1) = \epsilon$ closure $(\delta(A,1))$

$= \{q_1\ q_2\}$ — B

$\delta'(B,0) = \epsilon$ closure $(\delta(B,0))$

$= \phi$

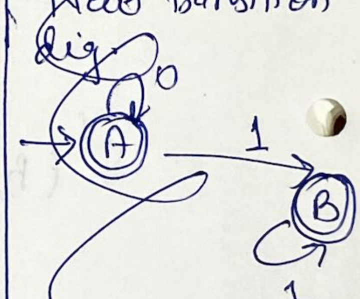
$\delta'(B,1) = \epsilon$ closure $(\delta(B,1))$

$= \{q_1 q_2\}$ — B

$\delta'(C,0) = \epsilon$ closure $(\delta(C,0)) = \phi$

$\delta'(C,1) = \epsilon$ closure $(\delta(C,1)) = \phi$

New transition dig.



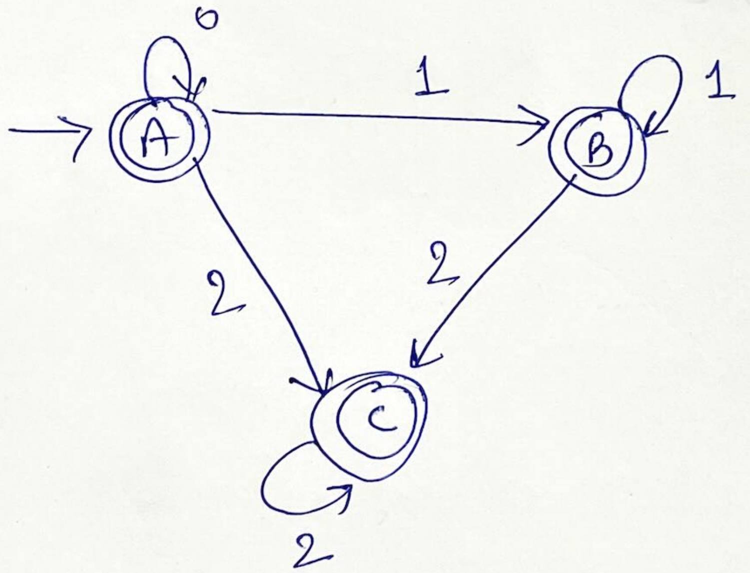
	0	1
→*A	A	B
*B	ϕ	B

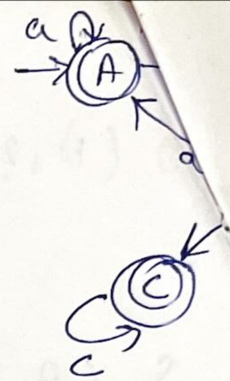
$\delta'(A,2) = \epsilon\text{ closure}(\delta(A,2))$
$= \{q_2\}$ — C

$\delta'(B,2) = \epsilon\text{ closure}(\delta(B,2))$
$= \{q_2\}$ — C

$\delta'(C,2) = \epsilon\text{ closure}(\delta(C,2))$
$= \{q_2\}$

	0	1	2
$\rightarrow$ *A	A	B	C
*B	ϕ	B	C
*C	ϕ	ϕ	C



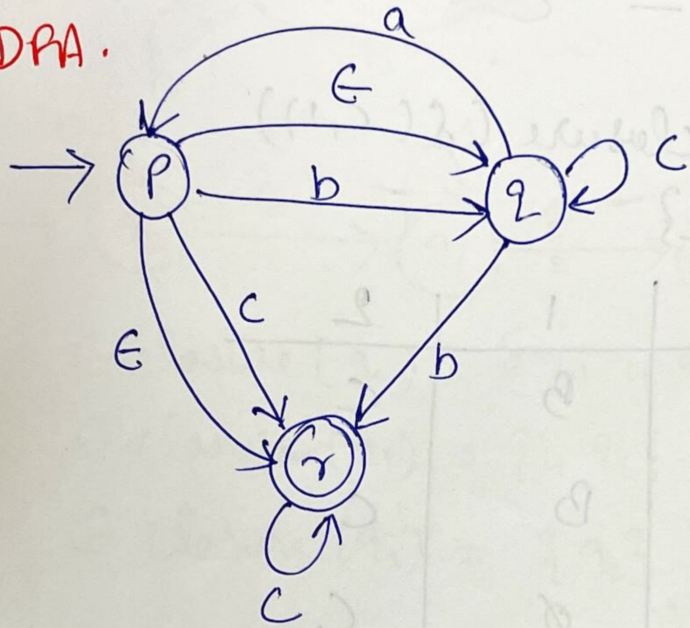


Q:-

	ε	a	b	c
p	q, r	φ	q	r
q	φ	p	r	q
* r	φ	φ	φ	r

Convert it ~~to DFA~~ NFA with ε moves to DFA.

Soln:-



ε closure (p) =

NFA with ε moves to DFA

Steps for converting NFA with ε moves to DFA.

Step 1: ~~Take~~ Find the ε-closure of all given states of NFA.

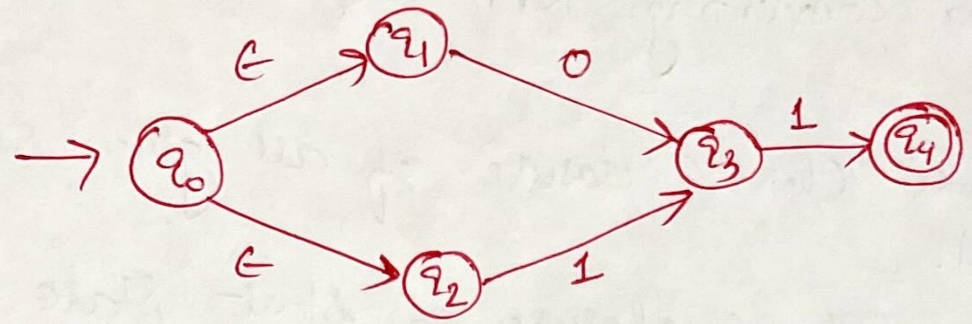
Step 2: Take the ε closure of start state of NFA as the start state of DFA.

Step 3: Find the new transitions of DFA wrt to this start state.

Step 4: Include the new transitions of previous step as a new state for DFA.

Step 5: Repeat step 3-4 till we are getting the new combinations.

Q: Convert the NFA with ϵ moves into its equivalent DFA.



Sol^n: Let's find out ϵ closure of each state.

ϵ closure $(q_0) = \{q_0, q_1, q_2\}$ — A

ϵ closure $(q_1) = \{q_1\}$

ϵ closure $(q_2) = \{q_2\}$

ϵ closure $(q_3) = \{q_3\}$

ϵ closure $(q_4) = \{q_4\}$

	0	1	ϵ
$\rightarrow q_0$	–	–	q_1, q_2
q_1	q_3	–	–
q_2	–	q_3	–
q_3	–	q_4	–
$*q_4$	–	–	–

$\delta'(A, 0) = \epsilon$ closure $(\delta((q_0 q_1, q_2), 0))$

$= \epsilon$ closure $(q_3) = q_3$ — B

$\delta'(A, 1) = \epsilon$ closure $(\delta((q_0 q_1 q_2), 1))$

$= \epsilon$ closure $(q_3) = q_3$ — B

$\delta'(B, 0) = \epsilon$ closure $(\delta((q_3), 0))$

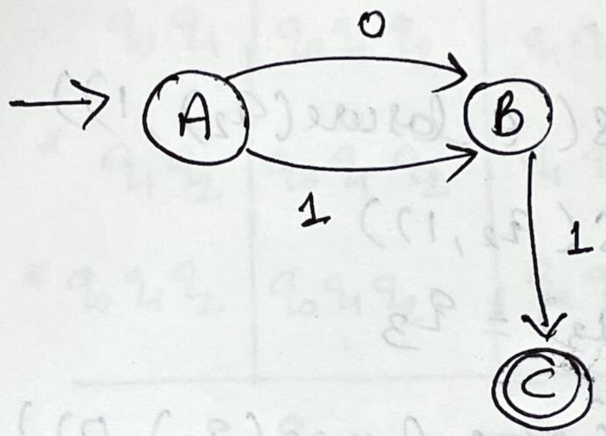
$= \epsilon$ closure $(\cancel{q_3}) = \emptyset$

$\delta'(B, 1) = \epsilon$ closure $(\delta(q_3, 1))$

$= \epsilon$ closure $(q_4) = q_4$ — C

$$\delta'(C,0) = \epsilon\,closure(\delta(q_4,0))$$
$$= \epsilon\,closure(\phi) = \phi$$

$$\delta'(C,1) = \epsilon\,closure(\delta(q_4,1))$$
$$= \epsilon\,closure(\phi) = \phi$$



II^{nd} Method:-

Step 1: ϵ closure of all the states

$\epsilon\,closure(q_0) = q_0q_1q_2$

$\epsilon\,closure(q_1) = q_1$

$\epsilon\,closure(q_2) = q_2$

$\epsilon\,closure(q_3) = q_3$

$\epsilon\,closure(q_4) = q_4$

Step 2:

$$\delta'(q_0,0) = \epsilon\,closure(\delta(\epsilon\,closure(q_0),0))$$
$$= \epsilon\,closure(\delta((q_0q_1q_2),0))$$
$$= \epsilon\,closure(q_3) = q_3$$

$$\delta'(q_0,1) = \epsilon\,closure(\delta(\epsilon\,closure(q_0),1))$$
$$= \epsilon\,closure(\delta((q_0q_1q_2),1))$$
$$= \epsilon\,closure(q_3) = q_3$$

$$\delta'(q_1,0) = \epsilon\,closure(\delta(\epsilon\,closure(q_1),0))$$
$$= \epsilon\,closure(\delta(q_1,0))$$
$$= q_3$$

$\delta'(q_1, 1) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_1), 1))$
$= \epsilon\text{closure}(\delta(q_1, 1)) = \phi$

$\delta'(q_2, 0) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_2), 0))$
$= \epsilon\text{closure}(\delta(q_2, 0))$
$= \phi$

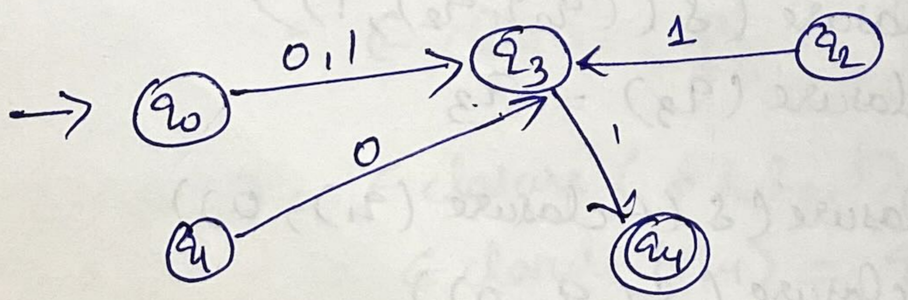
$\delta'(q_2, 1) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_2), 1))$
$= \epsilon\text{closure}(\delta(q_2, 1))$
$= \epsilon\text{closure}(q_3) = q_3$

$\delta'(q_3, 0) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_3), 0))$
$= \epsilon\text{closure}(\delta(q_3, 0))$
$= \phi$

$\delta'(q_3, 1) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_3), 1))$
$= \epsilon\text{closure}(\delta(q_3, 1))$
$= \epsilon\text{closure}(q_4) = q_4$

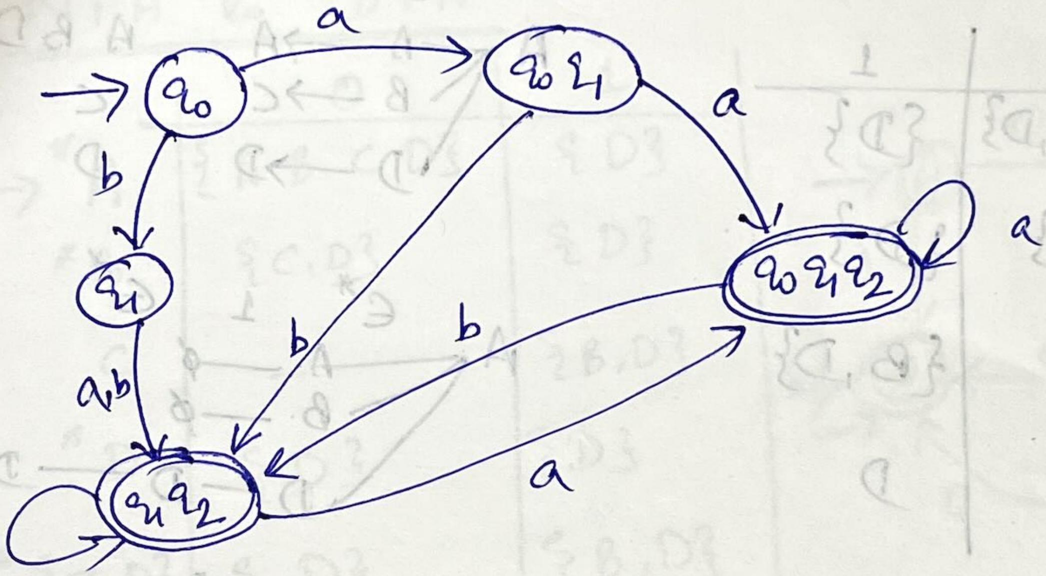
$\delta'(q_4, 0) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_4), 0))$
$= \epsilon\text{closure}(\delta(q_4, 0)) = \phi$

$\delta'(q_4, 1) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_4), 1))$
$= \epsilon\text{closure}(\delta(q_4, 1)) = \phi$

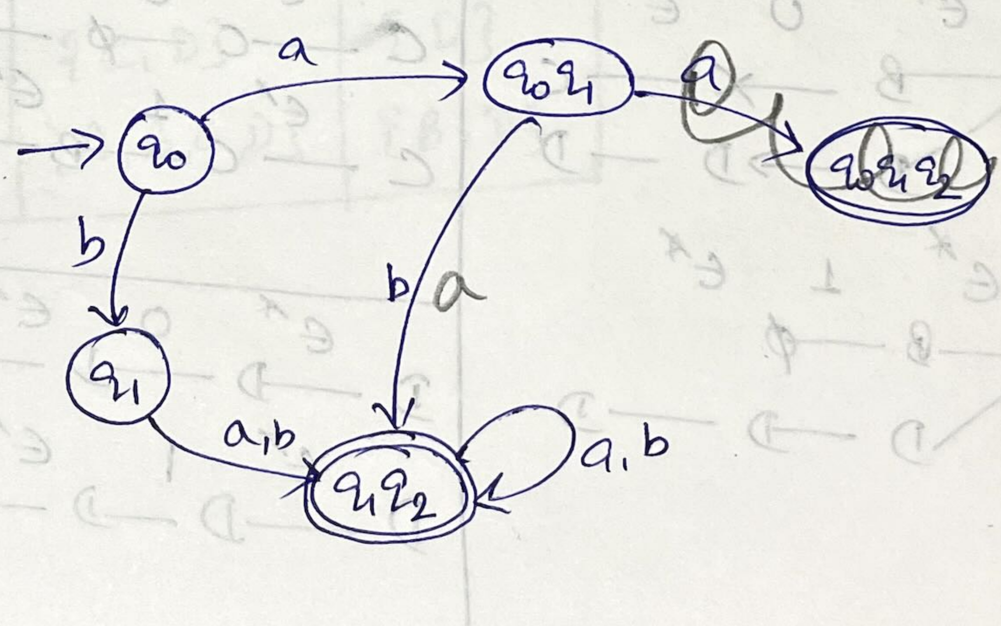


	a	b
$\rightarrow q_0$	q_0q_1	q_1
q_1	q_1q_2	q_1q_2
$*q_2$	$q_0q_1q_2$	q_1q_2
q_0q_1	$q_0q_1q_2$	q_1q_2
$*q_1q_2$	$q_0q_1q_2$	q_1q_2
$*q_0q_1q_2$	$q_0q_1q_2$	q_1q_2

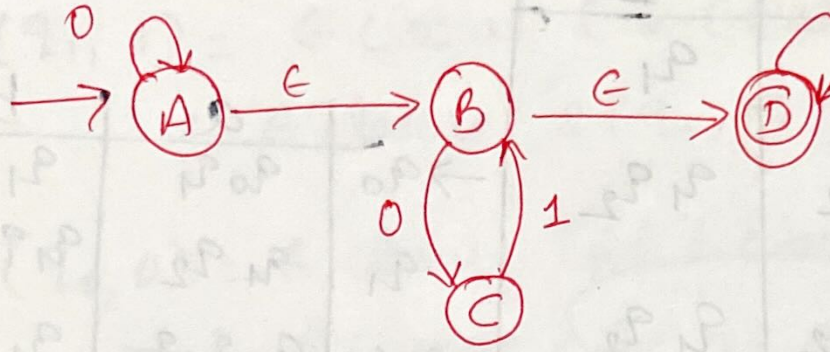
	a	b
$\rightarrow q_0$	q_0q_1	q_1
q_1	q_1q_2	q_1q_2
q_0q_1	$\cancel{q_0q_1q_2}$ q_1q_2	q_1q_2
$*q_1q_2$	q_1q_2	q_1q_2



or



Q:



ϵ closure $(A) = \{A, B, D\}$

ϵ closure $(B) = \{B, D\}$

ϵ closure $(C) = \{C\}$

ϵ closure $(D) = \{D\}$

Q1

Q2

Q3

Q4

	0	1
$\rightarrow$ *A	{A, B, C, D}	{D}
*B	{C, D}	{D}
C	—	{B, D}
*D	D	D

	ϵ^*	0	ϵ^*
A	A	A	A B D
	B	C	C
	D	D	D

	ϵ^*	1	ϵ^*
A	A	ϕ	
	B	ϕ	
	D	D	D

	ϵ^*	0	ϵ^*
B	B	$\times$	C
	D	D	D

	ϵ^*	1	ϵ^*
B	B	ϕ	
	D	D	D

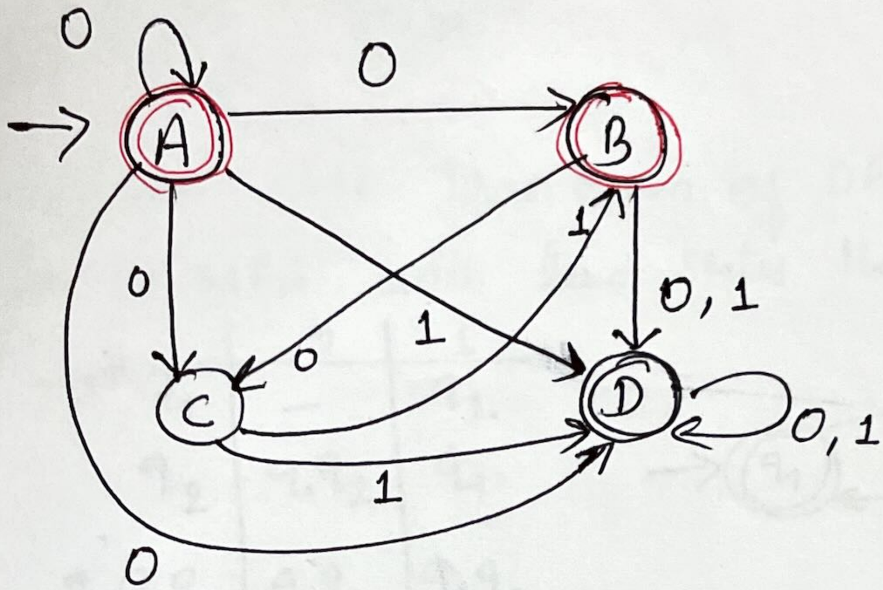
	ϵ^*	0	ϵ^*
C	C	ϕ	ϕ

	ϵ^*	1	ϵ^*
C	C	B	BD

	ϵ^*	0	ϵ^*
D	D	D	D

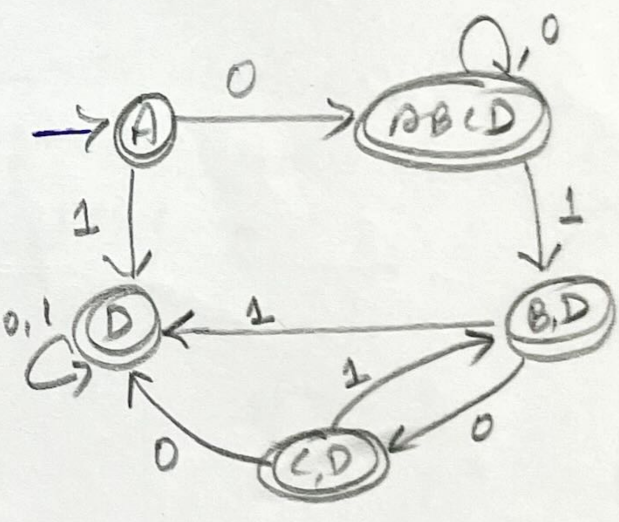
	ϵ^*	1	ϵ^*
D	D	D	D

∈ NFA to NFA



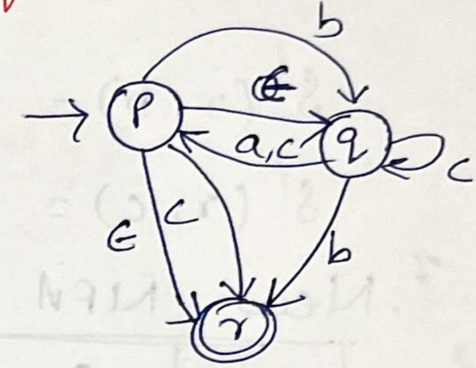
NFA to DFA

	0	1
→ *A	{A B C D}	{D}
*B	{C, D}	{D}
C	–	{B, D}
*D	{D}	{D}
*{C D}	{D}	{B, D}
*{B, D}	{C, D}	{D}
{A B C D}	{A, B, C, D}	{B, D}



Q: Convert the given ϵ NFA to equivalent DFA.

States	ϵ	a	b	c
$\rightarrow p$	q,r	–	q	r
q	–	p	r	p,q
$*r$	–	–	–	–



Solⁿ:- Step 1:- ϵ closure of all the states.

ϵ closure $(p) = \{p, q, r\}$

ϵ closure $(q) = \{q\}$

ϵ closure $(r) = \{r\}$

Step 2:- $\delta'(p, a) = \epsilon \text{ closure}(\delta(\epsilon \text{ closure}(p), a))$

$= \epsilon \text{ closure}(\delta((p, q, r), a))$

$= \epsilon \text{ closure}(p) = \{p, q, r\}$

$\delta'(p, b) = \epsilon \text{ closure}(\delta(\epsilon \text{ closure}(p), b))$

$= \epsilon \text{ closure}(q, r) = \{q, r\}$

$\delta'(p, c) = \epsilon \text{ closure}(\delta(\epsilon \text{ closure}(p), c))$

$= \epsilon \text{ closure}(p, q, r) = \{p, q, r\}$

$\delta'(q, a) = \epsilon \text{ closure}(\delta(\epsilon \text{ closure}(q), a))$

$= \{p, q, r\}$

$\delta'(q, b) = \{r\}$

$\delta'(q, c) = \{p, q, r\}$

$\delta'(r, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(r), a))$

$= \phi$

$\delta'(r, b) = \phi$

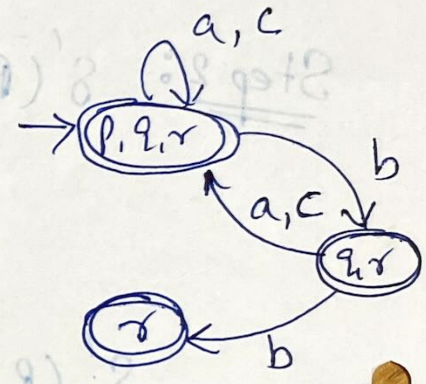
$\delta'(r, c) = \phi$

New NFA without ϵ moves :-

	a	b	c
→ p	p, q, r	q, r	p, q, r
q	p, q, r	r	p, q, r
* r	—	—	—

DFA

	a	b	c
→ p	p, q, r	q, r	p, q, r
q	p, q, r	r	p, q, r
* r	—	—	—
* q, r	p, q, r	r	p, q, r
* p, q, r	p, q, r	q, r	p, q, r

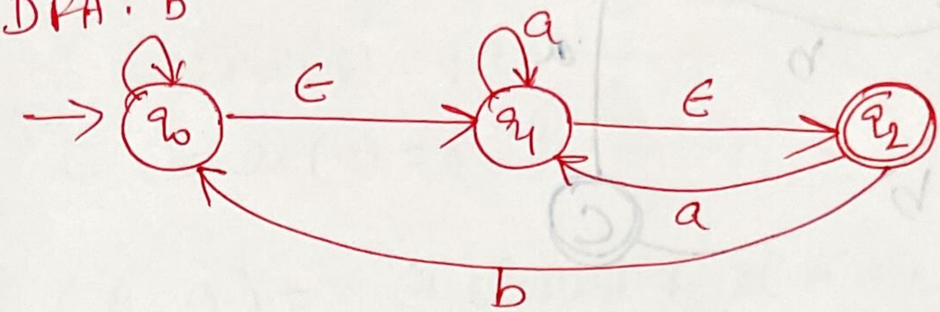


DFA from the given ENFA

New Reduced DFA :-

	a	b	c
→ p, q, r*	p, q, r	q, r	p, q, r
q	p, q, r	r	p, q, r
* r	—	—	—
* q, r	p, q, r	r	p, q, r

Q2:- Convert the following NFA with ϵ to equivalent DFA.



Soln:- Step 1: ϵ closure of all states

ϵ closure $(q_0) = \{q_0, q_1, q_2\}$ — A^*

ϵ closure $(q_1) = \{q_1, q_2\}$ — B^*

ϵ closure $(q_2) = \{q_2\}$ — C^*

$\delta'(A, a) = \epsilon$ closure $(\delta(A, a))$

$= \epsilon$ closure $(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$

$= \epsilon$ closure $(q_1) = q_1 q_2$ — B

$\delta'(A, b) = \epsilon$ closure $(\delta(A, b))$

$= \epsilon$ closure $(q_0) = q_0 q_1 q_2$ — A

$\delta'(B, a) = \epsilon$ closure $(\delta(B, a))$

$= \epsilon$ closure $(q_1) = q_1 q_2$ — B

$\delta'(B, b) = \epsilon$ closure $(\delta(B, b))$

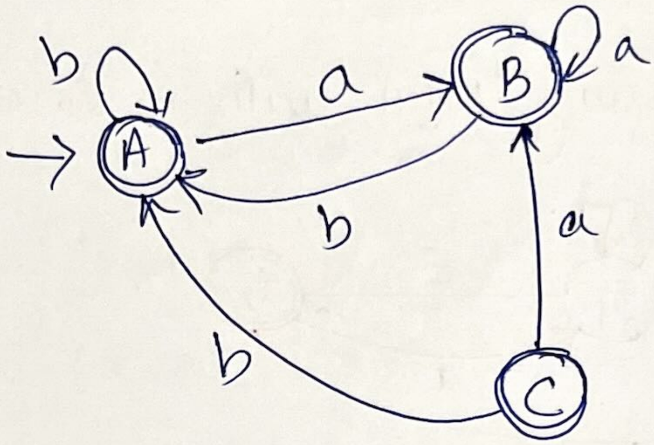
$= \epsilon$ closure $(q_0) = q_0 q_1 q_2$ — A

$\delta'(C, a) = \epsilon$ closure $(\delta(C, a))$

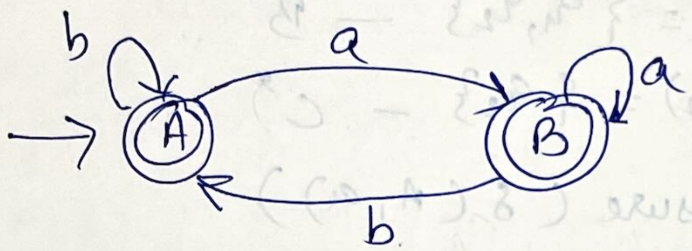
$= \epsilon$ closure $(q_1) = q_1 q_2$ — B

$\delta'(C, b) = \epsilon$ closure $(\delta(C, b))$

$= \epsilon$ closure $(q_0) = q_0 q_1 q_2$ — A



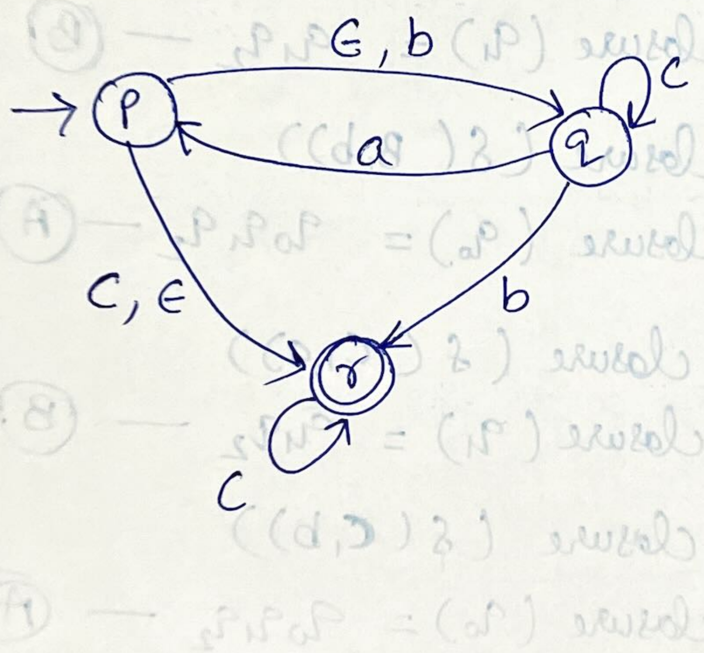
⇒ Reduced DFA



Q:- Convert the NFA with ϵ moves to DFA.

	ϵ	a	b	c
→ p	{q, r}	–	q	r
q	–	p	r	q
* r	–	–	–	r

Solⁿ:-



ϵ closure of the states P, q & r.

ϵ closure (P) = {P, q, r} — A^*

ϵ closure (q) = {q} — B

ϵ closure (r) = {r} — C^*

$\delta'(A, a) = \epsilon$ closure ($\delta(A, a)$)
$= \epsilon$ closure (P) = {P, q, r} - A

$\delta'(A, b) = \epsilon$ closure ($\delta(A, b)$)
$= \epsilon$ closure (q, r) = {q, r} - D

$\delta'(A, c) = \epsilon$ closure ($\delta(A, c)$)
$= \epsilon$ closure (q, r) = {q, r} - D^*

$\delta'(B, a) = \epsilon$ closure ($\delta(B, a)$)
$= \epsilon$ closure (P) = {P, q, r} — A

$\delta'(B, b) = \epsilon$ closure ($\delta(B, b)$)
$= \epsilon$ closure (r) = {r} — C

$\delta'(B, c) = \epsilon$ closure ($\delta(B, c)$)
$= \epsilon$ closure (q) = {q} — B

$\delta'(C, a) = \epsilon$ closure ($\delta(C, a)$)
$= \epsilon$ closure (ϕ) = ϕ

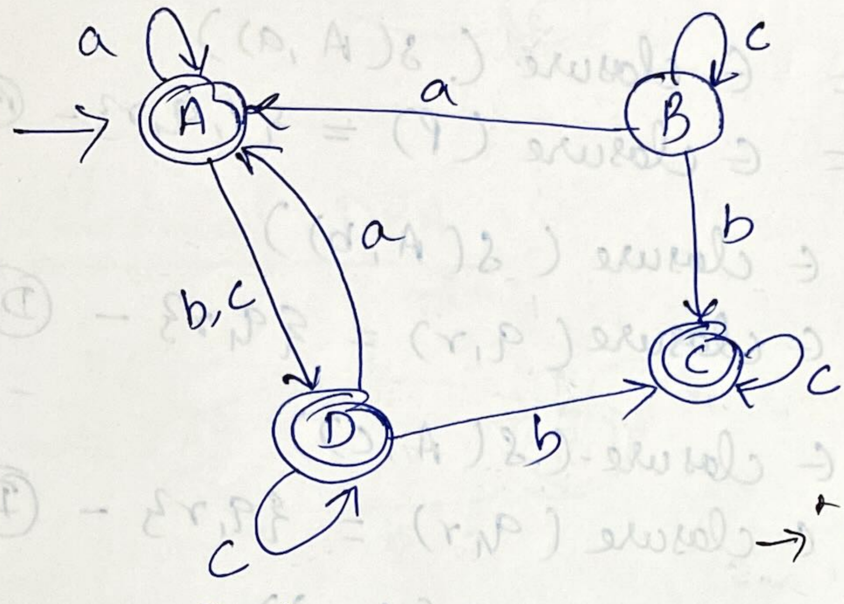
$\delta'(C, b) = \epsilon$ closure ($\delta(C, b)$)
$= \epsilon$ closure (ϕ) = ϕ

$\delta'(C, c) = \epsilon$ closure ($\delta(C, c)$)
$= \epsilon$ closure (r) = r — C

$\delta'(D, a) = \epsilon$ closure ($\delta(D, a)$)
$= \epsilon$ closure (P) = {P, q, r} - A

$\delta'(D, b) = \epsilon\text{ closure}(\delta(D, b))$
$= \epsilon\text{ closure}(r) = \{r\}$ - C

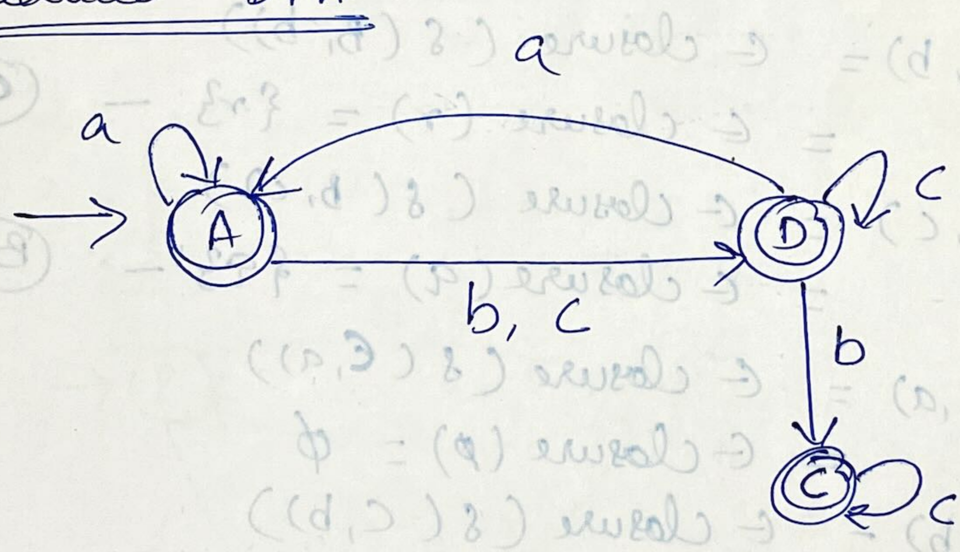
$\delta'(D, c) = \epsilon\text{ closure}(\delta(D, c))$
$= \epsilon\text{ closure}(q, r) = \{q, r\}$ - D



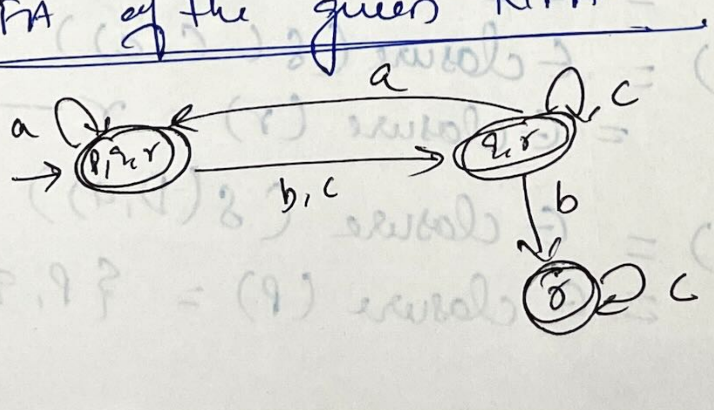
	a	b	c
$\rightarrow$ *A	A	D	D
B	A	C	B
*C	–	–	C
*D	A	C	D

DFA

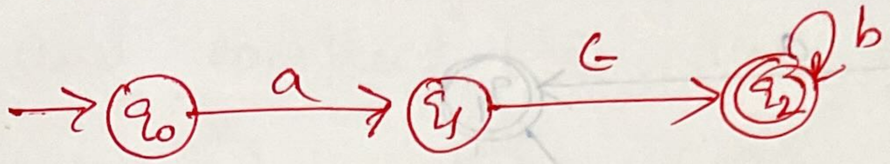
Reduced DFA



DFA of the given NFA



Q:- Convert εNFA to NFA without ε.



Solⁿ:- ε closure of all states:-

$\epsilon\text{ closure}(q_0) = \{q_0\}$

$\epsilon\text{ closure}(q_1) = \{q_1 q_2\}$

$\epsilon\text{ closure}(q_2) = \{q_2\}$

$\delta'(q_0, a) = \epsilon\text{ closure}(\cancel{\epsilon\text{ clos}}\ \delta(\epsilon\text{ closure}(q_0), a))$

$= \epsilon\text{ closure}(\delta(q_0, a))$

$= \epsilon\text{ closure}(q_1) = \{q_1, q_2\}$

$\delta'(q_0, b) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_0), b))$

$= \epsilon\text{ closure}(\delta(q_0, b))$

$= \phi$

$\delta'(q_1, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), a))$

$= \epsilon\text{ closure}(\delta(q_1 q_2, a))$

$= \phi$

$\delta'(q_1, b) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), b))$

$= \epsilon\text{ closure}(\delta(q_1 q_2, b))$

$= q_2$

$\delta'(q_2, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_2), a))$

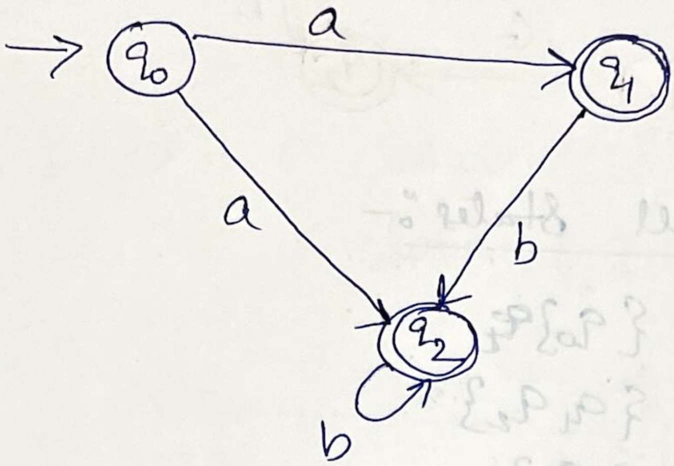
$= \epsilon\text{ closure}(\delta(q_2, a))$

$= \phi$

$\delta'(q_2, b) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_2), b))$

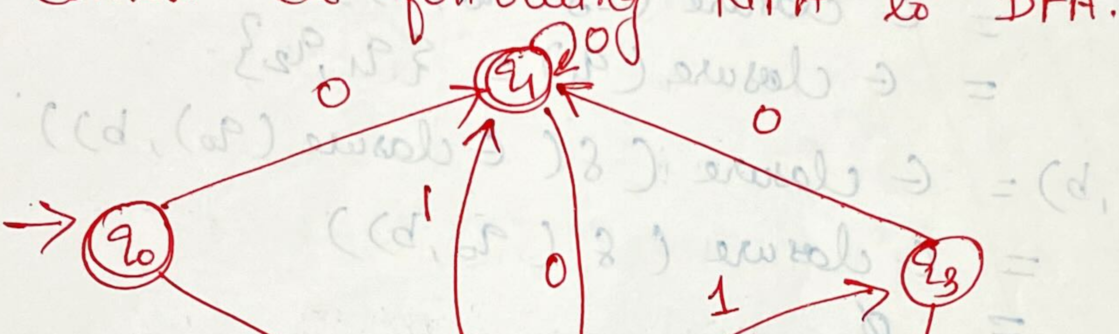
$= \epsilon\text{ closure}(\delta(q_2, b))$

$= q_2$



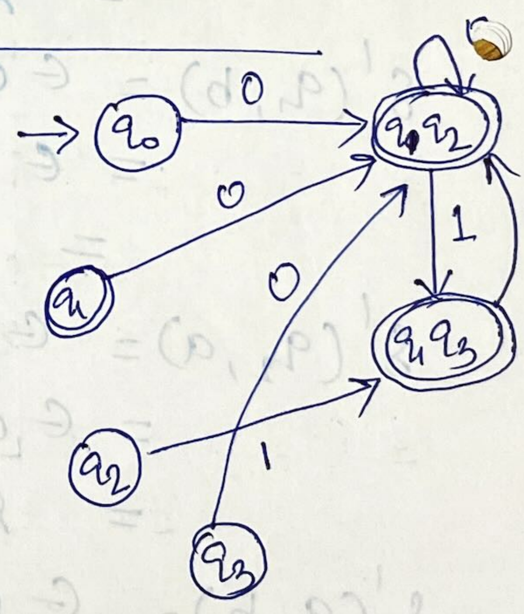
NFA without ϵ Moves.

Q:- Convert the following NFA to DFA.

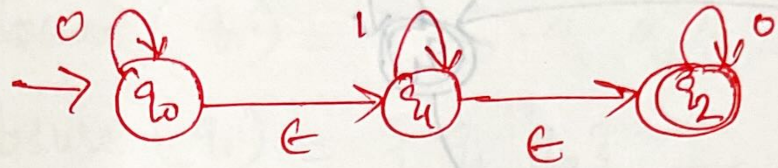


Soln:-

	0	1
$\rightarrow q_0$	q_1q_2	—
$*q_1$	q_1q_2	—
q_2	—	q_1q_3
q_3	q_1q_2	—
$*q_1q_2$	q_1q_2	q_1q_3
$*q_1q_3$	q_1q_2	—



Q:- Consider the following NFA with ϵ moves and construct the DFA for it.



Soln:- ϵ closure of all the states:-

ϵ closure $(q_0) = \{q_0\ q_1\ q_2\}$ - A

ϵ closure $(q_1) = \{q_1\ q_2\}$

ϵ closure $(q_2) = \{q_2\}$

$$\delta'(q_0, 0) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_0), 0))$$
$$= \epsilon\text{ closure}(\delta(q_0 q_1 q_2, 0))$$
$$= \{q_0\ q_1\ q_2\}$$

$$\delta'(q_0, 1) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_0), 1))$$
$$= \epsilon\text{ closure}(\delta(q_0 q_1 q_2, 1))$$
$$= \{q_1\ q_2\}$$

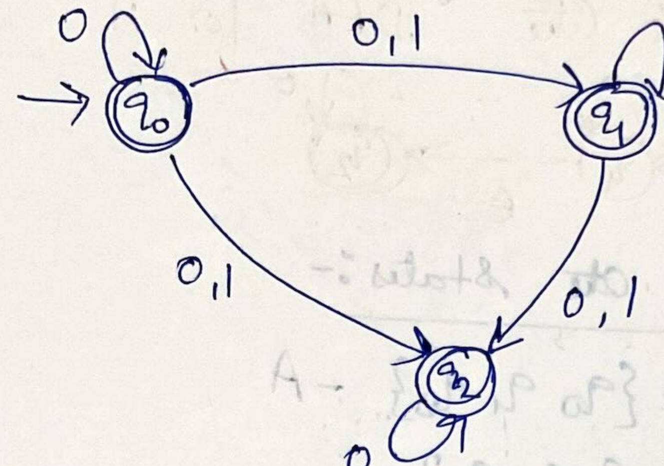
$$\delta'(q_1, 0) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), 0))$$
$$= \epsilon\text{ closure}(q_2) = q_2$$

$$\delta'(q_1, 1) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), 1))$$
$$= \epsilon\text{ closure}(\delta(q_1 q_2, 1))$$
$$= q_1 q_2$$

$$\delta'(q_2, 0) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_2), 0))$$
$$= \epsilon\text{ closure}(q_2) = q_2$$

$$\delta'(q_2, 1) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_2), 1))$$
$$= \epsilon\text{ closure}(\phi) = \phi$$

NFA without ε moves

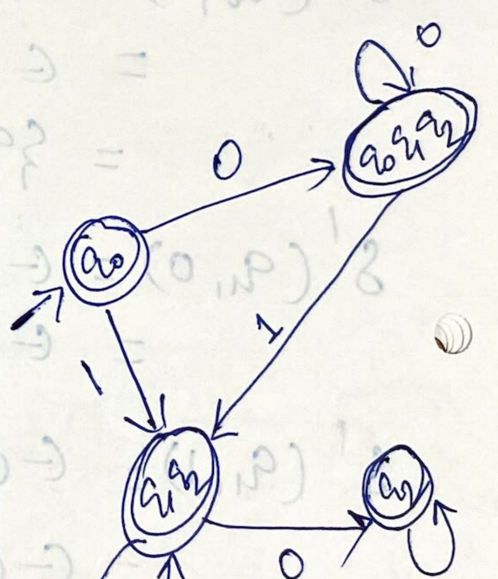


Transition table

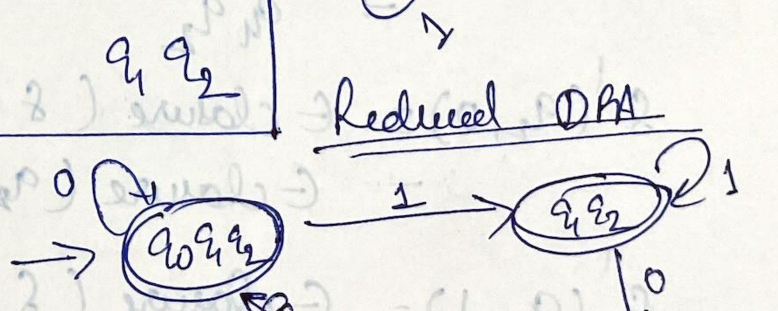
	0	1
→ * q_0	$q_0q_1q_2$	q_1q_2
* q_1	q_2	q_1q_2
* q_2	q_2	–

Transition Table for DFA

	0	1
→ * q_0	$q_0q_1q_2$	q_1q_2
* q_1	q_2	q_1q_2
* q_2	q_2	–
* q_1q_2	q_2	q_1q_2
* $q_0q_1q_2$	$q_0q_1q_2$	q_1q_2



Reduced DFA



Another Way

ϵ closure $(q_0) = \{q_0\ q_1\ q_2\}$ — A

ϵ closure $(q_1) = \{q_1\ q_2\}$ — B

ϵ closure $(q_2) = \{q_2\}$ — C

$\delta'(A,0) = \epsilon$ closure $(\delta(A,0))$

$= \{q_0\ q_1\ q_2\}$ — A

$\delta'(A,1) = \epsilon$ closure $(\delta(A,1))$

$= \{q_1\ q_2\}$ — B

$\delta'(B,0) = \epsilon$ closure $(\delta(B,0))$

$= \epsilon$ closure $(q_2) = q_2$ — C

$\delta'(B,1) = \epsilon$ closure $(\delta(B,1))$

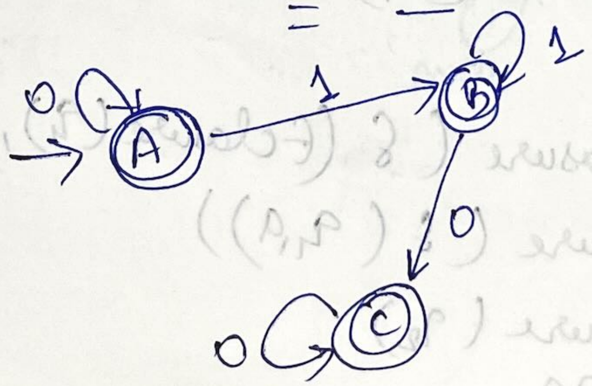
$= \{q_1\ q_2\}$ — B

$\delta'(C,0) = \epsilon$ closure $(\delta(C,0))$

$= q_2$ — C

$\delta'(C,1) = \epsilon$ closure $(\delta(C,1))$

$=$ —



Q:- Convert the following NFA to an equivalent DFA.

State	a	b	ϵ
$\rightarrow q_0$	$q_0 q_1$	q_1	{}
q_1	q_2	$q_1 q_2$	{}
$* q_2$	$\{q_0\}$	q_2	q_1

Solⁿ:- Step 1: ϵ closure of all states :-

ϵ closure $(q_0) = q_0$

ϵ closure $(q_1) = q_1$

ϵ closure $(q_2) = q_1 q_2$

Step 2: New Transitions

$\delta'(q_0, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_0), a))$

$= \epsilon\text{ closure}(\delta(q_0, a))$

$= \epsilon\text{ closure}(q_0 q_1)$

$= q_0 q_1$

$\delta'(q_0, b) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_0), b))$

$= \epsilon\text{ closure}(\delta(q_0, b))$

$= \epsilon\text{ closure}(q_1)$

$= q_1$

$\delta'(q_1, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), a))$

$= \epsilon\text{ closure}(\delta(q_1, a))$

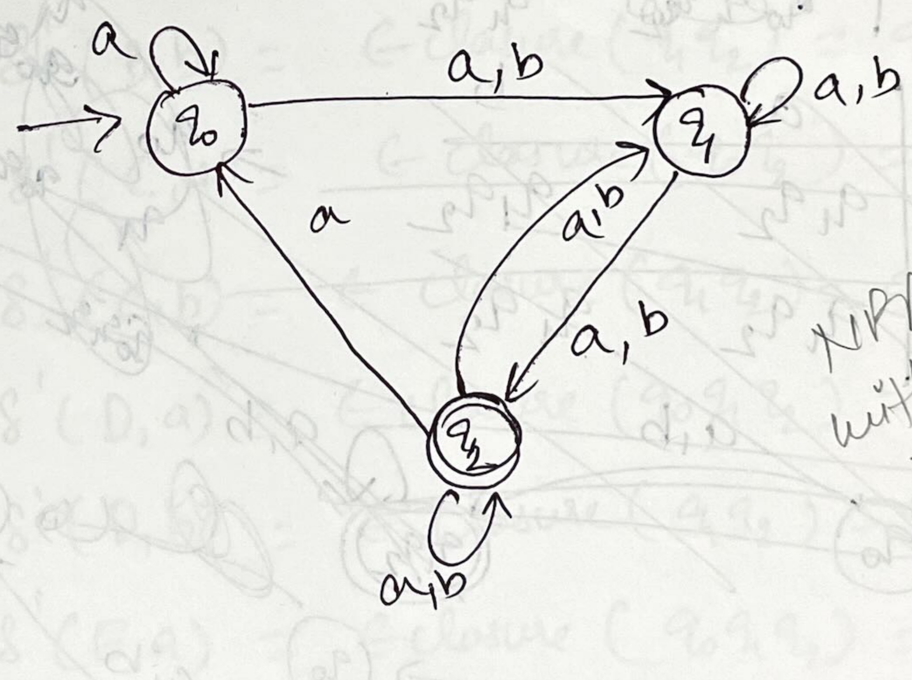
$= \epsilon\text{ closure}(q_2)$

$= q_1 q_2$

$$\delta'(q_1, b) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_1), b))$$
$$= \epsilon\text{closure}(\delta(q_1 q_2, b))$$
$$= \epsilon\text{closure}(q_1 q_2) = q_1 q_2$$

$$\delta'(q_2, a) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_2), a))$$
$$= \epsilon\text{ closure}(\delta(q_1 q_2, a))$$
$$= \epsilon\text{closure}(q_2, q_0)$$
$$= q_0 q_1 q_2$$

$$\delta'(q_2, b) = \epsilon\text{closure}(\delta(\epsilon\text{closure}(q_2), b))$$
$$= \epsilon\text{ closure}(\delta(q_1 q_2, b))$$
$$= \epsilon\text{closure}(q_1 q_2)$$
$$= q_1 q_2$$



NFA without ϵ moves

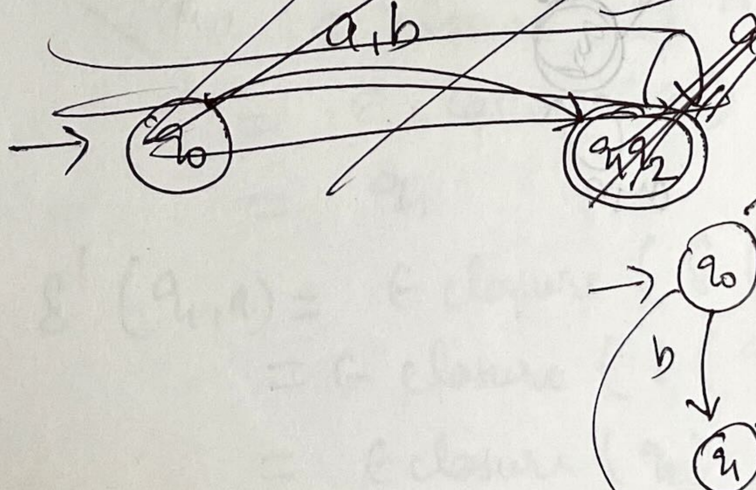
	a	b
$\rightarrow q_0$	q_0q_1	q_1
q_1	q_1q_2	q_1q_2
$*q_2$	$q_0q_1q_2$	q_1q_2
q_0q_1	$q_0q_1q_2$	q_1q_2
$*q_1q_2$	$q_0q_1q_2$	q_1q_2
$*q_0q_1q_2$	$q_0q_1q_2$	q_1q_2

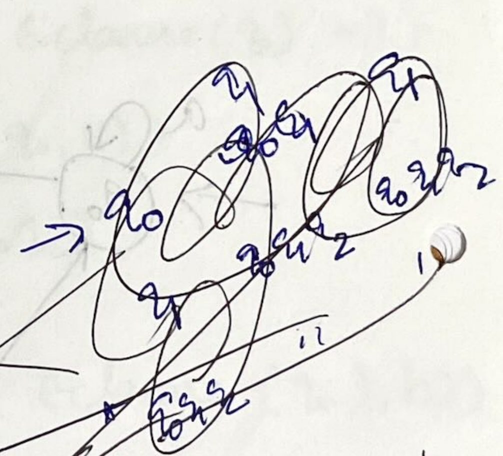
q_1q_2 · (Not possible bcoz final state can't be merged with non final)

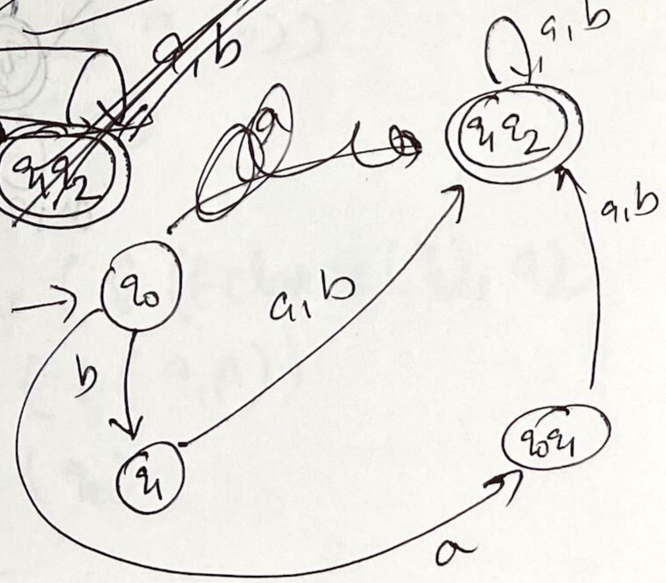
	a	b
$\rightarrow q_0$	q_0q_1	q_1
q_1	q_1q_2	q_1q_2
$*q_1q_2$	q_1q_2	q_1q_2
q_0q_1	$q_0q_1q_2$	q_1q_2

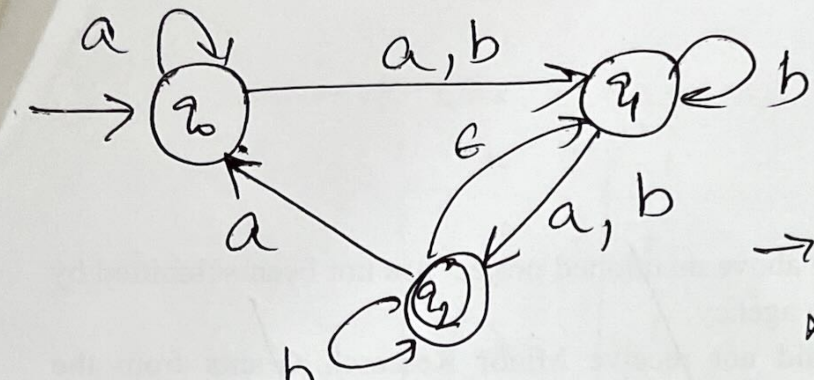
q_1q_2

	a	b
$\rightarrow q_0$	q_1q_2	q_1q_2
$*q_1q_2$	q_1q_2	q_1q_2









	a	b
A	D	B
→ B	C	C
* C	E	C
D	E	C
* E	E	C

ϵ Clo.(q_0) $\quad q_0$ — A

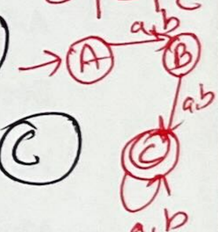
(q_1) $\quad q_1$ — B

q_2 $\quad q_1 q_2$ — C

	a	b
→ A	C	B
B	C	C
* C	C	C

	a	b
→ A	D	B
B	C	C
* C	C	C
D	C	C

	a	b
→ A	B	B
B	C	C
* C	C	C



$\delta'(A, a) = \epsilon \text{ closure}(\delta(A, a))$

$= \epsilon \text{ closure}(q_0 q_1) = q_0 q_1$ — D

$\delta'(A, b) = \epsilon \text{ closure}(q_1) = q_1$ — B

$\delta'(B, a) = \epsilon \text{ closure}(q_2) = q_1 q_2$ — C

$\delta'(B, b) = \epsilon \text{ closure}(q_1 q_2) = q_1 q_2$ — C

$\delta'(C, a) = \epsilon \text{ closure}(q_0 q_2) = q_0 q_1 q_2$ — E

$\delta'(C, b) = \epsilon \text{ closure}(q_1 q_2) = q_1 q_2$ — C

$\delta'(D, a) = \epsilon \text{ closure}(q_0 q_1 q_2) = q_0 q_1 q_2$ — E

$\delta'(D, b) = \epsilon \text{ closure}(q_1 q_2) = q_1 q_2$ — C

$\delta'(E, a) = \epsilon \text{ closure}(q_0 q_1 q_2) = q_0 q_1 q_2$ — E

$\delta'(E, b) = \epsilon \text{ closure}(q_1 q_2) = q_1 q_2$ — C

Q: Convert the following NFA to DFA.

	0	1
→ p	pq	p
q	rs	t
r	pr	t
* s	—	—
* t	—	—

Sol^n:

	0	1
→ p	pq	p
q	rs	t
r	pr	t
* s	—	—
* t	—	—
pq	pqrs	pt
* rs	pr	t
pr	pqr	t
pqr		
pr		
pqr		