

Linear fitter and uncertainties of parameters.

If N measurements result in N pairs of numbers x_i and y_i , and we expect that according to the physical process, y is the linear function of x given by $y = mx + b$, the unknown parameters m and b can be found as:

$$\Delta = N \sum x_i^2 - \left(\sum x_i \right)^2; \quad m = \frac{N \sum_{i=1}^N x_i y_i - \sum_{i=1}^N x_i \sum_{i=1}^N y_i}{\Delta}; \quad b = \bar{y} - m\bar{x} = \frac{\sum_{i=1}^N y_i - m \sum_{i=1}^N x_i}{N}. \quad (1)$$

If $s_{y,x}^2 = \frac{1}{N-2} \sum [y_i - (b + mx_i)]^2$ is the variance of $y(x)$, then the standard deviation s_m of the estimated slope is a square root of $s_m^2 = N \frac{s_{y,x}^2}{\Delta}$; (2a) and the standard deviation s_b of a y -intercept is the square root of $s_b^2 = \frac{s_{y,x}^2 \sum x_i^2}{\Delta}$ (2b).

We recommend creating a code in any programming language to simplify calculations for any number of measurements in your future experiments.

Finally, your linear fit is the function $y = mx + b$, with $slope = m \pm s_m$, and $y-intercept = b \pm s_b$, given by equations (1) and (2).

The coefficient of determination R^2 is calculated by Excel as follows:

$$R^2 = 1 - \frac{(N-2)s_{y,x}^2}{\sum (y_i - \bar{y})^2} = 1 - \frac{(N-2)s_{y,x}^2}{\sum \left(y_i - \frac{\sum y_i}{N} \right)^2}$$