Simulating an Electromagnetic Field on a Finite Domain

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Special thanks to Mansi Bezbaruah and Seth Gerberding

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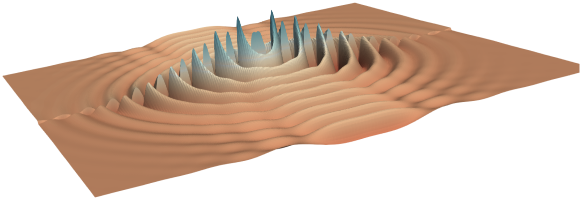
Outline

Motivation

- Effective and accurate implementation
- 3D Problem on a 2D Mesh
- Introduction of a source term
- Infinite domain on a finite computational domain
- Discontinuity terms



Governing Equations

$$\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon}\left(\frac{\partial H_z}{\partial y}\right)$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon}\left(\frac{\partial H_z}{\partial x}\right)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$$

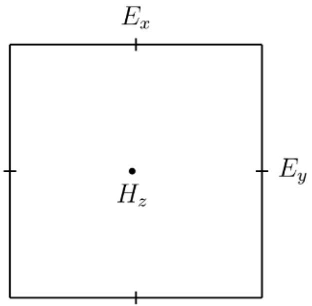
$E_{x,y}(t)$: Electric field (x,y-direction), $H_z(t)$: Magnetic field (z-direction)
ϵ : Electric permittivity, μ : Magnetic permeability, $\epsilon, \mu \in \mathbb{R}$

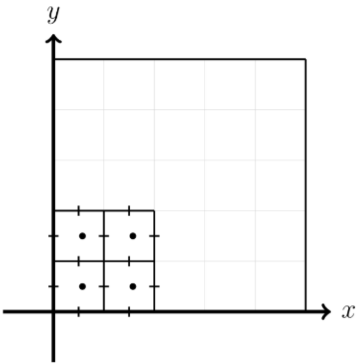
Yee's Scheme (FDTD)

- Centered finite-difference method
- Second-order accurate (in space and time)
- Staggered spacial grid
- Leapfrog time-marching

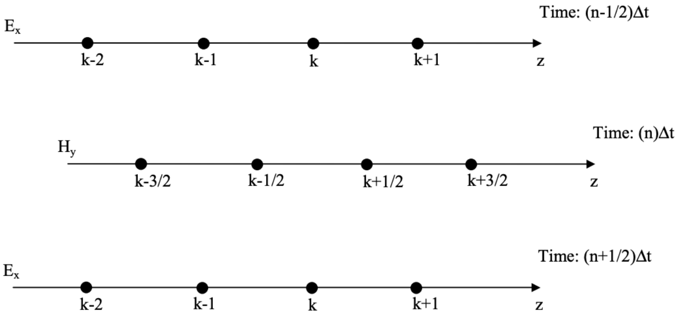
Staggered Grids

E_x and E_y values “live” on the edges, H_z “lives” in the center.





Time-Marching



URL: `https://my.ece.utah.edu/~ece6340/LECTURES/lecture%2014/FDTD.pdf`

Discretized Equations

$$E_x\Big|_{i,j}^{n+1} = E_x\Big|_{i,j}^{n} + \frac{\Delta t}{\epsilon \Delta y}\left[H_z\Big|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} - H_z\Big|_{i,j-\frac{1}{2}}^{n+\frac{1}{2}}\right]$$

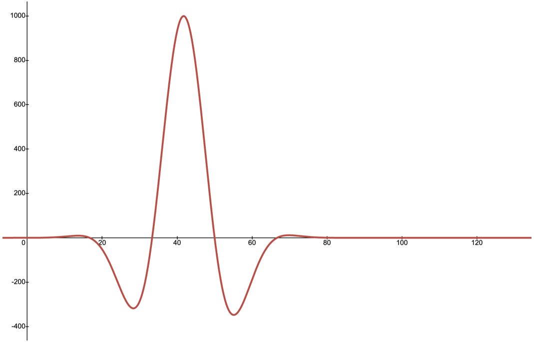
$$E_y\Big|_{i,j}^{n+1} = E_y\Big|_{i,j}^{n} - \frac{\Delta t}{\epsilon \Delta x}\left[H_z\Big|_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - H_z\Big|_{i-\frac{1}{2},j}^{n+\frac{1}{2}}\right]$$

$$H_z\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} = H_z\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{\Delta t}{\mu}\left[\frac{E_x\Big|_{i+\frac{1}{2},j+1}^{n} - E_x\Big|_{i+\frac{1}{2},j}^{n}}{\Delta y} - \frac{E_y\Big|_{i+1,j+\frac{1}{2}}^{n} - E_y\Big|_{i,j+\frac{1}{2}}^{n}}{\Delta x}\right]$$

Source Term

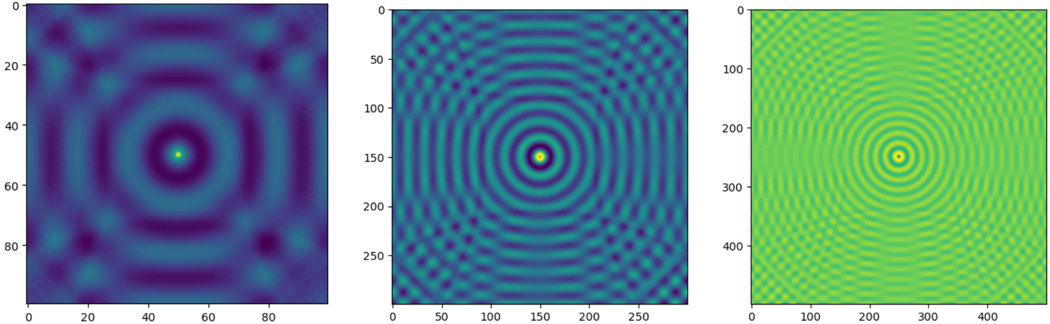
Including a source term that acts on the center of the domain given by:

$$S(t) = 10^3 \cdot \sin(0.06\pi \cdot t)e^{-0.005*(t-42)^2}$$



Doing so induces the dynamics in the problem.

Resolution Comparison



(a) Mesh Size: $\mathbb{R}^{100\times100}$, $t = 170$ (b) Mesh Size: $\mathbb{R}^{300\times300}$, $t = 510$ (c) Mesh Size: $\mathbb{R}^{500\times500}$, $t = 850$

Figure: Mesh Resolution and Refinement Comparison

For Loops vs NumPy Arrays

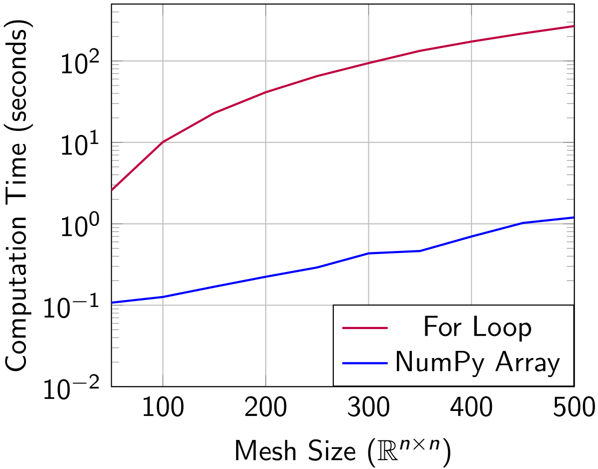


Figure: Comparison of computation time between naive for-loops and vectorized NumPy arrays, logarithmic scale ($t = 800$) .

Outline

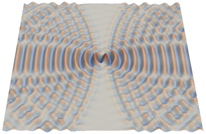
Introduction

Wave Modeling

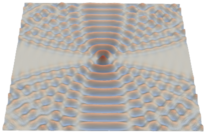
Circular PML

Split PML

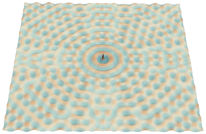
Wave With No Boundary



E_x



E_y



H_z

E_x, E_y, H_z with no boundary conditions.

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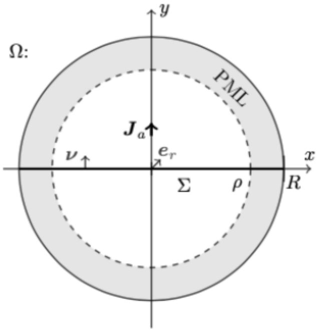
Split PML

Circular PML



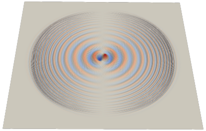
$$\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon}\left(\frac{\partial H_z}{\partial y}\right)$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon}\left(\frac{\partial H_z}{\partial x}\right)$$

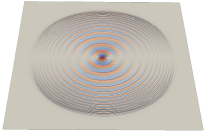
$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$$

URL: `https://www.dealii.org/current/doxygen/deal.II/step_81.html`

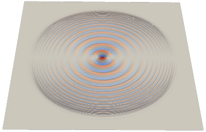
Circular PML Implementation



E_x



E_y



H_z

E_x, E_y, H_z with circular PML simulations.

Jump Conditions

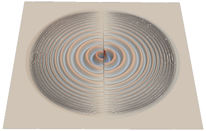
$$H_z(0^+, y, t) - H_z(0^-, y, t) = j(y, t)$$

$$E_y(0^+, y, t) - E_y(0^-, y, t) = 0$$

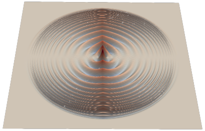
$$DE_y(x, 0, t) - \frac{j}{\tau} = \frac{\partial j}{\partial t}$$

j : Current on the graphene sheet

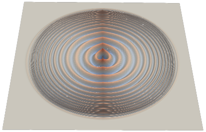
Circular PML with Graphene Sheet



E_x



E_y



H_z

E_x, E_y, H_z with circular PML and graphene sheet.

Outline

Split Field PML Governing Equations

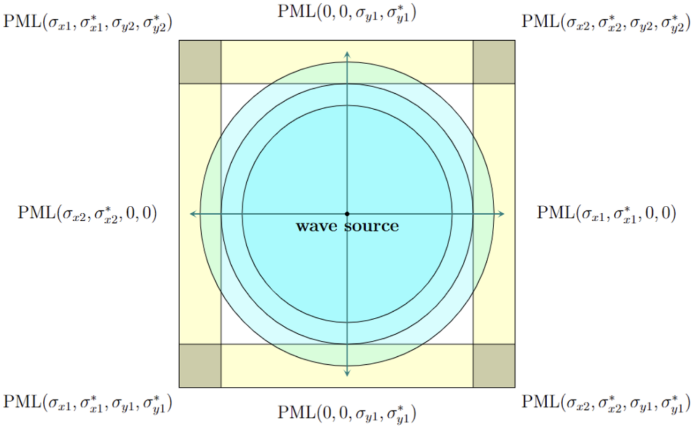
$$\epsilon \frac{\partial E_x}{\partial t} + \sigma_y E_y = \frac{\partial H_z}{\partial y} \qquad \mu \frac{\partial H_{zx}}{\partial t} + \sigma_x^* H_{zx} = -\frac{\partial E_y}{\partial x}$$

$$\epsilon \frac{\partial E_y}{\partial t} + \sigma_x E_x = -\frac{\partial H_z}{\partial x} \qquad \mu \frac{\partial H_{zy}}{\partial t} + \sigma_y^* H_{zy} = \frac{\partial E_y}{\partial y}$$

$$H_{zx} + H_{zy} = H_z$$

σ : Absorption constant, $\sigma \neq 0$

Split Field PML



Split PML Model

Discretized Equations

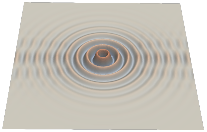
$$E_x\Big|_{i+\frac{1}{2},j}^{n+1} = \left(\frac{2\epsilon - \sigma_y \Delta t}{2\epsilon + \sigma_y \Delta t}\right) E_x\Big|_{i+\frac{1}{2},j}^{n} + \left(\frac{2\Delta t}{(2\epsilon + \sigma_y \Delta t)\Delta y}\right)\left[H_z\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_z\Big|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}}\right]$$

$$E_y\Big|_{i+\frac{1}{2},j}^{n+1} = \left(\frac{2\epsilon - \sigma_x \Delta t}{2\epsilon + \sigma_x \Delta t}\right) E_y\Big|_{i+\frac{1}{2},j}^{n} - \left(\frac{2\Delta t}{(2\epsilon + \sigma_x \Delta t)\Delta x}\right)\left[H_z\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_z\Big|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}}\right]$$

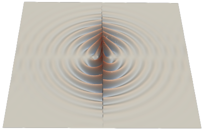
$$H_{zx}\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} = \left(\frac{2\mu - \sigma_x^* \Delta t}{2\mu + \sigma_x^* \Delta t}\right) H_{zx}\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n-\frac{1}{2}} - \left(\frac{2\Delta t}{(2\mu + \sigma_x^* \Delta t)\Delta x}\right)\left[E_y\Big|_{i+1,j+\frac{1}{2}}^{n} - E_y\Big|_{i,j+\frac{1}{2}}^{n}\right]$$

$$H_{zy}\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} = \left(\frac{2\mu - \sigma_y^* \Delta t}{2\mu + \sigma_y^* \Delta t}\right) H_{zy}\Big|_{i+\frac{1}{2},j+\frac{1}{2}}^{n-\frac{1}{2}} + \left(\frac{2\Delta t}{(2\mu + \sigma_y^* \Delta t)\Delta y}\right)\left[E_x\Big|_{i+\frac{1}{2},j+1}^{n} - E_x\Big|_{i+\frac{1}{2},j}^{n}\right]$$

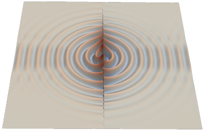
Split PML with Graphene Sheet



E_x



E_y



H_z

E_x, E_y, H_z with split PML and graphene sheet.

Conclusion

- Numpy arrays provide faster computation speed than nested for loops
- Both the circular and the split PML succeeded in simulating an infinite domain on a finite computational domain with a jump discontinuity
- Learned new tools and techniques, such as Paraview for visualization
- Put finished code on Github

Thank you!