CSE2403-Discrete Mathematics

Problem Sheet-3

Topic: **Sets, Relations and Functions**

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Date: 23/05/2022

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1. Prove that (A-C) $\cap$ (C-B)= $\emptyset$ analytically, where A, B, and C are sets. Verify graphically

2. If A, B and C are sets, prove analytically that A $\times$ (B $\cap$ C)= (A $\times$ B) $\cap$ (A $\times$ C)

3. If A, B, C and D are sets,
   Prove that analytically that (A $\cap$ B) $\times$ (C $\cap$ D) = (A $\times$ C) $\cap$ (B $\times$ D). Give an example to support this result.

4. Find the sets A and B, if

   (a) A - B ={1, 3, 7, 11}, B - A = {2, 6, 8} and A $\cap$ B = {4, 9}.

   (b) A - B ={1, 2, 4}, B - A = {7, 8} and A $\cup$ B = {1, 2, 4, 5, 7, 8, 9}.

5. Prove the following analytically or graphically:

   (a) A - B = A $\cap$ $\bar{B}$

   (b) (A $\cap$ B) $\cup$ (A $\cup$ $\bar{B}$) = A

   (c) (A $\cup$ B) $\cap$ (A $\cup$ $\emptyset$) = A

   (d) A - (A $\cap$ B)= A - B

   (e) (A $\cap$ B) $\cup$ (B - A) = B

   (f) (A - B) $\cup$ (B - A)= (A $\cup$ B) - (A $\cap$ B)

   (g) (A - B)- C = (A - C) - (B - C)

   (h) A $\cap$ (B - C) = (A $\cap$ B)-(A $\cap$ C)

   (i) (A - B) - C = (A - C) - (B - C)

6. List the ordered pairs in the relation R from A = $\{0, 1, 2, 3, 4\}$ to B = $\{0, 1, 2, 3\}$ where a R b if and only if i) a = b, ii) a + b = 4, iii) a greater than b, iv) a divides b, v) *gcd* (a, b)= 1, vi) *lcm* (a, b)= 2.

7. The relation R on the set A = $\{1, 2, 3, 4, 5\}$ is defined by a R b if and only if 3 divides (a-b).
   i) List the elements of R and R^{-1}
   ii)Find the domain and range of R
   iii)Find the domain and range of R^{-1}
   iv)List the elements of complement of R

8. If R = (1, 2), (2, 4), (3, 3) and S = (1, 3), (2, 4), (4, 2), find i) R $\cup$ S, ii) R $\cap$ S, iii) R - S, iv) S - R. Also verify that the dom (R $\cup$ S)= dom (R) $\cup$ dom (S) and range (R $\cap$ S) $\subseteq$ range (R) $\cap$ range (S).

9. If R = $\{$(a, b) : a $\equiv$ b (mod 3)$\}$ and S = $\{$(a, b) : a $\equiv$ b (mod 4)$\}$ are relations on the set of integers, the find i) R $\cup$ S, ii) R $\cap$ S, iii) R - S, iv) S - R.

10. Determine whether the relation R on the set of all integers is reflexive, symmetric, and antisymmetric and/or transitive, where a R b if and only if i) a $\neq$ b, ii) ab$\geq$ 0, iii) ab$\geq$ 1, iv) a is multiple of b, v) a $\equiv$ b (mod 7), vi) $\mid$ a - b $\mid$ = 1, vii) a = b^2, viii) a $\geq$ b^2.

11. Let R be the relation represented by the matrix

$$M_R = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

    Find the matrix representing i) R^{-1} , ii) $\bar{R}$, iii) R^2

12. Let R and S be the relations on a set A represented by the matrices
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$$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$M_S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

    Find the matrices that represents 5

    (a) R $\cup$ S
    (b) R $\cap$ S
    (c) R o S
    (d) S o R

(e) R $\bigoplus$ S

13. Find the transitive closure of the following relations on a set A = {a, b, c, d, e}, by using Warshall's algorithm.
    (a) {(a, c), (b, d), (c, a), (d, b), (e, d)}
    (b) {(b, c), (b, e), (c, e), (d, a), (e, b), (e, c)}
    (c) {(a, b), (a, c), (a, e), (b, a), (b, c), (c, a), (c, b), (d, a), (e, d)}
    (d) {(a, e), (b, a), (b, d), (c, d), (d, a), (d, c), (e, a), (e, b), (e, c), (e, e)}

14. How many elements are in $A_1 \cup A_2$ if there 12 elements in A_1, 18 elements in A_2, and
    (a) $A_1 \cap A_2 = \emptyset$?
    (b) $|A_1 \cap A_2| = 1$?
    (c) $|A_1 \cap A_2| = 6$?
    (d) $A_1 \subset A_2$?

15. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 1000 elements in A_2, and 10000 elements in A_3 if
    (a) $A_1 \subset A_2$ and $A_2 \subset A_3$
    (b) the sets are pairwise disjoint
    (c) there are two elements common to each pair of sets and one element in all the three sets.

16. Prove that the inverse of a function f, if exists, is unique.

17. If f: A→B and g: B→C are invertible functions, then show that gof is invertible and $(gof)^{-1} = f^{-1}og^{-1}$.

18. If A = {x∈ $\Re$: x $\neq$ 1/2} and f: A→ $\Re$ is defined by $f(x) = \frac{4x}{2x-1}$
    (a) Find the range of f
    (b) Show that f is invertible
    (c) dom(f^{-1})
    (d) range (f^{-1})
    (e) Find the formula of f^{-1}