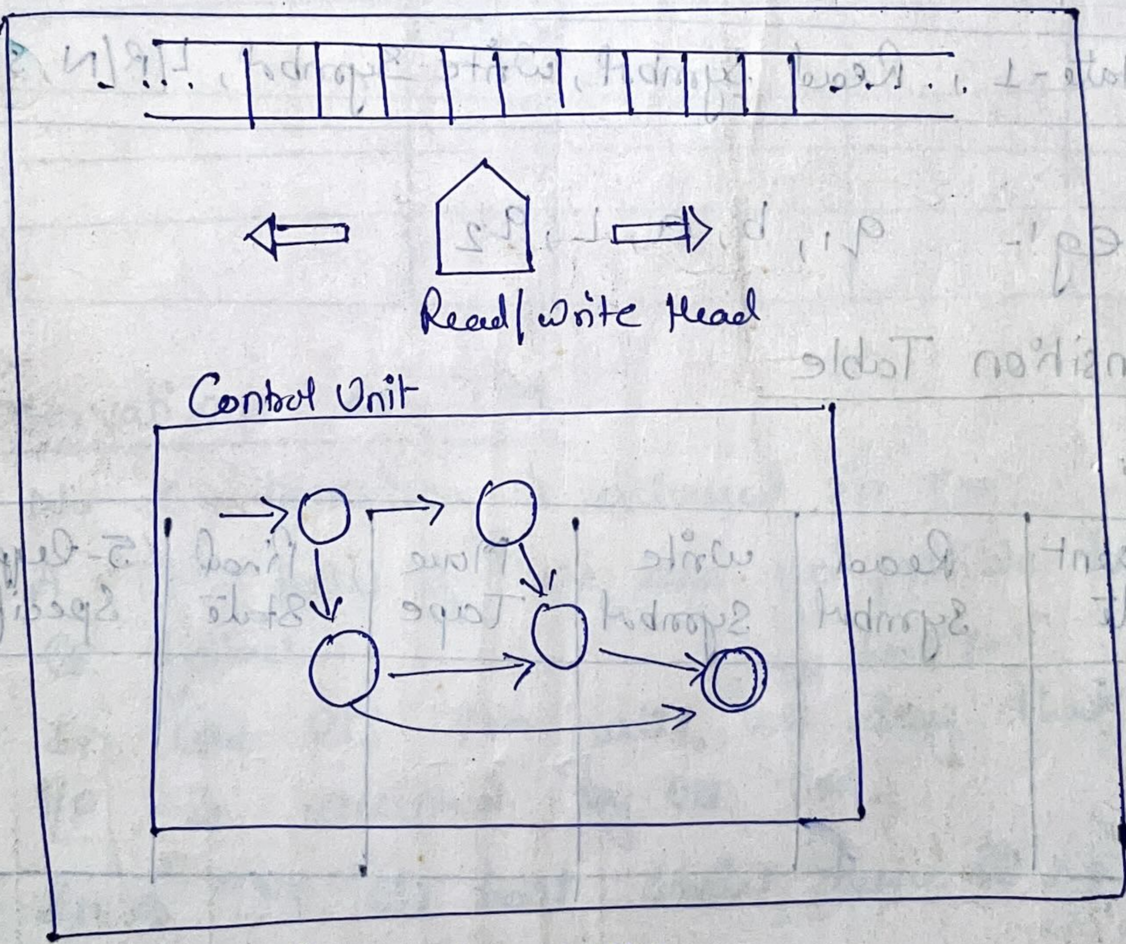
TURING MACHINE

A Turing Machine is a kind of State machine. At any time, the machine is in any One of the finite no. of states. Instructions for turing machine include the specification of conditions, under which the machine will make transitions from one state to other.

Components of Turing Machine

- Tape
- Head
- Control Unit



Components of TM

Description of TM :-

① The transition diagram :-

Form 1: Read Symbol (a) → Write Symbol b, move left (L) or NoMove (N)

$$(p) \xrightarrow{a \to b, L} (q)$$

Form 2: Read Symbol (a) / Write Symbol (b), move left (L) or move Right (R) or NoMove (N)

$$(p) \xrightarrow{a/b,\ R} (q)$$

② 5-tuple Specification :-

State-1, Read Symbol, Write Symbol, L/R/N, State 2

eg :- q_1, b, a, L, q_2

③ Transition Table

Form 1:

Current State	Read Symbol	Write Symbol	Move Tape	Final State	5-tuples Specification

Current State	q_1			...	q_n		
Tape Symbol	Write Symbol	Move Tape	Next State		Write Symbol	Move Tape	Next State

Form 3 :-

States \ Tape Symbol	a_1	a_2	a_3	...	a_n
	<action>				
				...	

④ Observations on TM

ⓐ No ∈- transition is allowed in TM.

ⓑ A TM halts if there are no possible transitions to follow.

ⓒ In case the TM halts in the halt state, we say that the i/p is accepted by the TM.

ⓓ In a TM, the halt states have no outgoing transitions.

ⓔ Infinite loop in a TM

Because of infinite loop -

i) the final state cannot be reached.

ii) the m/c never halts.

iii) the i/p is not accepted.

Elements of TM :-

Formally, a turing Machine M is represented as follows :- 7-tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, h)$$

Q - finite set of states

Σ - finite set of i/p

Γ - set of tape characters

δ - transition function - $(Q \times \Gamma)$ to $(Q \times \Gamma \times \{L, R, N\})$

q_0 - initial state

B - blank character

h - halt state $h \subseteq Q$

Transitions of TM

The transition of a m/c is represented as,

$$\delta(q_i, a_k) = (q_j, a_l, x)$$

$q_i \in Q$

$a_k, a_l \in \Gamma$

x is L, R or N

stantaneous Description of a TM

The complete state of a TM, at any point during a computation, may be described as:-

ⓐ the name of the state that in which the machine is.

ⓑ the symbols on the tape and

ⓒ the cell that is currently being scanned.

A description of these three data is called instantaneous description (ID) or configuration of a TM. A simple way to represent such a description is shown below:-

B	a_1	a_2	b	a_3	a_4	B

↑

q_1

Current State

Moves of TM

TM can

- Move to the left
- Move to the right
- No Move.

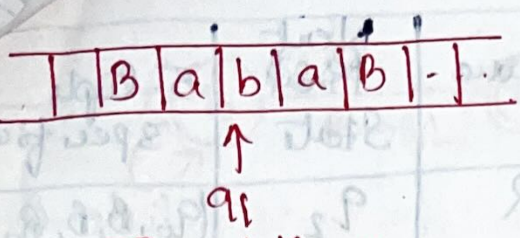
String Classes in TM

ⓐ Accept

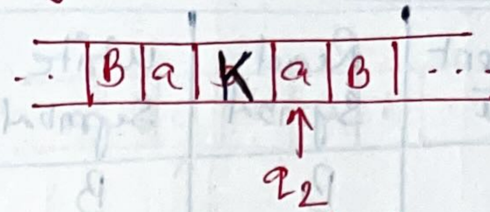
ⓑ Loop

ⓒ Reject

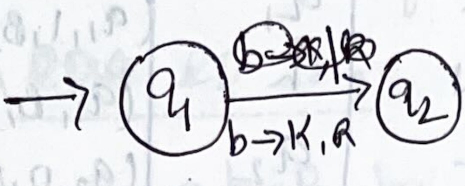
Take the transition dig.

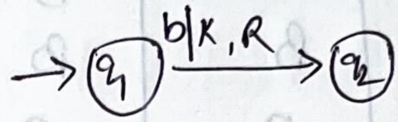


Tape at time 1

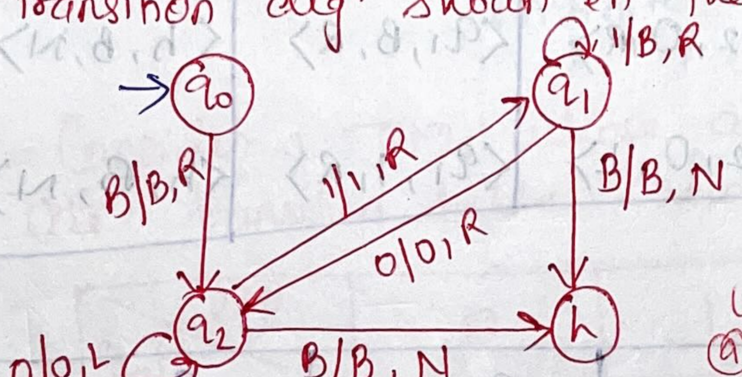


Tape at time 2





Q: Consider a TM, whose task is described in the transition dig. shown in the fig.



Write down Transition functions also.

Write ID for the string q_0B1010B

Transition functions :-

$\delta(q_0, B) = (q_2, B, R)$

$\delta(q_1, 0) = (q_2, 0, R)$

$\delta(q_1, 1) = (q_1, B, R)$

$\delta(q_1, B) = (h, B, N)$

$\delta(q_2, 0) = (q_2, 0, L)$

$\delta(q_2, 1) = (q_1, 1, R)$

$\delta(q_2, B) = (h, B, N)$

The transition table is as follows :-

(a) Table Form 1 :-

Current State	Read Symbol	Write Symbol	Move	Next State	5-tuple specification
q_0	B	B	R	q_2	(q_0, B, B, R, q_2)
q_1	0	0	R	q_2	$(q_1, 0, 0, R, q_2)$
q_1	1	B	R	q_1	$(q_1, 1, B, R, q_1)$
q_1	B	B	N	h	(q_1, B, B, N, h)
q_2	0	0	L	q_2	$(q_2, 0, 0, L, q_2)$
q_2	1	1	R	q_1	$(q_2, 1, 1, R, q_1)$

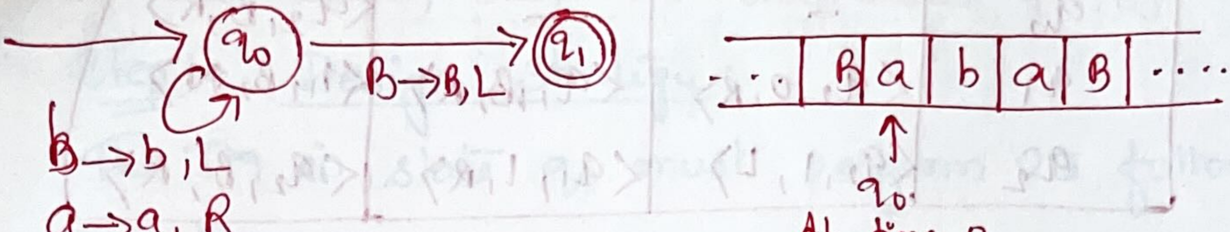
(b) Table Form 2 :-

States \ Tape Symbol	0	1	B
q_0	–	–	$\langle q_2, B, R \rangle$
q_1	$\langle q_2, 0, R \rangle$	$\langle q_1, B, R \rangle$	$\langle h, B, N \rangle$
q_2	$\langle q_2, 0, L \rangle$	$\langle q_1, 1, R \rangle$	$\langle h, B, N \rangle$

(c) Table Form 3 :-

Current States	q_0			q_1			q_2		
Tape Symbol	Write	Move	Next State	Write	Move	Next State	Write	Move	Next State
0	–	–	–	0	R	q_2	0	L	q_2
1	–	–	–	B	R	q_1	1	R	q_1
B	B	R	q_2	B	N	h	B	N	h

...sider a TM, whose action is described in fig.



At time 0

find further transitions

TM ~~PDA~~ ID is :-

$q_0 abaB \vdash a q_0 baB$

$\vdash q_0 abaB$

$\vdash a q_0 baB$

$\vdash q_0 abaB$

$\vdots$

Infinite loop

Q:- Consider a TM, whose action is described in the transition table shown in table ~~15.7~~

States \ I/P	0	1	B
q_0	—	—	q_2, B, R
q_1	$(q_2, 0, R)$	(q_1, B, R)	(h, B, N)
q_2	$(q_2, 0, L)$	$(q_1, 1, R)$	(h, B, N)

TM ~~PDA~~ ID for the string $w = 1010$

$q_0 1010B \vdash B q_2 1010B \vdash B q_1 010B$

$\vdash B0 q_2 10B$ ~~$\vdash B0 q_2 10B$~~

$\vdash B101 q_1 0B \vdash B1010 q_2 B \vdash B1010Bh$

Q1.-

States \ I/P	0	1	B
q_0	–	–	$\langle q_2, B, R \rangle$
q_1	$\langle q_2, 0, R \rangle$	$\langle q_1, B, R \rangle$	$\langle h, B, N \rangle$
q_2	$\langle q_2, 1, L \rangle$	$\langle q_1, 1, R \rangle$	$\langle q_2, B, R \rangle$

$w = 0101$

Ans:- $q_0 B1010B \vdash^{*} B1B01Bh$ — (Answer)

$q_0 B1010B \vdash B q_2 1010B$

$\vdash B1 q_1 010B$

$\vdash B10 q_2 10B$

$\vdash B101 q_1 0B$

$\vdash B1010 q_2 B$

$\vdash B1010$

$q_0 B0101B \vdash B q_2 0101B$

$\vdash q_2 B1101B$

$\vdash B q_2 1101B$

$\vdash B1 q_1 101B$

$\vdash B1B q_1 01B$

$\vdash B1B0 q_2 1B$

$\vdash B1B01 q_1 B$

$\vdash B1B01Bh$

sign a TM that erases all non-blank symbols on the tape, over the alphabet $\{a, b\}$.

Soln: **Step 1: Design Strategy:-**

The TM in state q_0 must perform the following operations:-

(a) On input symbol a, replace a by B, move the tape head towards right and stay at q_0.

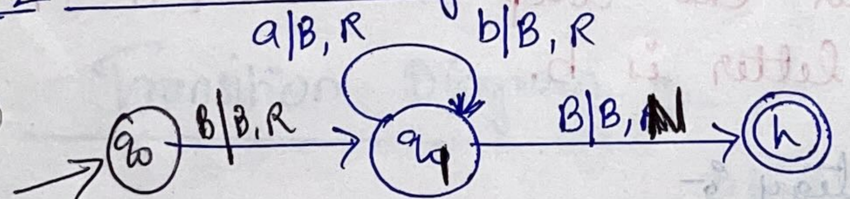
(b) On input symbol b, replace a by B, move the tape head towards right and stay at q_0.

(c) On input symbol B, replace B by B, change the state to h, and do not move tape head.

Thus, the TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, h)$

where δ is given by,

Step 2: Transition Diagram:-



Step 3: Transition Table:-

Tape Symbol / States	a	b	B
q_0	$\langle q_0, B, R\rangle$	$\langle q_0, B, R\rangle$	$\langle q_0, B, R\rangle$
h	–	–	Accept

Step 4:- TM action for the string w= ...

B	a	b	a	b	B

↑ q_0

ID is : ~~Bq_0babB~~

$q_0 ababBB \vdash Bq_0babB$

$\vdash BBq_0abB$

$\vdash BBBq_0bB$

$\vdash BBBBq_0B$

$\vdash BBBBBh$

Since, the final state h is reached, the string abab is accepted.

Q:- Design a TM that accepts the language of all strings, over the alphabet $\Sigma = \{a, b\}$, whose second letter is b.

Soln:- ~~Step~~ Design strategy :-

Step 1:- ⓐ On input a, change to state q_1, replace a by a and move the tape head towards right.

ⓑ On input b, change to state q_1, replace b by b and move the tape head towards right.

:- In state q_1 on input symbol b, replace b by b and move the tape head towards right and move to state q_2.

Step 3 :- In state q_2

(a) On input symbol a, replace a by a and move the tape head towards right. & remain in q_2.

(b) On input symbol b, replace b by b and move the tape head towards right. & remain in q_2.

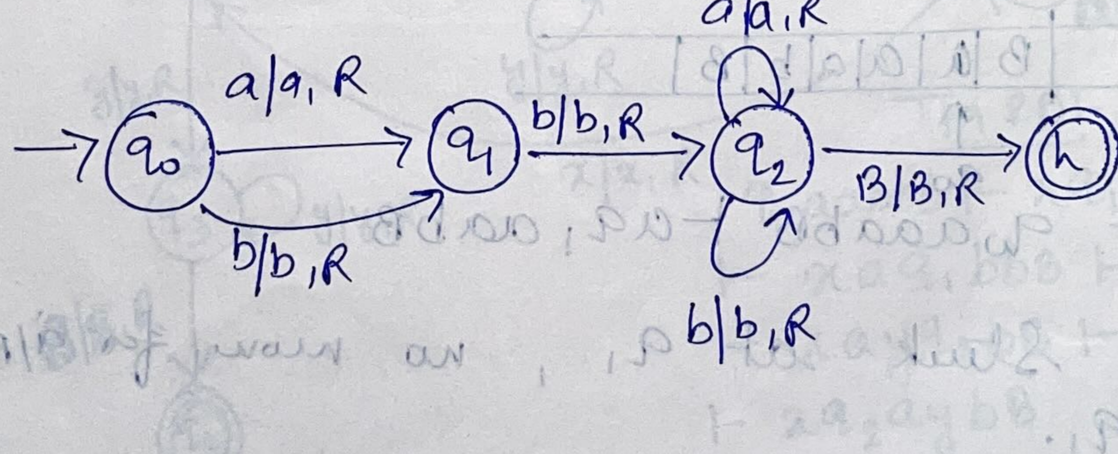
(c) On input symbol B, change to state h, and do not move the tape head.

Thus, the TM is

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, h)$$

$$M = (\{q_0, q_1, q_2, h\}, \Sigma = \{a, b\}, \{a, b, B\}, \delta, q_0, B, h\}$$

Transition Diagram :-



Transition Table :-

States \ Tape symbol	a	b	B
q_0	$\langle q_1, a, R\rangle$	$\langle q_1, b, R\rangle$	–
q_1	–	$\langle q_2, b, R\rangle$	
q_2	$\langle q_2, a, R\rangle$	$\langle q_2, b, R\rangle$	$\langle h, B, R\rangle$
h	–	–	Accept

* TM action for the string $w = aba$

B	a	b	a	B

$\uparrow q_0$ (under the first a)

ID is: $q_0abaB \vdash aq_1baB$

$\vdash abq_2aB$

$\vdash abaq_2B$

$\vdash abaBh$

Since the final state h is reached, the string aba is accepted.

* TM action for the string $w = aaab$

B	a	a	a	b	B

$\uparrow q_0$ (under the first a)

ID is :- $q_0aaabB \vdash aq_1aabB$

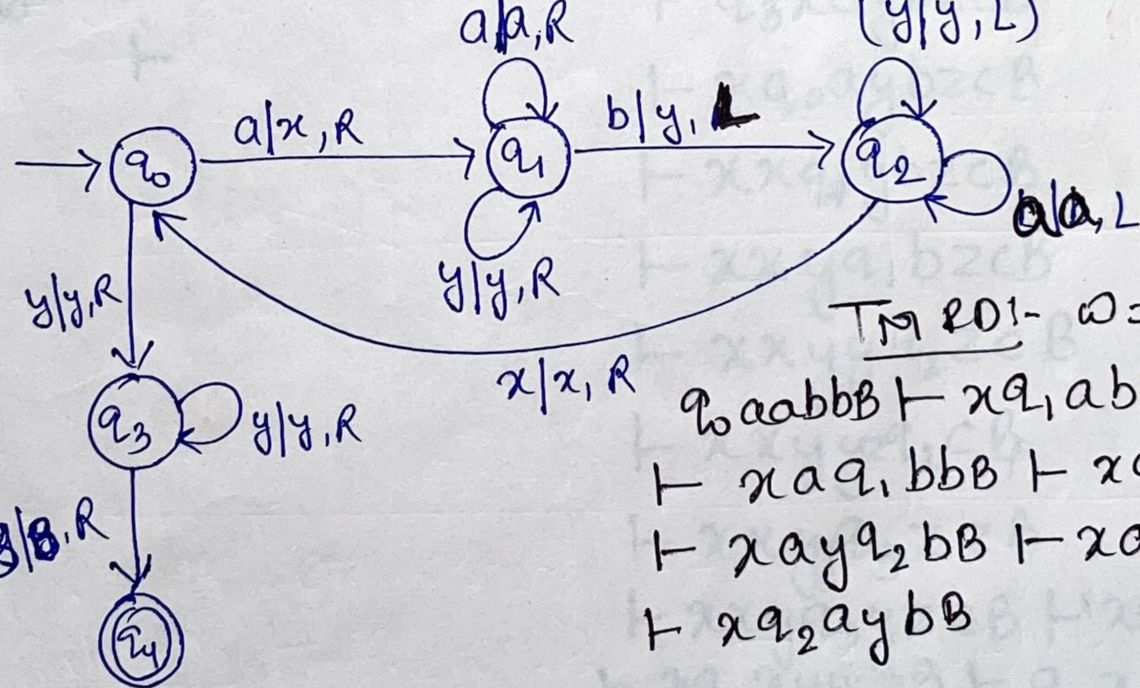
Stuck at q_1, no move for i/p a at q_1.

So, string is not accepted.

Design a TM which accepts all strings of the form $a^n b^n$ for $n \geq 1$.

Sol^n:- $a^n b^n$ means a will be followed by b. by same numbers.

Design Strategy:- Starting at the leftmost 'a' replacing it with x, the R/W head travels right to find the leftmost 'b', replacing it by y. after that R/W head will go left again and will replace leftmost 'a' by x, then moves to leftmost 'b' to replace it by y and so on. Travelling back and forth this way, we match each 'a' with corresponding 'b'. If after some, no a's + b's remain, then string must be in L and move to final state.



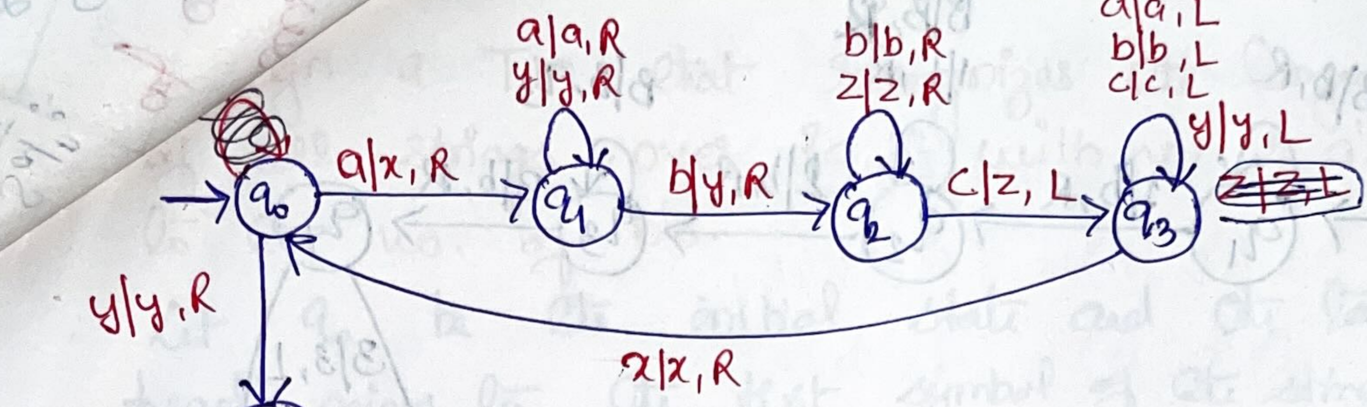
TM ID:- ω = aabb

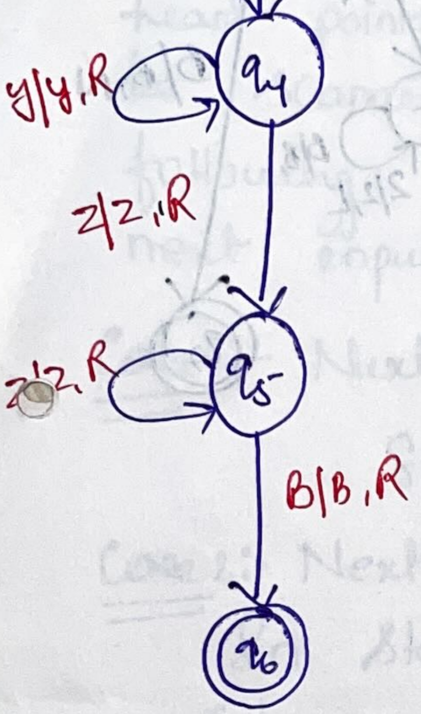
$q_0 aabbB \vdash x q_1 abbB$

$\vdash xa q_1 bbB \vdash x q_2 aybB$

$\vdash xay q_2 bB \vdash xa q_2 ybB$

$\vdash x q_2 aybB$





TM ID for string $w = aabbcc$

$q_0 aabbccB \vdash x q_1 abbccB$

$\vdash x a q_1 bbccB$

$\vdash x a y q_2 bccB$

$\vdash x a y b q_2 ccB$

$\vdash x a y q_3 b z c B$

$\vdash x a q_3 y b z c B$

$\vdash x q_3 a y b z c B$

$\vdash q_3 x a y b z c B$

$\vdash x q_0 a y b z c B$

$\vdash x x q_1 y b z c B$

$\vdash x x y q_1 b z c B$

$\vdash x x y y q_2 z c B$

$\vdash x x y y z q_2 c B$

$\vdash x x y y q_3 z z B$

$\vdash x x y q_3 y z z B \vdash x x q_3 y y z z B$

$\vdash x q_3 x y y z z B \vdash q_3 x x y y z z B$

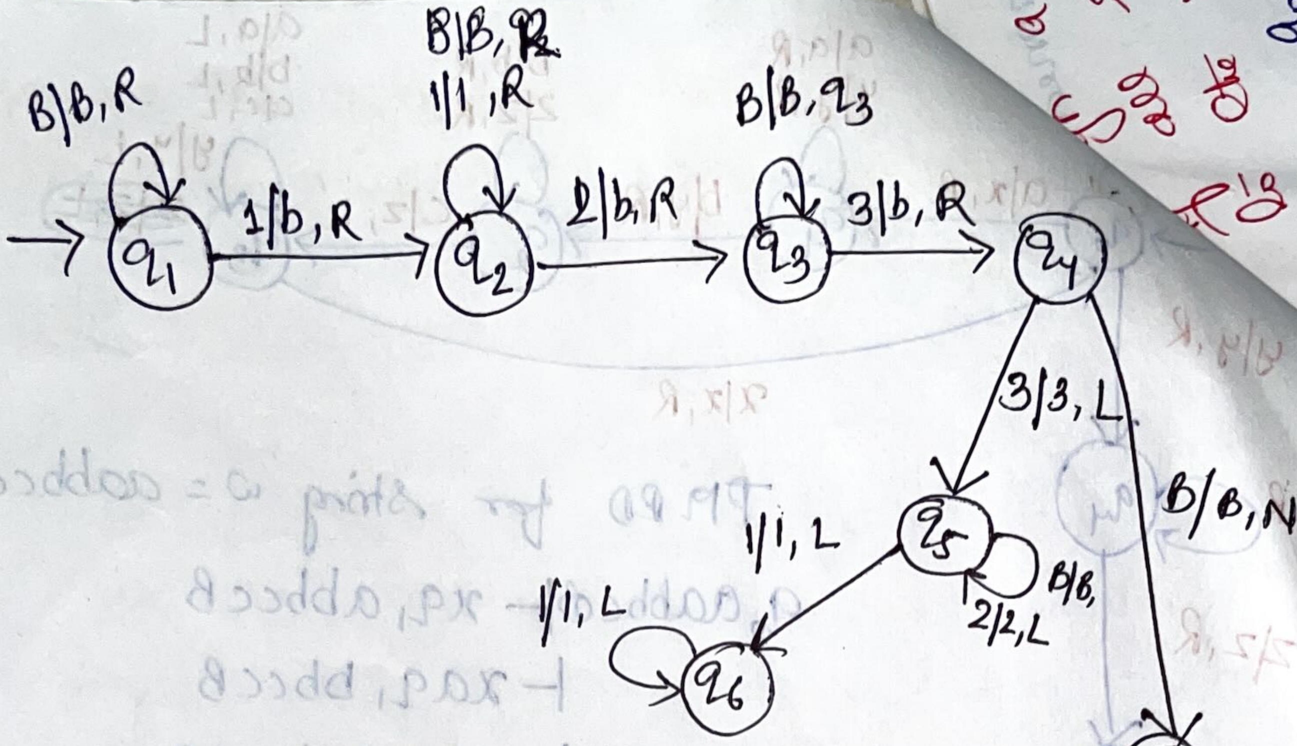
$\vdash x q_0 x y y z z B \vdash x x q_0 y y z z B$

$\vdash x x y q_4 y z z B \vdash x x y y q_4 z z B$

$\vdash x x y y z q_5 z B \vdash x x y y z z q_5 B \vdash x x y y z z B$

$q_0 abcB \vdash x q_1 bcB$

$\vdash$



B|B,R
1|b,R
B|B,R
1|1,R
2|b,R
B|B,q3
3|b,R
q1
q2
q3
q4
3|3,L
B|B,N
q5
1|1,L
B|B,
2|2,L
1|1,L
q6
q7

...sign a TM that recognizes the language L of all strings over $\{a,b\}$ with no. of a's equal to the no. of b's.

Let q_0 be the initial state and the tape head points to the first symbol of the string to be scanned, which can either be a or b. The following cases are considered, based on the next input symbol to be scanned.

Case 1:- Next i/p symbol to be scanned is B.

$$\delta(q_0, B) = (h, B, R)$$

Case 2:- Next i/p symbol to be scanned is a.

In state q_0 on i/p symbol a, skip all subsequent symbols till the symbol b is encountered. Then, come back to the next leftmost symbol and repeat any of the three cases based on the next symbol to be scanned.

$$\delta(q_0, y) = (q_0, y, R)$$
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_2, y) = (q_2, y, L)$$

Case 3:- Next i/p symbol to be scanne

$$\delta(q_0, y) = (q_0, y, R)$$

$$\delta(q_0, b) = (q_3, x, R)$$

$$\delta(q_3, b) = (q_3, b, R)$$

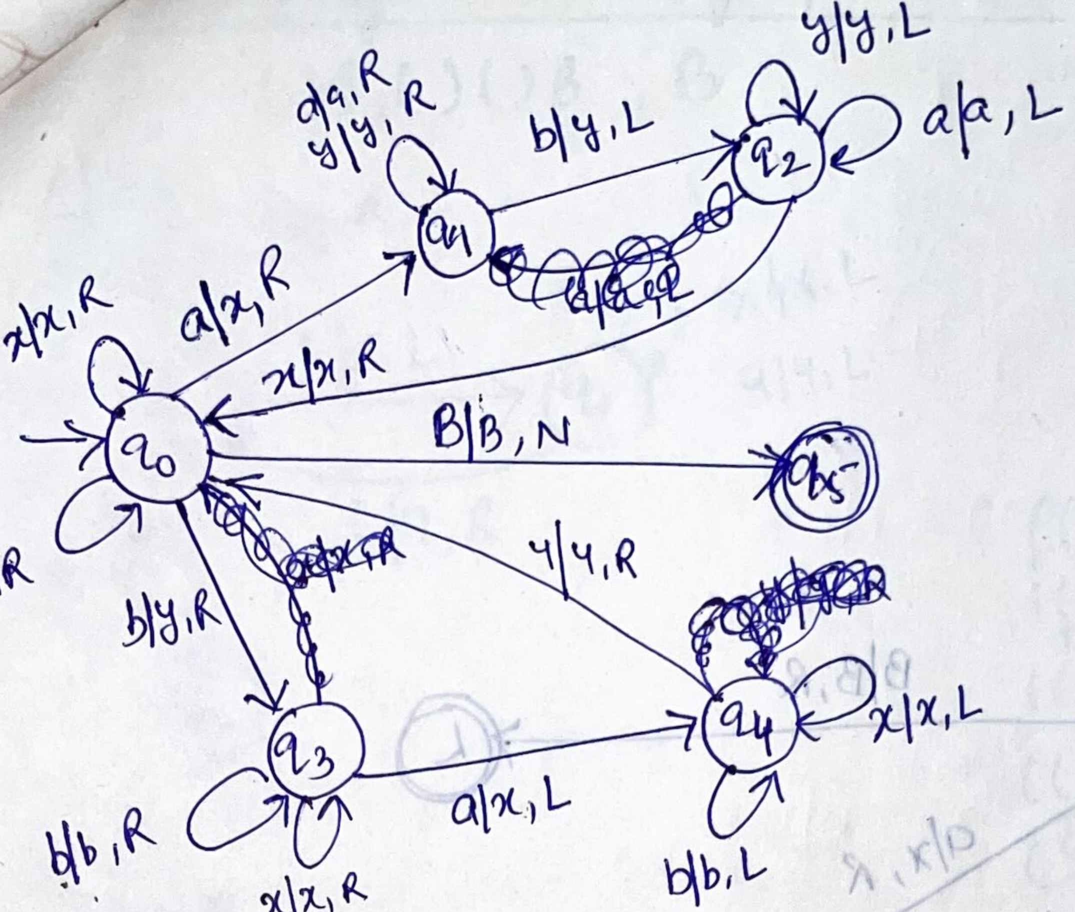
$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, a) = (q_4, y, L)$$

$$\delta(q_4, y) = (q_4, y, L)$$

$$\delta(q_4, b) = (q_4, b, L)$$

and $\delta(q_4, x) = (q_0, x, R)$



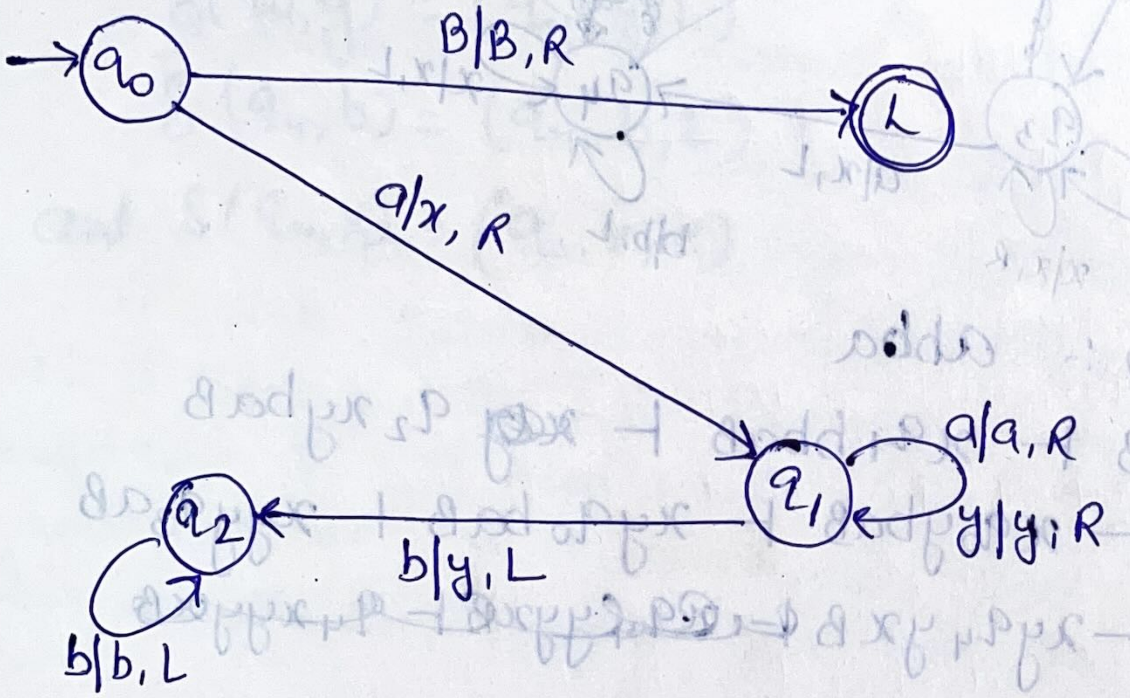
TM ID :- abba

$q_0 abbaB \vdash x q_1 bbaB \vdash \cancel{xq}\, q_2 xybaB$

$\vdash x q_0 ybaB \vdash xy q_0 baB \vdash xyy q_3 aB$

$\vdash xy q_4 yxB \cancel{\vdash x q_4 yyxB \vdash q_4 xyyxB}$

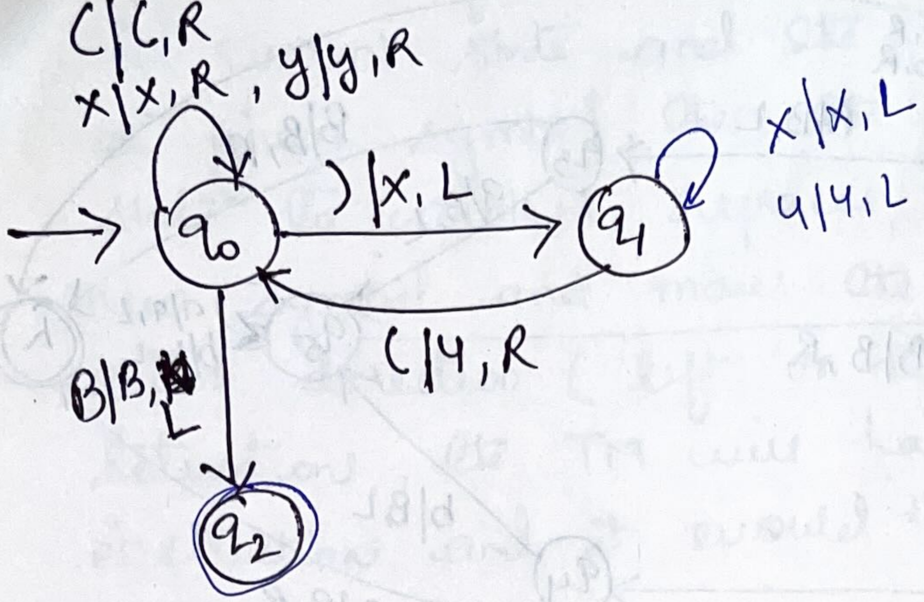
$\vdash xyy q_4 xB \vdash$



B|B, R
a|x, R
a|a, R
y|y, R
b|y, L
b|b, L

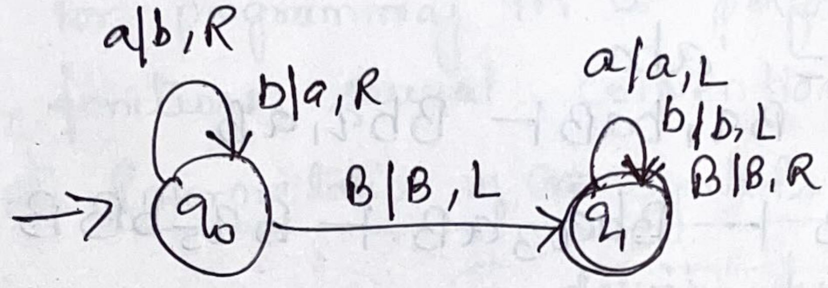
well formedness of parenthesis.

( )B, ( )( )B , B



( )( )
x

(( )) B
(( )) B
(( )) B
((x) B
(4 x) B
(4 x) B
(4 x x B
(4 x x B
(4 x x B
4



{ ( ) }
{ ( ) }
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{ (x }
{ 4 x }
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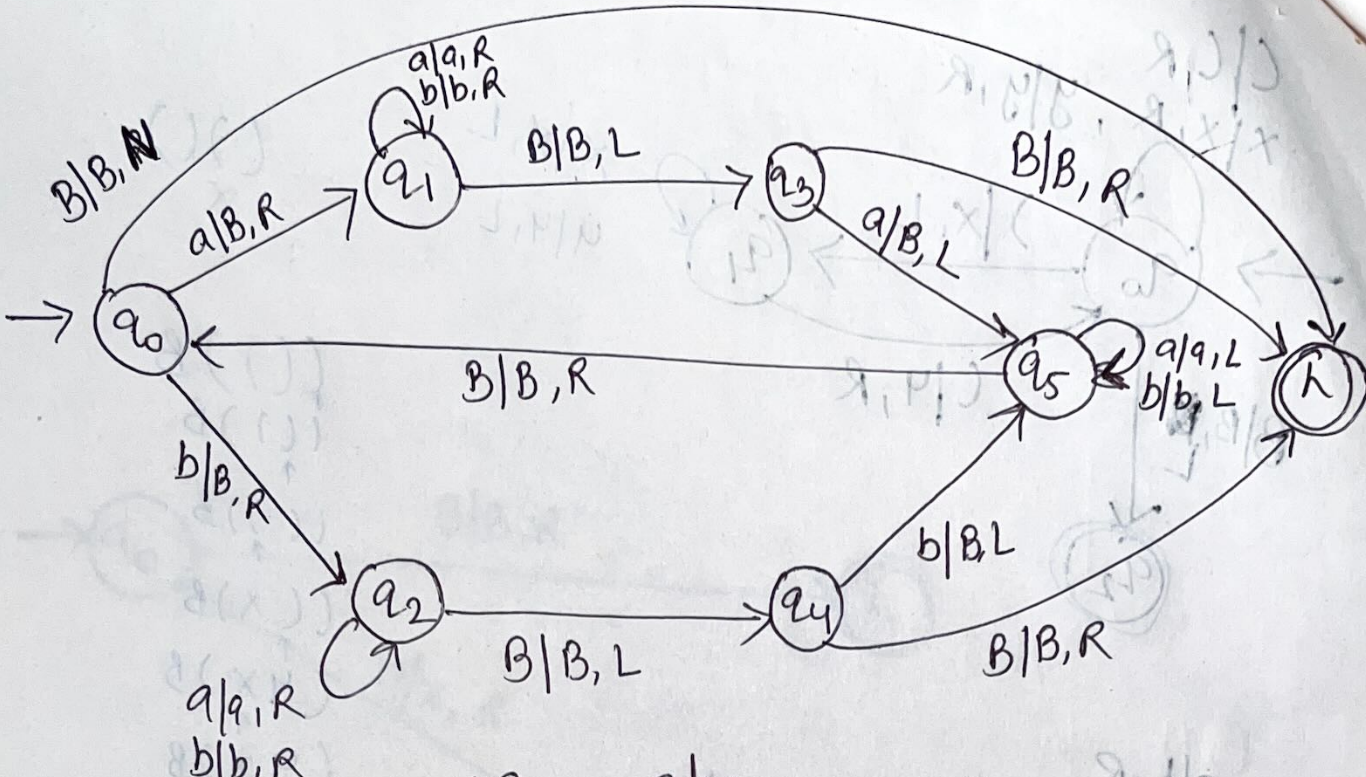
4 4 x x

{) } ({) (}
{ x

Q: Design a TM that accepts all the pal... over the alphabet {a,b}.



w = aba

TM ID: $q_0abaB \vdash Bq_1baB \vdash Bbq_1aB$

$\vdash Bbaq_1B \vdash Bbq_3aB \vdash Bq_5bBB$

$\vdash q_5BbBB$

$\vdash Bq_0bBB \vdash BBq_2BB$

$\vdash Bq_4BBB \vdash BBhBB$

---

w = bab

TM ID: $q_0bab \vdash Bq_2abB \vdash Baq_2bB$

$\vdash Babq_2B \vdash Baq_4bB \vdash Bq_5aBB$

$\vdash q_5BaBB \vdash Bq_0aBB \vdash BBq_1BB$

$\vdash Bq_3BB \vdash BBhBB$

TM as a Computer of functions

Instructions of TM means that if the TM is in current state and the head is pointing to a given symbol, then the TM goes from one state to another, replaces the symbol by a new symbol and moves the head in the given direction (left or right). In some situations, the TM will have no well-defined instructions and it would halt in such a situation.

For programming TM to perform some mathematical functions, several conventions are commonly used.

ⓐ Interpretation of the symbols recorded on the tape:-

To describe how to interpret the ones and zeros appearing on the tape as no., If the no. is represented in unary notation. It means that the non negative integer n is represented by using successive 1's.

2 – 11   3 – 111 ...

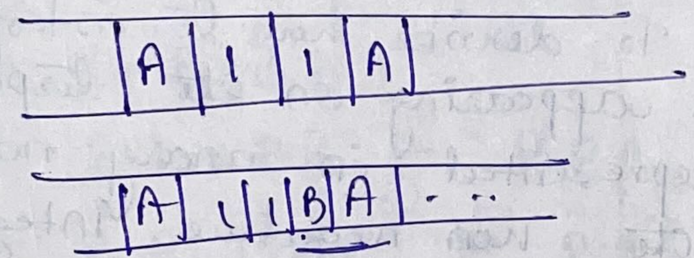
ⓑ If a function $f(n_1, n_2, \ldots n_k)$ has to be computed,

Each seq^n of 1 separated by from the previous one by a Blank symbol or special symbol. With the tape head initially located at the rightmost bit of the first i/p a to middle special symbol. or leftmost bit of the first i/p.

The TM is said to have computed $m = f(n_1, n_2, \ldots n_k)$ when it halts and the tape consists of the final result. The head is positioned at the rightmost or leftmost bit of result.

$$f(n) = n+2$$

$$n = 2$$



TM to compute the function $m = \text{multiply}(n_1, n_2)$
$= n_1 \times n_2$.

If $n_1 = 3$ and $n_2 = 2$, ~~then the tape is the~~

Solⁿ: Before Computation

B	A	1	1	1	B	1	1	A	B	...

$\underbrace{1\,1\,1}_{n_1}$ $\underbrace{1\,1}_{n_2}$

After Computation the tape will look like

B	A	1	1	1	1	1	1	A	B	...

Q:- Construct a TM that finds the difference of two natural no.

$$SUB(n_1, n_2) = \begin{cases} n_1 - n_2 & \text{if } n_1 > n_2 \\ 0 & \text{if } n_1 < n_2 \end{cases}$$

Suppose $n_1 = 3$, $n_2 = 2$

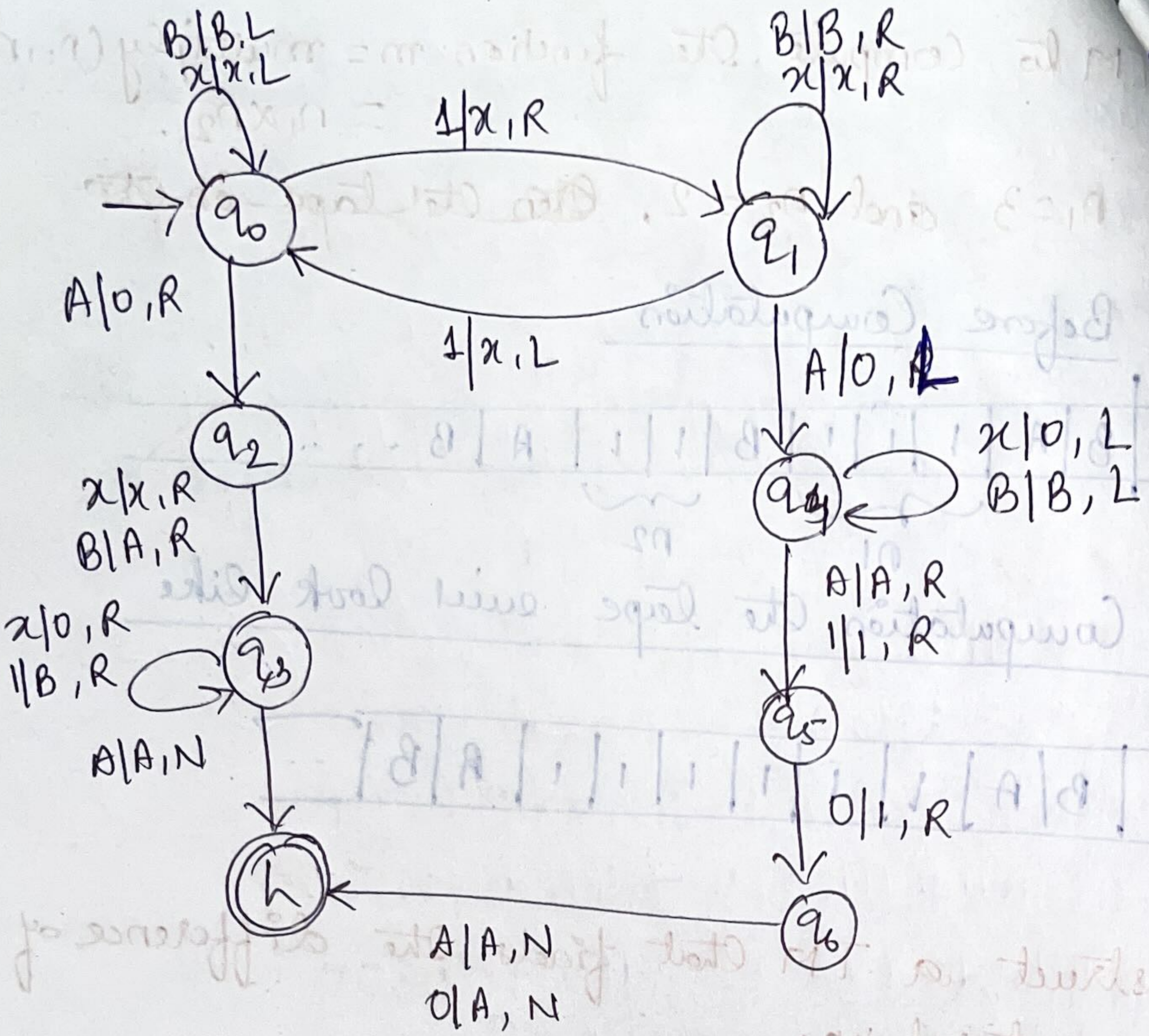
Initial Configuration of tape

A	1	1	1	B	1	1	A	...

Thus, the TM is

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, h)$

$M = ($



ID ~~for~~ $n_1=3+n_2=2$

ID is:- $A111q_0B11A \vdash A11q_01B11A$

$\vdash A11xq_0B11A \vdash$ ~~$A11q_0xB11A$~~

~~$\vdash A1q_0$~~ $\vdash A11xBq_111A$

$\vdash A11xq_0Bx1A \vdash$ ~~$A11q_0xBx1A$~~

$\vdash A1q_01xBx1A \vdash A1xq_1xBx1A$

$\vdash A1xxq_1Bx1A \vdash A1xxBq_1x1A$

$\vdash A1xxBxq_11A \vdash A1xxBq_0xxA$

$\vdash A1xxq_0BxxA \vdash A1xq_0xBxxA$

$\vdash A1q_0xxBxxA \vdash Aq_01xxBxxA$

$\vdash Axq_1xxBxxA \vdash Axxq_1xBxxA$

$\vdash Axxxq_1BxxA \vdash AxxxBxxq_1A \vdash AxxxBxq_4x0$

~~$\vdash AxxxBxx0q_4b$~~ $\vdash AxxxBq_4x00 \vdash Aq_4x00B000$

$\vdash q_4A000B000 \vdash Aq_5000B000B \vdash Aq_600B000$

$\vdash A1Ah$...

$n_1 = 4 \quad n_2 = 1$

A	1	1	1	1	B	1	A

$\uparrow q_0$

ID is:-

$A1111q_0B1A \vdash A111q_01B1A$

$\vdash A111xq_1B1A \vdash A111xBq_11A$

$\vdash A111xq_0BxA \vdash A111q_0xBxA$

$\vdash A11q_01xBxA \vdash A11xq_1xBxA$

$\vdash A11xxq_1BxA \vdash A11xxBq_1xA$

$\vdash A11xxBxq_1A \vdash A11xxBq_4x0$

$\vdash A11xxq_4B00 \vdash A11xq_4xB00$

$\vdash A11q_4x0B00 \vdash A1q_4100B00$

$\vdash A11q_5 00B00 \vdash A111q_60B00$

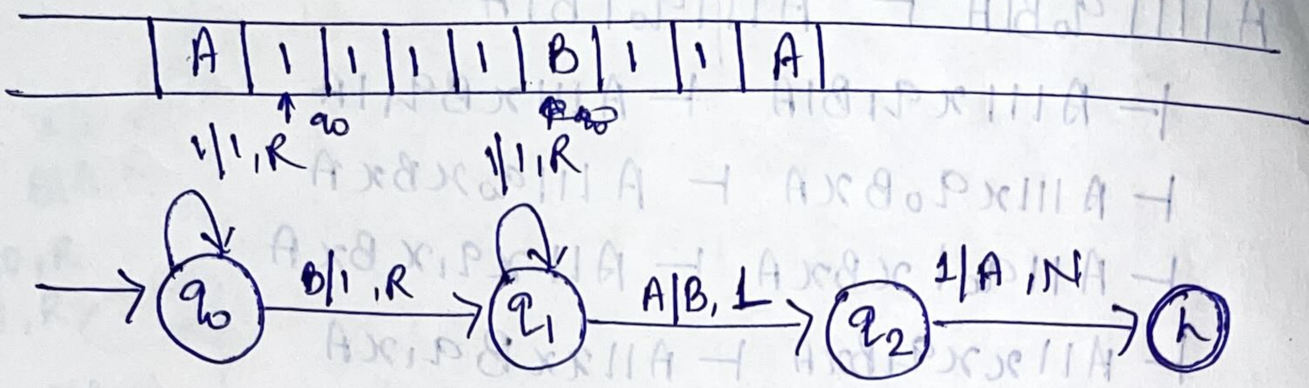
$\vdash A111Ah \ldots$

So the difference between two no. is 3.

Q:- Construct a TM that finds the sum of natural numbers.

$M = Sum(n_1 + n_2) = n_1 + n_2$

Suppose $n_1 = 4$ & $n_2 = 2$



<u>TM ID is :-</u>

$Aq_0 1111B11A \vdash A1q_0 111B11A$

$\vdash A11q_0 11B11A \vdash A111q_0 1B11A$

$\vdash A1111q_0 B11A \vdash A11111q_1 11A$

$\vdash A111111q_1 1A$

$\vdash A1111111q_1 A$

$\vdash A111111q_2 1B$

$\vdash A111111AhB$

$n_1 = 3$; $n_2 = 3$

<u>TM ID is :-</u> $Aq_0 111B111A \vdash A1q_0 11B111A$

$\vdash A11q_0 1B111A \vdash A111q_0 B111A$

$\vdash A1111q_1 111A \vdash A11111q_1 11A$

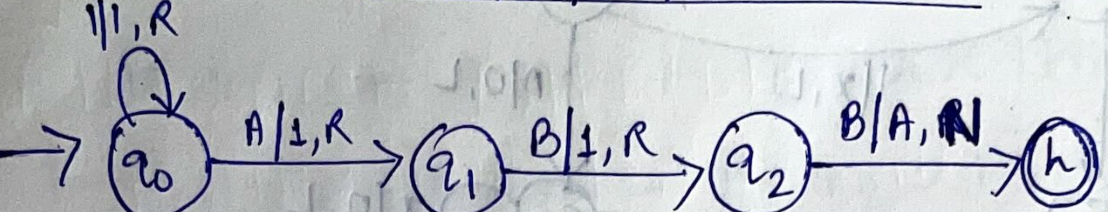
$\vdash A111111q_1 1A \vdash A1111111q_1 A$

$\vdash A111111q_2 1B \vdash A111111AhB$

So sum of n_1 & n_2 is 6.

Construct a TM to find $f(n) = n+2$

A	1	1	1	A	B



TM ID :- $n = 3$ $f(3) = 5$

$Aq_0111AB \vdash A1q_011AB$
$\vdash A11q_01AB \vdash A111q_0AB \vdash A1111q_1B$
$\vdash A11111q_2B \vdash A11111Ah$

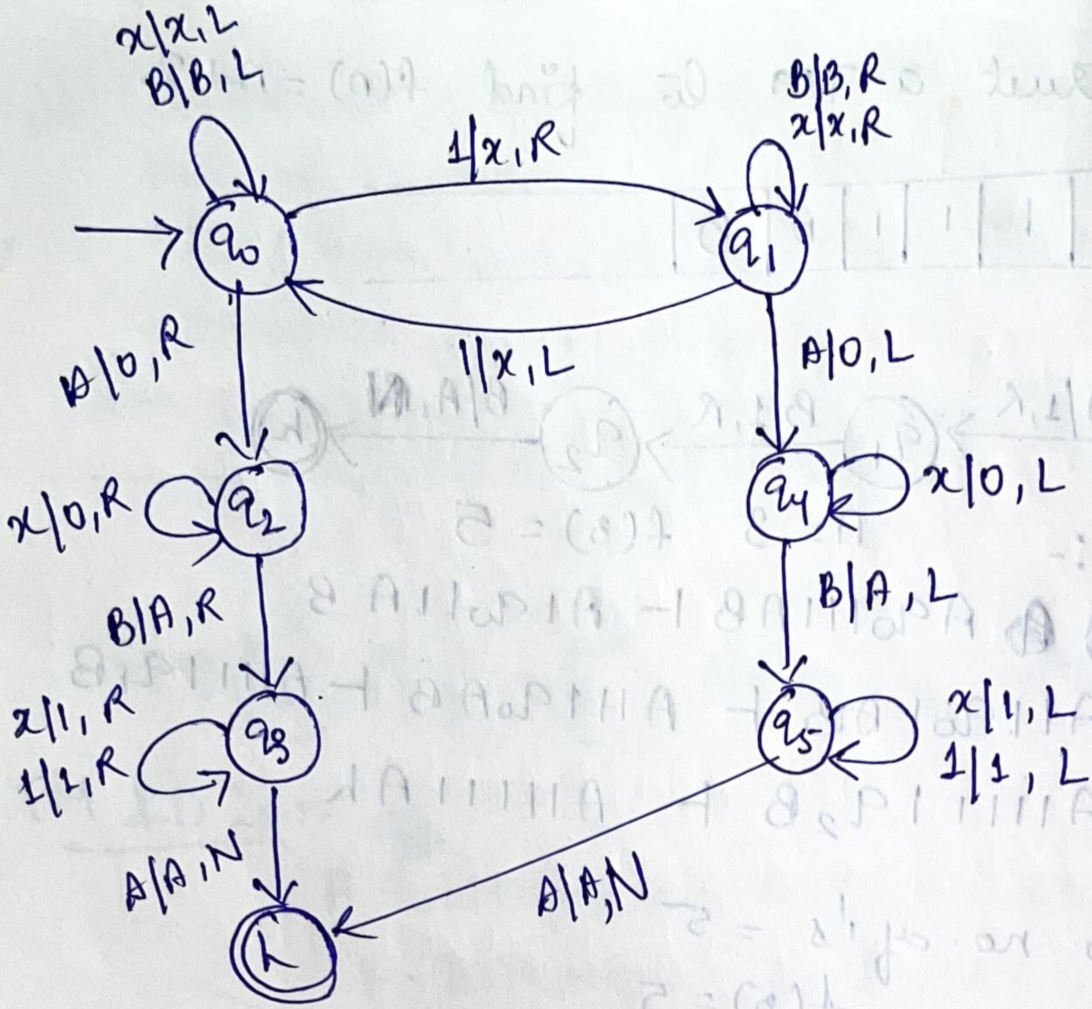
So no. of 1's $= 5$
$f(3) = 5$

Q: Design a TM to compute $\max(n_1, n_2)$.

$$\max(n_1, n_2) = \begin{cases} n_1 & \text{if } n_1 \geq n_2 \\ n_2 & \text{if } n_2 < n_1 \end{cases}$$

Suppose $n_1 = 1$ $n_2 = 2$

A	1	B	1	1	A



TMID is:- $A q_0 1 B 1 1 A \vdash A x q_1 B 1 1 A$

$\vdash A x B q_1 1 1 A$

$\vdash A x q_0 B x 1 A$

$\vdash A q_0 x B x 1 A \vdash q_0 A x B x 1 A$

$\vdash 0 q_2 x B x 1 A \vdash 0 0 q_2 B x 1 A$

$\vdash 0 0 A q_3 x 1 A \vdash 0 0 A 1 q_3 1 A$

$\vdash 0 0 A 1 1 q_3 A$

$\vdash 0 0 A 1 1 A h$

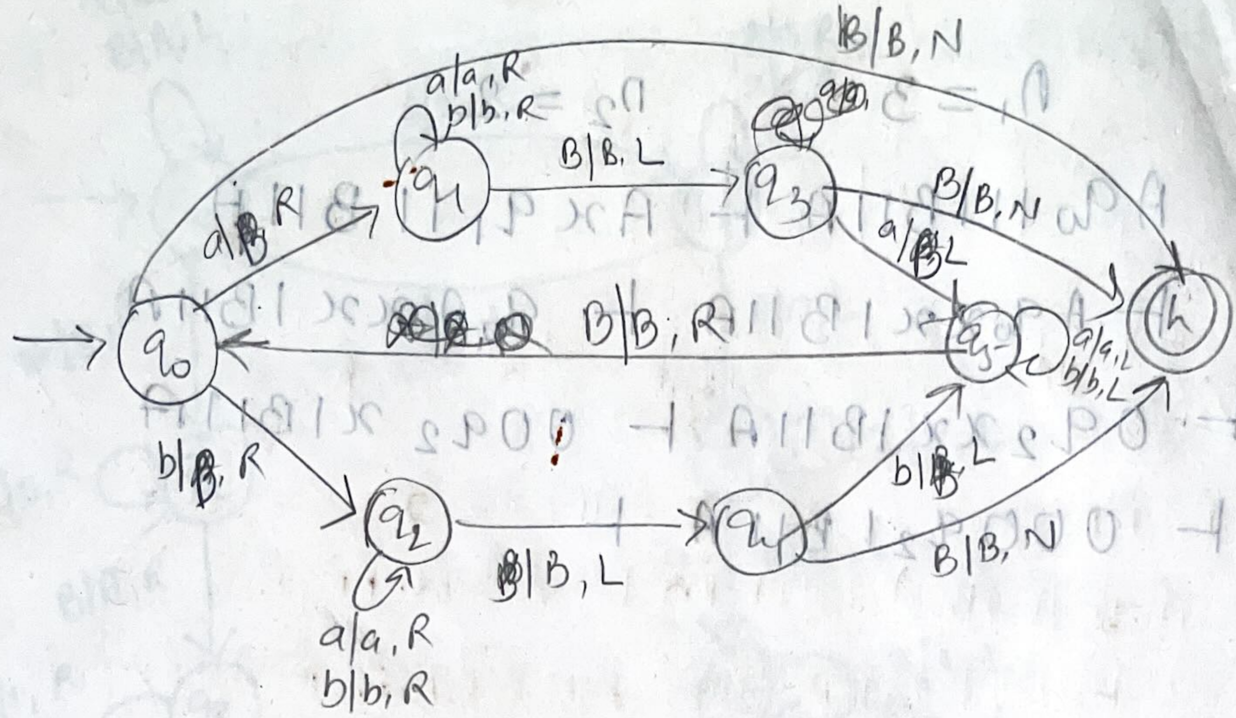
$n_1 = 3 \qquad n_2 = 2$

$A q_0 111B11A \vdash A x q_1 11B11A$

$\vdash A q_0 xx1B11A \vdash q_0 A xx1B11A$

$\vdash 0 q_2 xx1B11A \vdash 00 q_2 x1B11A$

$\vdash 000 q_2 1B11A \vdash$



$q_0\,ababaB \vdash \cancel{a}\,q_1\,babaB$

$\vdash \cancel{a}\,\cancel{B}aba\,q_1\,B \vdash \cancel{a}\,baba\,q_3\,\cancel{a}B$

$\vdash \cancel{a}\,ba\,q_5\,b\cancel{a}B \vdash q_5\,\cancel{a}\,bab\,\cancel{a}B$

$\vdash \cancel{a}\,q_0\,bab\,\cancel{a}B \vdash BB\,q_2\,ab\,\cancel{B}\,B$

$\vdash BBab\,q_2\,BB \vdash BB$