

## Linear fitter and uncertainties of parameters.

If  $N$  measurements result in  $N$  pairs of numbers  $x_i$  and  $y_i$ , and we expect that according to the physical process,  $y$  is the linear function of  $x$  given by  $y = mx + b$ , the unknown parameters  $m$  and  $b$  can be found as:

$$\Delta = N \sum x_i^2 - (\sum x_i)^2; \quad m = \frac{N \sum_{i=1}^N x_i y_i - \sum_{i=1}^N x_i \sum_{i=1}^N y_i}{\Delta}; \quad b = \bar{y} - m \bar{x} = \frac{\sum_{i=1}^N y_i - m \sum_{i=1}^N x_i}{N}. \quad (1)$$

If  $s_{y,x}^2 = \frac{1}{N-2} \sum [y_i - (b + mx_i)]^2$  is the variance of  $y(x)$ , then the standard deviation  $s_m$  of the estimated slope is a square root of  $s_m^2 = N \frac{s_{y,x}^2}{\Delta}$ ; (2a) and the standard deviation  $s_b$  of a  $y$ -intercept is the square root of  $s_b^2 = \frac{s_{y,x}^2 \sum x_i^2}{\Delta}$  (2b).

We recommend creating a code in any programming language to simplify calculations for any number of measurements in your future experiments.

Finally, your linear fit is the function  $y = mx + b$ , with  $\text{slope} = m \pm s_m$ , and  $y\text{-intercept} = b \pm s_b$ , given by equations (1) and (2).

The coefficient of determination  $R^2$  is calculated by Excel as follows:

$$R^2 = 1 - \frac{(N-2)s_{y,x}^2}{\sum (y_i - \bar{y})^2} = 1 - \frac{(N-2)s_{y,x}^2}{\sum \left(y_i - \frac{\sum y_i}{N}\right)^2}$$