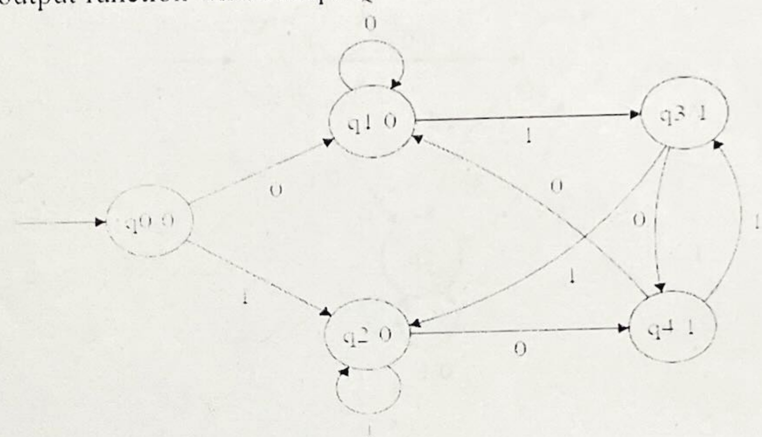
FINITE AUTOMATA WITH OUTPUTS

Mealy and Moore Machines

Moore Machines: Moore machines are finite state machines with output value and its output depends only on present state. It can be defined as (Q, $q0$, $\sum$, O, δ, λ) where:

- Q is finite set of states.
- $q0$ is the initial state.
- $\sum$ is the input alphabet.
- O is the output alphabet.
- δ is transition function which maps $Q \times \sum \rightarrow Q$.
- λ is the output function which maps $Q \rightarrow O$.



In the moore machine shown in Figure, the output is represented with each input state separated by /. The length of output for a moore machine is greater than input by 1.

- **Input:** 11
- **Transition:** δ (q0,11)=> δ(q2,1)=>q2

Output: 000 (0 for q0, 0 for q2 and again 0 for q2)

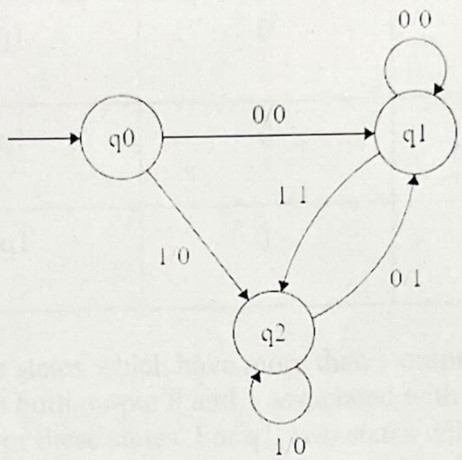
Transition Table for Moore Machine

	Input=0	Input=1	
Present State	**Next State**	**Next State**	**Output**
q0	q1	q2	0
q1	q1	q3	0
q2	q4	q2	0
q3	q4	q2	1

q4	q1	q3	1

Mealy Machines: Mealy machines are also finite state machines with output value and its out put depends on present state and current input. It can be defined as (Q, q0, ∑, O, δ, λ') where:

- Q is finite set of states.
- q0 is the initial state.
- ∑ is the input alphabet.
- O is the output alphabet.
- δ is transition function which maps Q×∑ → Q.
- 'λ' is the output function which maps Q×∑→ O.



In the mealy machine shown in Figure, the output is represented with each input symbol for each state separated by /. The length of output for a mealy machine is equal to the length of input.

- **Input:**11
- **Transition:** δ (q0,11)=> δ(q2,1)=>q2
- **Output:** 00 (q0 to q2 transition has Output 0 and q2 to q2 transition also has Output 0)

Transition Table for Mealy Machine

	Input=0		Input=1	
Present State	**Next State**	**Output**	**Next State**	**Output**
q0	q1	0	q2	0
q1	q1	0	q2	1
q2	q1	1	q2	0

Conversion from Mealy to Moore Machine

Let us take the transition table of mealy machine shown below.

	Input=0		Input=1	
Present State	**Next State**	**Output**	**Next State**	**Output**
q0	q1	0	q2	0
q1	q1	0	q2	1
q2	q1	1	q2	0

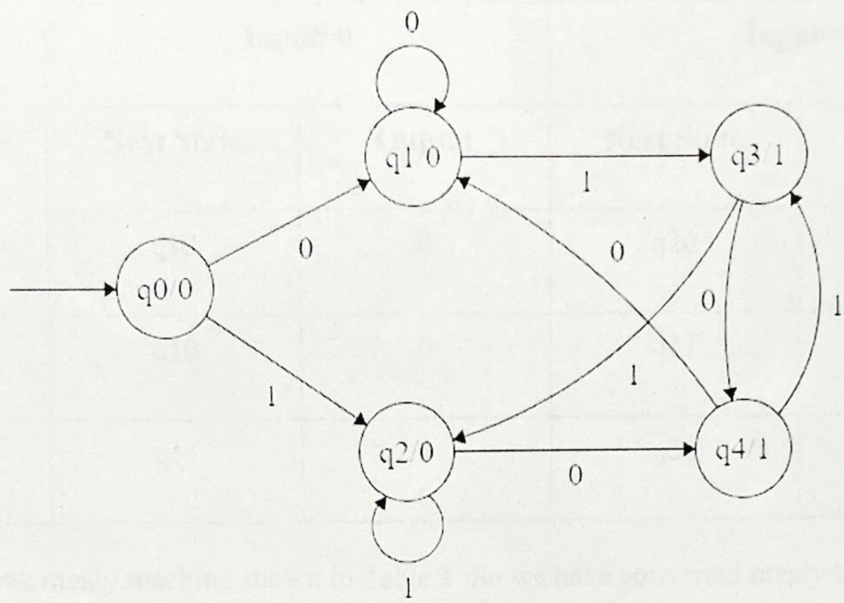
Step 1. First find out those states which have more than 1 outputs associated with them. q1 and q 2 are the states which have both output 0 and 1 associated with them.
Step 2. Create two states for these states. For q1, two states will be q10 (state with output 0) and q11 (state with output 1). Similarly for q2, two states will be q20 and q21.
Step 3. Create an empty moore machine with new generated state. For moore machine, Output w ill be associated to each state irrespective of inputs.
Step 4. Fill the entries of next state using mealy machine transition table shown in Table 1. For q0 on input 0, next
state is q10 (q1 with output 0). Similarly, for q0 on input 1, next state is q20 (q2 with output 0). F or q1 (both q10 and q11) on input 0, next state is q10. Similarly, for q1(both q10 and q11), next s tate is q21.. For q10, output will be 0 and for q11, output will be 1. Similarly, other entries can be filled.

	Input=0	Input=1	
Present State	**Next State**	**Next State**	**Output**
q0	q10	q20	$\emptyset$ ϵ
q10	q10	q21	0
q11	q10	q21	1
q20	q11	q20	0
q21	q11	q20	1

This is the transition table of moore machine shown in Figure.

Conversion from moore machine to mealy machine

Let us take the moore machine of Figure 1 and its transition table is shown in Table.



Step 1. Construct an empty mealy machine using all states of moore machine as shown in Table 4.

Step 2: Next state for each state can also be directly found from moore machine transition Table.

Step 3: As we can see output corresponding to each input in moore machine transition table. Use this to fill the Output entries. e.g.; Output corresponding to q10, q11, q20 and q21 are 0, 1, 0 and 1 respectively.

	Input=0		Input=1	
Present State	**Next State**	**Output**	**Next State**	**Output**
q0	q10	0	q20	0
q10	q10	0	q21	1
q11	q10	0	q21	1
q20	q11	1	q20	0
q21	q11	1	q20	0

	0	1	o/p
→ q_0	q_1	q_2	0
q_1	q_1	q_3	0
q_2	q_4	q_2	0
q_3	q_4	q_2	1
q_4	q_1	q_3	1

	Next State	o/p	Next State	o/p
→ q_0	q_1	0	q_2	0
q_1	q_1	0	q_3	1
q_2	q_4	1	q_2	0
q_3	q_4	1	q_2	0
q_4	q_1	0	q_3	1

Step 4: As we can see from table 6, q10 and q11 are similar to each other (same value of next st ate and Output for different Input). Similarly, q20 and q21 are also similar. So, q11 and q21 can be eliminated.

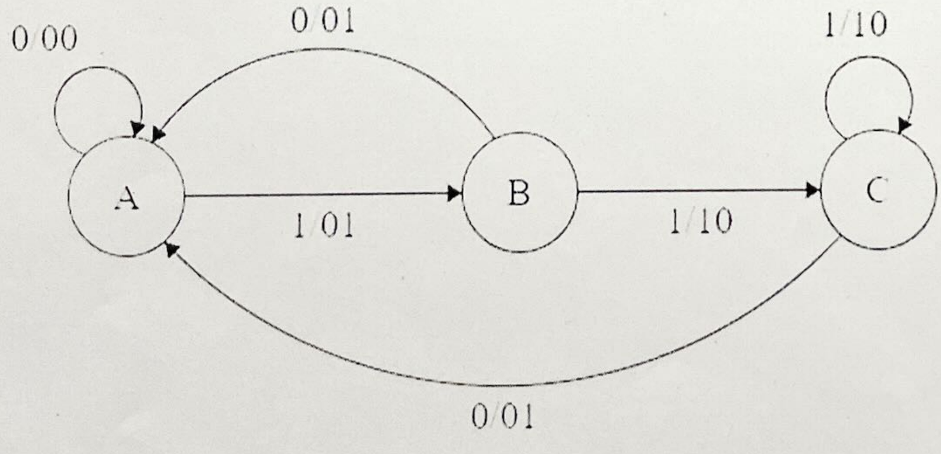
	Input=0		**Input=1**	
Present State	**Next State**	**Output**	**Next State**	**Output**
q0	q10	0	q20	0
q10	q10	0	q21	1
q20	q11	1	q20	0

Table 7

This is the same mealy machine shown in Table 1. So we have converted mealy to moore machin e and converted back moore to mealy.

Note: Number of states in mealy machine can't be greater than number of states in moore machine.

Example: The Finite state machine described by the following state diagram with A as starting s tate, where an arc label is x / y and x stands for 1-bit input and y stands for 2- bit output?



Outputs the sum of the present and the previous bits of the input.

1. Outputs 01 whenever the input sequence contains 11.
2. Outputs 00 whenever the input sequence contains 10.
3. None of these.

Solution: Let us take different inputs and its output and check which option works:

Input: 01
Output: 00 01 (For 0, Output is 00 and state is A. Then, for 1, Output is 01 and state will be B)
Input: 11
Output: 01 10 (For 1, Output is 01 and state is B. Then, for 1, Output is 10 and state is C)
As we can see, it is giving the binary sum of present and previous bit. For first bit, previous bit is taken as 0.

Q: Construct Moore machine to convert each occurrence of substring 100 by101.

Q: Design a Moore machine to determine residue mod 2 for each binary string treated as binary integer.

Q: Design a Moore machine to determine residue mod 3 for each binary string treated as binary integer.

Q: Design a Moore Machine for binary input Sequence ,if it ends in 101, output is „A", if itends in"110" output is "B" otherwise „C".

Q: Convert following Moore machine into Mealy Machine.

Present State	Next State		Output
	a=0	a=1	
→ q0			

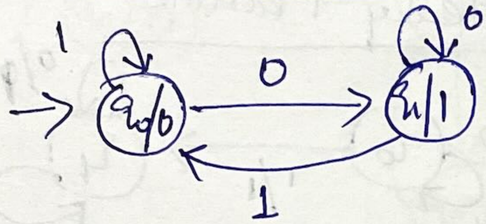
Q: Design a Mealy Machine and Moore Machine for 1's complement of a binary no.

Sol^n: i/p – Σ{1,0}   o/p :- {0,1}

i/p String:- 0110

o/p String:- 1001

Moore Machine



	0	1	o/p
→ q_0	q_1	q_0	0
q_1	q_1	q_0	1

i/p String: 10110

$\delta(q_0, 10110) \vdash \delta(q_0, 0110)$

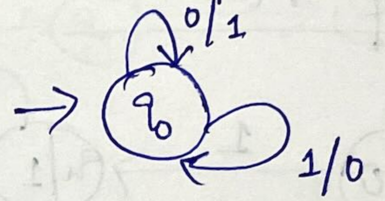
$\vdash \delta(q_1, 110)$

$\vdash \delta(q_0, 10)$

$\vdash \delta(q_0, 0)$

$\vdash q_1$

Mealy Machine



	0	o/p	1	o/p
→ q_0	q_0	1	q_0	1
q_1				

o/p

01001.

Q: Design a Mealy and Moore machine for 2's Compliment of a binary no.

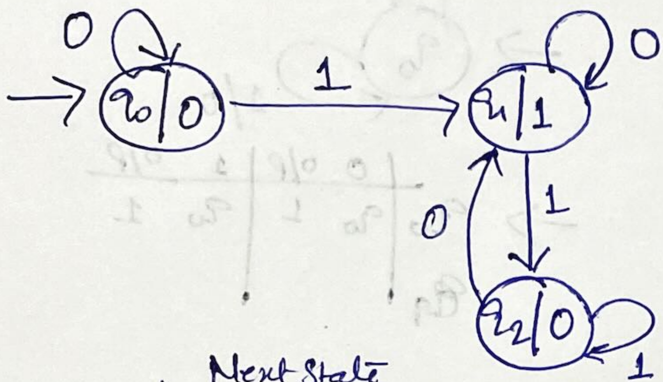
Soln: $\Sigma = \{1, 0\}$ $\qquad O = \{1, 0\}$

i/P String: 0110

O/P String: 1010

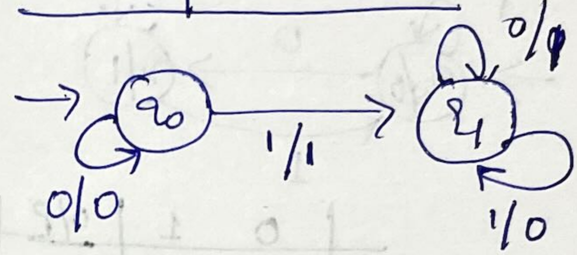
$$\begin{array}{r l} 0110 & \\ 1001 & \text{– 1's comp.} \\ 1 & \\ \hline 1010 & \text{2's comp.} \end{array}$$

~~Moore~~ Machine



	Next State 0	Next State 1	O/P
→ q_0	q_0	q_1	0
q_1	q_1	q_2	1
q_2	q_1	q_2	0

Mealy Machine



	0: Next State	0: O/P	1: Next State	1: O/P
→ q_0	q_0	0	q_1	1
q_1	q_1	1	q_1	0

i/P String: 101001 | O/P: 010111 $\qquad$ 0100100 | O/P: 1011100

~~$\delta(q_0, 101001) = \delta(q_1, 01001)$~~ $\qquad \delta(q_0, 0100100)$

101001 ⊢ 101001 $\qquad$ ⊢ 0100100

q_0 $\qquad$ $q_1\ q_2\ q_2\ q_1\ q_1$ $\qquad$ $q_1\ q_1\ q_1\ q_1\ q_0\ q_0\ q_0$

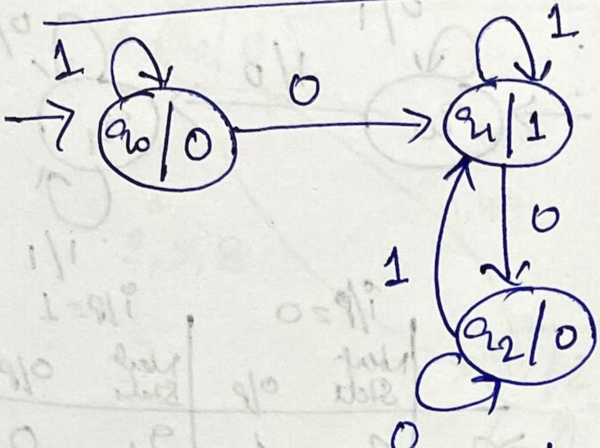
Q: Design a Mealy Machine and Moore Machine for increment of a binary no. by 1.

Sol^n:

i/p :- 9 ⇒ 1001   11 ⇒ 1011

o/p :- 10 ⇒ 1010   12 ⇒ 1100

Moore Machine



	0	1	o/p
→ q_0	q_1	q_0	0
q_1	q_2	q_1	1
q_2	q_2	q_1	0

i/p: 1011

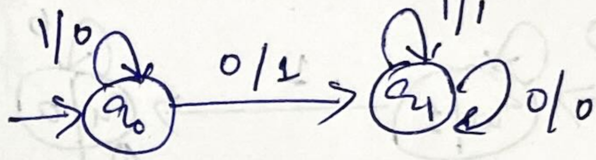
o/p:- 1100

1011   o/p

↑↑↑↑   1100

q_1 q_1 q_0 q_0 q_0

Mealy Machine



	i/p=0		i/p=0	
	Next State	o/p	Next State	o/p
→ q_0	q_1	1	q_0	0
q_2	q_1	0	q_1	1

i/p: 10100

o/p: 10101

10100   10101

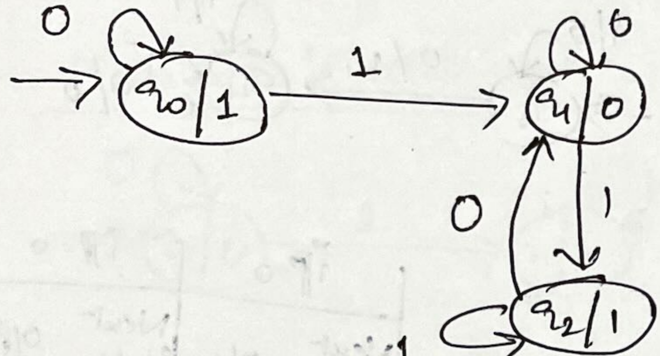
↑ ↑↑↑↑↑

q_1 q_2 q_1 q_2 q_1 q_0

Q:- Design a Mealy and Moore Machine to decrement a binary no. by 1.

Soln: i/p ⇒ 9   1001   12 ⇒ 1100
o/p ⇒ 8   1000   11 ⇒ 1011

Moore M/C



	0	1	o/p
→ q_0	q_0	q_1	1
q_1	q_1	q_2	0
q_2	q_1	q_2	1

i/p: 1100

1 1 0 0

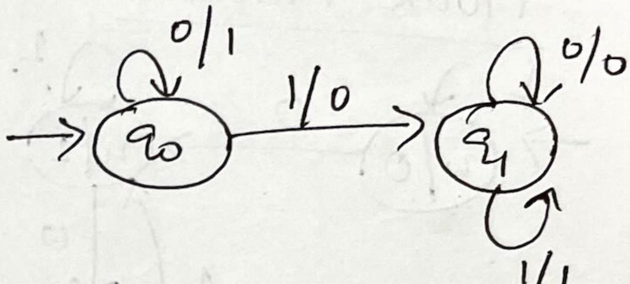
↑ ↑ ↑ ↑ ↑

q_2 q_1 q_0 q_0 q_0

o/p

1 0 1 1

Mealy M/C



	i/p = 0 Next State	o/p	i/p = 1 Next State	o/p
→ q_0	q_0	1	q_1	0
q_1	q_1	0	q_1	1

i/p 11001 ⇒ 25

1 1 0 0 1

↑ ↑ ↑ ↑ ↑ ↑

q_1 q_1 q_1 q_1 q_1 q_0

o/p

0
0
0
1
1

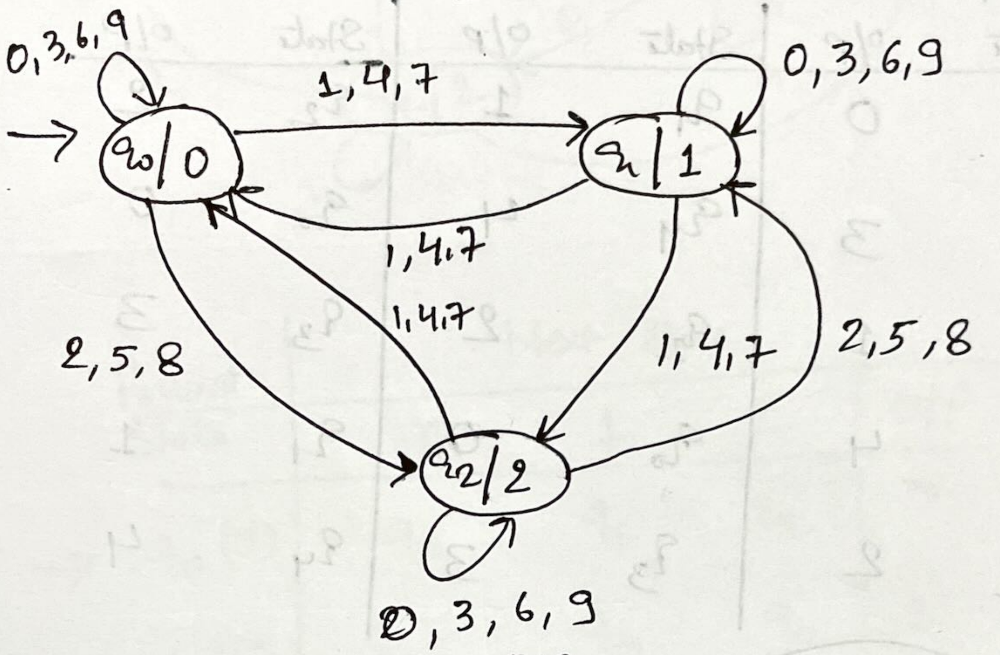
o/p:- 11000

⇒ 24

Q: Design a Moore machine for checking divisibility by 3 of a given decimal number.

Soln:- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$O = \{0, 1, 2\}$

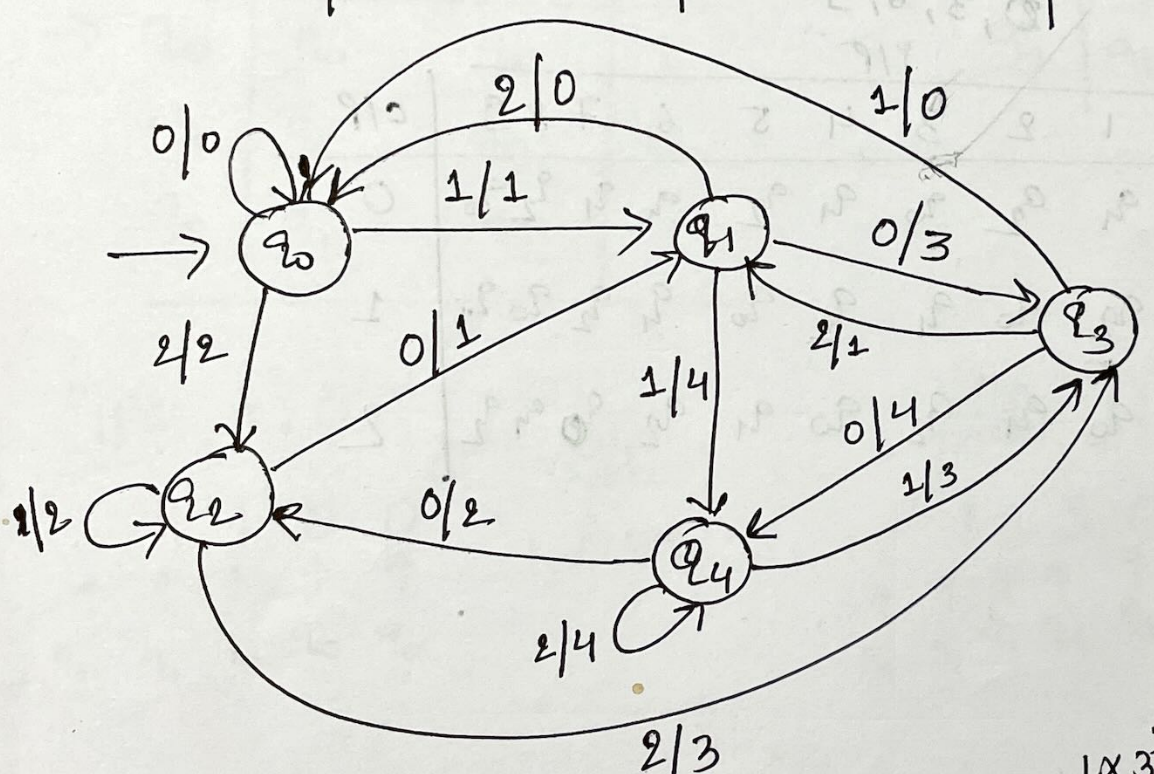


	i/p 0	1	2	3	4	5	6	7	8	9	o/p
→ q_0	q_0	q_1	q_2	q_0	q_1	q_2	q_0	q_1	q_2	q_0	0
q_1	q_1	q_2	q_0	q_1	q_2	q_0	q_1	q_2	q_0	q_1	1
q_2	q_2	q_0	q_1	q_2	q_0	q_1	q_2	q_0	q_1	q_2	2

Q: Give Mealy and Moore machine for the following $\Sigma = \{0, 1, 2\}$. Output is the remainders ~~of two~~ after division by 5. Input is treated as ternary (base 3).

Soln:

	i/p = 0		i/p = 1		i/p = 2	
	State	o/p	State	o/p	State	o/p
→(0) q_0	q_0	0	q_1	1	q_2	2
(1) q_1	q_3	3	q_4	4	q_0	0
(2) q_2	q_1	1	q_2	2	q_3	3
(10) q_3	q_4	4	q_0	0	q_1	1
(11) q_4	q_2	2	q_3	3	q_4	4

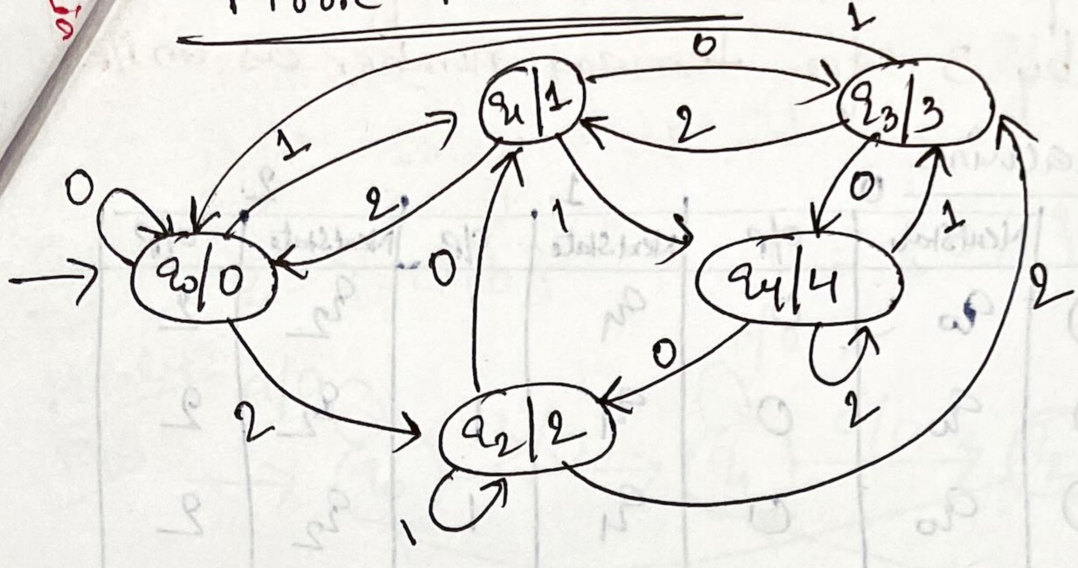


$10 = 1\times3^1 + 0\times3^0$
$= 3$
$11 = 1\times3^1 + 1\times3^0$
$= 3+1 = 4$

i/p :- 1002

$1\times3^3 + 0\times3^2 + 0\times3^1 + 2\times3^0$
$= 27 + 2 = 29$
$29 \bmod 5 = 4$

Moore Machine

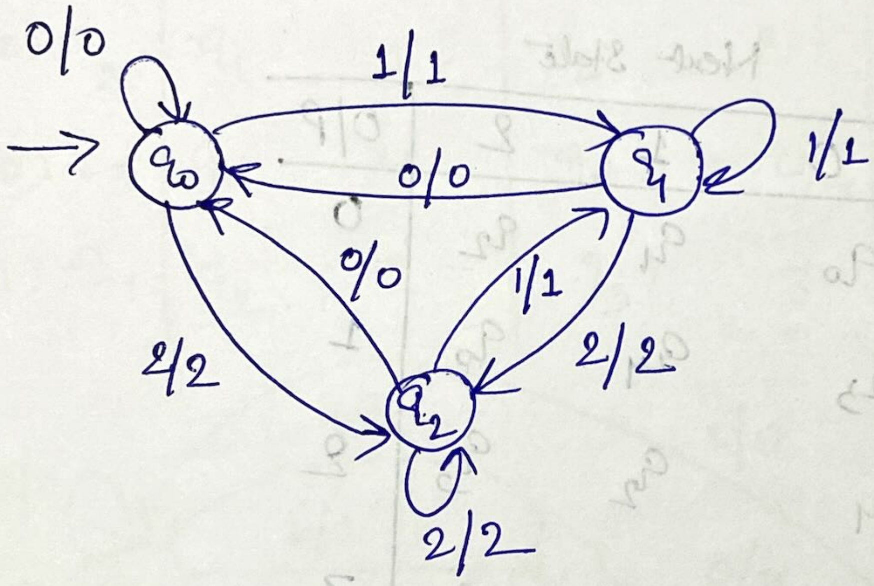


Present State	Next State 0	Next State 1	Next State 2	O/P
→ q_0(0)	q_0	q_1	q_2	0
q_1(1)	q_3	q_4	q_0	1
q_2(2)	q_1	q_2	q_3	2
q_3(10)	q_4	q_0	q_1	3
q_4(11)	q_2	q_3	q_4	4

Q: Design Mealy and Moore machine for divisibility by 3 for ternary number as an i/p.

Soln:- **Mealy Machine**

	0: Next State	0: O/P	1: Next State	1: O/P	2: Next State	2: O/P
→ q_0 (0)	q_0	0	q_1	1	q_2	2
q_1 (1)	q_0	0	q_1	1	q_2	2
q_2 (2)	q_0	0	q_1	1	q_2	2



012

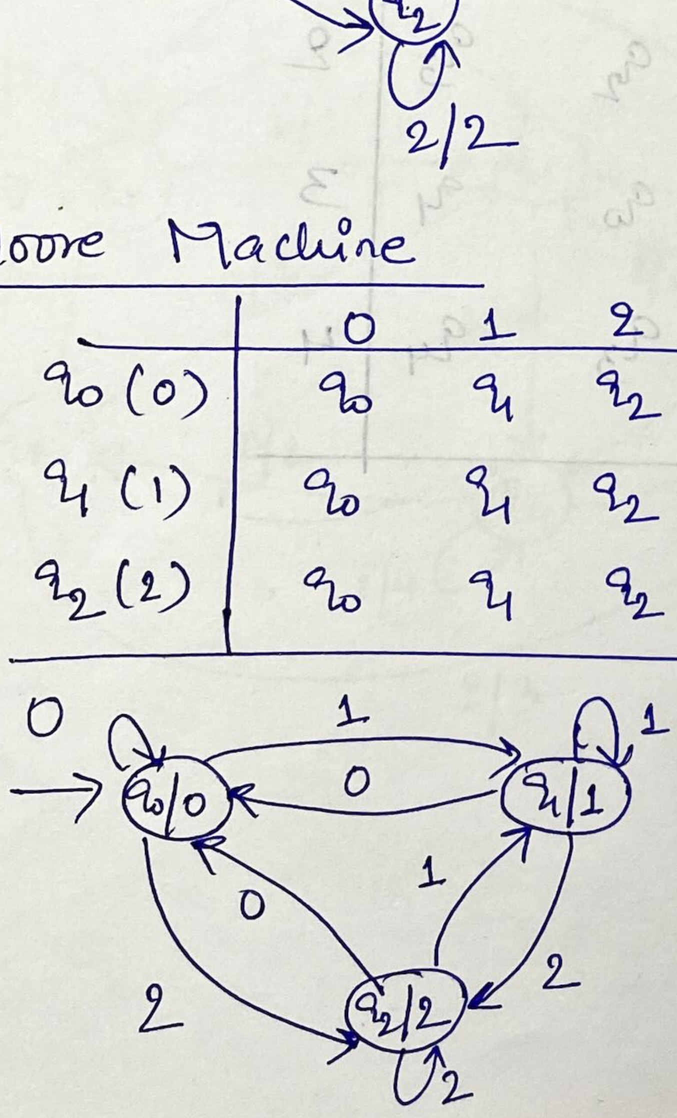
$0\times3^2+1\times3^1+2\times3^0$

$=0+3+2=5$

$5 \bmod 3 = 2$

Moore Machine

	0	1	2	O/P
→ q_0 (0)	q_0	q_1	q_2	0
q_1 (1)	q_0	q_1	q_2	1
q_2 (2)	q_0	q_1	q_2	2

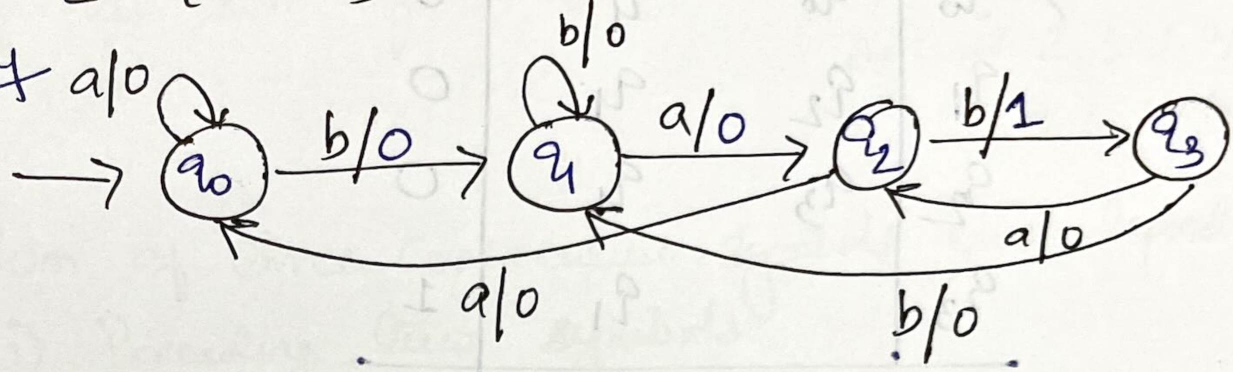


101 - A
110 - B
otherwise. C

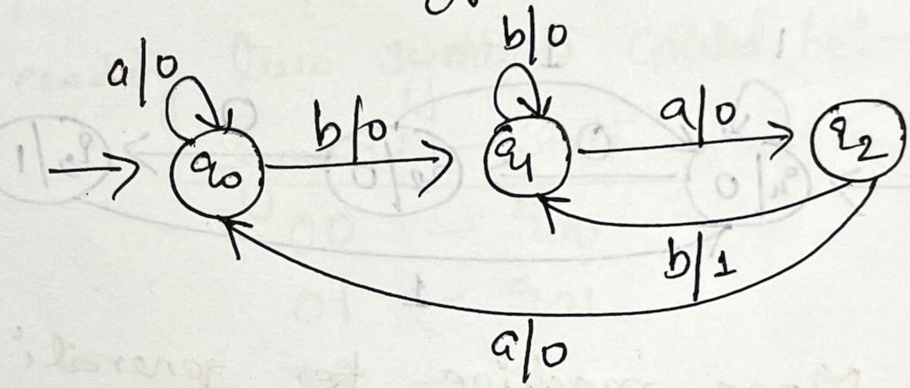
Q: Design a Mealy Machine and Moore machine which gives o/p of 1 if the input string ends in bab.

Solⁿ: $\Sigma = \{a, b\}$ $\quad O = \{0, 1\}$

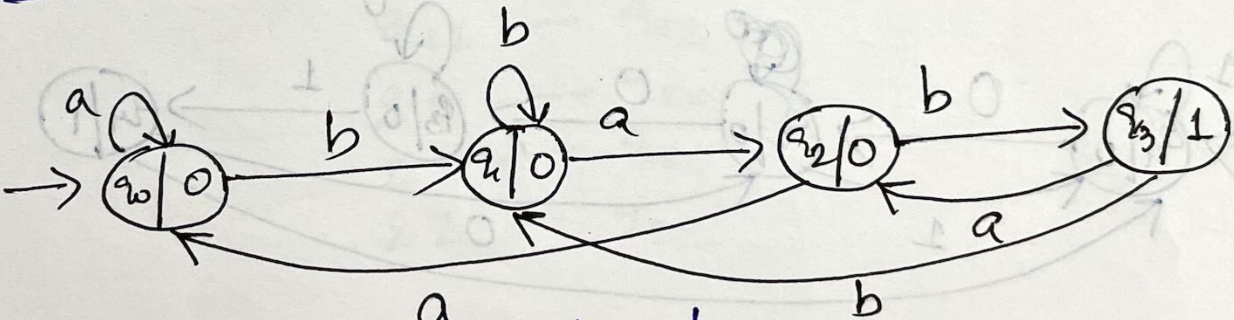
Mealy M/c



or



Moore Machine



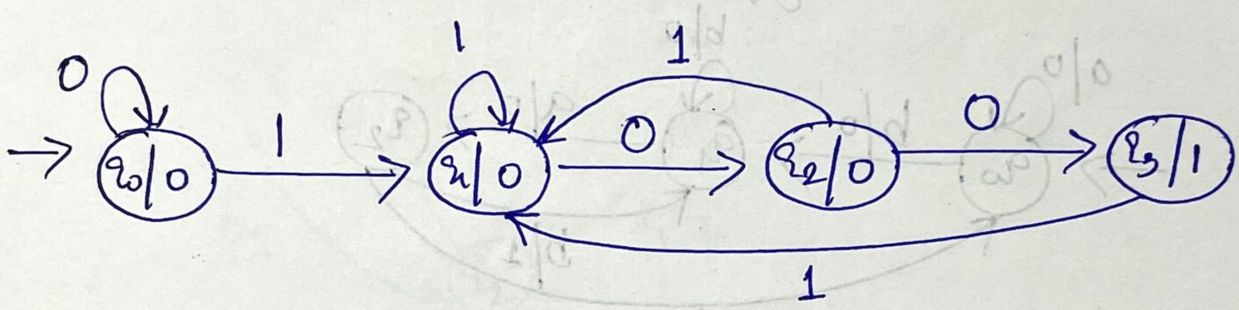
a

	a	b	
→ q_0	q_0	q_1	0
q_1	q_2	q_1	0
q_2	q_0	q_3	0
q_3	q_2	q_1	1

Q: Design a Moore machine for generating output 1 if input of binary sequence 1 is succeeded ~~preceded~~ by exactly two zeros.

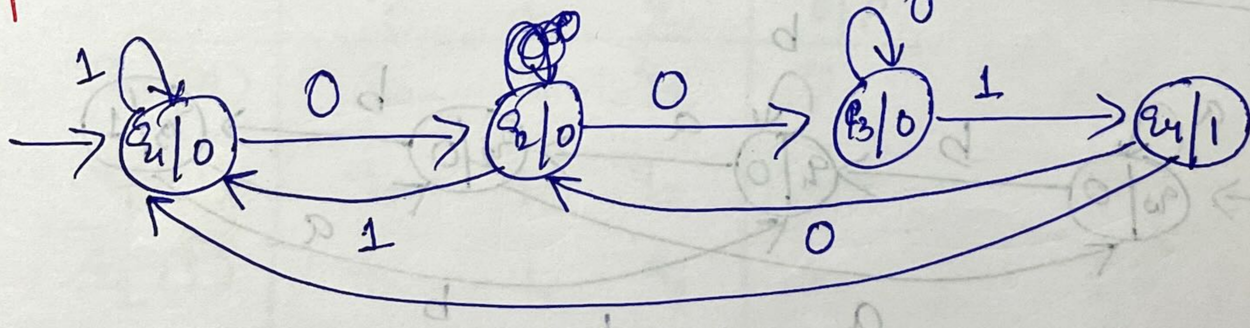
Solⁿ:

	0	1	O/P
$\rightarrow q_0$	q_0	q_1	0
q_1	q_2	q_1	0
q_2	q_3	q_1	0
q_3		q_1	1



Q: Design a Moore machine for generating output 1 if input of binary sequence 1 is preceded with exactly two zeros.

Solⁿ:-



Q:- Design a Moore machine for the following problem:-

Input alphabet = {0, 1, 2}

O/P alphabet = {a, b, c}

If the sum of 3 consecutive no. is 0, 1, 4 : O/P - a

" " " no. is 2, 3 : O/P - b

" " " is 5, 6 : O/P - c

Sol^n: Sum of three consecutive symbols will depend on

i) Preceding two symbols

ii) Next input.

Preceding two symbols could be:-

Symbols	State
00	q_{00}
01	q_{01}
02	q_{02}
10	q_{10}
11	q_{11}
12	q_{12}
20	q_{20}
21	q_{21}
22	q_{22}

Present State	Next State 0	Next State 1	Next State 2
→ q_{00}	q_{00}/a	q_{01}/a	q_{02}/b
q_{01}	q_{10}/a	q_{11}/b	q_{12}/b
q_{02}	q_{20}/b	q_{21}/b	q_{22}/a
q_{10}	q_{00}/a	q_{01}/b	q_{02}/b
q_{11}	q_{10}/b	q_{11}/b	q_{12}/a
q_{12}	q_{20}/b	q_{21}/a	q_{22}/c
q_{20}	q_{00}/b	q_{01}/b	q_{02}/a
q_{21}	q_{10}/b	q_{11}/a	q_{12}/c
q_{22}	q_{20}/a	q_{21}/c	q_{22}/c

O/P

$0,1,4 \Rightarrow a$

$2,3 \Rightarrow b$

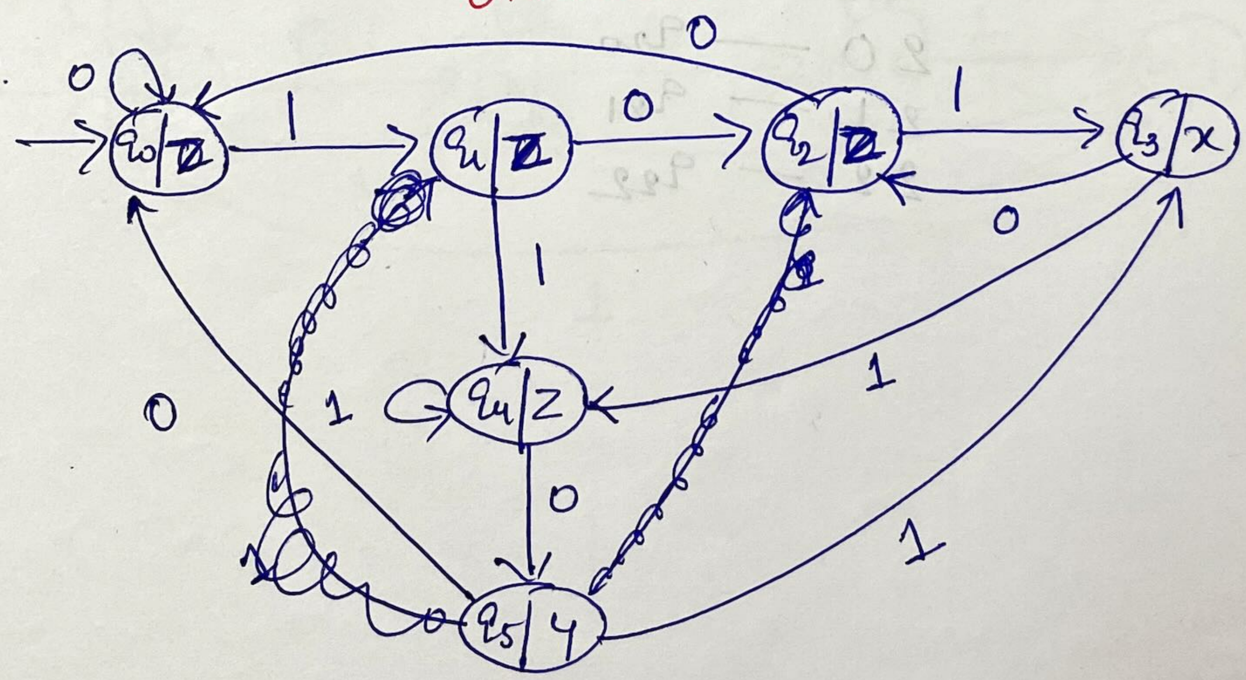
$5,6 \Rightarrow c$

Q:- Design a Mealy and Moore Machine for the i/p ending in 101 with output x

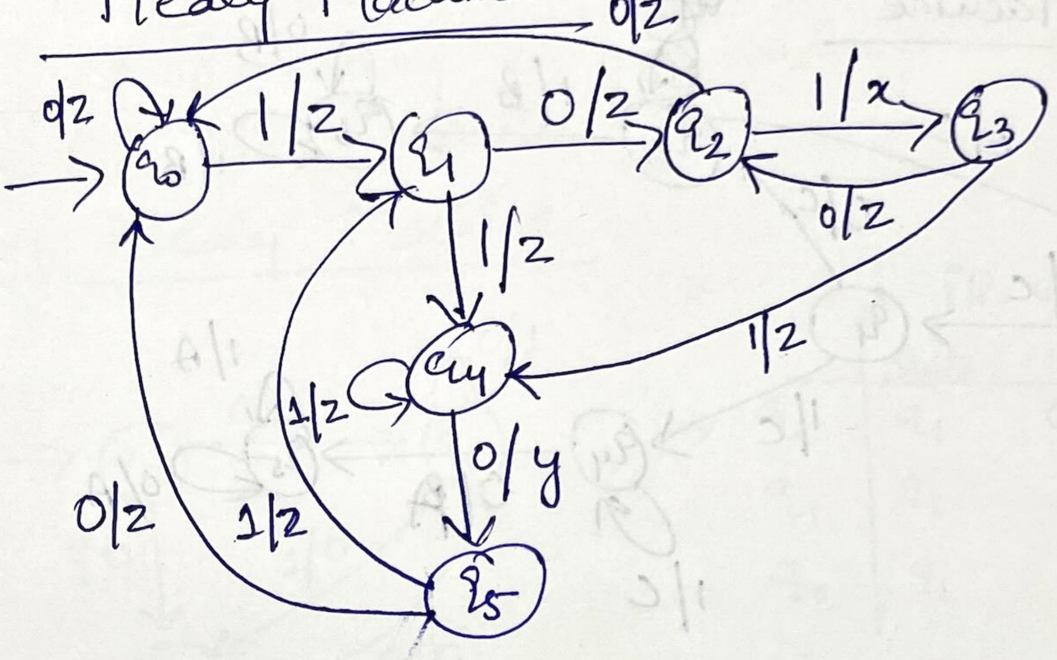
" " " 110 " " y

Other wise " z.

Solⁿ:

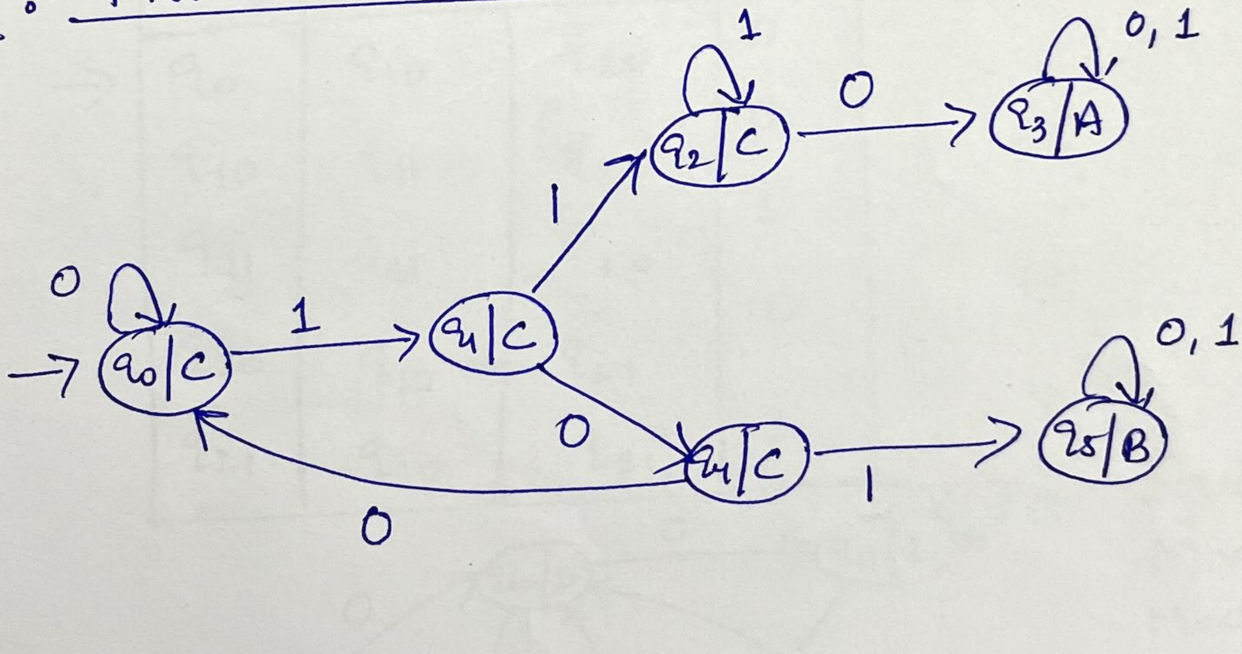


Mealy Machine

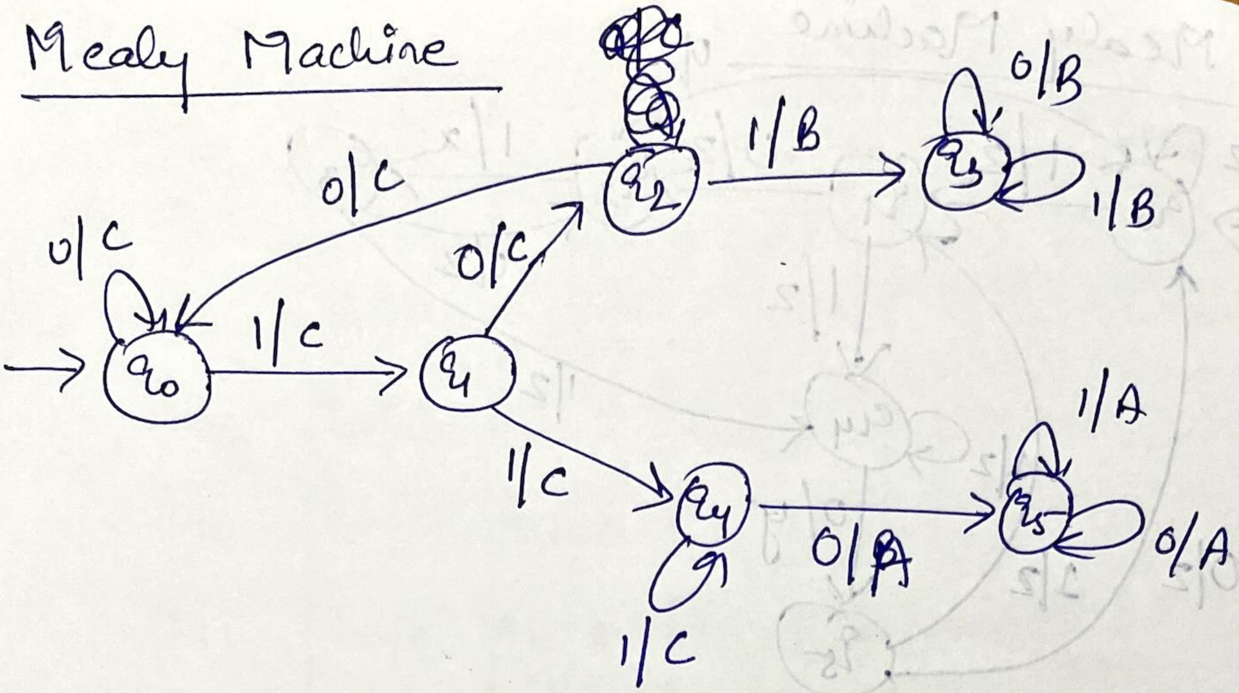


Q:- Design a Moore Machine and Mealy Machine for a binary input sequence such that if it has a substring 110 the machine outputs A, if it has a substring 101 machine outputs B, Otherwise C.

Solⁿ:- Moore Machine

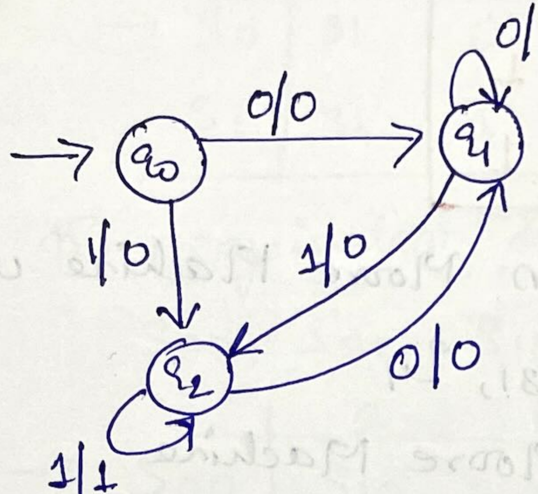


Mealy Machine



Q:- Construct a Mealy machine that accepts strings ending in '00' and '11'. Convert it to Moore machine.

Solⁿ:- Mealy Machine



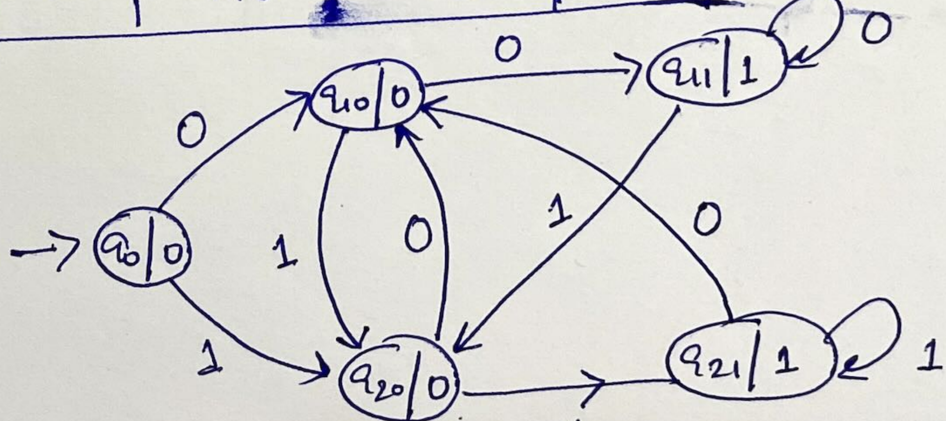
	i/p = 0		i/p = 1	
	State	o/p	State	o/p
→ q_0	q_1	0	q_2	0
q_1	q_1	1	q_2	0
q_2	q_1	0	q_2	1

Equivalent Moore Machine

For Moore Machine we will have states,

$q_0, q_{10}, q_{11}, q_{20}, q_{21}$.

	0	1	o/p
→ q_0	q_{10}	q_{20}	0
q_{10}	q_{11}	q_{20}	0
q_{11}	q_{11}	q_{20}	1
q_{20}	q_{10}	q_{21}	0
q_{21}	q_{10}	q_{21}	1



Equivalent Moore Machine

Q: Convert the following Mealy Machine to Moore Machine.

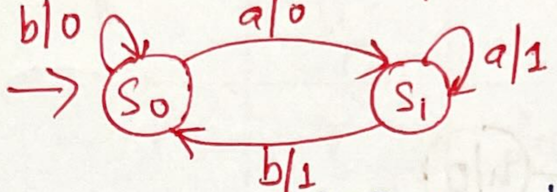
	a = 0		a = 1	
	State	O/P	State	O/P
→ q_1	q_1	1	q_2	0
q_2	q_4	1	q_4	1
q_3	q_2	1	q_3	1
q_4	q_3	0	q_1	1

Solⁿ: The equivalent states in Moore Machine will be $q_1, q_{20}, q_{21}, q_{30}, q_{31}, q_4$

Transition table for Moore Machine

	0	1	O/P
→ q_1	q_1	q_{20}	1
q_{20}	q_4	q_4	0
q_{21}	q_4	q_4	1
q_{30}	q_{21}	q_{31}	0
q_{31}	q_{21}	q_{31}	1
q_4	q_{30}	q_1	1

Convert the Mealy m/c into Moore m/c.



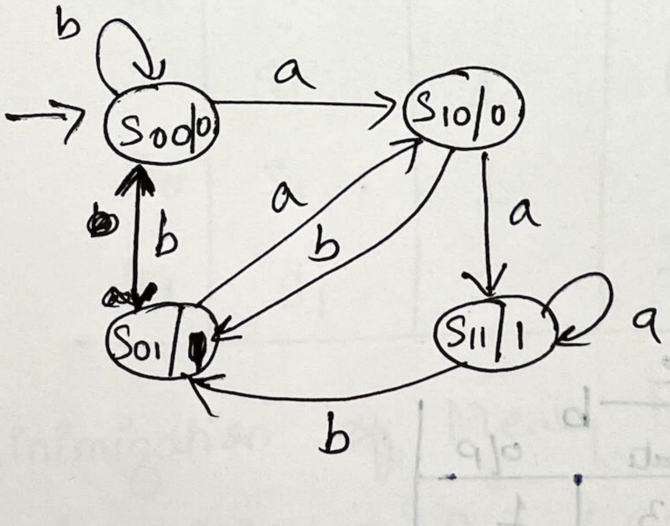
Sol^n:

	i/p = a		i/p = b	
	State	O/P	State	O/P
→ S_0	S_1	0	S_0	0
S_1	S_1	1	S_0	1

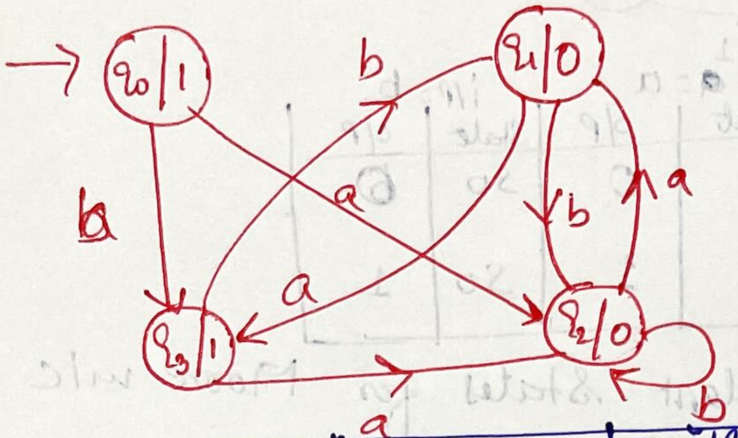
The equivalent States for Moore m/c

$S_{00}, S_{01}, S_{10}, S_{11}$

	a	b	O/P
→ S_{00}	S_{10}	S_{00}	0
S_{01}	S_{10}	S_{00}	1
S_{10}	S_{11}	S_{01}	0
S_{11}	S_{11}	S_{01}	1



Q: Convert the following Moore Machine into Mealy Machine.



Sol^n:

	i/p = a		i/p = b	
	Next State a	o/p	Next State b	o/p
→ q_0	q_2		q_3	
q_1				
q_2				
q_3				

	a	b	o/p
→ q_0	q_2	q_3	1
q_1	q_3	q_2	0
q_2	q_1	q_2	0
q_3	q_2	q_1	1

Mealy Machine Table

	a		b	
	State	o/p	State	o/p
→ q_0	q_2	0	q_3	1
q_1	q_3	1	q_2	0
q_2	q_1	0	q_2	0
q_3	q_2	0	q_1	0

Change the given Moore m/c to Mealy m/c.

	0	1	O/P
→ P_0	r	q_0	ϵ
P_1	r	q_0	1
q_0	P_1	S_0	0
q_1	P_1	S_0	1
r	q_1	P_1	0
S_0	S_1	r	0
S_1	S_1	r	1

Solⁿ:

	I/P = 0		I/P = 1	
	Next State	O/P	Next State	O/P
→ P_0	r	0	q_0	0
P_1	r	0	q_0	0
q_0	P_1	1	S_0	0
q_1	P_1	1	S_0	0
r	q_1	1	P_1	1
S_0	S_1	1	r	0
S_1	S_1	1	r	0

Minimization of Mealy Machine

$\underbrace{(P_0, P_1)}_{q_0}\ \underbrace{(q_0, q_1)}_{q_1}\ \underbrace{(S_0, S_1)}_{q_2}\ \underbrace{r}_{q_3}$

	i/p = 0		i/p = 1	
	Next State	o/p	Next State	o/p
→ q_0	q_3	0	q_1	0
q_1	q_0	1	q_2	0
q_2	q_2	1	q_3	0
q_3	q_1	1	q_0	1

Q. Convert the given Moore m/c to Mealy m/c.

	0	1	o/p
→ q_1	q_1	q_2	0
q_2	q_1	q_3	0
q_3	q_1	q_3	1

Soln:

Present State	i/p = 0		i/p = 1	
	Next State	o/p	Next State	o/p
→ q_1	q_1	0	q_2	0
q_2	q_1	0	q_3	1
q_3	q_1	0	q_3	1

	i/p = 0		i/p = 1	
	Next State	o/p	Next State	o/p
→ q_1	q_1	0	(q_2, q_3)	0
(q_2, q_3)	q_1	0	(q_2, q_3)	1

Convert the following Moore m/c to Mealy m/c.

	i/p a	i/p b	o/p
→ q_0	q_1	q_3	1
q_1	q_3	q_1	0
q_2	q_0	q_3	0
q_3	q_3	q_2	1

Solⁿ:

Present State	i/p = a		i/p = b	
	Next State	o/p	Next State	o/p
→ q_0	q_1	0	q_3	1
q_1	q_3	1	q_1	0
q_2	q_0	1	q_3	1
q_3	q_3	1	q_2	0

Present State	i/p = a		i/p = b	
	Next State	o/p	Next State	o/p
→ q_0	(q_1, q_3)		(q_1, q_3)	
(q_1, q_3)	(q_1, q_3)		(q_1, q_3)	
q_2				

Minimization is not possible bcoz transition property of (q_1, q_3) under b is not similar.

Equivalence of Mealy and Moore Machine

* From FA to Moore Machine

A finite automaton can be converted into a moore machine by introducing output alphabet $\Delta = \{0,1\}$, and defining the output function $\lambda(q)$ such that,

$$\lambda(q) = \begin{cases} 1 & \text{if } q \in F \\ 0 & \text{if } q \notin F \end{cases}$$

Q: Consider the FA given by table. Convert this FA into a Moore machine.

State	Next State	
	$a=0$	$a=1$
$\rightarrow q_0$	q_2	q_0
q_1	q_0	q_1
q_2	q_0	q_2

Solⁿ: Let $\Delta = \{1,0\}$ be the introduced alphabet.
$\lambda(q)$ is defined as $\lambda(q_0) = 1$, $\lambda(q_1) = \lambda(q_2) = 0$

State	Next State		Output
	$a=0$	$a=1$	
$\rightarrow q_0$	q_2	q_0	1
q_1	q_0	q_1	0
q_2	q_0	q_2	0

* From Moore Machine to Mealy Machine :-

We modify the acceptability of input string by a Moore Machine by neglecting the response of the Moore machine to input λ.

Let $M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ be a Moore Machine.

The equivalent Mealy Machine can be represented by, $M' = (Q, \Sigma, \Delta, \delta, \lambda', q_0)$.

The output function λ' can be obtained as,

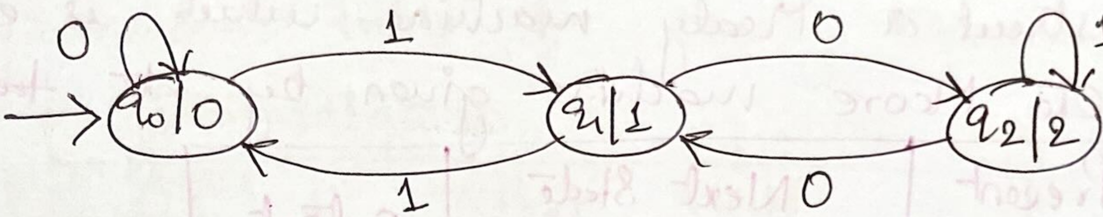
$$\boxed{\lambda'(q, a) = \lambda(\delta(q, a))}$$

for all states q and input symbol 'a' and the transition function is the same as that of the given Moore Machine.

Q:- The Moore Machine to determine residue mod 3 for binary number is given as,

Present State	Next State 0	Next State 1	Output
$\rightarrow q_0$	q_0	q_1	0
q_1	q_2	q_0	1
q_2	q_1	q_2	2

Convert it to equivalent Mealy Machine.



→ Transition dig. for the given problem.

The output function λ' can be obtained as,

$$\lambda'(q,a) = \lambda(\delta(q,a))$$

$$\lambda'(q_0,0) = \lambda(\delta(q_0,0)) = \lambda(q_0) = 0$$

$$\lambda'(q_0,1) = \lambda(\delta(q_0,1)) = \lambda(q_1) = 1$$

$$\lambda'(q_1,0) = \lambda(\delta(q_1,0)) = \lambda(q_2) = 2$$

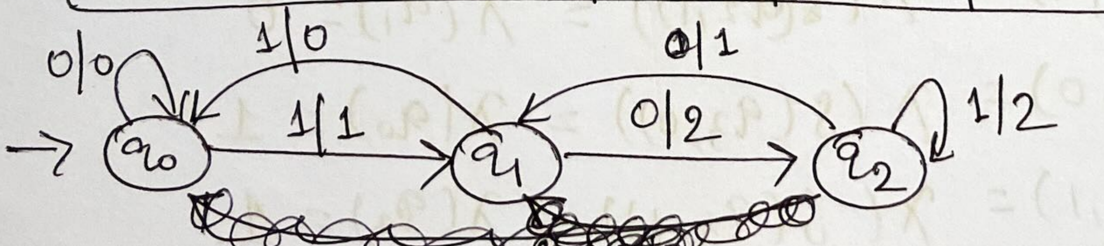
$$\lambda'(q_1,1) = \lambda(\delta(q_1,1)) = \lambda(q_0) = 0$$

$$\lambda'(q_2,0) = \lambda(\delta(q_2,0)) = \lambda(q_1) = 1$$

$$\lambda'(q_2,1) = \lambda(\delta(q_2,1)) = \lambda(q_2) = 2$$

Transition table for Mealy Machine can be drawn as:-

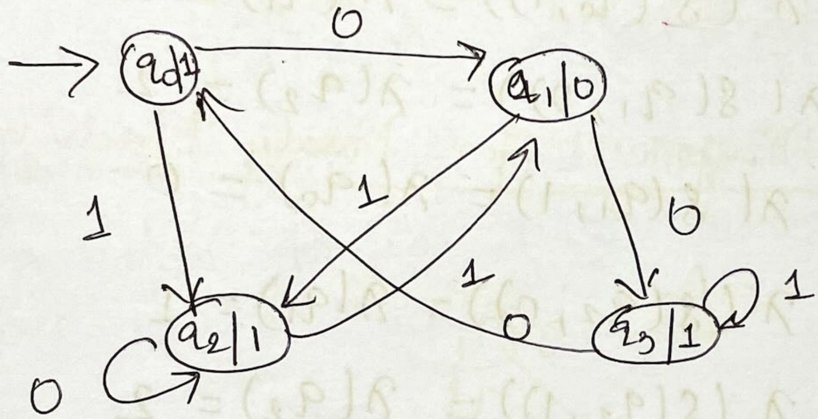
Present State	Next State 0		1	
	State	O/P	State	O/P
→ q_0	q_0	0	q_1	1
q_1	q_2	2	q_0	0
q_2	q_1	1	q_2	2



Q: Construct a Mealy machine which is equi[valent] to the Moore machine given by the table.

Present State	Next State a = 0	Next State a = 1	Output
$\rightarrow q_0$	q_1	q_2	1
q_1	q_3	q_2	0
q_2	q_2	q_1	1
q_3	q_0	q_3	1

Sol^n:



The output function λ' can be obtained as follows:-

$$\lambda'(q,a) = \lambda(\delta(q,a))$$

So, $\lambda'(q_0, 0) = \lambda(\delta(q_0,0)) = \lambda(q_1) = 0$

$\lambda'(q_0, 1) = \lambda(\delta(q_0,1)) = \lambda(q_2) = 1$

$\lambda'(q_1, 0) = \lambda(\delta(q_1,0)) = \lambda(q_3) = 1$

$\lambda'(q_1, 1) = \lambda(\delta(q_1,1)) = \lambda(q_2) = 1$

$\lambda'(q_2, 0) = \lambda(\delta(q_2,0)) = \lambda(q_2) = 1$

$\lambda'(q_2, 1) = \lambda(\delta(q_2,1)) = \lambda(q_1) = 0$

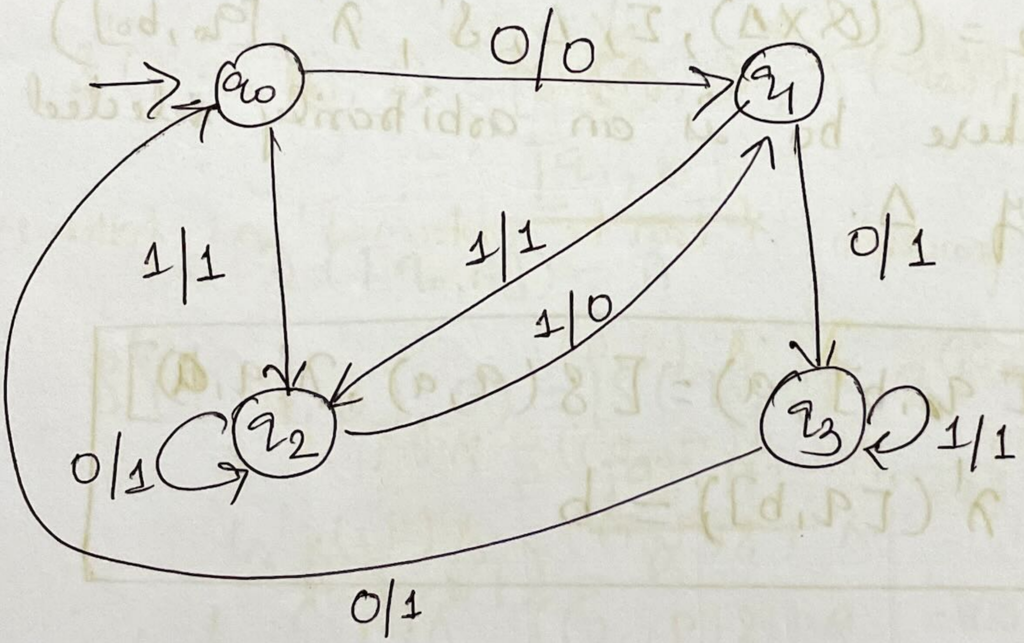
$\lambda'(q_3, 0) = \lambda(\delta(q_3,0)) = \lambda(q_0) = 1$

$\lambda'(q_3, 1) = \lambda(\delta(q_3,1)) = \lambda(q_3) = \underline{1}$

Transition table for Mealy Machine will be given as:-

Present State	Next State 0		Next State 1	
	State	O/P	State	O/P
$\rightarrow q_0$	q_1	0	q_2	1
q_1	q_3	1	q_2	1
q_2	q_2	1	q_1	0
q_3	q_0	1	q_3	1

<u>Transition Diagram for Mealy Machine</u>



From Mealy Machine to Moore Machine

To convert ~~mealy~~ moore machine to ~~moore~~ mealy machine state output symbols are distributed to input symbol paths. But while converting Mealy machine to moore machine we will create a separate state for every new output symbol and accordingly incoming and outgoing edges are distributed.

If given Mealy machine is

$M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$, then an equivalent Moore Machine will be,

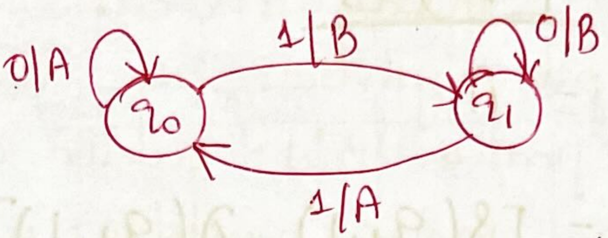
$$M_2 = ((Q \times \Delta), \Sigma, \Delta, \delta', \lambda', [q_0, b_0])$$

where b_0 is an arbitrarily selected member of Δ.

and

$$\boxed{\begin{aligned} \delta'([q, b], a) &= [\delta(q, a), \lambda(q, a)] \\ \lambda'([q, b]) &= b \end{aligned}}$$

Q: Convert the following Mealy machine into equivalent Moore Machine.



Sol^n: The states for Moore machine will be $Q \times \Delta$ i.e

$[q_0, A]$ $[q_0, B]$ $[q_1, A]$ $[q_1, B]$

Now, we will calculate δ' and λ' as follows:-

- $\delta'([q_0, A], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$

  $= \underline{[q_0, A]}$

  $\lambda'([q_0, A]) = A$

  $\delta'([q_0, A], 1) = [\delta(q_0, 1), \lambda(q_0, 1)]$

  $= \underline{[q_1, B]}$

  $\lambda'([q_0, A]) = A$

- $\delta'([q_0, B], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$

  $= \underline{[q_0, A]}$

  $\lambda'([q_0, B]) = B$

  $\delta'([q_0, B], 1) = [\delta(q_0, 1), \lambda(q_0, 1)]$

  $= \underline{[q_1, B]}$

  $\lambda'([q_0, B]) = B$

$$\delta'([q_1, A], 0) = [\delta(q_1, 0), \lambda(q_1, 0)]$$
$$= \underline{[q_1, B]}$$
$$\lambda'([q_1, A]) = A$$

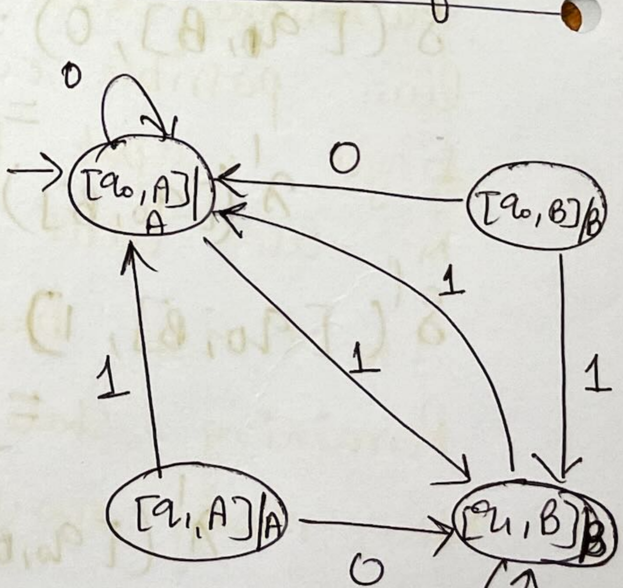
$$\delta'([q_1, A], 1) = [\delta(q_1, 1), \lambda(q_1, 1)]$$
$$= \underline{[q_0, A]}$$
$$\lambda'([q_1, A]) = A$$

$$\delta'([q_1, B], 0) = [\delta(q_1, 0), \lambda(q_1, 0)]$$
$$= \underline{[q_1, B]}$$
$$\lambda'([q_1, B]) = B$$

$$\delta'([q_1, B], 1) = [\delta(q_1, 1), \lambda(q_1, 1)]$$
$$= [q_0, A]$$
$$\lambda'([q_1, B]) = B$$

Transition table for Moore machine is as follows:-

Present State	Next State 0	Next State 1	O/P
$\rightarrow [q_0, A]$	$[q_0, A]$	$[q_1, B]$	A
$[q_0, B]$	$[q_0, A]$	$[q_1, B]$	B
$[q_1, A]$	$[q_1, B]$	$[q_0, A]$	A
$[q_1, B]$	$[q_1, B]$	$[q_0, A]$	B



Transition Graph for Mealy Machine

5. Consider the Mealy Machine given by the following table

Present State	Next State			
	0		1	
	State	o/p	State	o/p
→ q_1	q_3	1	q_2	0
q_2	q_1	1	q_4	1
q_3	q_2	0	q_1	1
q_4	q_4	1	q_3	0

Construct the Moore m/c equivalent to the following mealy m/c.

Solⁿ: Here, we will see the columns of Next-State field where for the respective state we have different outputs.

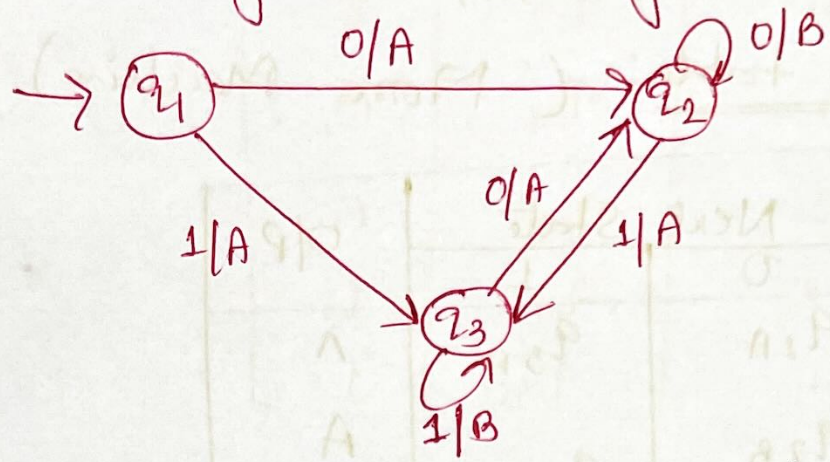
$q_3 - 1$, $q_3 - 0$
$q_2 - 0$
$q_1 - 1$
$q_4 - 1$

We have seen that for q_3 state we have two possible outputs 0 & 1, but there is single output for q_2, q_1, & q_4.
So, we will split state q_3 into q_{30} & q_{31} based upon output symbol.
Remaining states will remain same.
Now, mealy machine has 5 states q_1, q_2, q_{30}, q_{31} & q_4.
Next we will prepare Moore m/c transition table.

Present State	Next State 0	Next State 1	O/P
$\rightarrow q_1$	q_{31}	q_2	1
q_2	q_1	q_4	0
q_{30}	q_2	q_1	0
q_{31}	q_2	q_1	1
q_4	q_4	q_{30}	1

~~Now~~,
Here, we have added a new output column next to 'next state'. The value in this 'output' column are corresponding to 'present state' column.

Q: Construct a moore machine equivalent to the Mealy machine given-



Sol^n: Transition table for Mealy Machine

Present State	Next State			
	i/P=0	o/P	i/P=1	o/P
$\rightarrow q_1$	q_2	A	q_3	A
q_2	q_2	B	q_3	A
q_3	q_2	A	q_3	B

Now, we will check for output's of respective States from 'Next State'.

q_2 - A q_2 - B

q_3 - A q_3 - B

Here for both q_2 & q_3 we have 2 possible output's A & B.

So, new states are added

q_{2A} q_{2B} q_{3A} & q_{3B}

Now, we have total 5 states,

q_1, q_{2A}, q_{2B}, q_{3A}, q_{3B}.

Transition table:- (Moore Machine)

Present State	Next State 0	Next State 1	O/P
$\rightarrow q_1$	q_{2A}	q_{3A}	$\wedge$
q_{2A}	q_{2B}	q_{3A}	A
q_{2B}	q_{2B}	q_{3A}	B
q_{3A}	q_{2A}	q_{3B}	A
q_{3B}	q_{2A}	q_{3B}	B

The transition diagram for Moore m/c is above.

Equivalence of Mealy and Moore Machines

From Moore Machine to Mealy Machine

We modify the acceptability of input string by a Moore machine by neglecting the response of the moore machine to input λ.

Let $M = (Q, \Sigma, \Delta, \delta, q_0, \lambda)$ – be the Moore M. equivalent mealy machine will be,

$$M' = (Q, \Sigma, \Delta, \delta, \lambda', q_0)$$

The output λ' can be obtained as,

$$\lambda'(q, a) = \lambda(\delta(q, a))$$

for all states q and input symbol 'a'.

From Mealy Machine to Moore Machine

To convert mealy m/c to moore m/c state output symbols are distributed to input symbol paths. But while converting mealy m/c to moore m/c we will create a separate state for every new output symbol and according to incoming and outgoing edges are distributed.

If given mealy m/c $M_1 = (Q, \Sigma, \Delta, \delta^*, \lambda^*, q_0)$ then equivalent moore m/c $M_2 = ([Q \times \Delta], \Sigma, \Delta, \delta', \lambda', [q_0, b_0])$

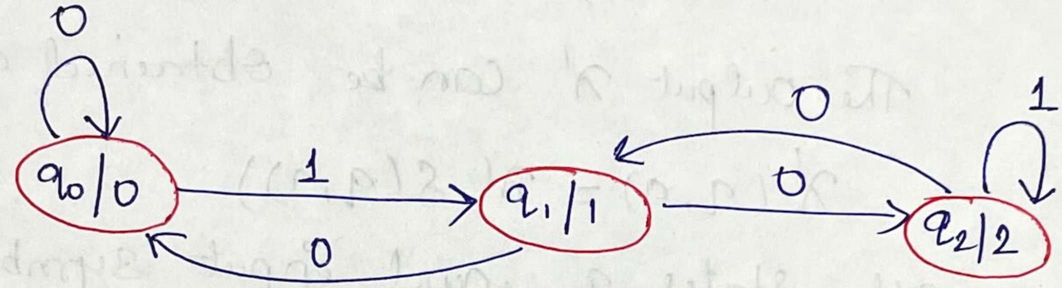
$$\delta'([q, \overset{\text{o/p set}}{b}], \overset{\text{i/p set}}{a}) = [\delta(q, a), \lambda(q, a)]$$

$$\lambda'([q, b]) = b$$

Q: The moore m/c ~~given~~ given is following, convert it to ~~following~~ equivalent mealy m/c.

Present State	Next State		Output
	0	1	
$\rightarrow q_0$	q_0	q_1	0
q_1	q_2	q_0	1
q_2	q_1	q_2	2

Solⁿ:



The output function λ' can be obtained by the following rule:-

$$\lambda'(q, a) = \lambda(\delta(q, a))$$

Now, we will obtain transition for every input symbol,

$$\lambda'(q_0, 0) = \lambda(\delta(q_0, 0)) = \lambda(q_0) = 0$$

$$\lambda'(q_0, 1) = \lambda(\delta(q_0, 1)) = \lambda(q_1) = 1$$

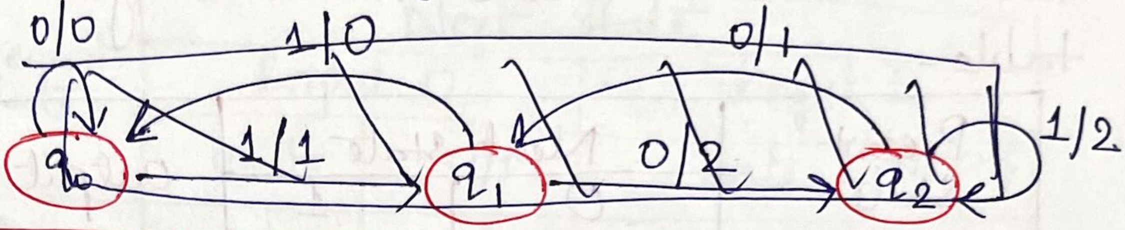
$$\lambda'(q_1, 0) = \lambda(\delta(q_1, 0)) = \lambda(q_2) = 2$$

$$\lambda'(q_1, 1) = \lambda(\delta(q_1, 1)) = \lambda(q_0) = 0$$

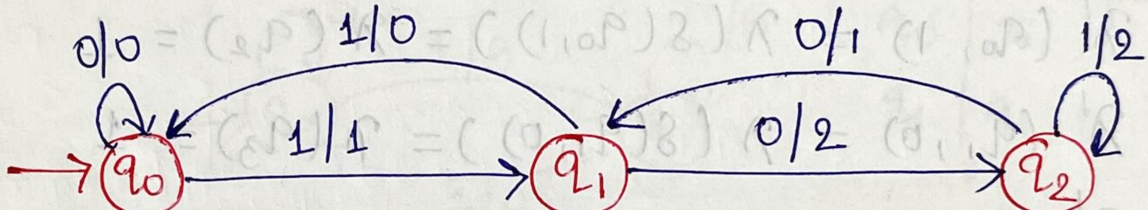
$$\lambda'(q_2, 0) = \lambda(\delta(q_2, 0)) = \lambda(q_1) = 1$$

$$\lambda'(q_2, 1) = \lambda(\delta(q_2, 1)) = \lambda(q_2) = 2$$

Equivalent mealy machine will be—



Present State	Next State			
	Input 0		Input 1	
	State	O/P	State	O/P
$\rightarrow q_0$	q_0	0	q_1	1
q_1	q_2	2	q_0	0
q_2	q_1	1	q_2	2



Present State	0	1	O/P
q_0	q_0	q_1	0
q_1	q_2	q_0	1
q_2	q_1	q_2	2

q_{01}

q_{00}

Q: Construct a mealy machine which is equivalent to the moore machine given by the table -

Present State	Next State 0	Next State 1	Output
→ q_0	q_1	q_2	1
q_1	q_3	q_2	0
q_2	q_2	q_1	1
q_3	q_0	q_3	1

Solⁿ: $\lambda'(q,a) = \lambda(\delta(q,a))$

$\lambda'(q_0,0) = \lambda(\delta(q_0,0)) = \lambda(q_1) = 0$

$\lambda'(q_0,1) = \lambda(\delta(q_0,1)) = \lambda(q_2) = 1$

$\lambda'(q_1,0) = \lambda(\delta(q_1,0)) = \lambda(q_3) = 1$

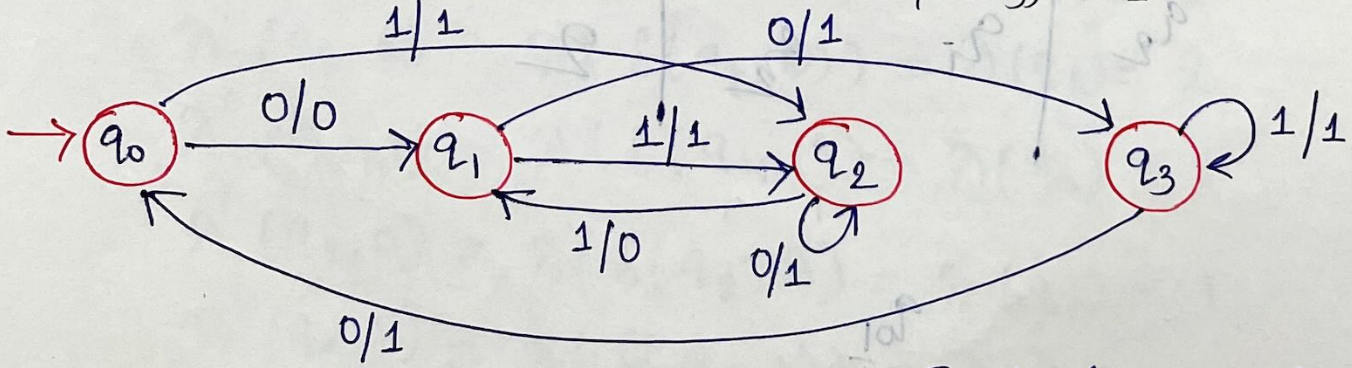
$\lambda'(q_1,1) = \lambda(\delta(q_1,1)) = \lambda(q_2) = 1$

$\lambda'(q_2,0) = \lambda(\delta(q_2,0)) = \lambda(q_2) = 1$

$\lambda'(q_2,1) = \lambda(\delta(q_2,1)) = \lambda(q_1) = 0$

$\lambda'(q_3,0) = \lambda(\delta(q_3,0)) = \lambda(q_0) = 1$

$\lambda'(q_3,1) = \lambda(\delta(q_3,1)) = \lambda(q_3) = 1$

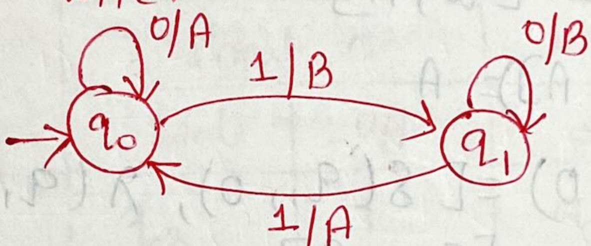


Equivalent Mealy m/c

Transition table for equivalent mealy m/c is:-

Present State	Next State			
	Input 0		Input 1	
	State	O/P	State	O/P
$\rightarrow q_0$	q_1	0	q_2	1
q_1	q_3	1	q_2	1
q_2	q_2	1	q_1	0
q_3	q_0	1	q_3	1

Q: Convert the following Mealy m/c into equivalent moore m/c.



	0		1	
q_0	q_0	A	q_1	B
q_1	q_1	B	q_0	A

↓

q_0	q_0	q_1	A
q_1	q_1	q_0	B

Sol^n:- The states for moore m/c will be $Q \times \Delta$ i.e

$[q_0, A]$, $[q_0, B]$, $[q_1, A]$, $[q_1, B]$

Now we will calculate δ' & λ' as follows:-

$$\delta'([q_0, A], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$$
$$= [q_0, A]$$

$$\lambda'([q_0, A]) = A$$

$$\delta'([q_0, A], 1) = [\delta(q_0, 1), \lambda(q_0, 1)]$$
$$= [q_1, B]$$

$$\lambda'([q_0, A]) = A$$

$$\delta'([q_0, B], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$$
$$= [q_0, A]$$
$$\lambda'([q_0, B]) = B$$

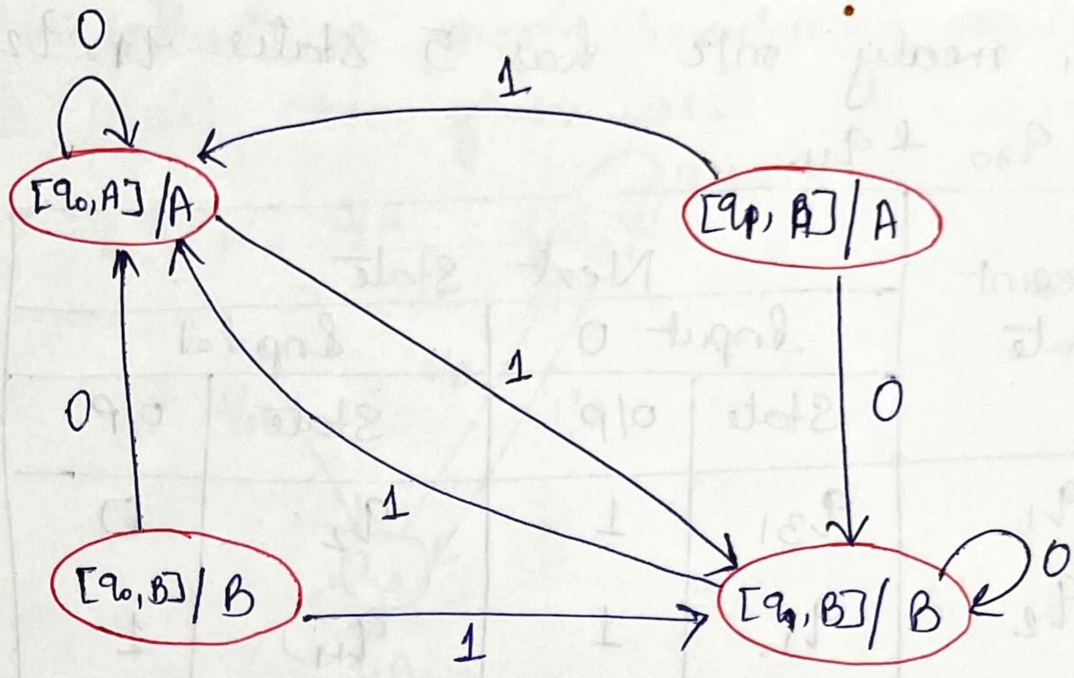
$$\delta'([q_0, B], 1) = [\delta(q_0, 1), \lambda(q_0, 1)]$$
$$= [q_1, B]$$
$$\lambda'([q_0, B]) = B$$

$$\delta'([q_1, A], 0) = [\delta(q_1, 0), \lambda(q_1, 0)]$$
$$= [q_1, B]$$
$$\lambda'([q_1, A]) = A$$

$$\delta'([q_1, A], 1) = [\delta(q_1, 1), \lambda(q_1, 1)]$$
$$= [q_0, A]$$
$$\lambda'([q_1, A]) = A$$

$$\delta'([q_1, B], 0) = [\delta(q_1, 0), \lambda(q_1, 0)]$$
$$= [q_1, B]$$
$$\lambda'([q_1, B]) = B$$

$$\delta'([q_1, B], 1) = [\delta(q_1, 1), \lambda(q_1, 1)]$$
$$= [q_0, A]$$
$$\lambda'([q_1, B]) = B$$



Q:- Consider the mealy m/c, & convert it to moore m/c.

Present State	Next State			
	Input 0		Input 1	
	State	O/P	State	O/P
→ q_1	q_3	1	q_2	0
q_2	q_1	1	q_4	1
q_3	q_2	0	q_1	1
q_4	q_4	1	q_3	0

Solⁿ: In this we will look into next column of those which are associated with more than one O/P. There is only one state q_3 associated with two O/P's 0 & 1.
So, we will introduce two new states here q_{30} & q_{31}.

Now, mealy m/c has 5 states q_1, q_2, q_{31}, q_{30} & q_4.

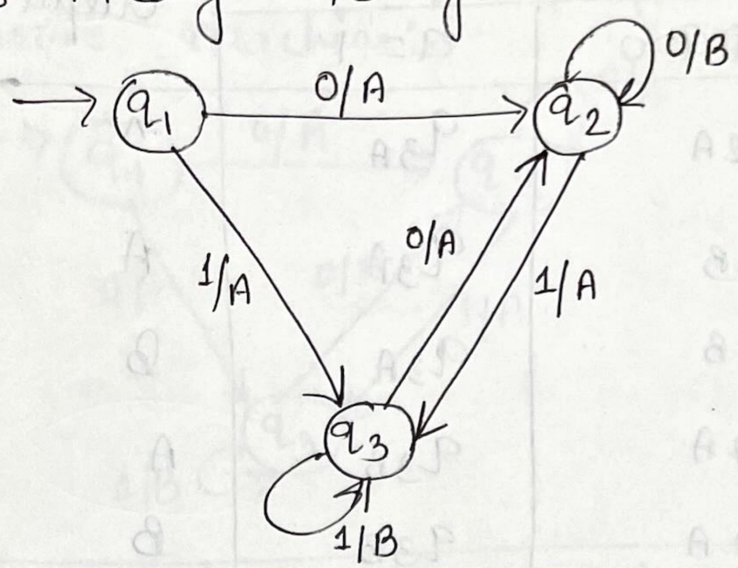
Present State	Next State			
	Input 0		Input=1	
	State	O/P	State	O/P
$\rightarrow q_1$	q_{31}	1	q_2	0
q_2	q_1	1	q_4	1
q_{31}	q_2	0	q_1	1
q_{30}	q_2	0	q_1	1
q_4	q_4	1	q_{30}	0

To transform this table into a moore m/c, we will introduce a new O/P column next to state column eliminating the previous mealy m/c O/P columns and the values in this O/P column are corresponding to 'present state' column.

Equivalent Moore M/C

Present state	Next State		Output
	a=0	a=1	
$\rightarrow q_1$	q_{31}	q_2	1
q_2	q_1	q_4	0
q_{30}	q_2	q_1	0
q_{31}	q_2	q_1	1
q_4	q_4	q_{30}	1

Q:- Construct a moore machine equivalent to Mealy m/c given as:-



Solⁿ:-

Present State	Next State			
	a = 0		a = 1	
	State	O/P	State	O/P
$\rightarrow q_1$	q_2	A	q_3	A
q_2	q_2	B	q_3	A
q_3	q_2	A	q_3	B

Here, state q_2 is having two O/P's A & B.

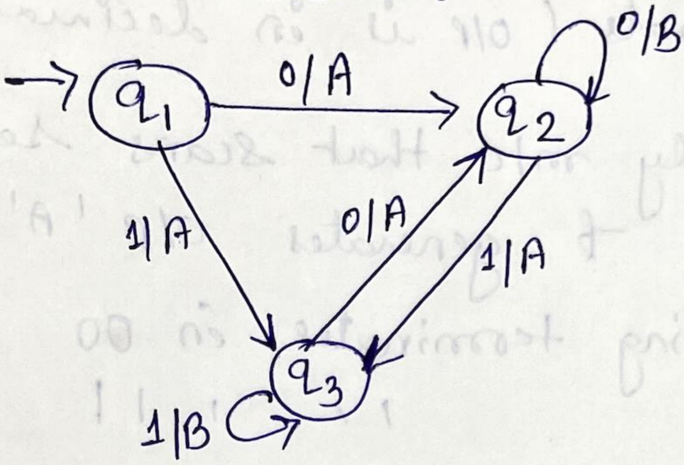
Present state	Next State			
	a = 0		a = 1	
	State	O/P	State	O/P
$\rightarrow q_1$	q_{2A}	A	q_{3A}	A
q_{2A}	q_{2B}	B	q_{3A}	A
q_{2B}	q_{2B}	B	q_{3A}	A
q_{3A}	q_{2A}	A	q_{3B}	B
q_{3B}	q_{2A}	A	q_{3B}	B

Present State	Next State		Output
	$a=0$	$a=1$	
$\rightarrow q_1$	q_{2A}	q_{3A}	^
q_{2A}	q_{2B}	q_{3A}	A
q_{2B}	q_{2B}	q_{3A}	B
q_{3A}	q_{2A}	q_{3B}	A
q_{3B}	q_{2A}	q_{3B}	B

Q: Convert the following mealy machine to moore machine:-

Repeat

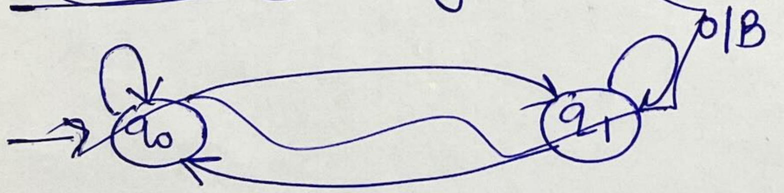
i)



ii)

Present State	Next State			
	Input = 0		Input = 1	
	State	O/P	State	O/P
$\rightarrow q_1$	q_3	1	q_2	0
q_2	q_1	1	q_4	1
q_3	q_2	0	q_1	1
q_4	q_4	1	q_3	0

Q:- ~~Moore~~ Mealy to ~~Mealy~~ Moore machine



Q: ~~Mealy~~ Moore to Mealy

$\rightarrow q_0$	q_1	q_2	1
q_1	q_3	q_2	0
q_2	q_2	q_1	1
q_3	q_0	q_3	1

Q:- Design a mealy m/c which has input as binary no. & output is the sum of the three consecutive symbols (O/P is in decimal).

Q:- Design a mealy m/c that scans sequence of inputs 0 & 1 & generates O/P 'A' if the

'A' if string terminates in 00

'B' " " " 11

C otherwise.

Q:- Design a