

Languages

"31"

Language :- A system suitable for the expression of certain ideas, facts or concepts including a set of symbols and rules for their implementation.

Formal Languages :-

1) Symbol :- A "Symbol" is an abstract or a user defined entity. Letters and digits are examples of frequently used symbols. Symbols cannot be formally defined like point in geometry.
e.g. 'a, b, c, d' are symbols.

2) Alphabet :- Let A denote a non empty set of symbols. Such a set A is called an alphabet. An alphabet need not be finite. But, here we will assume alphabet as a finite. So, an alphabet is a finite set of symbols.

e.g. $\{a, b, \dots, z\}$, $\{0, 1, 2, \dots\}$

3) String :- A "string" over an alphabet is a finite sequence of symbols from that alphabet, which is usually written next to one another not separated by commas.

e.g. if $\Sigma = \{a, b\}$ then $u = abba$

$v = abbab$ strings on Σ .

and if $\Sigma = \{0, 1\}$ then 00110011 is the string over
Length of the String:-

The length of a string is its length as a sequence. So, the length of the string 'w' is denoted by $|w|$, is the no. of symbols in the string.

$$u = abba \quad |u| = 5$$

$$v = abba \quad |v| = 4$$

The string of zero length is called as "empty string". This is denoted by λ or ϵ or \emptyset .

Introduction to defining language

Suppose,

$$\Sigma = \{0\}$$

$$L_1 = \{0, 00, 000, 0000, \dots\}$$

This could also be written as,

$$L_1 = \{0^n : n = 1, 2, 3, \dots\}$$

Concatenation :- In concatenation two strings are written down side by side to form a new longer string.

Some basic properties of concatenation are as follows:-

The set of all strings (including λ) on the alphabet set Σ is denoted by Σ^* and the set of non-empty strings by Σ^+ .

Thus, $\Sigma = \{a, b\}$

$$\Sigma^* = \{ \lambda, a, b, ab, aab, abb, aabb, \dots \}$$

- 1) Concatenation on a set Σ^* is associative
since for each u, v, w in Σ^* , $u(vw) = (uv)w$.
- 2) Identity Element:- The empty string is an identity element for the operation i.e

$$\lambda u = u \lambda = u \quad \forall u \in \Sigma^*$$

- 3) Σ^* has left and right cancellation.
for, u, v & w in Σ^* .

$$uww = vw \Rightarrow u = v \quad (\text{left cancellation})$$

$$uw = vw \Rightarrow u = v \quad (\text{right cancellation})$$

- 4) For, u, v in Σ^* , we have

$$|uv| = |u| + |v|.$$

That is, the length of the concatenation of two strings is the sum of the individual lengths.

- 5) x^i is concatenated with x^j is the string x^{i+j} .

Substring:-

Suppose $\Sigma = \{a, b\}$

Consider any string $u = 0011010$ on an alphabet Σ .

Any sequence $w = 0110$ is called a substring of u .

Prefix and Suffix :-

Given two words u and v over an alphabet Σ , if they have same length and the same symbols at the same positions.

We say u is a prefix of v if there is word w over Σ such that $w = uv$ and we say that u is a proper prefix of v if $u \neq \lambda$, $u \neq v$ and u is a prefix of v .

That is λ is always a prefix of any word ' u' including itself and a word u is always prefix of itself and if $w = uv$ the v is the suffix of u .

e.g. $w = "0011"$

Prefixes — $\lambda, 0, 00, 001, 0011$

Suffix — $1, 11, 011, 0011$

Reverse of a String:-

If $w = w_1 w_2 \dots w_n$ where $w_i \in \Sigma$ the reverse of w is $w^R = w_n w_{n-1} \dots w_1$.
If ' w ' be a word in language L , then $\text{reverse}(w)$ is the same string of letters traversed backward called the reverse of w , even if this backward string is not a word in language L .

Eg:- $\text{reverse}(a_1 a_2 a_3 a_4 a_5) = a_5 a_4 a_3 a_2 a_1$,

$\text{reverse}(2361) = 1632$

Transpose is also called reverse operation.

* Palindrome :-

Palindrome of a string is a property of a string in which string can be ~~read~~ read same from left to right as well as from right to left.

* Lexicographic Ordering:-

The lexicographic ordering of a string is the same as the dictionary ordering, except that shorter strings precede longer strings.

Lexicographic ordering $\{0,1\}$ is

$\{ \lambda, 0, 1, 00, 11, 01, 10, 111, 000, 101, 010, 011, \dots \}$

Q1: $L_1 = \{x, xy, x^2\}$

(V_N, Σ, S, P)
 (Q, Σ, S, q_0, F)

$$L_2 = \{y^2, xyx\}$$

Find $\textcircled{a} L_1 L_2$ $\textcircled{b} L_2^2$ ~~\textcircled{c}~~ $\textcircled{c} L_2 L_1$

$\textcircled{a} L_1 L_2 = \{x, xy, x^2\} \{y^2, xyx\}$

~~$\{xy^2, xyy^2, x^2y^2, x^2yx, xyxyx, x^3yx^2\}$~~

$$= xy^2, \cancel{xy^3}, x^2y^2, x^2yx, xyxyx, x^3yx^2$$

$$L_2 L_1 = \{y^2, xyx\} \{x, xy, x^2\} = \{y^2x, xyx^2, y^2xy, y^2x^2, xyx^2y, xyx^3\}$$

$\textcircled{b} L_2^2 = \{y^2, xyx\} \{y^2, xyx\}$

$$= \{y^4, xyxy^2, y^2xyx, xyx^2yx\}$$

Q2: Let $L = \{ab, aa, baa\}$ which of the following
 are in L^*

$\textcircled{a} abaabaaaabaa \quad \textcircled{b} aaabbaaaa$

$\textcircled{c} baaaaabaaaab \quad \textcircled{d} baaaaabaa$

$\textcircled{a} \underline{abaabaaaabaa} \in L^*$

$\textcircled{b} \underline{aaaabaaa} \in L^*$

$\textcircled{c} \underline{baaaa} \underline{baaaa} b \notin L^*$

$\textcircled{d} \underline{baaaa} \underline{abaa} \in L^*$

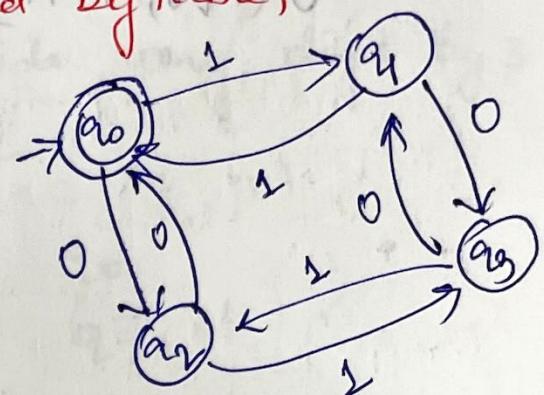
$L = \{a, b\}^*$
$L^* = \{\epsilon, a, b, ab, ba, abb, baa, abba\}$

45.23
45.32
9.88
9.29

Acceptability of a string by FA

Q:- For the FSM, M described by table,

State	Input	
	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2



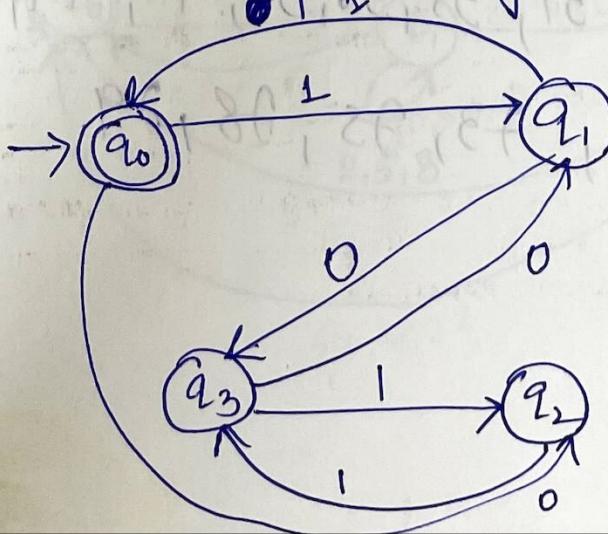
Find the strings among the following strings which are accepted by M!

- (a) 101101 (b) 11111* (c) 000000 * (d) 1101011

Soln:- $\delta(q_0, 101101) \Rightarrow \delta(q_1, 01101)$
 $\Rightarrow \delta(q_3, 1101)$
 $\Rightarrow \delta(q_2, 101)$
 $\Rightarrow \delta(q_3, 01)$
 $\Rightarrow \delta(q_1, 1) \Rightarrow q_0$

* (e) 0011011
 (f) 11010010

q_0 is the final state which we got after processing the provided input. So, the string 101101 is accepted by M.



ii) 11111

$$\delta(q_0, 11111) \vdash \delta(q_1, 1111)$$

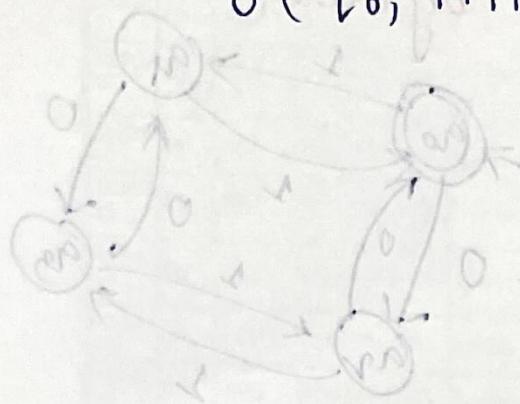
$$\vdash \delta(q_0, 111)$$

$$\vdash \delta(q_1, 11)$$

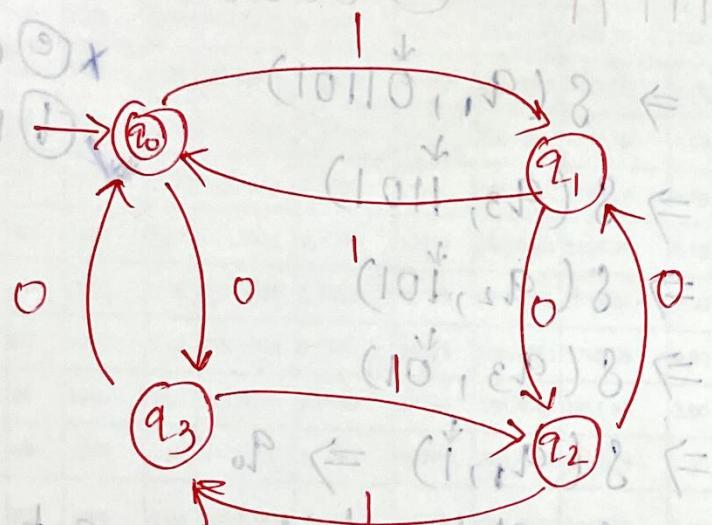
$$\vdash \delta(q_0, 1)$$

$\vdash q_1$ Non final State

Not acceptable



Q1: Check



~~10110~~

~~101011~~

a) 1101011 ~~no~~

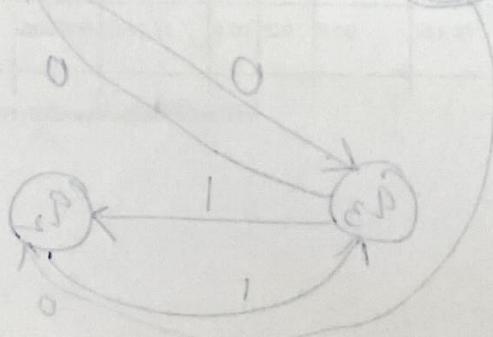
b) 0011011 ~~no~~

c) 11010010 ~~yes~~

1-80

58

1, 11, 24, 25, 31, 35, 36, 32, 42, 44, 50, 53, 54,
58, 60, 69, 73, 75, 78, 79.



Q: Construct a DFA for accepting the decimal no. which is divisible by 3.

Soln: Always whenever we divide any digit by 3 we get remainders 0, 1 or 2.

For 0, 3, 6, 9 — rem 0 q_0
 1, 4, 7 — rem 1 q_1
 2, 5, 8 — rem 2 q_2

$$M = \{ Q, \Sigma, \delta, q_0, F \}$$

$$\delta = \{ q_0, q_1, q_2 \}$$

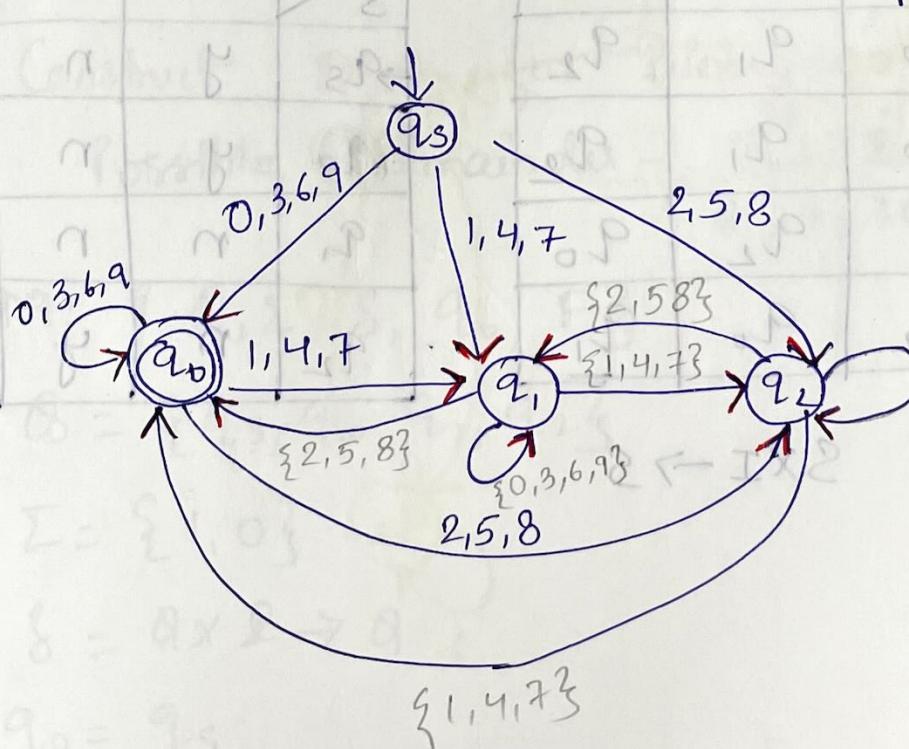
$$\Sigma = \{ 0, 1, \dots, 9 \}$$

$$S = Q \times \Sigma \rightarrow \delta$$

$$q_0 = q_s$$

$$F = q_0$$

	0	1
$\xrightarrow{*} q_0 (00)$	q_0	q_1
$q_1 (01)$	q_2	q_0
$q_2 (10)$	q_1	q_2



Suppose the no. is 256542

$$S(q_3, 2) \rightarrow q_2$$

$$S(q_2, 5) \rightarrow q_1$$

$$S(q_1, 6) \rightarrow q_1$$

$$S(q_1, 5) \rightarrow q_0$$

$$S(q_0, 4) \rightarrow q_1$$

$$S(q_1, 2) \rightarrow q_0$$

It reaches to final state q_0 so it is divisible by 3.

Transition Table

S \ I	{0, 3, 6, 9}	{1, 4, 7, 2}	{2, 5, 8}
$\rightarrow q_3$	q_0	q_1	q_2
q_0	q_0	q_1	q_2
q_1	q_1	q_2	q_0
q_2	q_2	q_0	q_1

S \ I	0, 3, 6, 9	1, 4, 7	2, 5, 8
$\rightarrow q_3$	y	n	n
q_0	y	n	n
q_1	n	n	y
q_2	n	y	n

STFI SXI \rightarrow S

Q: Give DFA for accepting a language of string over the alphabet {0,1} such that they contain a substring "1001".

$$M = \{ Q, \Sigma, S, q_0, F \}$$

$$Q = \{q_3, q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{0, 1\}$$

$$S = Q \times \Sigma^* \rightarrow Q$$

$$q_0 = q_s$$

q_s - Start State

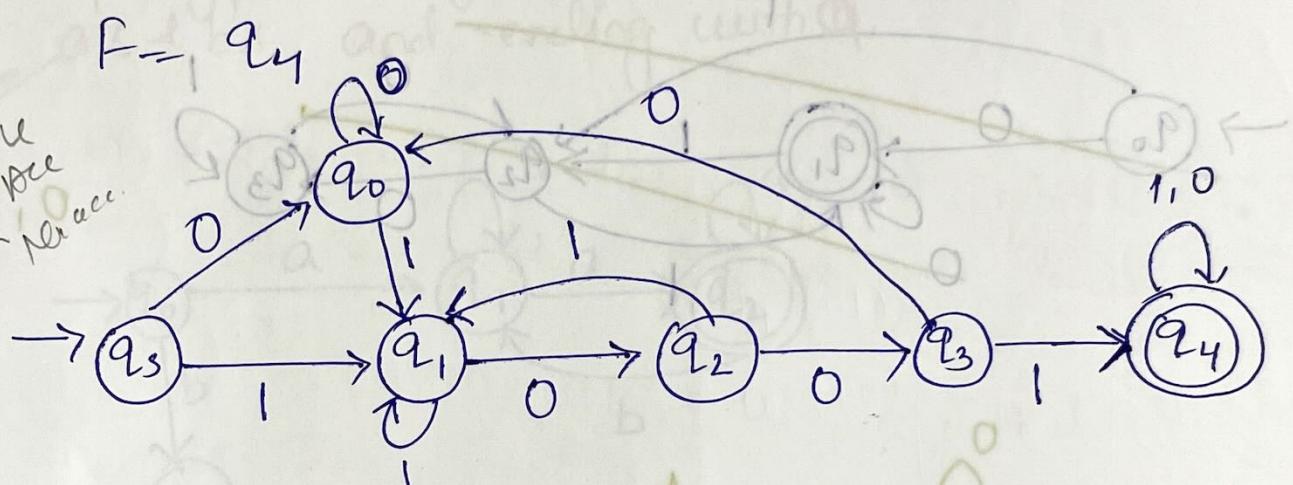
q_0 - ends with 0

q_1 - " " 1

q_2 - " " 10

q_3 - " " 100

q_4 - " " 1001



Q: Construct DFA for Binary no. divisible by 3.

Soln: Possible Remainder - 0, 1, 2
0, 1, 10

$$M = \{ Q, \Sigma, S, q_0, F \}$$

$$Q = \{q_3, q_0, q_1, q_2\}$$

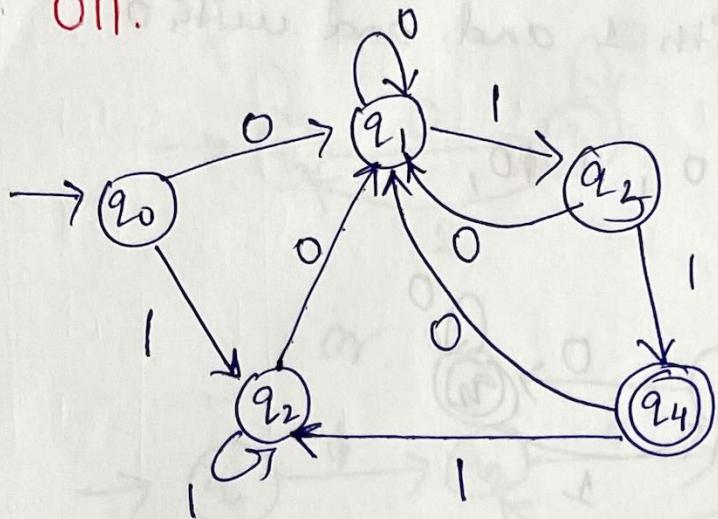
$$\Sigma = \{1, 0\}$$

$$S = Q \times \Sigma^* \rightarrow Q$$

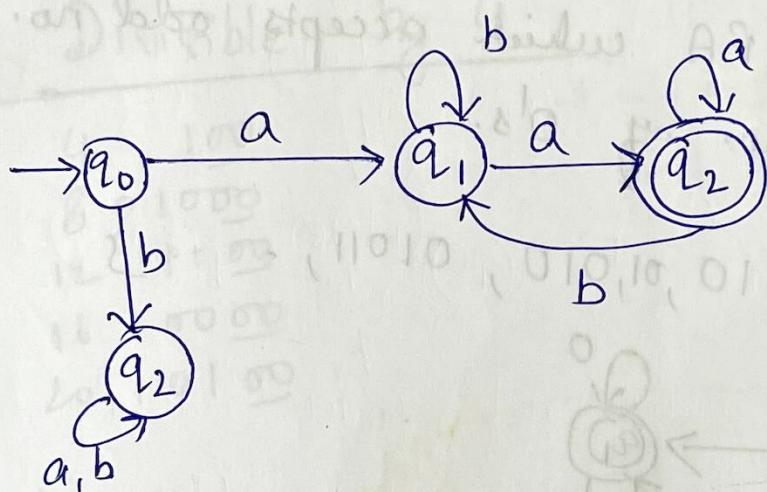
$$q_0 = q_s$$

$$F = q_0$$

Ex. DFA to accept string of 0's & 1's ending with 011.



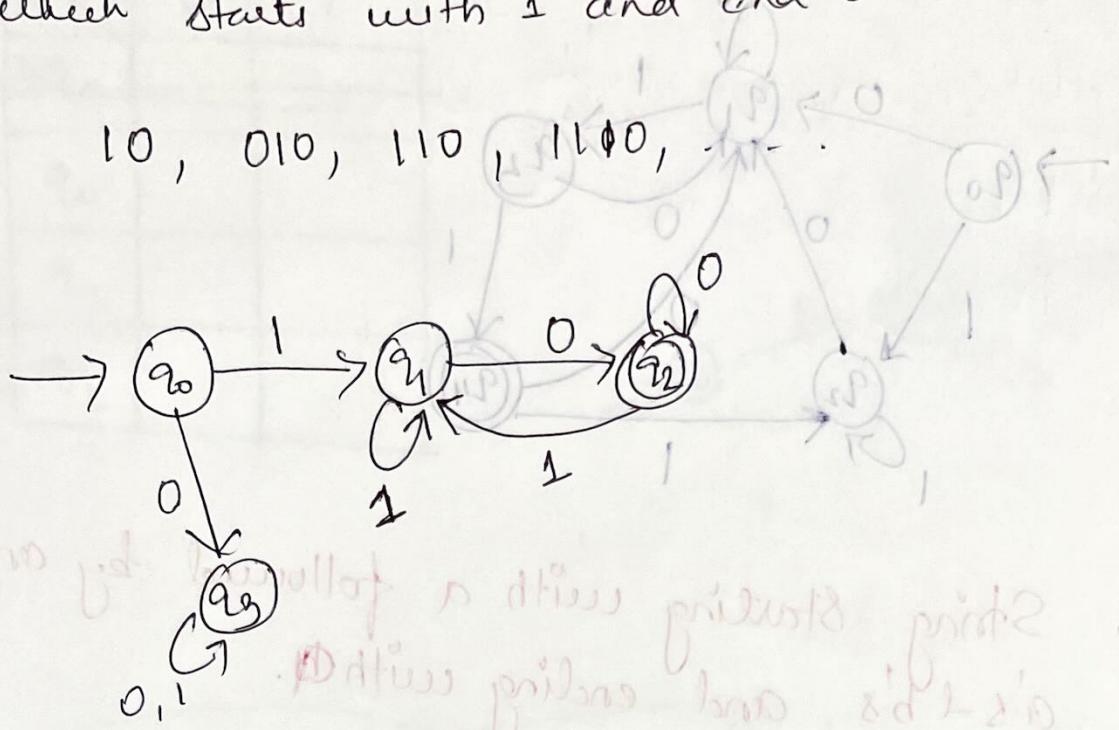
\Leftrightarrow String starting with a followed by any no. a's & b's and ending with 011.



Q1: Design a FA which accepts strings which starts with 1 and end with 0.

Sol^{No}:

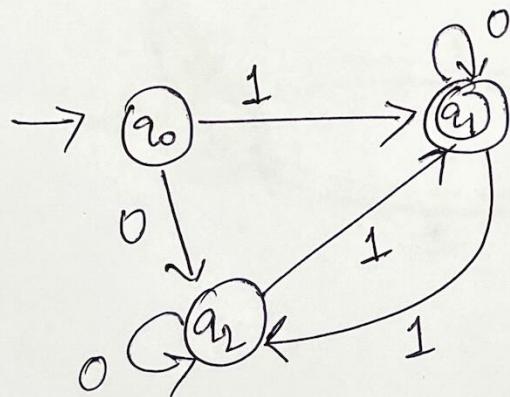
$$L = \{10, 010, 110, 1110, \dots\}$$



Q2: Design an FA which accepts odd no. of 1's and any no. of 0's.

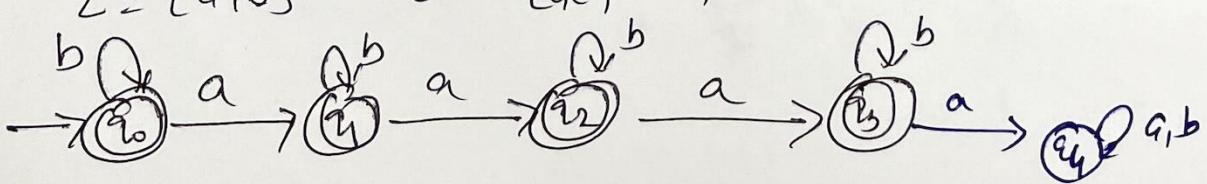
Sol^{No}:

$$L = \{1, 10, 01, 010, 0101, \dots\}$$

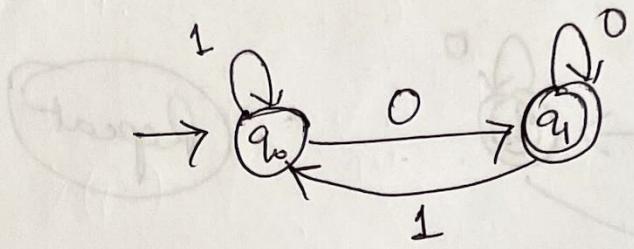


Q3: Design an FA which has atleast 3 a's and any no. of b.

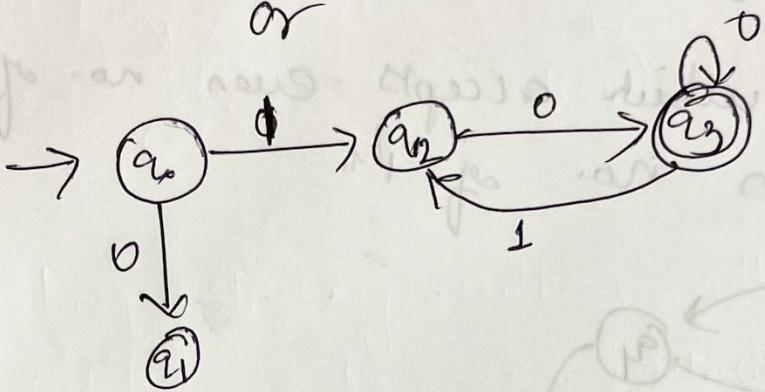
$$\Sigma = \{a, b\} \quad L = \{aa, aa, aaaa, \epsilon, aabb, b, \dots\}$$



Divisible by 2

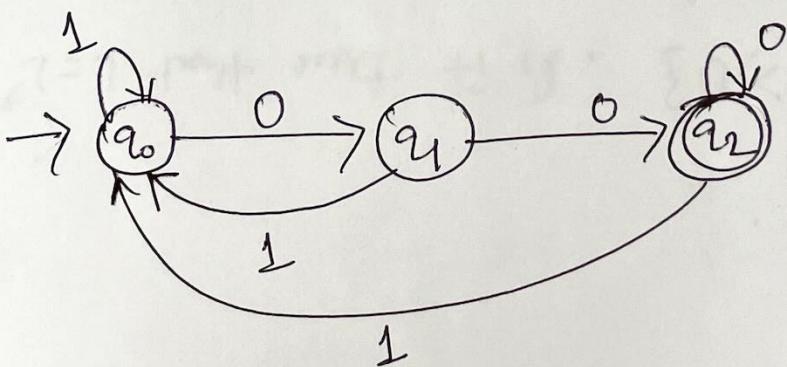


or

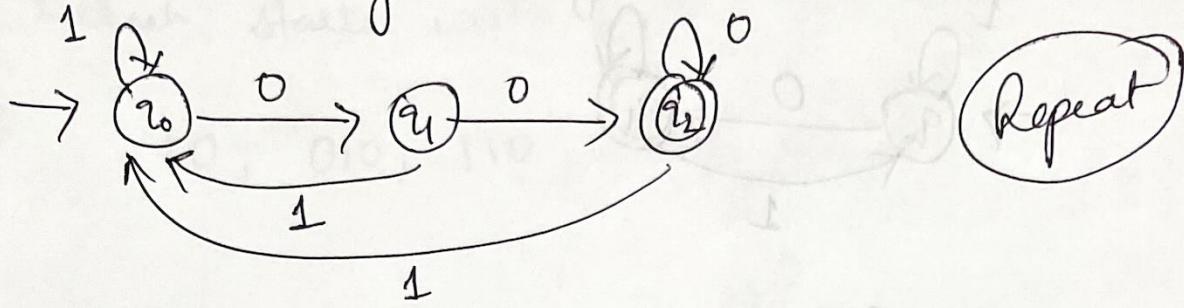


Divisible by 4

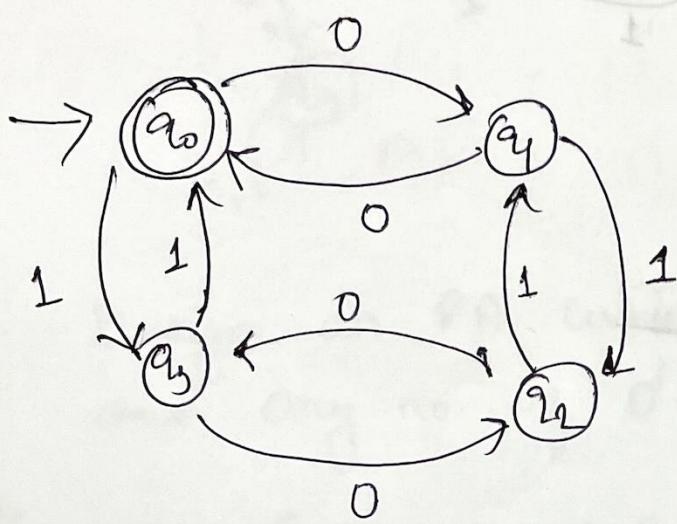
4	100
8	1000
12	1100
16	10000
20	10100



Q: Design an FA to accept the string that always ends with 00.



Q: Design FA which accepts even no. of 0's and even no. of 1's.



q_0

q_1

q_2

q_3

$$\begin{aligned} n=0 \quad m=1 &\Rightarrow b \\ n=1 \quad m=1 &\Rightarrow ab \end{aligned}$$

Q1:-
a) $L = a^n b^m; n \geq 0, m \leq 4$

b) odd no. of 1's followed by an even no. of 0's.
 $100, 10000, 11100\ldots$

c) String over 0 & 1, no. of 0's divisible by 3
and no. of 1's divisible by 5.
 $\begin{array}{c} 00011111 \\ 00000011111111 \end{array}$

Q2:- a) Write strings over a and b such that
the fourth symbol from right end is a
and fifth symbol is b?
 $\begin{array}{c} ba \text{ or } a \\ \text{or } b \\ b \end{array}$

b) Strings of a's and b's containing no more
than 3 a's.
 $b, ab, ba, aa, aab, bas, abs\ldots$

c) Which are the strings of a's and b's whose
length is divisible by 3?
 $aba, aag, bas, abb\ldots$

d) Write the set of all strings over a and b
containing symbol a at second position
from the right end?
 $\begin{array}{c} a \text{ or } \\ a \\ b \end{array}$

e) Write strings over 0 & 1 with a substring 010.
 $010, 0010, 00101, 001010\ldots$

f) Set of all strings over 0 & 1 whose decimal
no. is not divisible by 5.

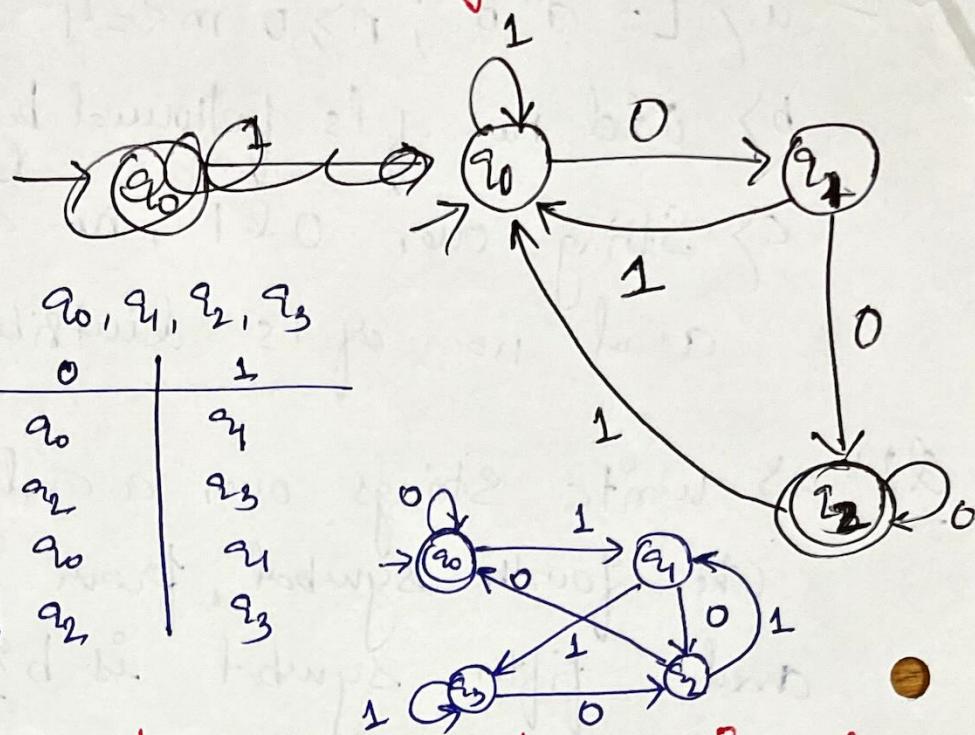
decimal no. divisible by 5.

Q.: Design an automaton that recognizes numbers divisible by 4.

Sol^{n.o.}

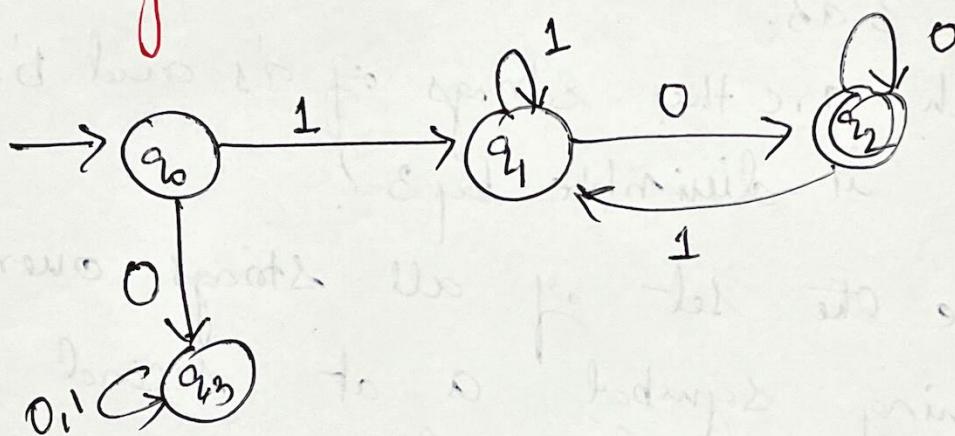
4	100
8	1000
12	1100
16	10000
...	

	0	1
$q_0(00)$	q_0	q_1
$q_1(01)$	q_2	q_3
$q_2(10)$	q_0	q_1
$q_3(11)$	q_2	q_3



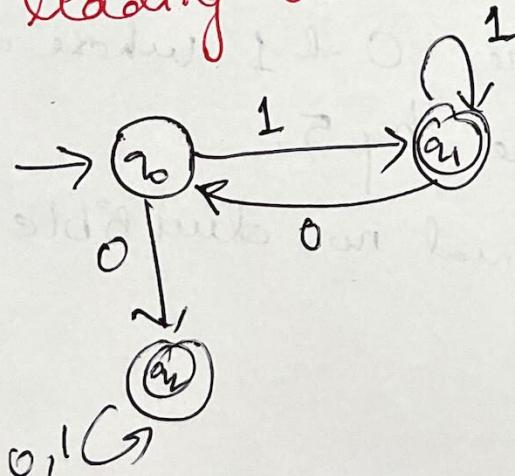
Q.: Design a DFA for even numbers without leading 0's.

Sol^{n.o.}



Q.: Design a DFA for odd numbers without leading 0's.

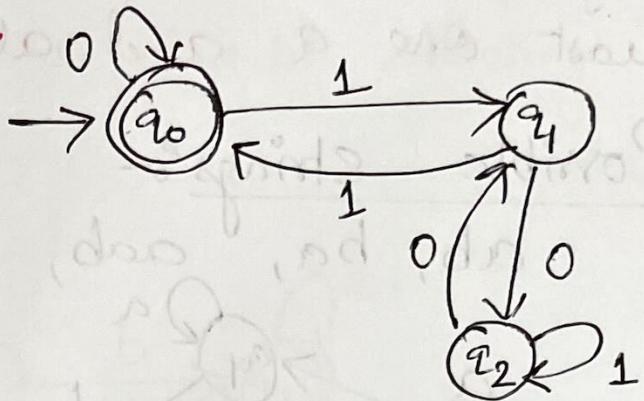
Sol^{n.o.}



Q_b: Design a DFA that accepts all binary numbers divisible by 3.

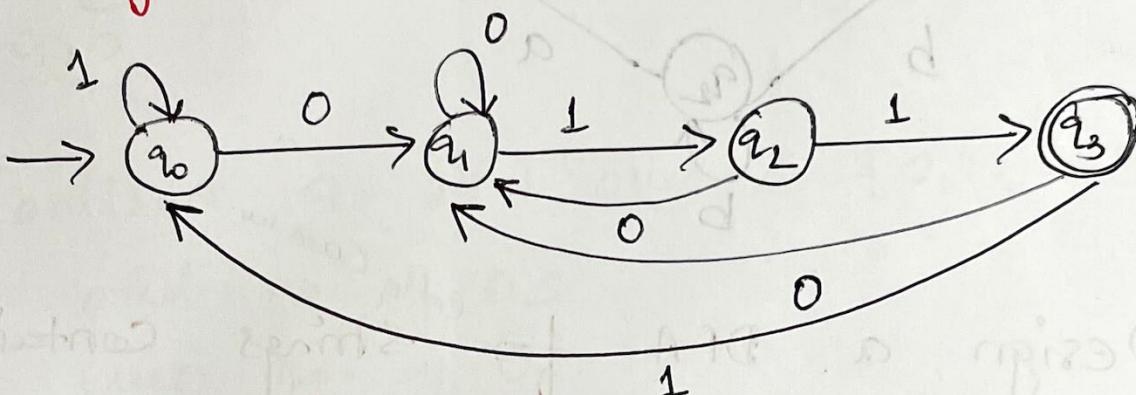
Sol^{n_o}:

$$\begin{array}{l} 3 - 11 \\ 6 - 110 \\ 9 - 1001 \\ 12 - 1100 \\ \dots \end{array}$$



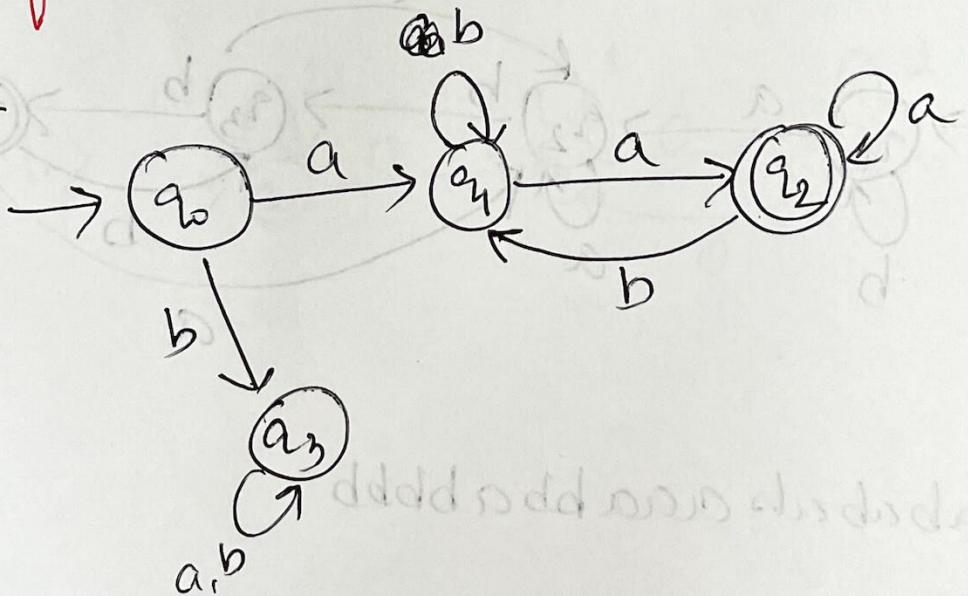
Q_b: Design a DFA to accept strings of 0's and 1's ending with 011.

Sol^{n_o}:



Q_b: String starting with a followed by any no. of a's and b's and ending with a.

Sol^{n_o}:



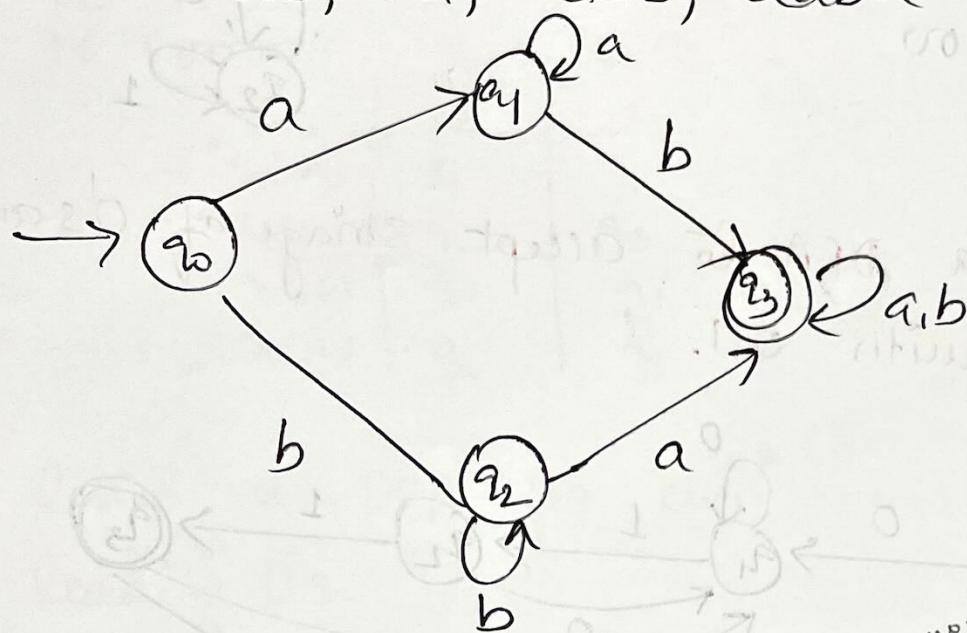
$$\begin{aligned}
 A &- 8 \\
 SB &- 4 \\
 AJP &- 4 \\
 BT &- 3 \\
 \hline
 &\frac{15}{4}
 \end{aligned}$$

Q:- Design a DFA for strings containing atleast one a and atleast one b.

Sol:-

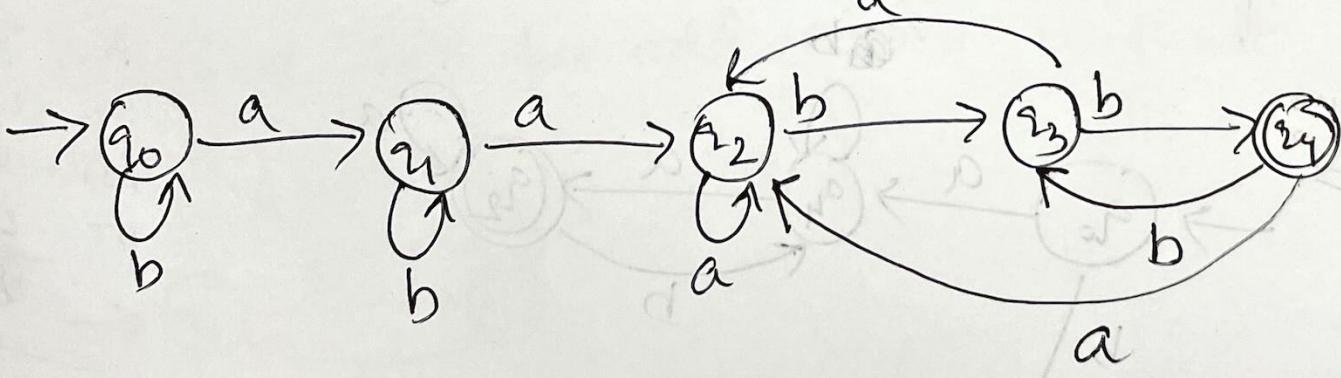
Possible strings:-

ab, ba, aab, aaba



Q:- Design a DFA for strings containing atleast two a's and ending with an even no. of b's.

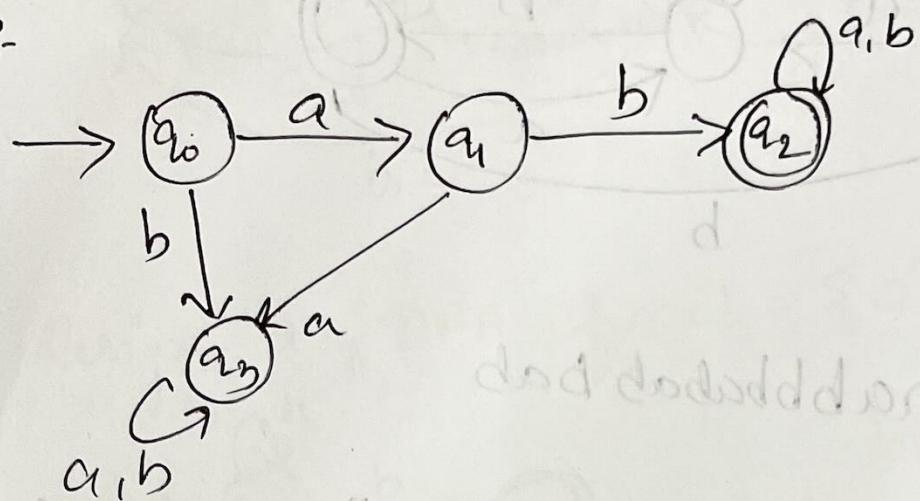
Sol:- aabb, aaabb, aaabbbb, baaabbbb



e.g. ababab aaabbabbbb

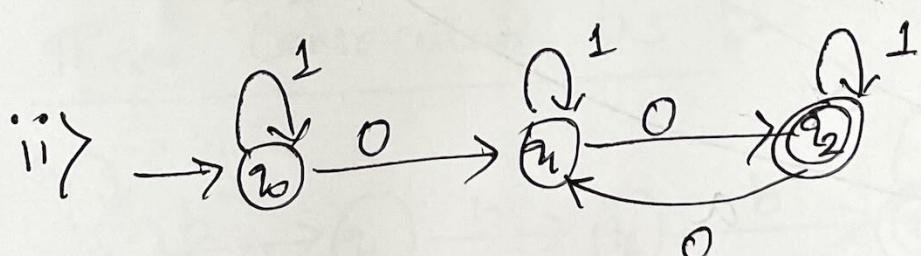
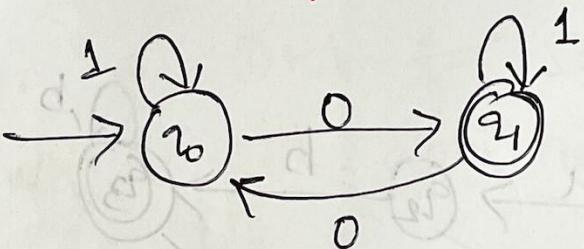
Q₆- Design a DFA which accepts the set
of all strings on $\Sigma = \{a, b\}$ starting with the
prefix ab.

Sol^{n.o.}-



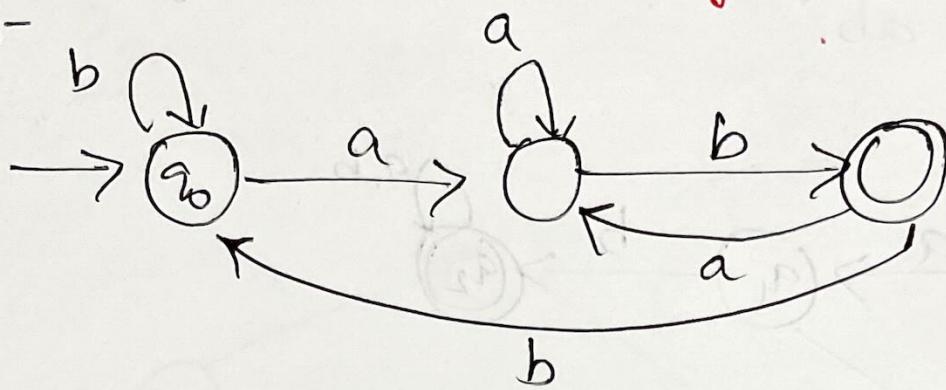
Q₇- Construct a DFA over $\Sigma = \{0, 1\}$ accepting
 i) odd no. of 0's
 ii) even no. of 0's.

Sol^{n.o.}- i)



Q.:- Design a DFA accepting all string over $\Sigma = \{a, b\}$ ending in ab.

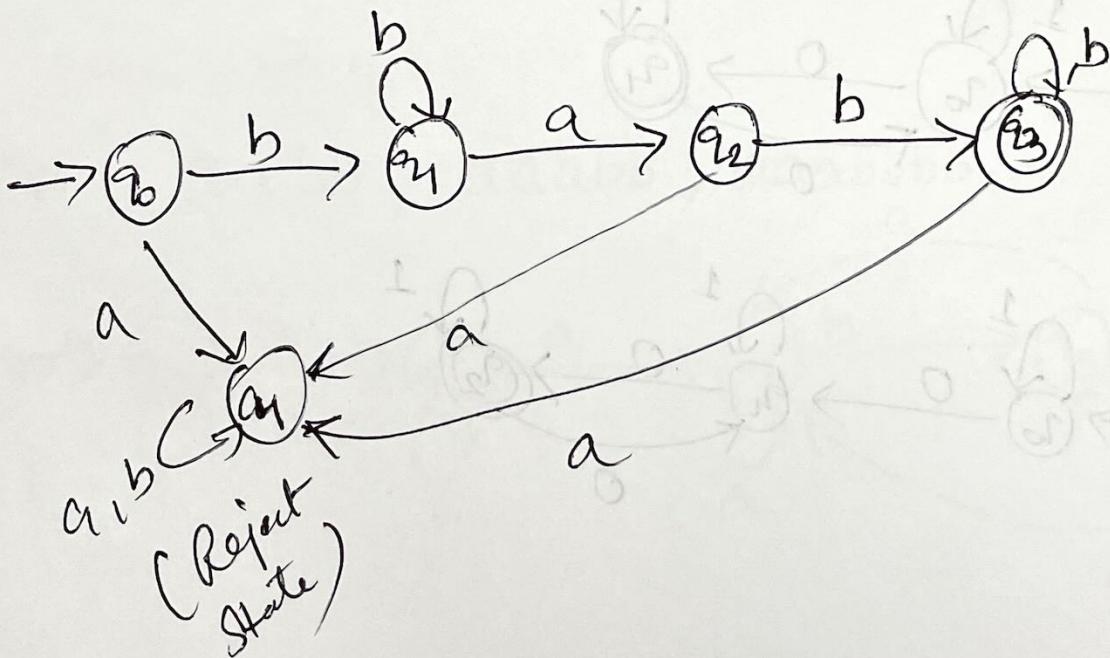
Sol:-



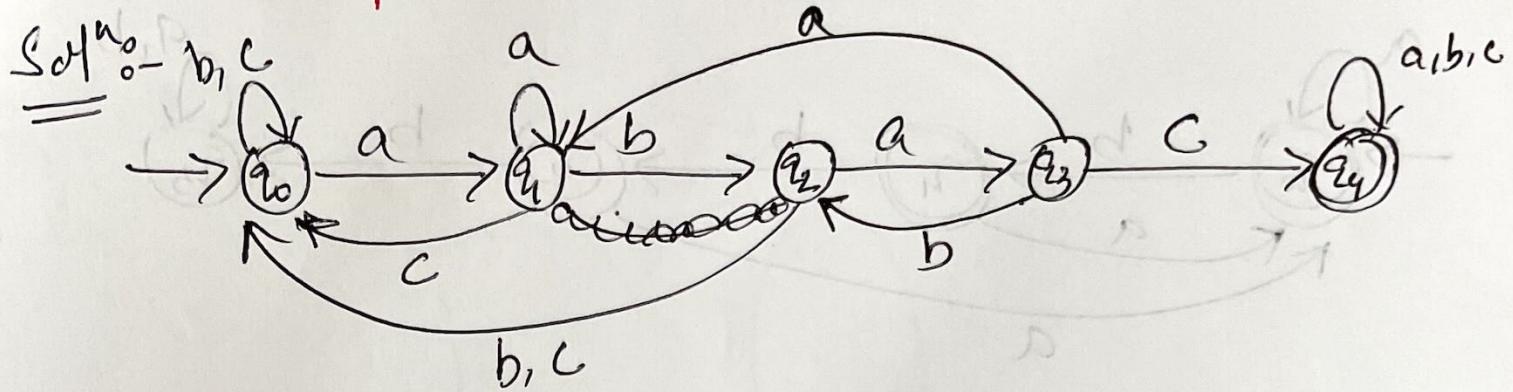
e.g.- aabbabab bab

Q.:- Construct a DFA which will recognize the language $L = \{b^m a b^n : m, n > 0\}$.

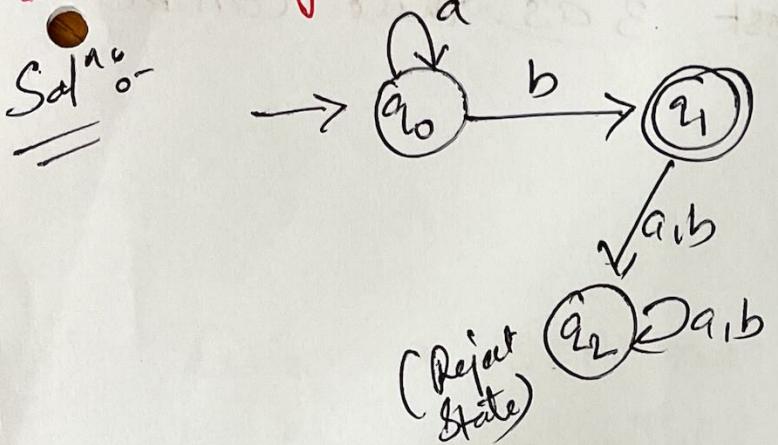
Sol:- $L = \{bab, bbab, babb, bbb a bbb, \dots\}$



Q:- Design a DFA for strings containing the pattern abac over $\Sigma = \{a, b, c\}$

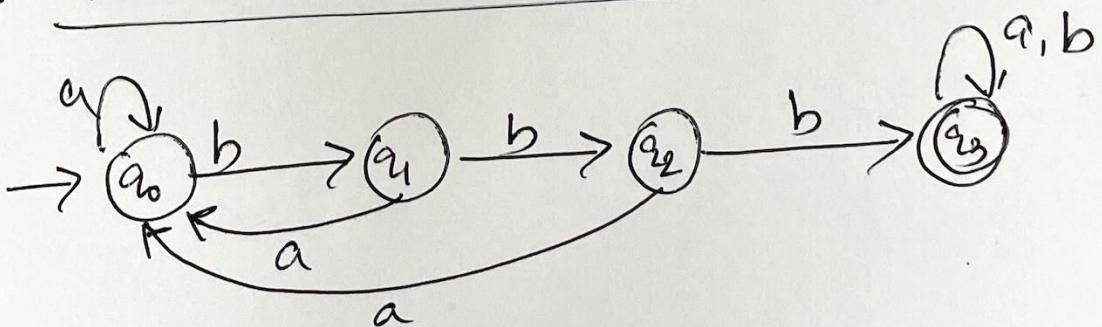


Q:- Design a DFA for $L = \{a^n b : n \geq 0\}$

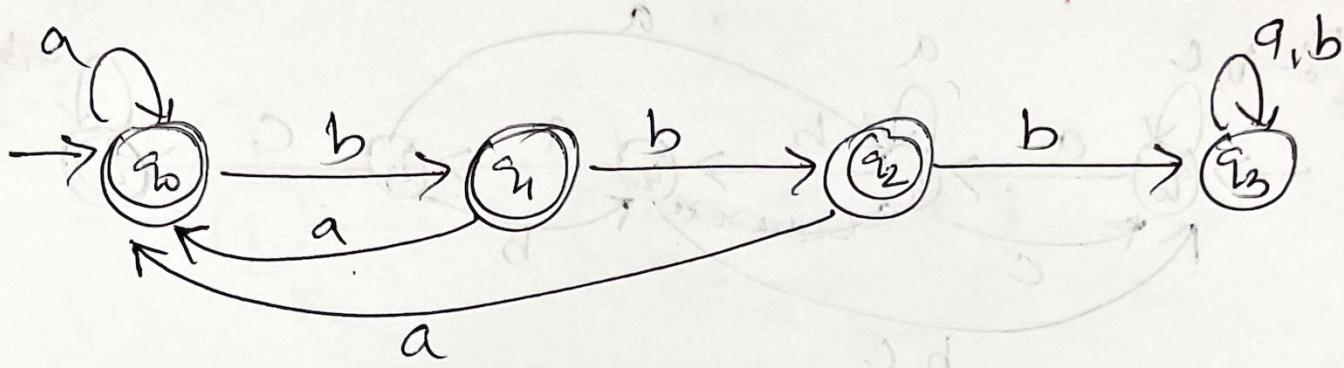


Q:- Design a DFA for set of strings over $\Sigma = \{a, b\}$ which does not contain three consecutive b's.

Sol:- Three consecutive b's :-

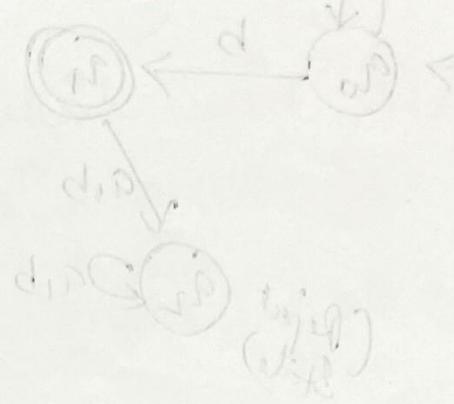


Not Containing three consecutive b's.

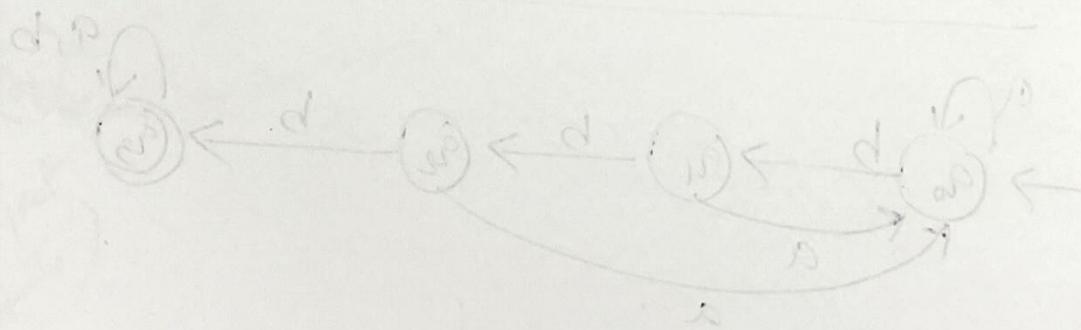


Q.:- Design a DFA with input symbols a and b that contains at most 3 a's. There can be any no. of b's.

Sol:-

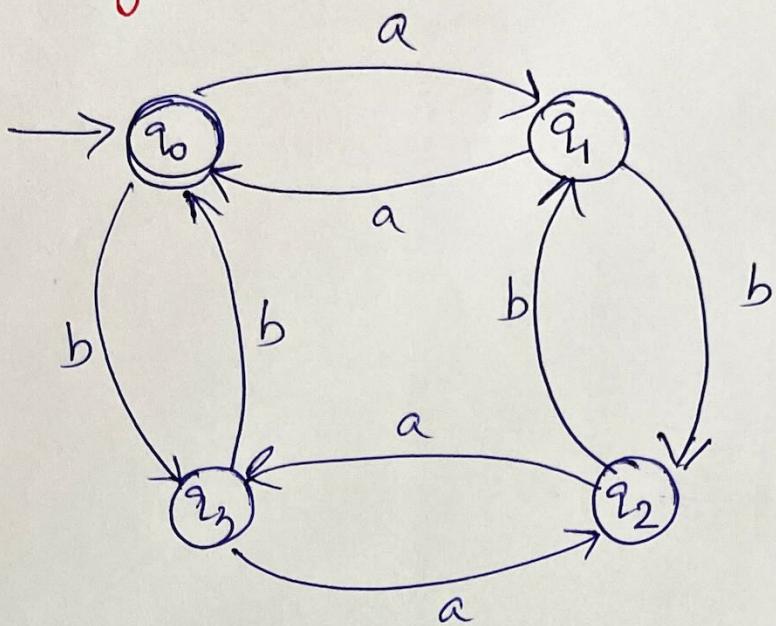


Two words for the set of strings accepted
are (aaab) and (aaaa)



Q₀: Design a DFA over the input symbol $\Sigma = \{a, b\}$ having even no. of a and b.

Solⁿ:-



Q₀: Design a DFA over the input symbol $\Sigma = \{a, b\}$ having even no. of a and even no. of b.
ii) even no. of a and odd no. of b.

Solⁿ:- ii) odd no. of a and even no.

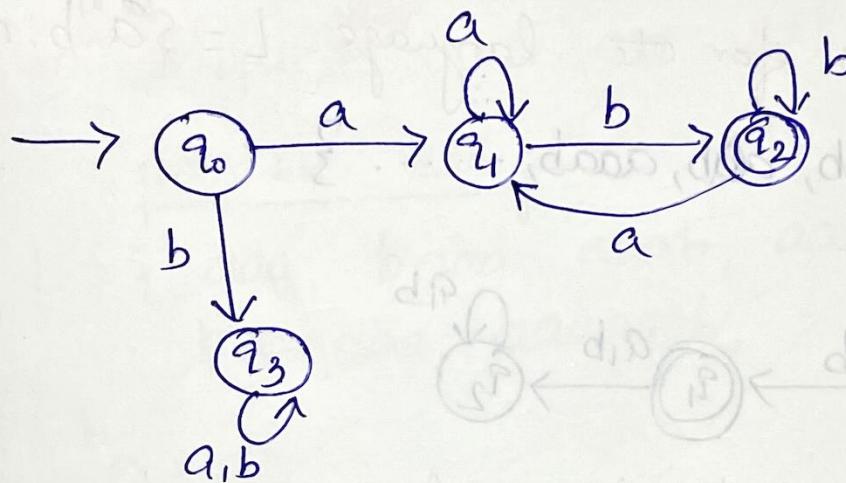
Q. 0: Determine the DFA with the

a) Set of integers

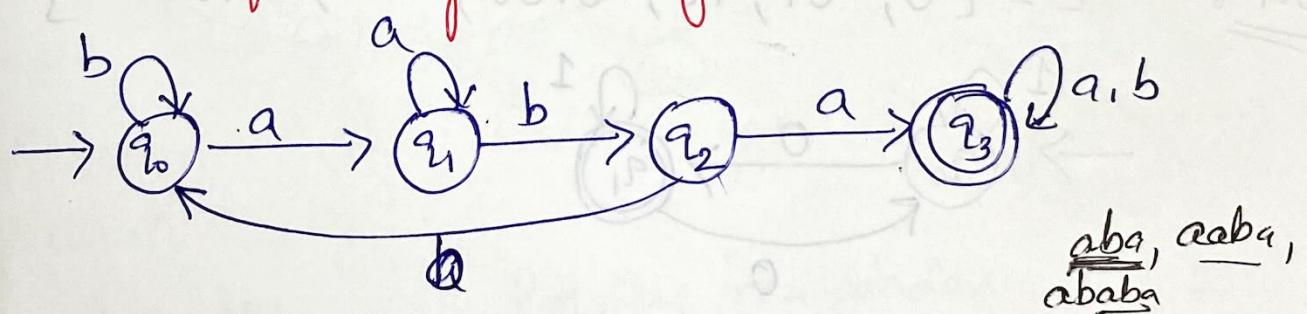
$$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$



b) Set of strings beginning with 'a' and ending with 'b'.

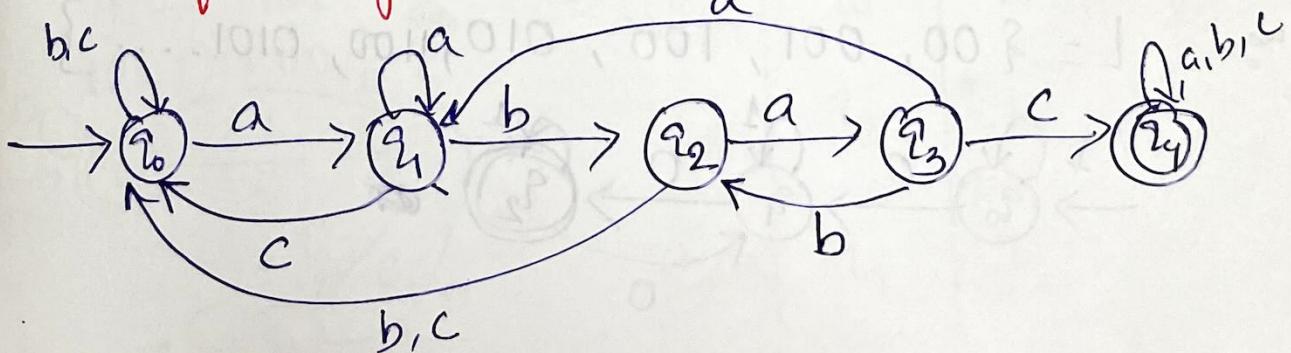


c) Set of strings having 'aba' as a subword.



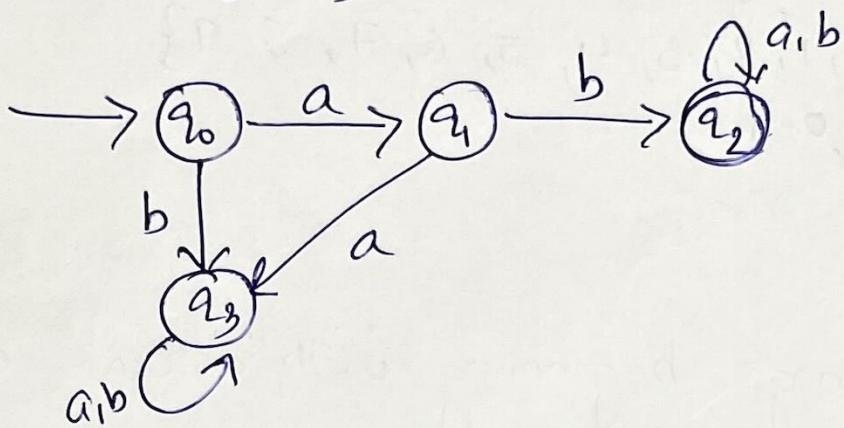
aba, aaba,
ababa

d) Set of strings which contains the pattern abac-



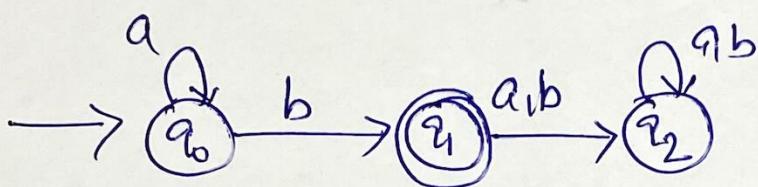
c) Set of all strings starting with the prefix 'ab'. $\Sigma = \{a, b\}$

Sol:-



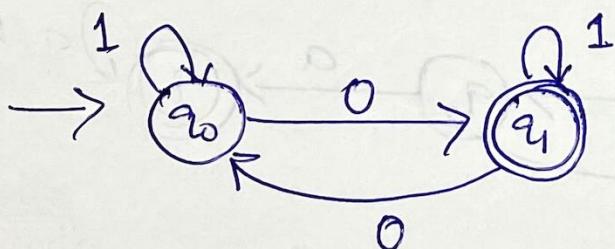
f) Set of strings for the language $L = \{a^n b : n \geq 0\}$

Sol:- $L = \{b, ab, a^2b, aaab, \dots\}$



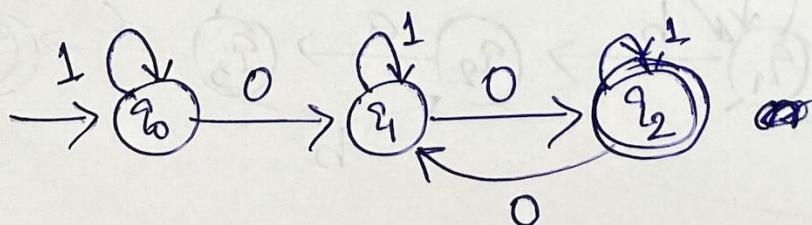
g) Set of strings having odd no. of 0's over $\{0, 1\}$

Sol:- $L = \{0, 01, 10, 0100, 1000, 0010, \dots\}$



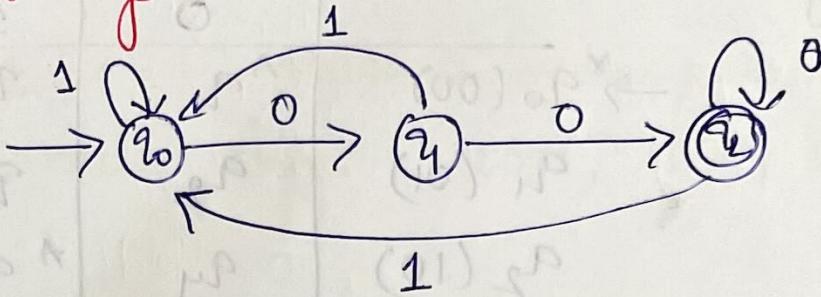
h) Set of strings having even no. of 0's over $\{0, 1\}$

Sol:- $L = \{00, 001, 100, 010, 1100, 0101, \dots\}$



Q.: Design DFA to accept the string that always ends with 00.

Solⁿ:

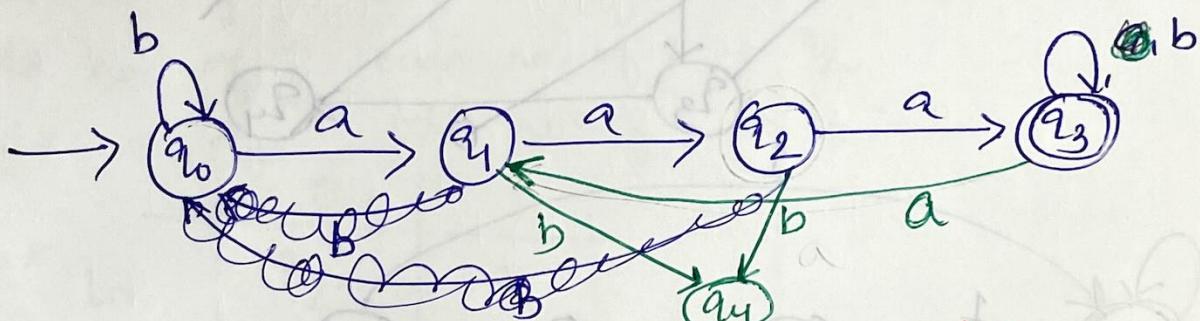


Q.: Design DFA to accept the language L , where $L = \{ \text{strings in which 'a' appears in triplets} \}$ over the set $\Sigma = \{a, b\}$

Solⁿ:

possible strings :-

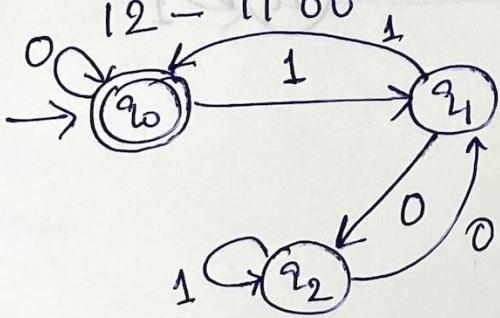
$$L = \{ \text{aaa, baaa, aaab, aaaaa, aaabb, baaaaaa, aaaaaab, ... } \}$$



Q.: Design DFA which accepts binary numbers divisible by 3.

Solⁿ:

$$\begin{array}{l|l} 3 & 101 \\ 6 & 110 \\ 9 & 1001 \\ 12 & 1100 \end{array}$$



Possible remainders = 0, 1, 2

	0	1
$q_0(00)$	* q_0	q_1
$q_1(01)$	q_2	* q_0
$q_2(10)$	q_1	q_2

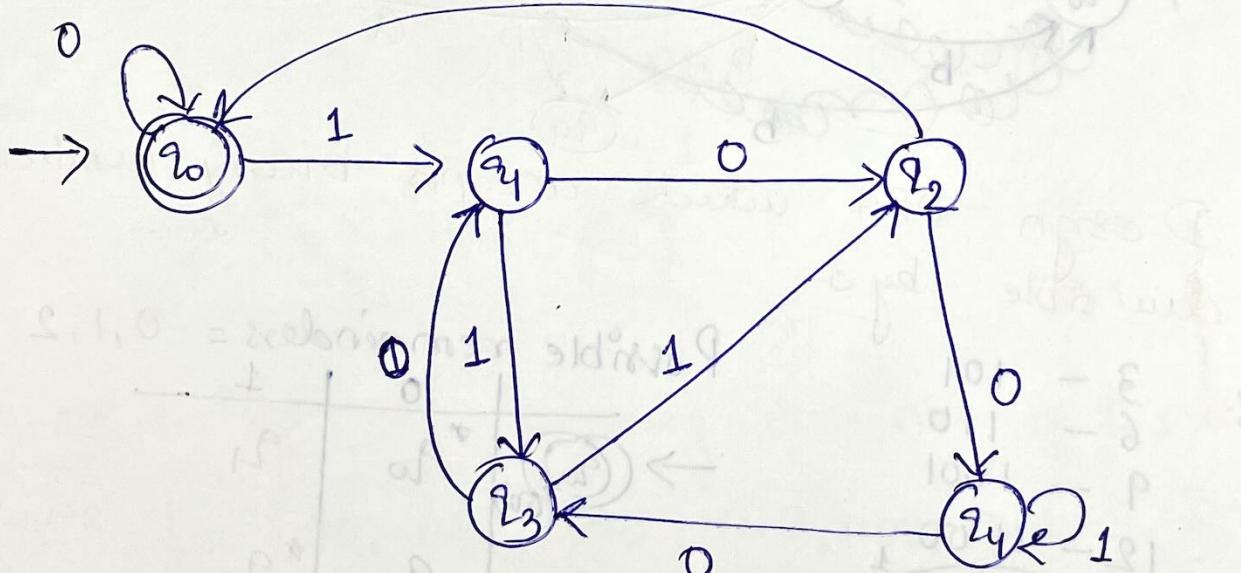
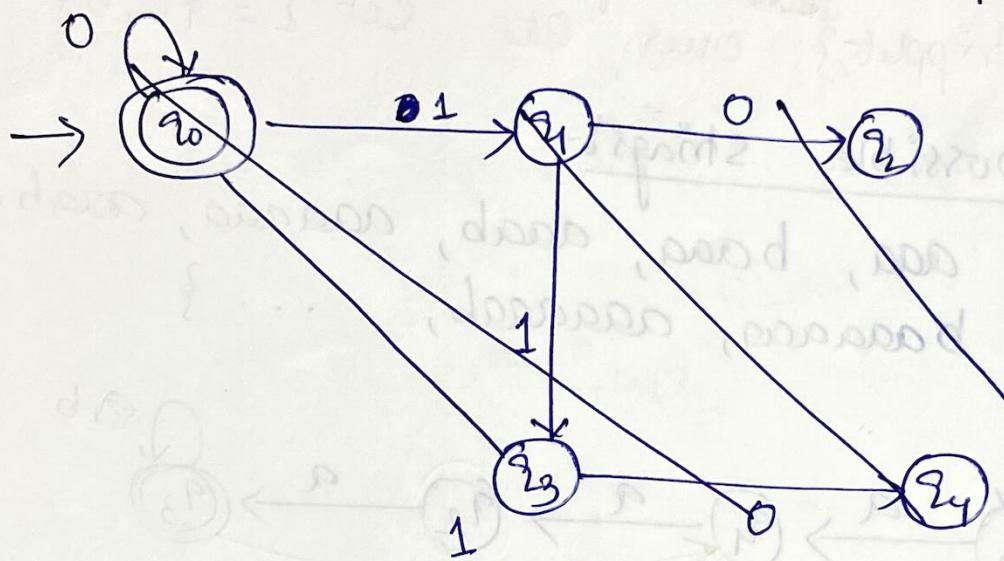
$$\begin{array}{l} 22 - 10110 \\ 23 - 10111 \\ 24 - 11000 \end{array}$$

Q: Design DFA which accepts binary numbers divisible by 5.

Solⁿ:

5	101
10	1010
15	1111
20	10100
25	10001
30	11110

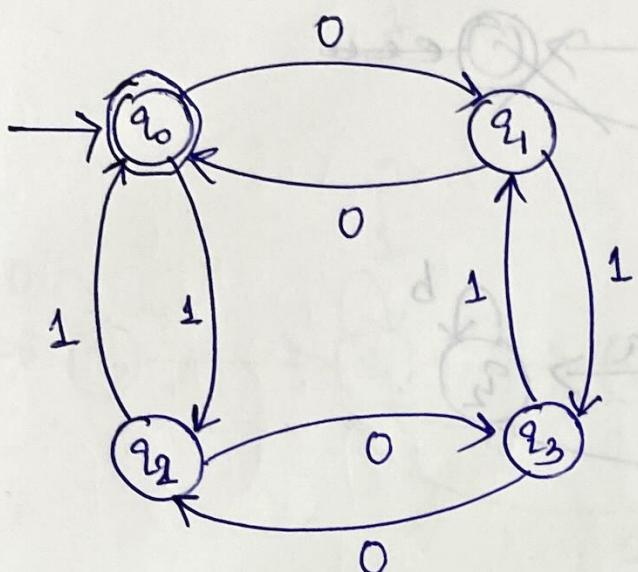
	0	1
$\rightarrow q_0(00)$	q_0	q_1
$q_1(01)$	q_2	q_3
$q_2(10)$	q_4	q_0
$q_3(11)$	q_1	q_2
$q_4(100)$	q_3	q_4



Q. Design DFA which accepts even no of 0's and even no. of 1's.

0 1 2 1 4 2 3 7 2 1 3 4 7 5 3 2 7

Solⁿ Possible strings:- 0011, 1100, 1010, 0100....



Even no. of 0 even no. of 1 - q_0

even no. of 0 odd no. of 1 - q_1

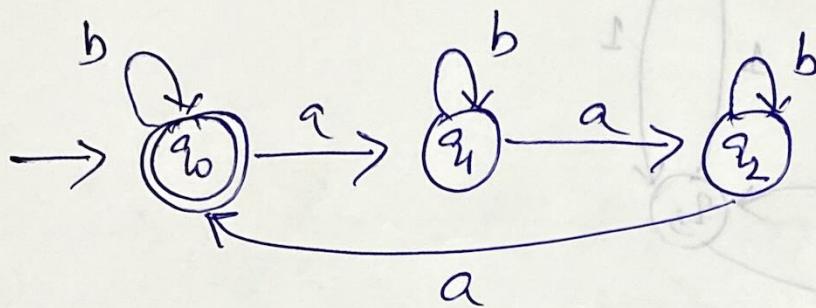
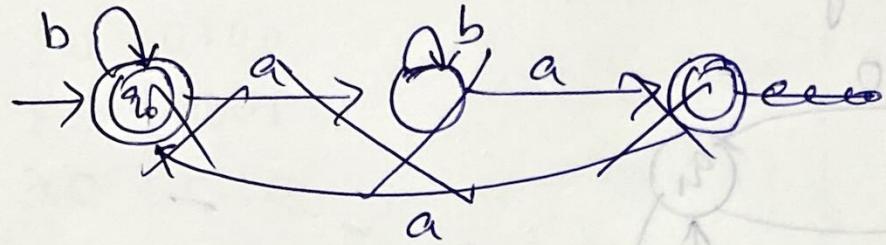
odd no. of 0 even no. of 1 - q_2

odd no. of 0 odd no. of 1 - q_3

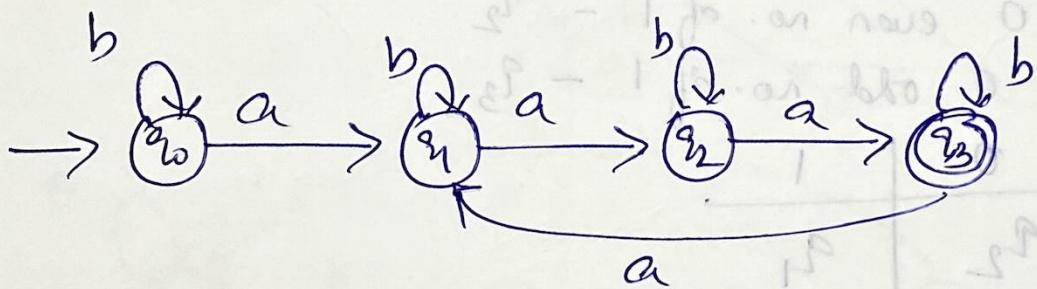
	0	1
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Q: Design DPA to accept strings in L'' , such that the total no. of 'a's in them are divisible by 3.

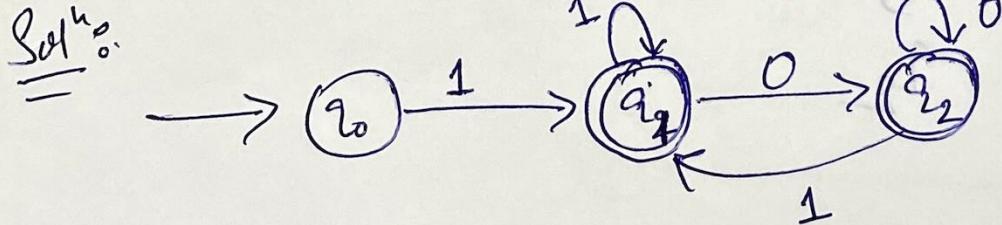
Solⁿ: e.g. aaa, baaa, aaab, ...



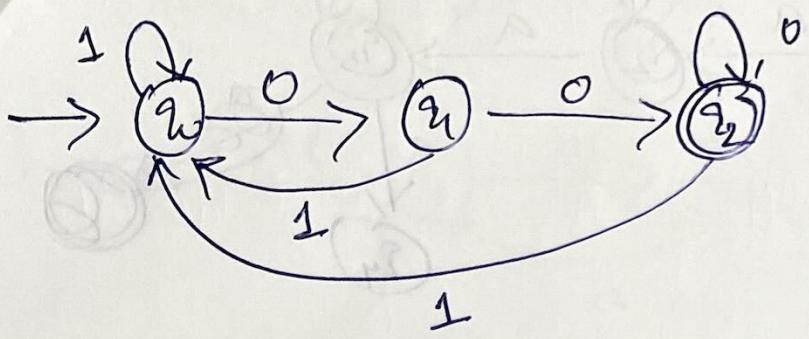
Or



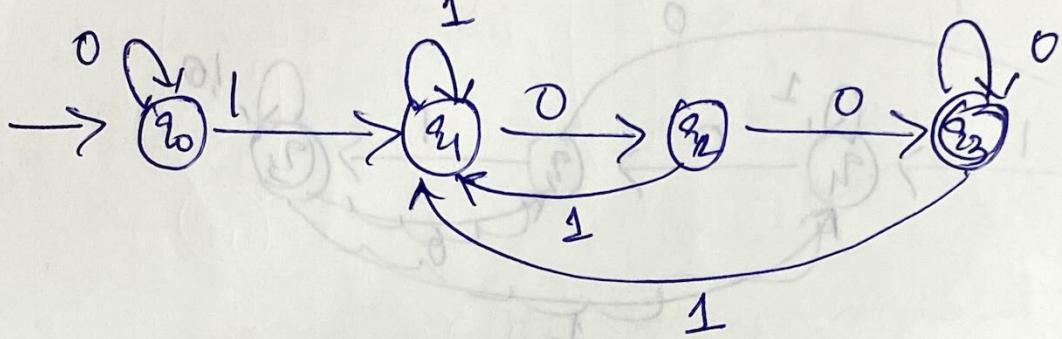
Q: Design a DPA to accept odd and even no. represented using binary notation.



Q. Design a DFA for divisibility by 4.

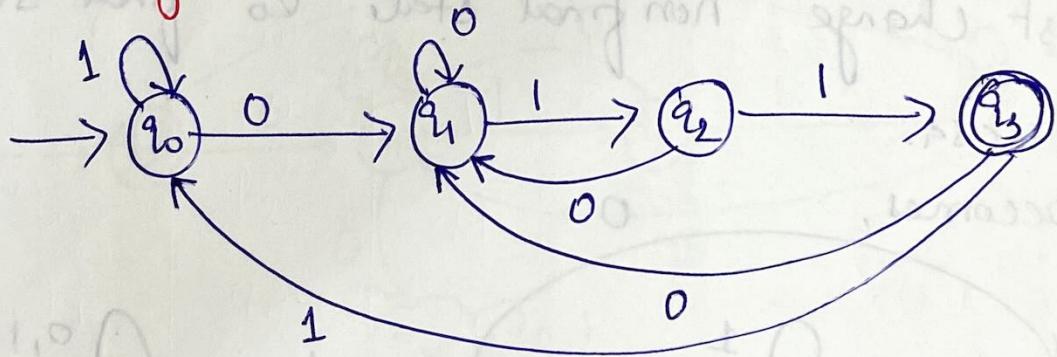


Or



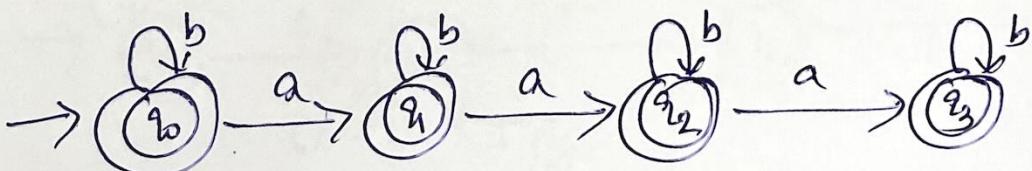
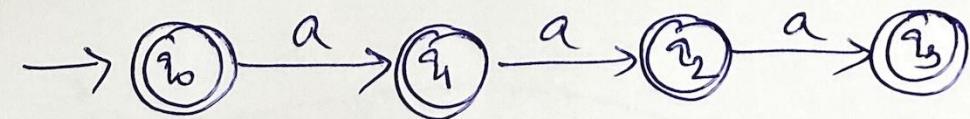
Q. Design a DFA to accept the string of 0's and 1's ending with 011.

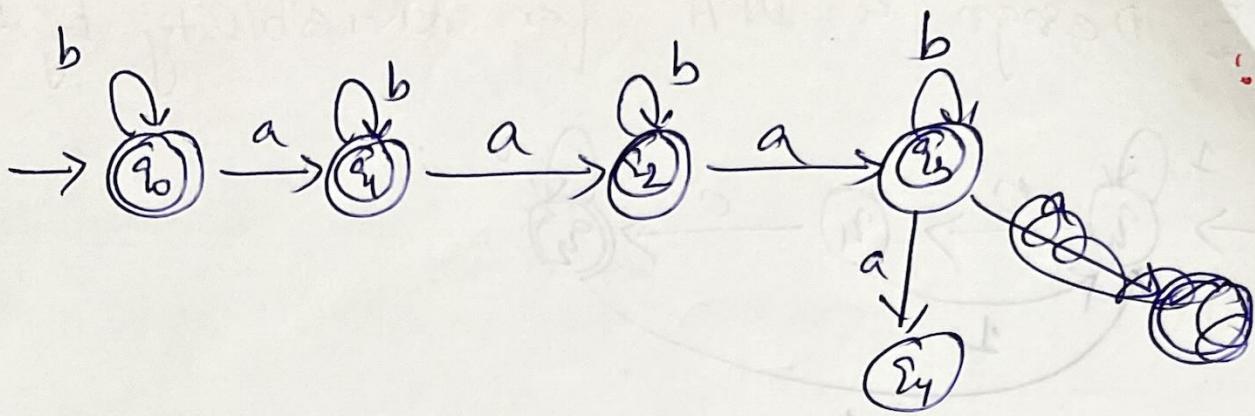
Sol^{n.o.}



Q. Design a DFA with I/P symbols a and b that contains almost 3a's. There can be any no. of b's.

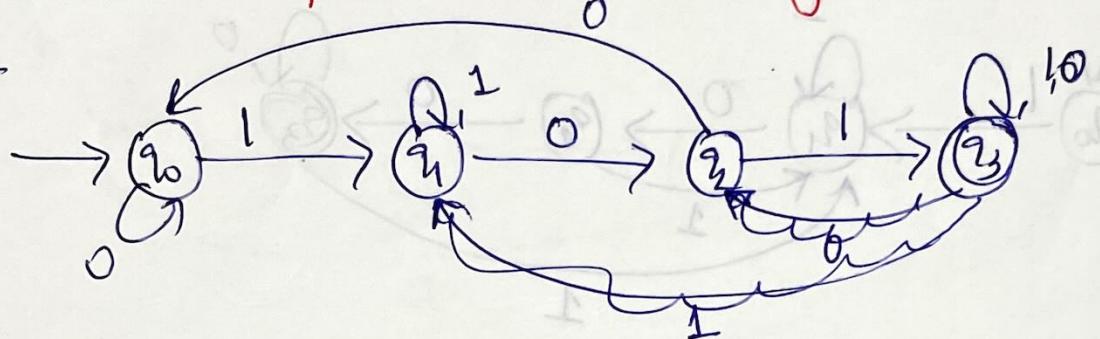
Sol^{n.o.}





~~Q.:-~~ Construct a DFA that accepts all the strings on $\{0, 1\}$ except those containing the substring 101.

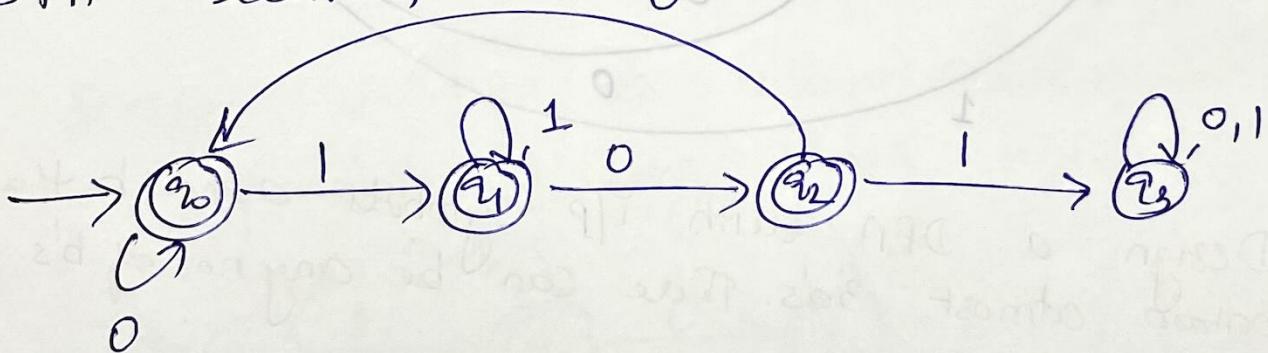
Sol:-



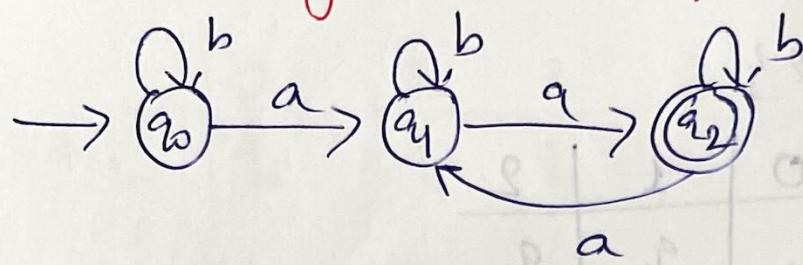
DFA accepting 101 as substring

Now, just change non final state q_0 final state and vice versa.

DFA becomes,



E: DFA to accept strings over $\Sigma(a, b)$
Containing even no. of 'a's.



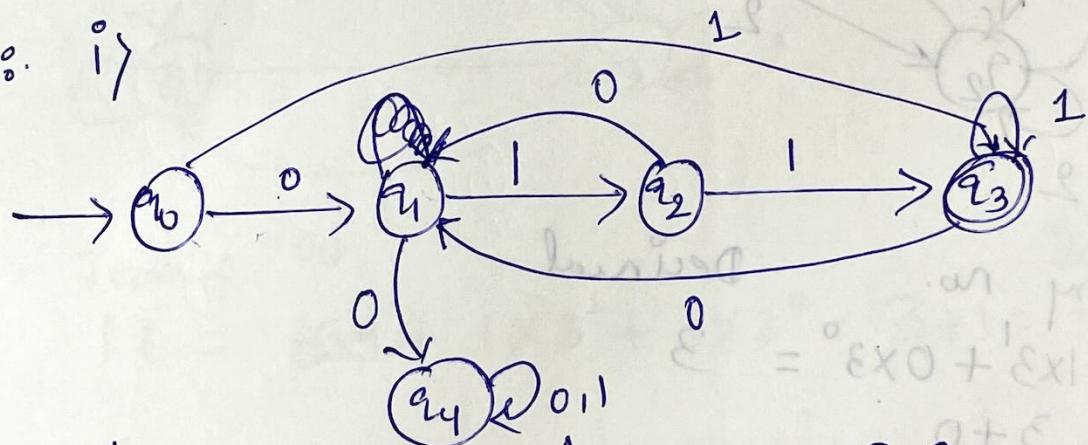
abaaab

Q: Design a DFA to accept

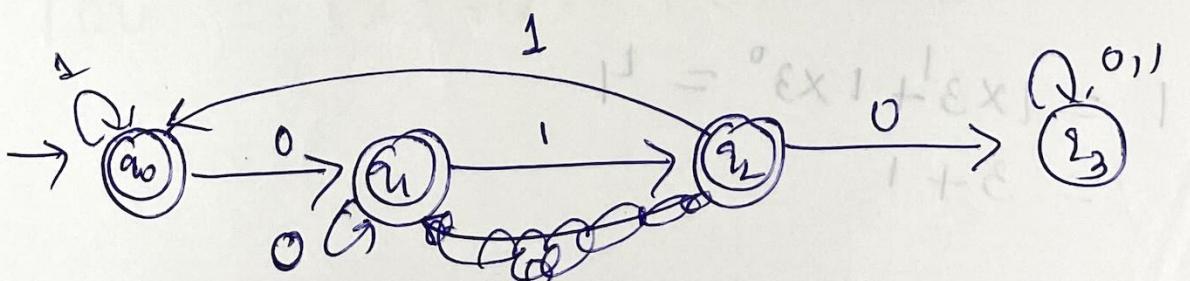
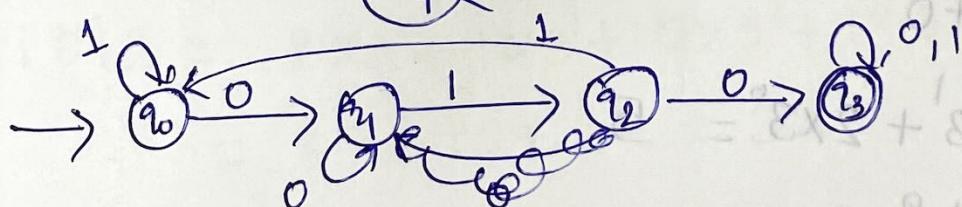
i) Binary strings in which every 0 is followed by 1.

ii) Strings over the binary alphabet that do not contain the substring 010.

Sol: i)



ii)



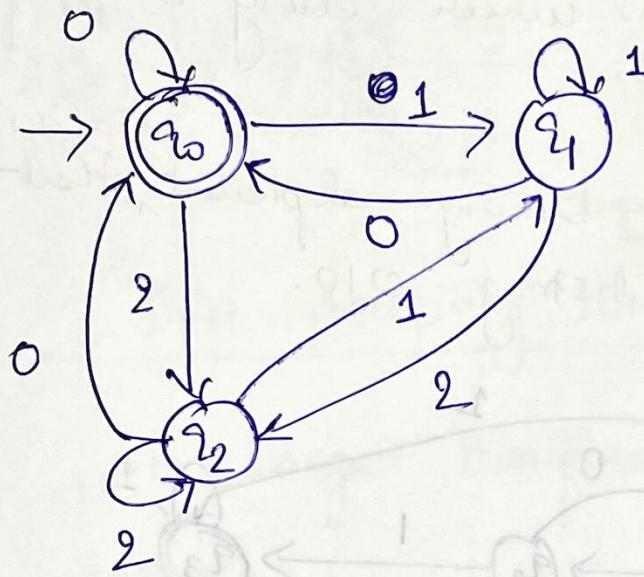
Q.: Design a DRA for ternary no. division by 3.

Soln: $Z = \{0, 1, 2\}$

	0	1	2	
0	q_0	q_0	q_1	q_2
1	q_1	q_0	q_1	q_2
2	q_2	q_0	q_1	q_2

3	101	2
3	33	0
3	11	2
3	3	0
3	1	2

$(10202)_3$



Ternary no. Decimal

$$10 = 1 \times 3^1 + 0 \times 3^0 = 3 \\ = 3 + 0$$

$$12 = 1 \times 3^1 + 2 \times 3^0 = 5 \\ = 3 + 2$$

$$11 = 1 \times 3^1 + 1 \times 3^0 = 4 \\ = 3 + 1$$

$$10 = 0 \\ 01 = 0 \times 3^1 + 1 \times 3^0 = 1$$

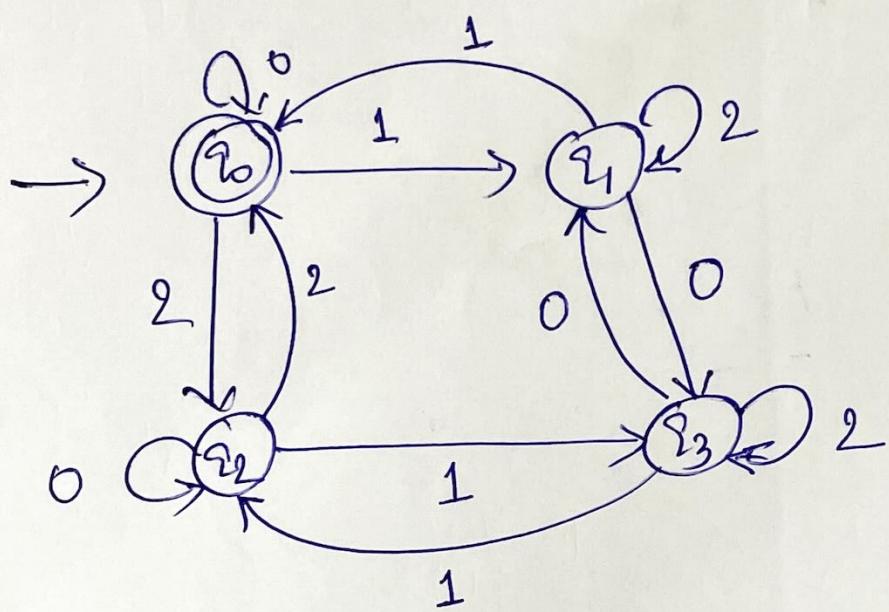
$$1 \times 3^4 + 0 \times 3^3 + 2 \times 3^2 + 0 \times 3^1 + 2 \times 3^0$$

$$81 + 0 + 18 + 0 + 2 \\ = 101$$

$$00 = 0 \\ \frac{1}{2}$$

Q. - DFA for divisibility by 4 for ternary no.
 $\Sigma = \{0, 1, 2\}$

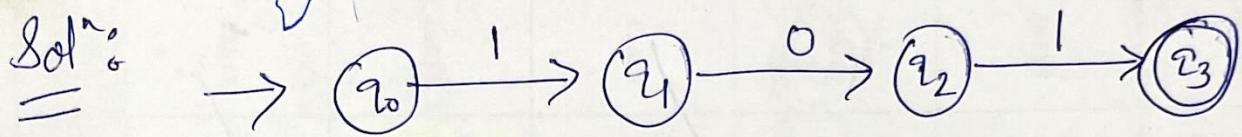
	0	1	2
$\rightarrow *q_0$	q_0	q_1	q_2
q_1	q_3	$*q_0$	q_1
q_2	q_2	q_3	$*q_0$
q_3	q_1	q_2	q_3



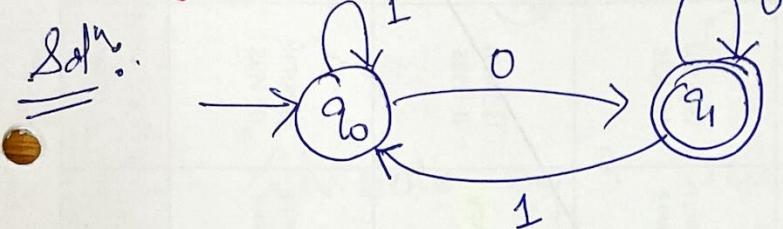
$$1012 = 2 \times 3^0 + 1 \times 3^1 + 0 \times 3^2 + 1 \times 3^3 \\ = 2 + 3 + 0 + 27 = 32$$

$$100 = 1 \times 3^2 + 0 \times 3^1 + 0 \times 3^0 = 9$$

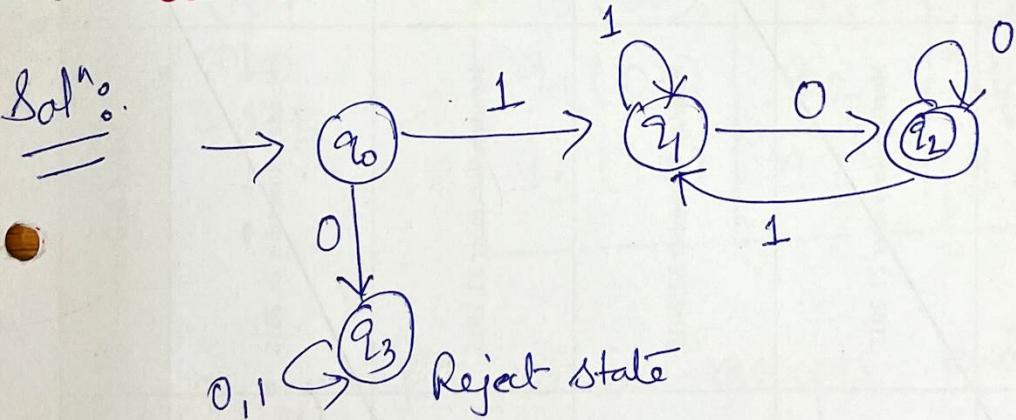
Q6: Design a FA which accepts the only input 101 over the input set $Z = \{0, 1\}$.



Q7: Design a FA which checks whether the given binary no. is even.

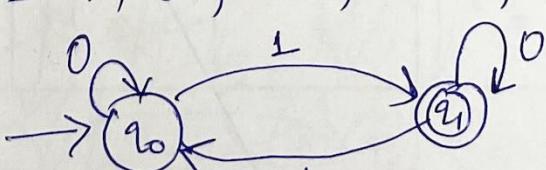


Q8: Design FA which accepts only those strings which starts with 1 and ends with 0.

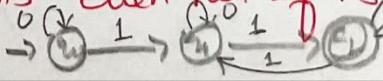


Q9: Design FA which accepts odd no. of 1's and any no. of 0's.

Sol^{n.o.}: Strings = 1, 01, 010, 0111, 1011, 10, ... 1101110

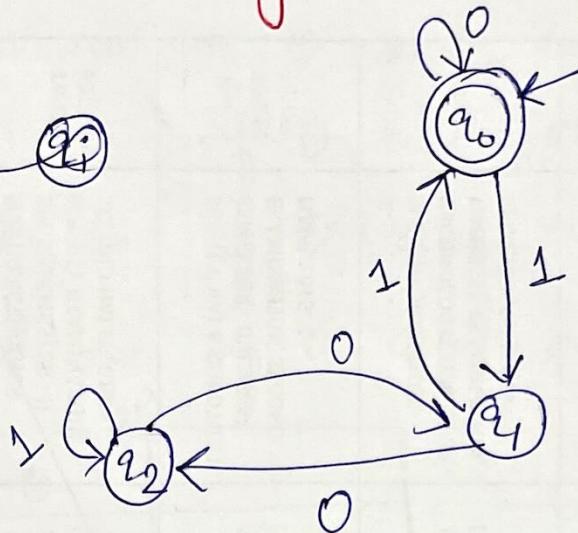


Q10: Design FA which accepts even no. of 1's and any string of 0's.



Q6 Design FA which checks whether a given binary no. is divisible by three.

Soln:-



{	18 - 10010
	21 - 10101
	22 - 10110
	23 - 10111



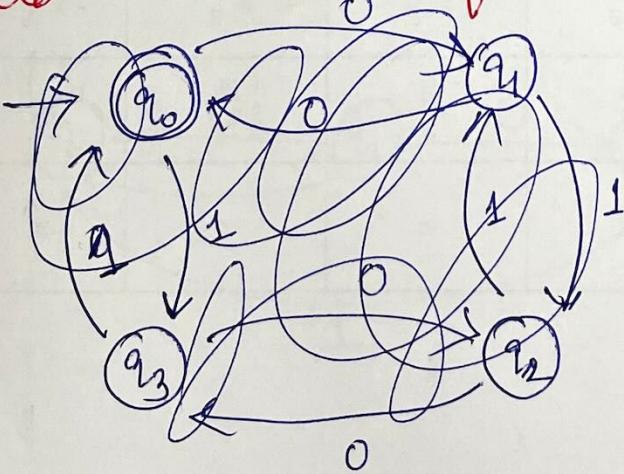
No. divisible by 3 will have three remainders 0, 1, 2. so, we will have three states

q_0 , q_1 & q_2 wst remainders.

	0	1
$q_0(00)$	* q_0	q_1
$q_1(01)$	q_2	* q_0
$q_2(10)$	q_1	q_2

Q7 Design FA which accepts even no. of 0's and even no. of 1's.

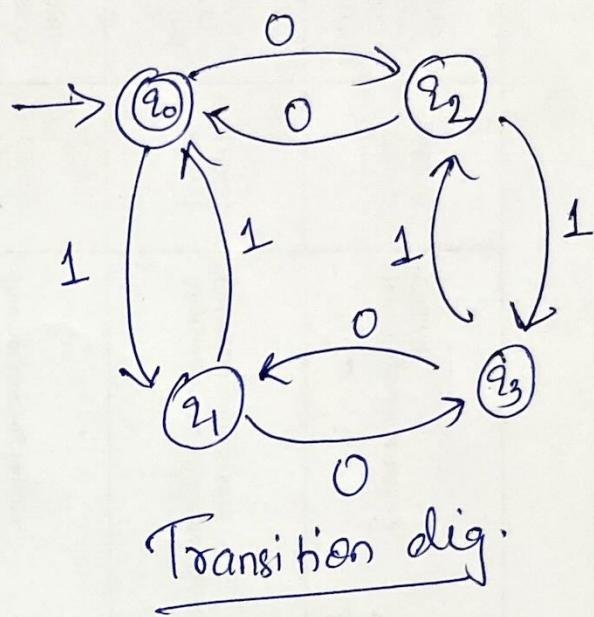
Soln:-



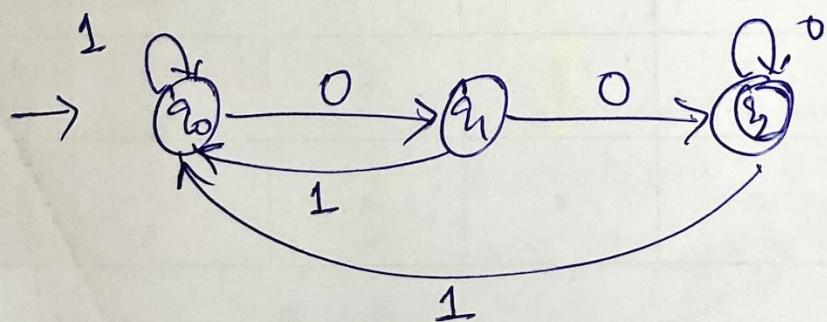
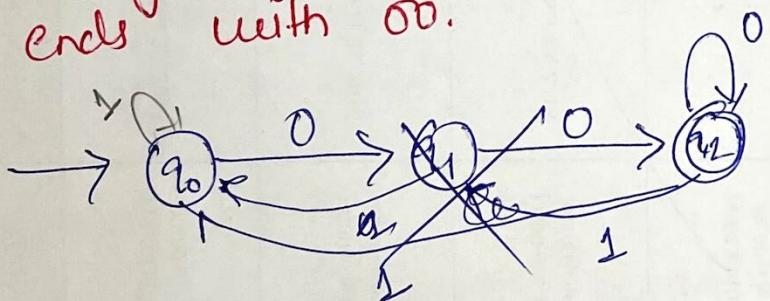
This FA will have four different possible combinations :-

- i) even no. of 0 and even no. of 1. - q_0
- ii) even no. of 0 and odd no. of 1 - q_1
- iii) odd no. of 0 and even no. of 1 - q_2
- iv) odd no. of 0 and odd no. of 1 - q_3

States	0	1
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2



Q. Design FA to accept the string that always ends with 00.

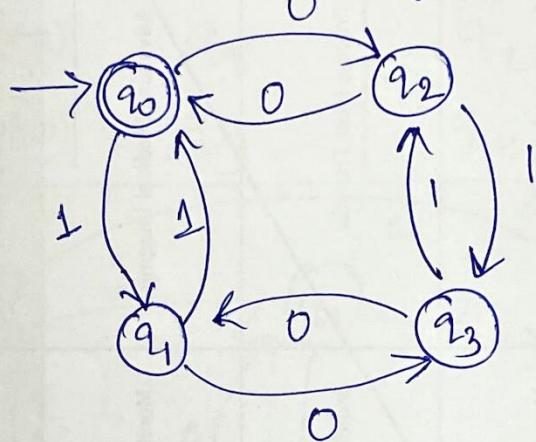


Q.: Consider the finite automaton transition table shown below with $F = \{q_0\}$.

States	Inputs	
q_0	0	q_1
q_1	0	q_0
q_2	0	q_3
q_3	0	q_2

Find the language accepted by the finite automata.

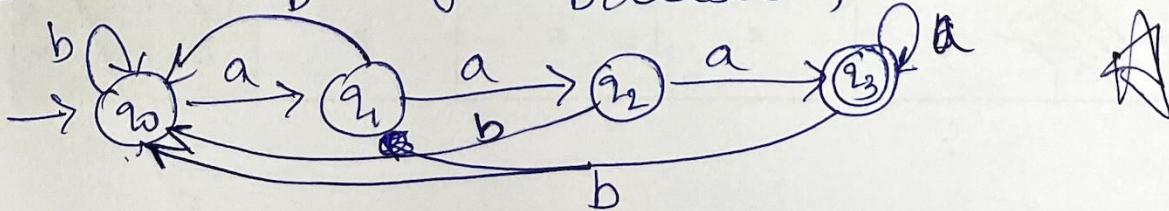
Sol.: The transition dig.:-



It shows that the string containing even no. of 1's and 0's are accepted.

Q.: Design FA to accept L , where $L = \{ \text{strings in which a language always appears in triplets} \}$ over the set $\Sigma = \{a, b\}$.

Sol.: Possible string values - aaa, baaa, aaab, baab, baaabaaa, baaaaaaa ...



Q:- Design a DFA which checks whether the given binary no. is divisible by 5.

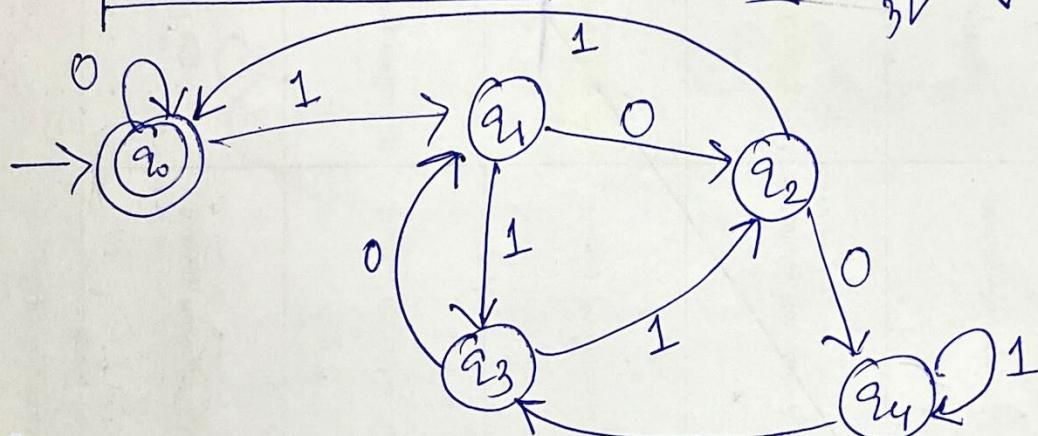
Sol:- The no. divisible by 5 will have 5 remainders.

0, 1, 2, 3, 4.

$$q_0 = 0, q_1 = 1, q_2 = 2, q_3 = 3, q_4 = 4$$

	0	1
$q_0(00)$	q_0	q_1
$q_1(01)$	q_2	q_3
$q_2(10)$	q_4	q_0
$q_3(11)$	q_1	q_2
$q_4(100)$	q_3	q_4

$$32 \xrightarrow{1} 16 \xrightarrow{0} 8 \xrightarrow{1} 4 \xrightarrow{1} 2 \xrightarrow{0} 0$$

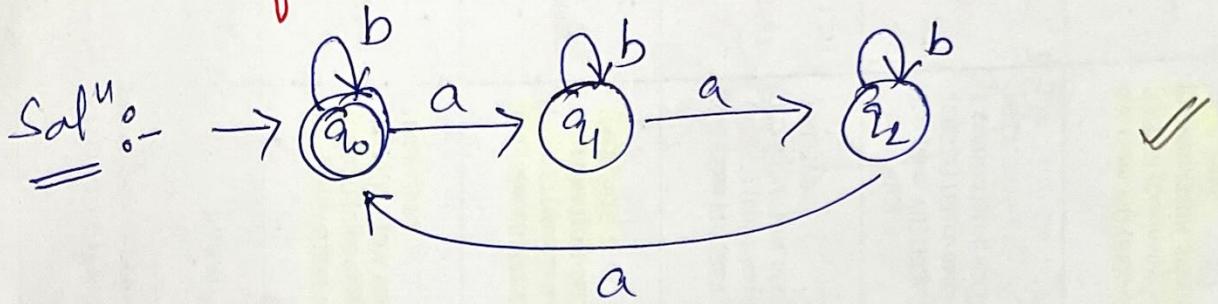


30

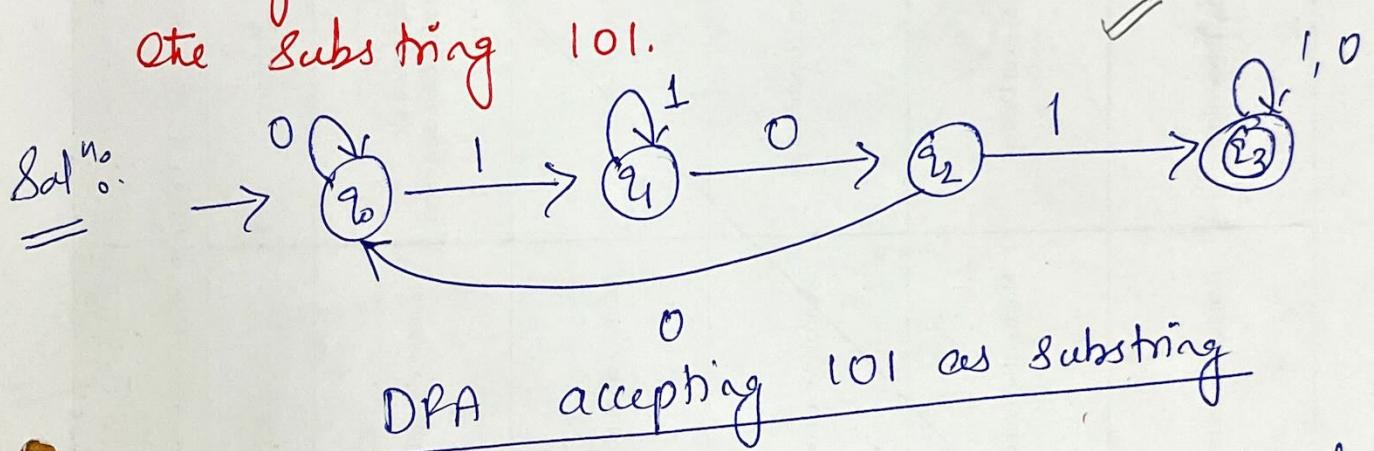
$$\begin{array}{r}
 16 \ 12 \ 8 \ 4 \ 2 \ 1 \\
 \hline
 1 \ 1 \ 0 \ 0 \ 1 \ 0
 \end{array}
 \quad \delta(q_0, 110010) = \delta(q_1, 10010) \\
 = \delta(q_3, 0010) \\
 = \delta(q_1, 010) \\
 = \delta(q_2, 10) \\
 = \delta(q_0, 0) \\
 = q_0$$



Q.: Design FA to accept L where all the strings in L are such that the total no. of 'a's in them are divisible by 3.

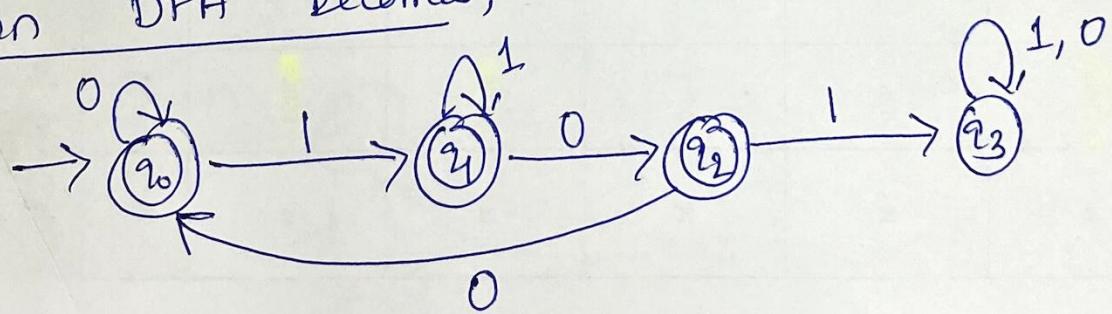


Q.: Construct a DFA that accepts all the strings on $\{0, 1\}$ except those containing the substring 101.



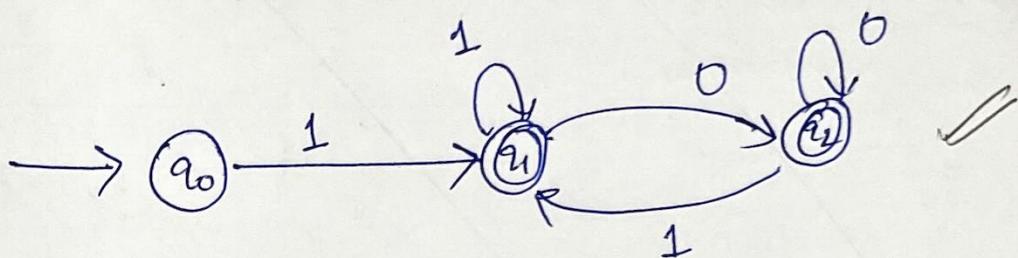
Now, just change non final states to final state and make final state as non-final state.

Then DFA becomes,



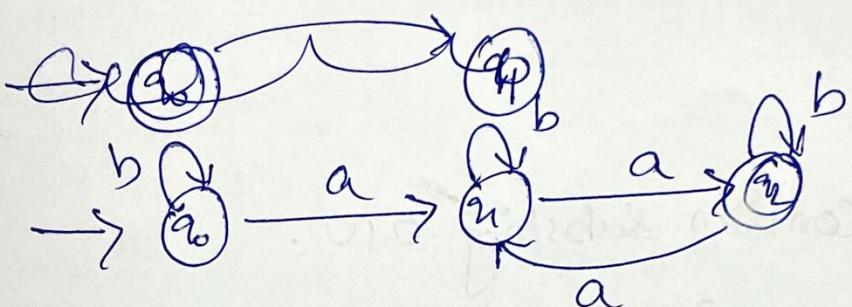
Q:- Design DFA to accept odd and even no. represented using binary notation.

Sol:-



Q:- Design DFA to accept strings over the alphabet $\Sigma = \{a, b\}$ containing even no. of 'a's. May-14

Sol:-



- Q:- Design a DFA to accept
- i) Binary strings in which every 0 is followed by 11.
 - ii) Strings over the binary alphabet that do not contain the substring 010.

