CSE2403-Discrete Mathematics

Problem Sheet-1

Topic: **Mathematical Logic**

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Date: 23/05/2022

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1. Consider the following propositions:
   P: Mathematicians are generous
   Q: Siddhesh hate algebra
   Write the compound propositions symbolized by:
   i) $P \wedge \neg Q$
   ii) $\neg(P \wedge Q)$
   iii)$\neg P \rightarrow Q$
   iv)$\neg P \leftrightarrow \neg Q$

2. Consider the following propositions:
   P: Today is Monday
   Q: I will go to Movie
   Write the compound propositions symbolically:
   i) If today is Monday, then I will not go to Movie
   ii) I will go to Movie and today is not Monday
   iii) If and only if today is not Monday then I will go to Movie

3. Consider the following propositions:
   P: Bats are blind
   Q:Gnats eat grass
   R: Ants have long teeth
   Write the compound propositions symbolically:
   i) If bats are blind then gnats donot eat grass.
   ii) If and only if bats are blind or gnats eats grass then ants don't have long teeth.
   iii) Ants don't have long teeth and, if bats are blind, then gnats donot eat grass.
   iv) Bats are blind or gnats eat grass and, if gnats don't eat grass, then ants don't have long teeth.

4. Construct the truth table for the following propositions
   i) $(P \vee (Q \vee (\neg P \wedge \neg R)))$
   ii) $(P \rightarrow R) \rightarrow (Q \rightarrow R)$
   iii) $(P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q)$
   iv)$\neg((\neg Q \rightarrow \neg P) \wedge (Q \rightarrow \neg R))$
   v) $((P \rightarrow Q) \rightarrow R) \rightarrow S$
   vi)$\neg(P \vee (Q \wedge R)) \leftrightarrow ((P \vee Q) \wedge (P \rightarrow R))$

5. Without using truth table, Prove that the following propositions are equivalent
   i) $(P \rightarrow Q)$

ii)$(P \wedge \neg Q) \rightarrow \neg P$
iii) $(P \wedge \neg Q) \rightarrow Q$

6. Find the truth tables for the following propositions. Are any of them equivalent
   i) $(P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q)$
   ii) $R \rightarrow \neg P$
   iii)$P \rightarrow \neg R$
   iv)$\neg((\neg Q \rightarrow \neg P) \wedge (Q \rightarrow \neg R))$

7. Determine which of the following compound propositions are tautologies and which of them are contradiction, using trurh tables:
   i)$\neg Q \wedge (P \rightarrow Q) \rightarrow \neg P$
   ii) $\neg(Q \rightarrow R) \wedge R \wedge (P \rightarrow Q)$
   iii)$((P \vee Q) \wedge (P \rightarrow R) \wedge (Q \wedge R)) \rightarrow R$

8. Without using truth tables, Prove the following:
   i)$(\neg P \vee Q) \wedge (P \wedge (P \wedge Q)) \Leftrightarrow (P \wedge Q)$
   ii) $P \rightarrow (Q \rightarrow P) \Leftrightarrow \neg P \rightarrow (P \rightarrow Q)$
   iii) $\neg P \rightarrow (Q \rightarrow R) \Leftrightarrow Q \rightarrow (P \vee R)$
   iv) $\neg(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$

9. Prove the following implications by using truth tables:
   i)$(P \rightarrow (Q \rightarrow S)) \wedge (\neg R \vee P) \wedge Q \Rightarrow (R \rightarrow S)$
   ii) $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R) \Rightarrow R$
   iii) $((P \vee \neg P) \rightarrow Q)) \rightarrow ((P \vee \neg P) \rightarrow R)) \Rightarrow Q \rightarrow R$
   iv) $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow (P \rightarrow R)$

10. Prove the following implications,without using truth tables:
    i)$(P \rightarrow (Q \rightarrow S)) \wedge (\neg R \vee P) \wedge Q \Rightarrow (R \rightarrow S)$
    ii) $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R) \Rightarrow R$
    iii) $((P \vee \neg P) \rightarrow Q)) \rightarrow ((P \vee \neg P) \rightarrow R)) \Rightarrow Q \rightarrow R$
    iv) $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow (P \rightarrow R)$