CSE2403-Discrete Mathematics

Problem Sheet-4

Topic: **Recurrence Relations** Subject Instructor : **Suresh Badarla**

Date: 23-05-2022

1. **Find the recurrence relation for the following sequences**
   (a) $a_n = 3.2^n$, $n \geq 1$
   (b) $a_n = 6.(\text{-}5)^n$, $n \geq 0$
   (c) $a_n = 2n{+}9$, $n \geq 1$
   (d) $a_n = \text{A}(2)^n + \text{B}(3)^n$, $n \geq 0$
   (e) $a_n = \text{A}(3)^n + \text{B}(\text{-}4)^n$, $n \geq 0$
   (f) $a_n = n^2 - n$, $n \geq 1$

2. **Solve the following homogeneous recurrence relations**
   (a) $a_n = a_{n-1} + a_{n-2}$, $a_0 = 2$, $a_1 = 7$
   (b) $a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1$, $a_1 = 6$
   (c) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, $a_0 = 2$, $a_1 = 5$, $a_2 = 15$
   (d) $a_{n+2} - 6a_{n+1} + 9a_n = 0$, $a_0 = 1$, $a_1 = 4$
   (e) $a_n = -\,3a_{n-1} - 3a_{n-2} - a_{n-3}$, $a_0 = 0$, $a_1 = \text{-}2$, $a_2 = \text{-}1$
   (f) $a_n - 10a_{n-1} + 9a_{n-2}$, $a_0 = 3$, $a_1 = 11$
   (g) $a_n - 4a_{n-1} - 11a_{n-2} + 30a_{n-3}$, $a_0 = 0$, $a_1 = \text{-}35$, $a_2 = \text{-}85$
   (h) $a_n - 8a_{n-1} + 16a_{n-2} = 0$, $a_0 = 16$, $a_1 = 80$
   (i) $a_n - 7a_{n-2} + 6a_{n-3} = 0$, $a_0 = 8$, $a_1 = 6$, $a_2 = 22$

3. **Solve the following non homogeneous recurrence relations**
   (a) $a_n - 3a_{n-1} = 2n$, $a_0 = 3$
   (b) $a_n = 5a_{n-1} - 6a_{n-2} + (7)^n$, $a_0 = 1$, $a_1 = 6$
   (c) $a_n - 5a_{n-1} + 6a_{n-2} = 2$, $a_0 = 1$, $a_1 = -1$
   (d) $a_{n+1} - a_n = 3n^2 - n$, $a_0 = 3$
   (e) $a_n - 4a_{n-1} + 4a_{n-2} = 3n + (2)^n$, $a_0 = 1$, $a_1 = 1$
   (f) $a_n - 7a_{n-1} + 10a_{n-2} = 8n + 6$, $a_0 = 1$, $a_1 = 2$
   (g) $a_{n+2} - 6a_{n+1} + 9a_n = 3(2)^n + 7(3)^n$, $a_0 = 1$, $a_1 = 4$
   (h) $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1$, $a_0 = 1$, $a_1 = 2$

4. **Using generating function, Solve the following recurrence relations**
   (a) $a_n = 3a_{n-1}$, $n \geq 1$, $a_0 = 2$
   (b) $a_n = 3a_{n-1} + 1$, $n \geq 1$, $a_0 = 1$
   (c) $a_n - 7a_{n-1} + 10a_{n-2} = 0$, $n \geq 2$, $a_0 = 10$, $a_1 = 41$
   (d) $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$, $a_0 = 0$, $a_1 = 1$
   (e) $a_{n+1} - 2a_n = (4)^n$, $n \geq 0$, $a_0 = 1$
   (f) $a_n = 4a_{n-1} + 3n(2)^n$, $n \geq 1$, $a_0 = 4$
   (g) $a_{n+2} - 8a_{n+1} + 15a_n = 0$, $n \geq 0$, $a_0 = 2$, $a_1 = 8$
   (h) $a_n = a_{n-1} + 2a_{n-2} = 0$, $n \geq 2$, $a_0 = 3$, $a_1 = 1$

5. Show that the set G = {1, -1, i, -i} consisting of the fourth roots of unity is a commutative group under multiplication.

6. Show that (Q^+, $*$) is an abelian group where $*$ is defined by $a * b = \frac{ab}{2}$, $\forall\ a, b \in Q$

7. Show that $(R - \{-1\}, *)$ is an abelian group, where $*$ is defined by $a * b = a + b + ab$, $\forall$ $a, b \in R$

8. Show that (R-{-1/2}, $*$) is an abelian group, where $*$ is defined by $a * b = a + b + 2ab$, $\forall\ a, b \in R$

9. Let G denote the set of all 2x2 matrices of the form $[a_{ij}]$, where $a_{ij} = x$, $\forall i, j$, and $x \in R^*$. Prove that G is a group under matrix multiplication.

10. (a) Prove that the identity element of a group is unique.
    (b) Prove that the inverse element of a group is unique.
    (c) Prove that, in a group the only idempotent element is identity element.
    (d) In a group (G, $*$), the left and right cancellation laws are hold good.

11. Let (G, $*$) be a group. If $a, b \in G$, then show that $(a * b)^{-1} = b^{-1} * a^{-1}$

12. Prove that a group (G, $*$) is an abelian group *iff* $(a * b)^2 = a^2 * b^2$, $a, b \in G$.

13. If every element of a group G has its own inverse, then show that G is abelian. Is the converse true?

14. If (G $*$) is an abelian group, then show that $(a*b)^n = a^n * b^n$, $a, b \in G$ and n is a positive integer

15. Prove that the intersection of two subgroups of group is also a subgroup of the group.Is the union of two subgroups of a group a subgroup? Justify your answer.

16. Prove the following
    (a) Homomorphism preseves identity
    (b) Homomorphism preseves inverse
    (c) Let f be a homorphism from (G, $*$) into (G$'$, Δ), then f (G) is a subgroup of G$'$.
    (d) The kernel of a homorphism f from a group (G, $*$) into (G$'$, Δ) is a subgroup of G.
    (e) If f be a homorphism from (G, $*$) into (G$'$, Δ), then $ker f = \{e\}$ *iff* f is one-to-one.