Normal Forms for CFG

In a CFG, the RHS of a production can be any string of variable and terminals. When the production in G satisfy certain restrictions then 'G' is said to be in 'normal form'. Much complex ~~productions in to~~ CFG gets reduced to simple forms after modifying it or re-representing it using these normal forms. Two important normal forms for CFG are:-

i) CHOMSKY NORMAL FORM (CNF)

ii) GREIBACH NORMAL FORM (GNF)

Chomsky Normal Form (CNF):-

A CFG G is said to be in CNF if every production is of the form either

$A \rightarrow a$ or $A \rightarrow BC$ and $S \rightarrow \Lambda$ if $\Lambda \in L(G)$.

Reduction to CNF:-

Let $G = (V_N, \Sigma, P, S)$ be a CFG. A context free grammar is converted into Chomsky Normal form (CNF) by using following steps:-

Step 1:- Elimination of null and Unit production & useless

Step 2:- Elimination of Nonterminals from right hand side.

Step 3:- Restricting the no. of variables on the right hand side.

Greibach Normal form (GNF):-

A Context free grammar is said to be in Greibach Normal form (GNF) if every production is of the form

$$A \to a\alpha$$

where $a \in \Sigma$, $A \in V_N$ and $\alpha \in V_N^*$.

It shows that α is a string of zero or more variables.

Reduction to GNF:-

Step 1: Simplify the given grammar, G
i.e Eliminate Null Production
Unit Production
Useless Production

Step 2:- Use any combination of Rule 1 & Rule 2 to get CFG in GNF.

Rule 1: Let $A \to B\alpha$ be some A Production and
$B \to B_1 | B_2 | \dots | B_n$ be the B production
then we can write A production as follows:-

$$A \to B_1\alpha \mid B_2\alpha \mid \dots \mid B_n\alpha$$

Rule 2:- Let $A \to A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_n$ be some A production (which start with A) then,
$A \to B_1 \mid B_2 \mid \dots \mid B_n$ be the remaining A production (which do not start with A) then introduce a new variable B

B- Production $B \to \alpha_i \mid \alpha_i B$ $1 \le i \le r$
A - Production $A \to \beta_i \mid \beta_i B$ $1 \le i \le s$

Guideline for CFG to CNF Conversion

Step 1: Simplify the given G,
i.e Eliminate Null Production $\boxed{N|U|U}$
Unit Production
Useless Production

Step 2: Add to the solution the productions which are already in CNF.

Step 3: For the remaining non CNF production replace the terminals by some variable limit the no. of variable on RHS to 2.

Q: Reduce the following G to CNF,

$S \to aSaA \mid A$

$A \to abA \mid b$

Sol^n: **Step 1:** Here, we have Unit productions that we will have to remove,

Production	New Production
$A \to b$	$A \to b$
$A \to abA$	$A \to abA$
$S \to A$	$S \to b$
$S \to aSaA$	$S \to aSaA$
	$S \to abA$

New Productions:-

$S \to abA \mid aSaA \mid b$

$A \to b \mid abA$

a|a—

Step 3:

Production	Solution
$S \to b$	
$A \to b$	
$A \to abA$	$C_1 \to a \quad C_2 \to b$ $A \to C_1C_2A$ $C_3 \to C_2A$ $A \to C_1C_3$
$A \to C_1C_3$ $C_1 \to a$ $C_2 \to b$ $C_3 \to C_2A$ $S \to abA$	$C_1 \to a \quad C_2 \to b$ $S \to C_1C_2A$ $C_3 \to C_2A$ $S \to C_1C_3$
$S \to C_1C_3$	
$S \to aSaA$	$S \to C_1SC_1A$, $S \to C_1C_4$ $C_4 \to SC_1A \quad C_4 \to SC_5$ $C_5 \to C_1A$

CFG in CNF is :-

$S \to b \mid C_1C_3 \mid C_1C_4$

$A \to b \mid C_1C_3$

$C_1 \to a$

$C_2 \to b$

$C_3 \to C_2A$

$C_4 \to SC_5$

$C_5 \to C_1A$

Q: Express CFG in CNF,

$S \to aSb \mid aSbb \mid aa \mid a \mid bb \mid b$

Solⁿ: Production — Solution

Production	Solution
$S \to a$	$S \to a$
$S \to b$	$S \to b$
$S \to aa$	$C_1 \to a$, $S \to C_1C_1$
$S \to bb$	$C_2 \to b$, $S \to C_2C_2$
$S \to aSb$	$S \to C_1SC_2$, $C_3 \to SC_2$, $S \to C_1C_3$
$S \to aSbb$	$S \to C_1SC_2C_2$, ~~$S \to C_1C_3C_2$~~, ~~$C_4 \to C_3C_2$~~, ~~$S \to C_1C_4$~~, $C_4 \to SC_2C_2$, $C_4 \to SC_5$, $S \to C_1C_4$, $C_5 \to C_2C_2$, $C_4 \to SC_5$

CFG in CNF is

$S \to a \mid b \mid C_1C_1 \mid C_2C_2 \mid C_1C_3 \mid C_1C_4$

$C_1 \to a$

$C_2 \to b$

$C_3 \to SC_2$

$C_4 \to SC_5$

$C_5 \to C_2C_2$

$\left\{ \begin{array}{l} C_a \to a \quad C_b \to b \\ S \to C_aSC_b \mid C_aSC_bC_b \mid C_aC_a \mid a \mid C_bC_b \mid b \end{array} \right.$

$\left\{ \begin{array}{l} C_a \to a \quad C_b \to b \quad X \to C_aS \\ S \to XC_b \mid XC_bC_b \mid C_aC_a \mid a \mid C_bC_b \mid b \end{array} \right.$

$C_a \to a$ $C_b \to b$, $X \to C_aS$, $Y \to XC_b$

$S \to XC_b \mid YC_b \mid C_aC_a \mid a \mid C_bC_b \mid b$

Q:- Express CFG in CNF

$S \to aB \mid bA$

$A \to a \mid aS \mid bAA$

$B \to b \mid bS \mid aBB$

Sol^n: Production form for CNF:-

$A \to a$ or $A \to BC$

Production	Solution
$S \to aB$	$C_1 \to a$ $S \to C_1 B$
$S \to bA$	$C_2 \to b$ $S \to C_2 A$
$A \to a$	$A \to a$
$A \to aS$	$A \to C_1 S$
$A \to bAA$	$A \to C_2 AA$ $C_3 \to AA$ $\therefore A \to C_2 C_3$ $C_3 \to AA$
$B \to b$	$B \to b$
$B \to bS$	$B \to C_2 S$
$B \to aBB$	$B \to C_1 BB$, $C_4 \to BB$; $B \to C_1 C_4$, $C_4 \to BB$

CFG in CNF is

$S \to C_1 B \mid C_2 A$

$A \to a \mid C_1 S \mid C_2 C_3$

$B \to b \mid C_2 S \mid C_1 C_4$

$C_1 \to a$

$C_2 \to b$

$C_3 \to AA$

$C_4 \to BB$

Q: Convert the following CFG to CNF

$S \to uS$

$S \to [S \supset S]$

$S \to p$

$S \to q$

$G = (\{S\}, \{p, q, u, [, \supset,]\}, P, S)$

Soln:

Production	Solution
* $S \to p$	
* $S \to q$	
$S \to uS$	* $C_1 \to u$
* $S \to C_1S$	$S \to C_1S$
$S \to [S \supset S]$	* $C_2 \to [$
	* $C_3 \to \supset$
$S \to C_2 S C_3 S C_4$	* $C_4 \to]$
* $S \to C_2 C_5$	* $C_5 \to S C_3 S C_4$
	* $C_5 \to S C_6$
	* $C_6 \to C_3 C_7$
	* $C_7 \to S C_4$

CFG in CNF is

$S \to p \mid q \mid C_1S \mid C_2C_5$

$C_1 \to u$

$C_2 \to [$

$C_3 \to \supset$

$C_4 \to]$

$C_5 \to SC_6$

$C_6 \to C_3C_7$

$C_7 \to SC_4$

Q: $S \to aAbB$, $A \to aA \mid a$, $B \to bB \mid b$

CNF Form:- $A \to a$

$A \to BC$ or $A \to \wedge$

$\cancel{A \to aA}$ $A \to a$ ✓

$B \to b$

$S \to aAbB$

$C_a \to a$; $C_b \to b$

$S \to C_a A C_b B$ $\cancel{S \to AB}$

$A \to C_a A$

$B \to C_b B$

$S \to C_a A C_b B$

$\cancel{S \to C_a C_1}$

$\cancel{C_1 \to A C_b B}$

$\cancel{C_1 \to A C_2}$

$\cancel{C_2 \to C_b B}$

$\cancel{S \to C_a C_1}$
$\cancel{A \to C_a A \mid a}$
$\cancel{B \to C_b B \mid b}$
$\cancel{C_1 \to A C_2}$
$\cancel{C_2 \to C_b B}$
$\cancel{C_a \to a}$
$\cancel{C_b \to b}$

$S \to C_a A C_1$
$C_1 \to C_b B$
$A \to C_a A$
$B \to C_b B$
$C_a \to a$
$C_b \to b$

$S \to C_a C_2$
$C_2 \to A C_1$
$C_1 \to C_b B$
$A \to C_a A$
$B \to C_b B$
$C_a \to a$
$C_b \to b$

1.

$S \to ABA$
$A \to aA \mid bA \mid \epsilon$
$B \to bB \mid aA \mid \epsilon$

Convert the following grammar to CNF.

Solⁿ:

$S \to ABA \mid AB \mid BA \mid AA \mid B \mid A$
$A \to aA \mid bA \mid a \mid b$
$B \to bB \mid aA \mid a \mid b$

Removal of Null production

$S \to ABA \mid AB \mid BA \mid AA \mid bB \mid aA \mid a \mid b \mid aA \mid bA$
$A \to aA \mid bA \mid a \mid b$
$B \to bB \mid aA \mid a \mid b$

— Removal of Unit Production

$S \to ABA \mid AB \mid BA \mid AA \mid X_2B \mid X_1A \mid a \mid b \mid X_1A \mid X_2A$
$A \to X_1A \mid X_2A \mid a \mid b$
$B \to X_2B \mid X_1A \mid a \mid b$
$X_1 \to a$
$X_2 \to b$

$S \to X_3A \mid AB \mid BA \mid AA \mid X_2B \mid X_1A \mid a \mid b \mid X_1A \mid X_2A$
$A \to X_1A \mid X_2A \mid a \mid b$
$B \to X_2B \mid X_1A \mid a \mid b$
$X_1 \to a$
$X_2 \to b$
$X_3 \to AB$

$S \to ABC \mid BaB$

$A \to aA \mid BaC \mid aaa$

$B \to bBb \mid a \mid D$

$C \to CA \mid AC$

$D \to \epsilon$

Solⁿ: ~~Rlo~~ Useless

$S \to ABC \mid BaB \mid AC \mid Ba \mid aB \mid a$

$A \to aA \mid BaC \mid aaa \mid aC$

$B \to bBb \mid bb \mid a$

$C \to CA \mid AC$

} Removal of null production

No Unit

$S \to \cancel{ABC} \mid BaB \mid Ba \mid aB \mid a$

$A \to aA \mid aaa$

$B \to bBb \mid bb \mid a$

} Removal of useless

Not able to ~~reach~~ generate A & C

$S \to BaB \mid Ba \mid aB \mid a$

$B \to bBb \mid bb \mid a$

$C_1 \to a$

$C_2 \to b$

$S \to BC_1B \mid BC_1 \mid C_1B \mid a$

$B \to C_2BC_2 \mid C_2C_2 \mid a$

$C_1 \to a$

$C_2 \to b$

$C_3 \to C_1B$

$C_4 \to BC_2$

$S \to BC_3 \mid BC_1 \mid C_1B \mid a$

$B \to C_2C_4 \mid C_2C_2 \mid a$

$C_1 \to a$

$C_2 \to b$

$S \to BC_1B \mid C_1B \mid BC_1 \mid a$

$B \to C_2BC_2 \mid C_2C_2 \mid a$

$C_3 \to BC_1$

$C_4 \to C_2B$

$S \to C_3B \mid C_1B \mid BC_1 \mid a$

$B \to C_4C_2 \mid C_2C_2 \mid a$

$C_1 \to a$

$C_2 \to b$

$C_3 \to BC_1$

$C_4 \to C_2B$

$C_1 \to a$

$C_2 \to b$

$C_3 \to C_1B$

$C_4 \to BC_2$

$S \to BC_3 \mid C_1B \mid BC_1 \mid a$

$B \to C_2C_4 \mid C_2C_2 \mid a$

Q: $S \to bA \mid aB$
$A \to bAA \mid aS \mid a$
$B \to aBB \mid bS \mid b$

Find the equivalent grammar in CNF.

(Repeat)

Soln:

$S \to bA$, $C_1 \to b$ $\Big\}$ $S \to C_1 A$, $C_1 \to b$

$S \to aB$, $C_2 \to a$ $\Big\}$ $S \to C_2 B$, $C_2 \to a$

$A \to bAA$, $A \to C_1 AA$, $C_3 \to AA$ $\Big\}$ $A \to C_1 C_3$, $C_3 \to AA$

$A \to aS$ $\}$ $A \to C_2 S$

$A \to a$ — $A \to a$

$B \to aBB$, $B \to C_2 BB$, $C_4 \to BB$ $\Big\}$ $B \to C_2 C_4$, $C_4 \to BB$

$B \to bS$ $\}$ $B \to C_1 S$

$B \to b$ $\}$ $B \to b$

The reduced Grammar

$S \to C_1 A \mid C_2 B$
$A \to C_1 C_3 \mid C_2 S \mid a$
$B \to C_2 C_4 \mid C_1 S \mid b$
$C_1 \to b$
$C_2 \to a$
$C_3 \to AA$
$C_4 \to BB$

Q:- Reduce the following given grammar to CNF.

ⓐ $S \to absb \mid a \mid aAb$

$A \to bS \mid aAAb$

ⓑ $S \to aaaaS$ i.e $L = \{a^{4n} : n \geq 1\}$

$S \to aaaa$

ⓒ $S \to ABa$

$A \to aab$

$B \to AC$

ⓐ $S \to absb \mid a \mid aAb$

$A \to bS \mid aAAb$

$S \to absb$ $S \to C_1C_2SC_2$

$C_1 \to a$ ✓ $S \to C_1C_2C_3$

$C_2 \to b$ ✓ $S \to C_1C_4$ ✓

$C_3 \to SC_2$ ✓

$C_4 \to C_2C_3$ ✓

$S \to aAb$

$S \to C_1AC_2$ $S \to C_1C_5$ ✓

$C_5 \to AC_2$ ✓

$A \to bS$

$A \to C_2S$ ✓

$A \to aAAb$

$A \to C_1AAC_2$ $A \to C_1AC_5$

$C_6 \to AC_5$ ✓ $A \to C_1C_6$ ✓

New Rule:-

$S \to C_1C_4 \mid C_1C_5 \mid a$

$C_1 \to a$

$C_4 \to C_2C_3$

$C_2 \to b$

$C_3 \to SC_2$

$C_5 \to AC_2$

$A \to C_2S \mid C_1C_6$

$C_6 \to AC_5$

(b) $S \to aaaaS \mid aaaa$

$S \to aaaa$ $\quad S \to C_1C_1C_1C_1$

$C_1 \to a$ $\quad S \to C_2C_2$

$C_2 \to C_1C_1$

~~$S \to C_2C_2$~~ $\quad S \to aaaaS$

$S \to C_2C_2S$

$C_3 \to C_2S$ $\quad S \to C_2C_3$

New Rule :-

$S \to C_2C_2 \mid C_2C_3$

$C_2 \to C_1C_1$

$C_1 \to a$

$C_3 \to C_2S$

(c) $S \to ABa$

$A \to aab$

$B \to AC$

No Unit, No Null

$B \to AC$ is useless if we will remove this variable B, we have to remove $S \to ABa$ also, this grammar complete is giving no string as output.

$S \to ABC_a$

$A \to C_aC_ab$

$B \to AC$

$C_a \to a$

$S \to AC_1$

$C_1 \to BC_a$

$A \to C_aC_2$

$C_2 \to C_ab$

$B \to AC$

$C_a \to a$

Rule 1: $A \rightarrow B\alpha$

and $B \rightarrow \beta_1 | \beta_2 | \ldots | \beta_n$

Then we can convert A rule to GNF as,

$A \rightarrow \beta_1\alpha | \beta_2\alpha | \beta_3\alpha | \ldots | \beta_n\alpha$

& $B \rightarrow \beta_1 | \beta_2 | \beta_3 | \ldots | \beta_n$

Rule 2: $A \rightarrow A\alpha_1 | A\alpha_2 | \ldots | A\alpha_n | \beta_1 | \beta_2 | \beta_3 | \ldots | \beta_n$

Such that β_i doesn't start with A then equivalent GNF will be,

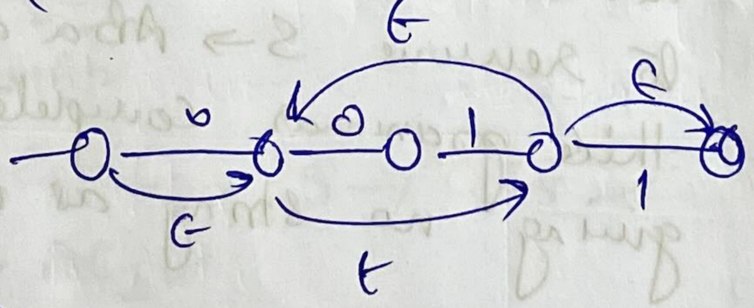
$A \rightarrow \beta_1 | \beta_2 | \beta_3 \ldots | \beta_n$

$A \rightarrow \beta_1 Z | \beta_2 Z | \ldots | \beta_n Z$

$Z \rightarrow \alpha_1 | \alpha_2 | \ldots | \alpha_n$

$Z \rightarrow \alpha_1 Z | \alpha_2 Z | \ldots | \alpha_n Z$

$(0+\epsilon)(01)^*(1+\epsilon)$



MongoDB →
add to path

Q: Convert the following grammar to GNF:-

S → ABA
A → aA | ∧
B → bB | ∧

The rule for GNF is

Non Terminal → one Terminal Any no. of Non Terminal

We first eliminate null production, unit production & useless production:-

S → ABA | AB | BA | AA | A | B
A → aA | a
B → bB | b

Now we will remove unit production

S → aABA | aAB | aAA | aA | aBA | aB | a
S → bBA | bA | bB | b
A → aA | a
B → bB | b

This grammar is in GNF.

S → ABA | bB | BA | AA | aA | a | bB | b

S → aABA | aBA | aAB | aB | bBA | bB |
aAA | aA | aA | a | bB | b

A → aA | a
B → bB | b

Q6.- $A \to \underset{\alpha_1}{A1} \mid \underset{\beta_1}{0B} \mid \underset{\beta_2}{2}$

$A \to Aa_1 \mid Aa_2 \mid Aa_3 \mid \dots \mid \beta_1 \mid \beta_2 \mid \beta_3 \mid \dots \beta_n$

then, equivalent GNF will be,

$A \to \beta_1 \mid \beta_2 \mid \beta_3 \mid \dots \mid \beta_n$

$A \to \beta_1 Z \mid \beta_2 Z \mid \beta_3 Z \mid \dots \mid \beta_n Z$

$Z \to a_1 \mid a_2 \mid a_3 \mid \dots \mid a_n$

$Z \to a_1 Z \mid a_2 Z \mid a_3 Z \mid \dots \mid a_n Z$

$A \to A1 \mid 0B \mid 2$

Here $\beta_1 = 0B$ $\beta_2 = 2$ $a_1 = 1$

$$\left\{ \begin{array}{l} A \to 0B \mid 2 \\ A \to 0BZ \mid 2Z \\ Z \to 1 \mid 1Z \end{array} \right.$$

$A_1 \to A_2 A_3$ — Convert the following into GNF.

$A_2 \to A_3 A_1 | b$

$A_3 \to A_1 A_2 | a$

Soln: $A_1 \to A_2 A_3$ ok

$A_2 \to A_3 A_1 | b$ ok

$A_3 \to A_1 A_2 | a$ — Needs to be Modified. ($i<j$ ×)

$A_3 \to A_2 A_3 A_2 | a$ $A_3 \to a$ $i<j$ Not Satisfied

$A_3 \to A_3 A_1 A_3 A_2 | b A_3 A_2 | a$

Here $i<j$ – No but $A_3 \to$ $i=j$

So,

$$A_3 \to A_3 \underbrace{A_1 A_3 A_2}_{\alpha} | \underbrace{b A_3 A_2}_{\beta_1} | \underbrace{a}_{\beta_2}$$

$A_3 \to b A_3 A_2 | a | b A_3 A_2 Z | a Z$

$Z \to A_1 A_3 A_2 | A_1 A_3 A_2 Z$

~~A_3 is in~~ GNF but Z is not. Modification required for Z.

$A_1 \to A_2 A_3$

$A_2 \to A_3 A_1 | b$

$A_3 \to b A_3 A_2 | a | b A_3 A_2 Z | a Z$

$Z \to A_1 A_3 A_2 | A_1 A_3 A_2 Z$

Substitute A_2 in A_1

$A_1 \to A_2 A_3$

$A_1 \to A_3 A_1 A_3 | b A_3$

$A_1 \rightarrow bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2ZA_1A_3 \mid aZA_1A_3 \mid b$

$A_2 \rightarrow bA_3A_2A_1 \mid aA_1 \mid bA_3A_2ZA_1 \mid aZA_1 \mid b$

$A_3 \rightarrow bA_3A_2 \mid a \mid bA_3A_2Z \mid aZ$ $\boxed{A_1A_3A_2}$

$Z \rightarrow bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid bA_3A_2ZA_1A_3A_3A_2 \mid aZA_1A_3A_3A_2 \mid bA_3A_3A_2 \mid bA_3A_2A_1A_3A_3A_2Z \mid aA_1A_3A_3A_2Z \mid bA_3A_2ZA_1A_3A_3A_2Z \mid aZA_1A_3A_3A_2Z \mid bA_3A_3A_2Z$

CFG to CNF, then to GNF

① $S \rightarrow AB$
$A \rightarrow BSB \mid BB \mid b$
$B \rightarrow a$

$S \rightarrow AB$
$A \rightarrow BR_1 \mid BB \mid b$
$B \rightarrow a$
$R_1 \rightarrow SB$

$A \rightarrow aR_1 \mid aB \mid b$
$B \rightarrow a$
$S \rightarrow aR_1B \mid aBB \mid bB$
$R_1 \rightarrow aR_1BB \mid aBBB \mid bBB$

$R_1 \rightarrow ABB$
$R_1 \rightarrow aR_1BB \mid aBBB \mid bBB$

② $S \rightarrow 01S \mid 01$
$S \rightarrow 10S \mid 10$
$S \rightarrow 00 \mid \lambda$

$A \rightarrow 0$ $B \rightarrow 1$
$S \rightarrow ABS \mid AB$
$S \rightarrow BAS \mid BA$
$S \rightarrow AA \mid$

$S \rightarrow AR_1 \mid AB$ $R_1 \rightarrow BS$
$S \rightarrow BR_2 \mid BA$ $R_2 \rightarrow AS$
$S \rightarrow AA$ $A \rightarrow 0$ $B \rightarrow 1$

$S \rightarrow 0R_1 \mid 0B$
$S \rightarrow 1R_2 \mid 1A$
$S \rightarrow 0A$
$R_1 \rightarrow 1S$
$R_2 \rightarrow 0S$
$A \rightarrow 0$ $B \rightarrow 1$

Q: $S \to AB$
$A \to BSB$
$A \to a$
$B \to b$

Bring the grammar G to GNF.
By first converting to CNF & then to GNF.

Soln: Step 1: Useless Production ✗
Null " ✗
Unit Production ✗

Step 2: Transform G into an equivalent G' in CNF.

$S \to AB$
$A \to BD_1$
$A \to a$
$B \to b$
$D_1 \to SB$

Step 3: Transform G' into an equivalent G'' in GNF.

Substitute
$S = A_1 \quad A = A_2 \quad B = A_3 \quad D_1 = A_4$

$A_1 \to A_2 A_3$	$A_1 \to A_2 A_3$	ok
$A_2 \to A_3 A_4$	$A_2 \to A_3 A_4 \mid a$	ok
$A_2 \to a$	$A_3 \to b$	ok
$A_3 \to b$	$A_4 \to A_1 A_3$	needs to modified
$A_4 \to A_1 A_3$		

Step 4: $A_4 \to A_1 A_3$
$A_4 \to A_2 A_3 A_3$
$A_4 \to A_3 A_4 A_3 A_3 \mid a A_3 A_3$
$A_4 \to b A_4 A_3 A_3 \mid a A_3 A_3$

Now,

$A_1 \to A_2 A_3$

$A_2 \to A_3 A_4 \mid a$

$A_3 \to b$

$A_4 \to b A_4 A_3 A_3 \mid a A_3 A_3$

Now, Substitute Backwards,

$A_1 \to A_2 A_3$

$A_1 \to A_3 A_4 A_3 \mid a A_3$

$$\boxed{\begin{aligned} &A_1 \to b A_4 A_3 \mid a A_3 \\ &A_2 \to b A_4 \mid a \\ &A_3 \to b \\ &A_4 \to b A_4 A_3 A_3 \mid a A_3 A_3 \end{aligned}}$$

Q: find a GNF grammar,

$S \to AA \mid 0$

$A \to SS \mid 1$

Solⁿ: Step 1: No Useless, No Unit & No Null.

Step 2: Convert it to equivalent G' in CNF.

Already in CNF.

Step 3: Convert it equivalent G' in GNF.

Substitute $S = A_1$ $A = A_2$

$A_1 \to A_2 A_2 \mid 0$ $\quad i<j$ ✓ No modification

~~A_2~~ $A_2 \to A_1 A_1 \mid 1$ $\quad i<j$ × Modification required

$A_2 \to A_1 A_1 \mid 1$

Substitute A_1

$A_2 \to A_2 A_2 A_1 \mid 0 A_1 \mid 1$

$A_2 \to \underbrace{A_2 A_2 A_1}_{\alpha} \mid \underbrace{0 A_1}_{\beta_1} \mid \underbrace{1}_{\beta_2}$ - left recursion needs to be resolved

Introduce New Variable Z

$A_2 \to 0A_1 \mid 1 \mid 0A_1 Z \mid 1Z$

$Z \to A_2 A_1 \mid A_2 A_1 Z$

For GNF A_2 is OK, Z needs to be modified

$Z \to A_2 A_1 \mid A_2 A_1 Z$

New Rule:

$A_1 \to A_2 A_2 \mid 0$

$A_2 \to 0A_1 \mid 1 \mid 0A_1 Z \mid 1Z$

$Z \to A_2 A_1 \mid A_2 A_1 Z$

Substitute to achieve GNF

$A_1 \to 0A_1 A_2 \mid 1A_2 \mid 0A_1 Z A_2 \mid 1ZA_2 \mid 0$

$A_2 \to 0A_1 \mid 1 \mid 0A_1 Z \mid 1Z$

$Z \to 0A_1 A_1 \mid 1A_1 \mid 0A_1 Z A_1 \mid 1ZA_1 \mid 0A_1 A_1 Z \mid 1A_1 Z \mid$
$0A_1 Z A_1 Z \mid 1ZA_1 Z$

$S \to 0SA \mid 1A \mid 0SZA \mid 1ZA \mid 0$

$A \to 0S \mid 1 \mid 0SZ \mid 12$

$Z \to 0SS \mid 1S \mid 0SZS \mid 1ZS \mid 0SSZ \mid 1SZ \mid 0SZSZ \mid 1ZSZ$

Q:- $A_1 \to A_1A_2 \mid a$

$A_2 \to A_1A_3 \mid b$

$A_3 \to A_1A_1 \mid a$

$A \to AB \mid a$

$B \to AC \mid b$

$C \to AA \mid a$

Solⁿ:- $A_1 \to A_1A_2 \mid a$

$\Rightarrow A_1 \to a \mid aZ$ ✓

$Z \to A_2 \mid A_2Z$

$Z \to A_1A_3 \mid b \mid A_1A_3Z \mid bZ$

$\to aA_3 \mid aZA_3 \mid aA_3Z \mid aZA_3Z \mid b \mid bZ$

$A_2 \to aA_3 \mid aZA_3 \mid b$

$A_3 \to aA_1 \mid aZA_1 \mid a$

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1) Lectures will be on Zoom, details of Zoom — Login with your name
2) Microsoft team - email id, google form, install mic team.
3) assignment, Quiz
4) Sub. of assi
5) Video lectures will also be shared.

Google form

install Mic. team

We will also shared Video lecture

Q: $S \to A$ ☆ First into CNF & then CNF grammar into GNF.

$A \to aBa \mid a$ CNF & into GNF.

$B \to bAb \mid b$

Soln:

Step 1: There are no useless variables.

There is no null productions.

Unit Production:- $S \to A$

$S \to aBa \mid a$
$A \to aBa \mid a$
$B \to bAb \mid b$
} New Rule

Step 2: Transform CFG into equivalent G' in CNF.

$S \to C_1 B C_1 \mid a$	$S \to C_1 D_1 \mid a$
$A \to C_1 B C_1 \mid a$	$A \to C_1 D_1 \mid a$
$B \to C_2 A C_2 \mid b$	$B \to C_2 D_2 \mid b$
$C_1 \to a$	$C_1 \to a$
$C_2 \to b$	$C_2 \to b$
	$D_1 \to B C_1$
	$D_2 \to A C_2$

} New Rule for G'

Step 3: Now Transform G' into equivalent G' in GNF.

Rename the variables to $S, A, B, C_1, C_2, D_1, D_2$ to

A_1, A_2 $S = A_1$; $A = A_2$; $B = A_3$; $C_1 = A_4$; $C_2 = A_5$

$D_1 = A_6$; $D_2 = A_7$.

$V_i \to V_j$
$i < j$

$A_1 \to A_4 A_6 \mid a$

$A_2 \to A_4 A_6 \mid a$

$A_3 \to A_5 A_7 \mid b$

$A_4 \to a$

$A_5 \to b$

$A_6 \to A_3 A_4$

$A_7 \to A_2 A_5$

GNF:-

$S \to a D_1 \mid a$
$A \to a D_1 \mid a$
$B \to b D_2 \mid b$
$C_1 \to a$
$C_2 \to b$
$D_1 \to b D_2 C_1 \mid b C_1$
$D_2 \to a D_1 C_2 \mid a C_2$

} In GNF format

$A_1 \rightarrow A_4 A_6 \mid \underline{a}$ $(\rightarrow a A_6 \mid a)$

$A_2 \rightarrow A_4 A_6 \mid \underline{a}$ $(a A_6 \mid a)$

$A_3 \rightarrow A_5 A_7 \mid \underline{b}$ $(b A_7 \mid b)$ Rules for A_1, A_2, A_3, A_4 & A_5 are fine w.r.t GNF we have to modify A_6 & A_7.

$A_4 \rightarrow \underline{a}$

$A_5 \rightarrow \underline{b}$

$A_6 \rightarrow A_3 A_4$

$A_7 \rightarrow A_2 A_5$

$A_6 \rightarrow A_3 A_4$

$A_6 \rightarrow A_5 A_7 A_4 \mid b A_4$

$A_6 \rightarrow b A_7 A_4 \mid b A_4$ ✓

$A_7 \rightarrow A_2 A_5$

$A_7 \rightarrow A_4 A_6 A_5 \mid a A_5$

$A_7 \rightarrow a A_6 A_5 \mid a A_5$ ✓

Now all the rules are sorted properly, according to the ordering constraint : if $V_i \rightarrow V_j$ .. then $i<j$

The new productions for G' in GNF are :-

$A_1 \rightarrow a A_6 \mid a$

$A_2 \rightarrow a A_6 \mid a$

$A_3 \rightarrow b A_7 \mid b$

$A_4 \rightarrow a$

$A_5 \rightarrow b$

$A_6 \rightarrow b A_7 A_4 \mid b A_4$

$A_7 \rightarrow a A_6 A_5 \mid a A_5$

Q:- Find a grammar in GNF equivalent to the grammar

$E \to E+T \mid T$

$T \to T*F \mid F$

$F \to (E) \mid a$

Soln: Step 1: Here we have to remove the Unit productions

$E \to T$ $\quad E \to E+T \mid T*F \mid (E) \mid a$

$T \to F$ $\quad T \to T*F \mid (E) \mid a$

$F \to (E) \mid a$

Step 2: Now, we will have to change it to CNF.

$E \to E+T$ $\quad A_1 \to +$

$E \to EA_1T$

$\underline{E \to EA_2}$ $\quad \underline{A_2 \to A_1T}$

$E \to T*F$ $\quad A_3 \to *$

$E \to TA_3F$

$\underline{E \to TA_4}$ $\quad \underline{A_4 \to A_3F}$

$E \to (E)$ $\quad A_5 \to ($ $\quad A_6 \to)$

$E \to A_5EA_6$

$E \to A_5A_7$ $\quad A_7 \to EA_6$

$E \to a$

$T \to T*F \to TA_3F \to TA_4$

$T \to (E) \to A_5EA_6 \to A_5A_7$

$T \to a$

$F \to (E)$

$F \to A_5 A_7 \mid a$

$E \to EA_2 \mid TA_4 \mid A_5 A_7 \mid a$

$T \to TA_4 \mid A_5 A_7 \mid a$

$F \to A_5 A_7 \mid a$

$A_1 \to +$

$A_2 \to A_1 T$

$A_3 \to *$

$A_4 \to A_3 F$

$A_5 \to ($

$A_6 \to)$

$A_7 \to EA_6$

$E \to EAT \mid TBF \mid CEC \mid a$

$T \to TBF \mid CEC \mid a$

$F \to CEC \mid a$

$A \to +$

$B \to *$

$C \to)$

Replace the variable A, B, C, F, T & E by $A_1, A_2, A_3, A_4, A_5, A_6$.

$A_1 \to +$

$A_2 \to *$

$A_3 \to)$

$A_4 \to \cancel{EAS}\ (A_6 A_3 \mid a$

$A_5 \to A_5 A_2 A_4 \mid (A_6 A_3 \mid a$

$A_6 \to A_6 A_1 A_5 \mid A_5 A_2 A_4 \mid (A_6 A_3 \mid a$

The productions of A_1, A_2, A_3 & A_4 are ok we have to modify A_5 & A_6.

$$A_5 \to A_5 \underbrace{A_2 A_4}_{\alpha} \mid \underbrace{(A_6 A_3}_{\beta_1} \mid \underbrace{a}_{\beta_2}$$

$$A_5 \to (A_6 A_3 \mid a \mid (A_6 A_3 Z_5 \mid a Z_5$$

$$Z_5 \to A_2 A_4 \mid A_2 A_4 Z_5$$

$$A_6 \to A_6 \underbrace{A_1 A_5}_{\alpha} \mid \underbrace{A_5 A_2 A_4}_{\beta_1} \mid \underbrace{(A_6 A_3}_{\beta_2} \mid \underbrace{a}_{\beta_3}$$

$$A_6 \to A_5 A_2 A_4 \mid (A_6 A_3 \mid a \mid A_5 A_2 A_4 Z_6 \mid (A_6 A_3 Z_6 \mid a Z_6$$

$$Z_6 \to A_1 A_5 \mid A_1 A_5 Z_6$$

Q: $A_1 \to A_2 A_3$

★ $A_2 \to A_3 A_1 \mid b$

$A_3 \to A_1 A_2 \mid a$

Soln:

$A_3 \to A_1 A_2$ $\qquad\qquad A_3 \to a$ ✓

$A_3 \to A_2 A_3 A_2$

$A_3 \to A_3 A_1 A_3 A_2$

New Production

$A_1 \to A_2 A_3$

$A_2 \to A_3 A_1 \mid b$

$A_3 \to \underbrace{A_3 A_1 A_3 A_2}_{\alpha} \mid \underbrace{b A_3 A_2}_{\beta_1} \mid \underbrace{a}_{\beta_2}$

$A_3 \to A_1 A_2$
$\to A_2 A_3 A_2$
$\to A_3 A_1 A_3 A_2$

$A_3 \to a \mid b A_3 A_2$

$A_3 \to aZ \mid b A_3 A_2 Z$

$Z \to A_1 A_3 A_2 \mid A_1 A_3 A_2 Z$

Now A_3 is in GNF:-

$A_2 \to A_3 A_1 \mid b$

$A_2 \to aA_1 \mid bA_3A_2A_1 \mid aZA_1 \mid bA_3A_2ZA_1 \mid b$

$A_1 \to aA_1A_3 \mid bA_3A_2A_1A_3 \mid aZA_1A_3 \mid bA_3A_2ZA_1A_3 \mid bA_3$

Substitute value of A_1 in Z

$Z \to aA_1A_3A_3A_2 \mid aA_1A_3A_3A_2Z \mid bA_3A_2A_1A_3A_3A_2 \mid bA_3A_2A_1A_3A_3A_2Z$

Q: Express CFG in GNF,

$S \to AA \mid ABA \mid BA$

$A \to aA \mid a$

$B \to bB \mid b$

$A \to a\gamma$

Soln: $A \to aA \mid a$

$B \to bB \mid b$

$S \to AA$ $S \to aAA \mid aA$

$S \to ABA$ $S \to aABA \mid aBA$

$S \to BA$ $S \to bBA \mid bA$

CFG in GNF

$S \to aAA \mid aA \mid aABA \mid aBA \mid bBA \mid bA$

$A \to aA \mid a$

$B \to bB \mid b$

Q:- Express CFG in GNF,

$A_1 \to A_2 A_2 \mid a$

$A_2 \to A_1 A_1 \mid b$

Repeat

Soln: Step 1:- No, unit production, useless production or null production.

~~Step 2~~ $A_1 \to A_2 A_2 \mid a \quad A_2 \to A_1 A_2 \mid b$

Step 2: $A_i \to a\gamma$ or $A_i \to A_j\gamma$ where $i<j$, we convert the A_i-production in the form $A_i \to A_j\gamma$ such that $i<j$.

Here A_1 productions are of the form,

$A_1 \to A_2 A_2 \mid a$ (i.e of the form $A_i \to A_j \gamma$, $i<j$ & $A_i \to a\gamma$)

– Required form

Production

$A_2 \to A_1 A_1 \mid b$

Here $A_2 \to b$ is in required for $A_i \to a\gamma$

But $A_2 \to A_1 A_1$ is not in required form ($A_i \to A_j\gamma$, $i<j$)

$A_2 \to A_2 A_2 A_1 \mid \cancel{a A_1}$

$A_2 \to a A_1$

$A_2 \to b$

Step 3:- Convert A_n- productions to the form $A_n \to a\gamma$. Here, productions of the form $A_n \to A_n \gamma$ are eliminated by following lemma:

$A_2 \to A_2 A_2 A_1$ is not in required form.

We will apply lemma to A_2 productions. Let Z_2 be the new variable.

Here $\alpha = A_2 A_1$

$A_2 \to a A_1 \quad A_2 \to b$

$A_2 \to a A_1 Z_2 \quad A_2 \to b Z_2$

$Z_2 \to A_2 A_1 \quad Z_2 \to A_2 A_1 Z_2$

Step 4: Modify A_i productions to the form $A_i \rightarrow a\gamma$ for $i = 1, 2, \ldots n-1$.

Here A_2 productions:-

$$A_2 \rightarrow aA_1 \mid b \mid aA_1Z_2 \mid bZ_2$$

— Required form

A_1 Productions

$A_1 \rightarrow A_2A_2$ $\qquad A_1 \rightarrow a$ ✓

$$A_1 \rightarrow aA_1A_2 \mid bA_2 \mid aA_1Z_2A_2 \mid bZ_2A_2 \mid a$$

Step 5: Modify Z_2 productions

Z_2 Productions

$Z_2 \rightarrow A_2A_1 \qquad Z_2 \rightarrow A_2A_1Z_2$

$$Z_2 \rightarrow aA_1A_1 \mid bA_1 \mid aA_1Z_2A_1 \mid bZ_2A_1$$

$$Z_2 \rightarrow aA_1A_1Z_2 \mid bA_1Z_2 \mid aA_1Z_2A_1Z_2 \mid bZ_2A_1Z_2$$

Hence, the equivalent grammar is:-

$$A_1 \rightarrow a \mid aA_1A_2 \mid bA_2 \mid aA_1Z_2A_2 \mid bZ_2A_2$$

$$A_2 \rightarrow aA_1 \mid b \mid aA_1Z_2 \mid bZ_2$$

$$Z_2 \rightarrow aA_1A_1 \mid bA_1 \mid aA_1Z_2A_1 \mid bZ_2A_1$$

$$Z_2 \rightarrow aA_1A_1Z_2 \mid bA_1Z_2 \mid aA_1Z_2A_1Z_2 \mid bZ_2A_1Z_2$$

Q: $S \to AA \mid 0$
$A \to SS \mid 1$

Solⁿ: $A \to SS \mid 1$

$A \to \underset{\alpha}{\underline{A\,AS}} \mid \underset{\beta}{\underline{0S}} \mid \underset{\beta}{\underline{1}}$

$Z \to AS \mid ASZ$

Repeat

$A \to 0S \mid 1 \mid 0SZ \mid 1Z$

$S \to 0SA \mid 1A \mid 0SZA \mid 1ZA \mid 0$

$A \to 0S \mid 1 \mid 0SZ \mid 1Z$

$Z \to AS \mid ASZ$

$Z \to 0SS \mid 1S \mid 0SZS \mid 1ZS \mid 0SSZ \mid 1SZ \mid 0SZSZ \mid 1ZSZ$

Q: $A_1 \to A_2 A_3 \mid a$
$A_2 \to A_3 A_1 \mid b$
$A_3 \to A_1 A_2 \mid a$

Solⁿ: $A_3 \to A_1 A_2 \mid a$

$A_3 \to A_2 A_3 A_2 \mid a A_2 \mid a$

$A_3 \to A_3 \underset{\alpha}{\underline{A_1 A_3 A_2}} \mid \underset{\beta_1}{\underline{b A_3 A_2}} \mid \underset{\beta_2}{\underline{a A_2}} \mid \underset{\beta_3}{\underline{a}}$

$B \to A_1 A_3 A_2 \mid A_1 A_3 A_2 B$

$A_3 \to bA_3A_2 \mid aA_2 \mid a \mid bA_3A_2B \mid aA_2B \mid aB$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_2 \rightarrow bA_3A_2A_1 \mid aA_2A_1 \mid aA_1 \mid bA_3A_2BA_1 \mid \overset{aA_2BA_1}{\cancel{aA_2A_1B}} \mid$

aZA_1 [illegible] $\mid b$

$A_1 \rightarrow A_2 A_3 \mid a$

$A_1 \rightarrow bA_3A_2A_1A_3 \mid aA_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2A_1BA_3 \mid$

$aA_2A_1BA_3 \mid aA_1BA_3 \mid bA_3 \mid a$

$B \rightarrow bA_3A_2A_1A_3A_3A_2 \mid aA_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid$

$bA_3A_2A_1BA_3A_3A_2 \mid aA_2A_1BA_3A_3A_2 \mid aA_1BA_3A_3A_2 \mid$

$bA_3A_3A_2 \mid aA_3A_2 \mid bA_3A_2A_1A_3A_3A_2B \mid$

$aA_2A_1A_3A_3A_2B \mid aA_1A_3A_3A_2B \mid bA_3A_2A_1BA_3A_3A_2B$

$\mid aA_2A_1BA_3A_3A_2B \mid aA_1BA_3A_3A_2B \mid bA_3A_3A_2B \mid$

aA_3A_2B

Q: $S \to ABC$

$A \to a \mid b$

$B \to Bb \mid aa$

$C \to aC \mid cC \mid ba$

Sol^n: $A \to a$

$A \to b$

$\cancel{B \to Bb}$ $S \to ABC$

$S \to aBC \mid bBC$

$C \to aC \mid cC \mid ba$ $\quad$ $C \to aC \mid cC \mid bC_1$

$C_1 \to a$

$B \to B\underset{\alpha}{\underline{b}} \mid \underset{\beta}{\underline{aa}}$

$Z \to aa \mid aaZ$

$B \to$

Step 1: No Unit, No Null & No Useless Production

Step 2: CNF for the given CFG,

$S \to AC_1 \quad C_1 \to BC$

$A \to a \mid b \quad C_2 \to b$

$B \to BC_2 \mid C_3C_3 \quad C_3 \to a$

$C_4 \to c$

$C \to C_3C \mid C_4C \mid C_2C_3$

Step 3: Equivalent GNF for the CFG.

Substitute $S = A_1 \quad A = A_2 \quad B = A_3 \quad C = A_4$

$C_1 = A_5 \quad C_2 = A_6 \quad C_3 = A_7 \quad C_4 = A_8$

New Rules:-

$A_1 \to A_2 A_5$ ok

$A_2 \to a \mid b \; (A_7 \mid A_6)$ ok

$A_3 \to A_3 A_6 \mid \underbrace{A_7 A_7}_{ok}$ $A_3 \to A_3 A_6$ to be modified

$A_4 \to A_7 A_4 \mid A_8 A_4 \mid A_6 A_7$ ok

$A_5 \to A_3 A_4$ Needs to be modified

$A_6 \to b$

$A_7 \to a$

$A_8 \to c$

Equivalent GNF:-

$A_3 \to A_3 A_6$