# Shocking Revelations: Numerical Analysis for the Balance System Applied to a Varying Chaplygin Gas

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## Introduction

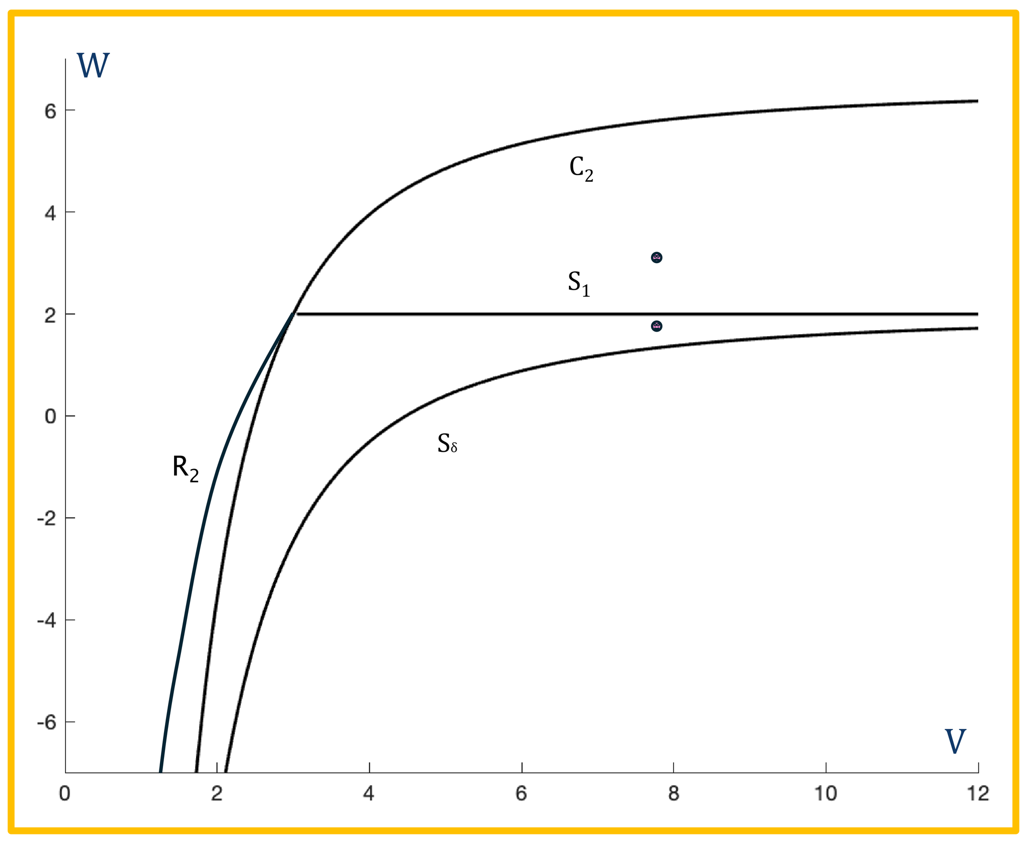
- Chaplygin gas is a cosmological model to connect dark matter and dark energy
- The given equation of state is:

$$p = A\rho^{\gamma}e^{nt}$$

- We combine this with a 2 x 2 Keyfitz -Kranzer type balance system in the form:

$$\begin{cases} \rho_t + (\rho\Phi(\rho,u))_x & = F(\rho,u) \\ (\rho u)_t + (\rho\Phi(\rho,u))_x & = G(\rho,u) \end{cases}$$

- With initial left and right state known




- Change to V,W variables
- Shock is a jump in states
- Rarefaction is a smooth transition from one state to another
- Delta-shock characterized by "blowup" in a variable
- Cases defined by sign of $k(\gamma+1)$, as well as the magnitude of γ
- Case 2 shown here, $-1 < \gamma < 0$, $k(\gamma+1) < 0$
- Consists of 4 curves, $S_1$, $C_2$, $R_2$, $S_\delta$
- $R_2$ lies extremely close to $C_2$

## Methods

- Numerical scheme used is local Lax-Friedrich's (LLF) method

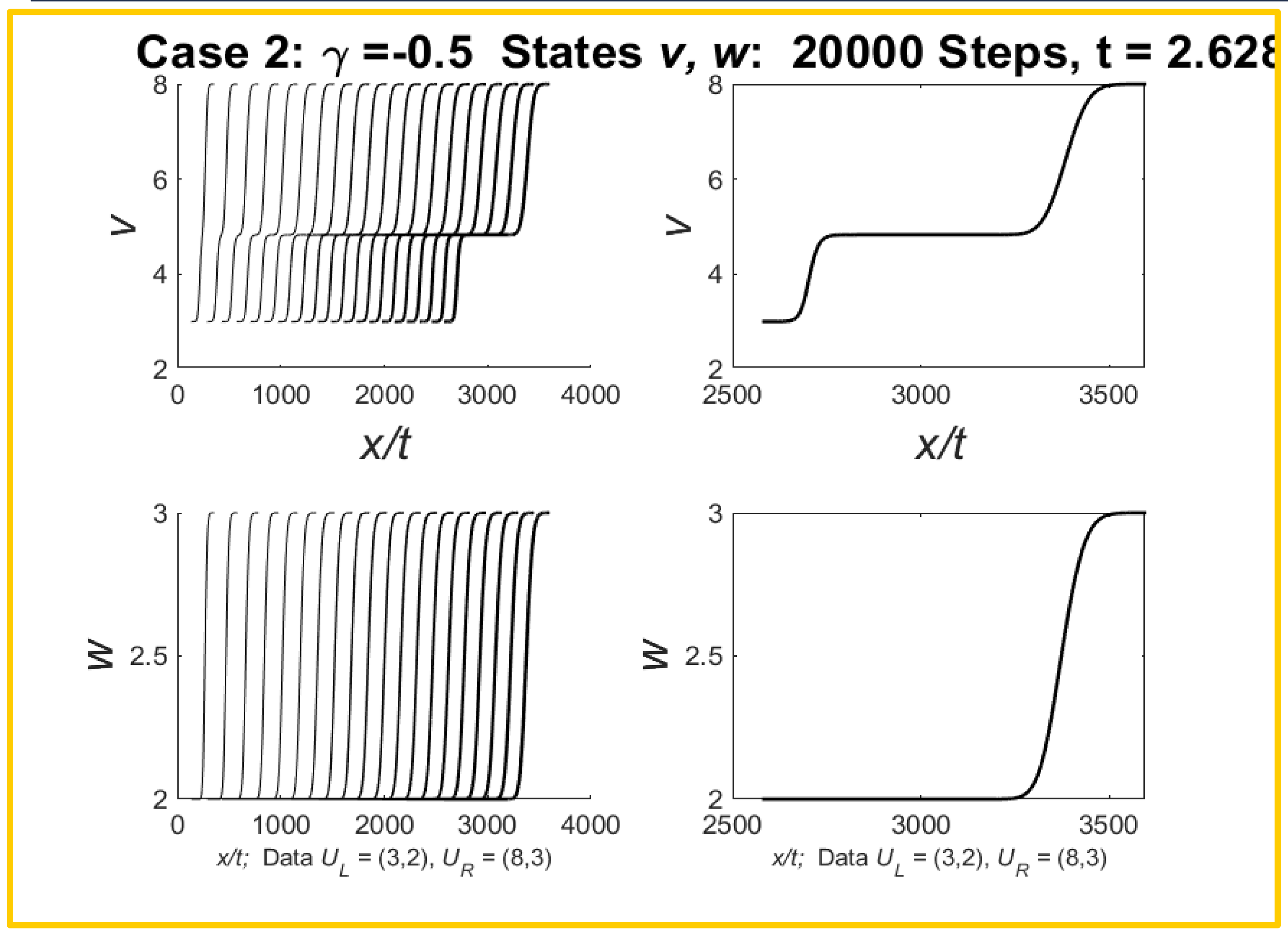
$$U_j^{n+1} = \frac{1}{2}(U_{j-1}^n + U_{j+1}^n) + \frac{CFL}{2\lambda}(F_{j+1}^n - F_{j-1}^n)$$

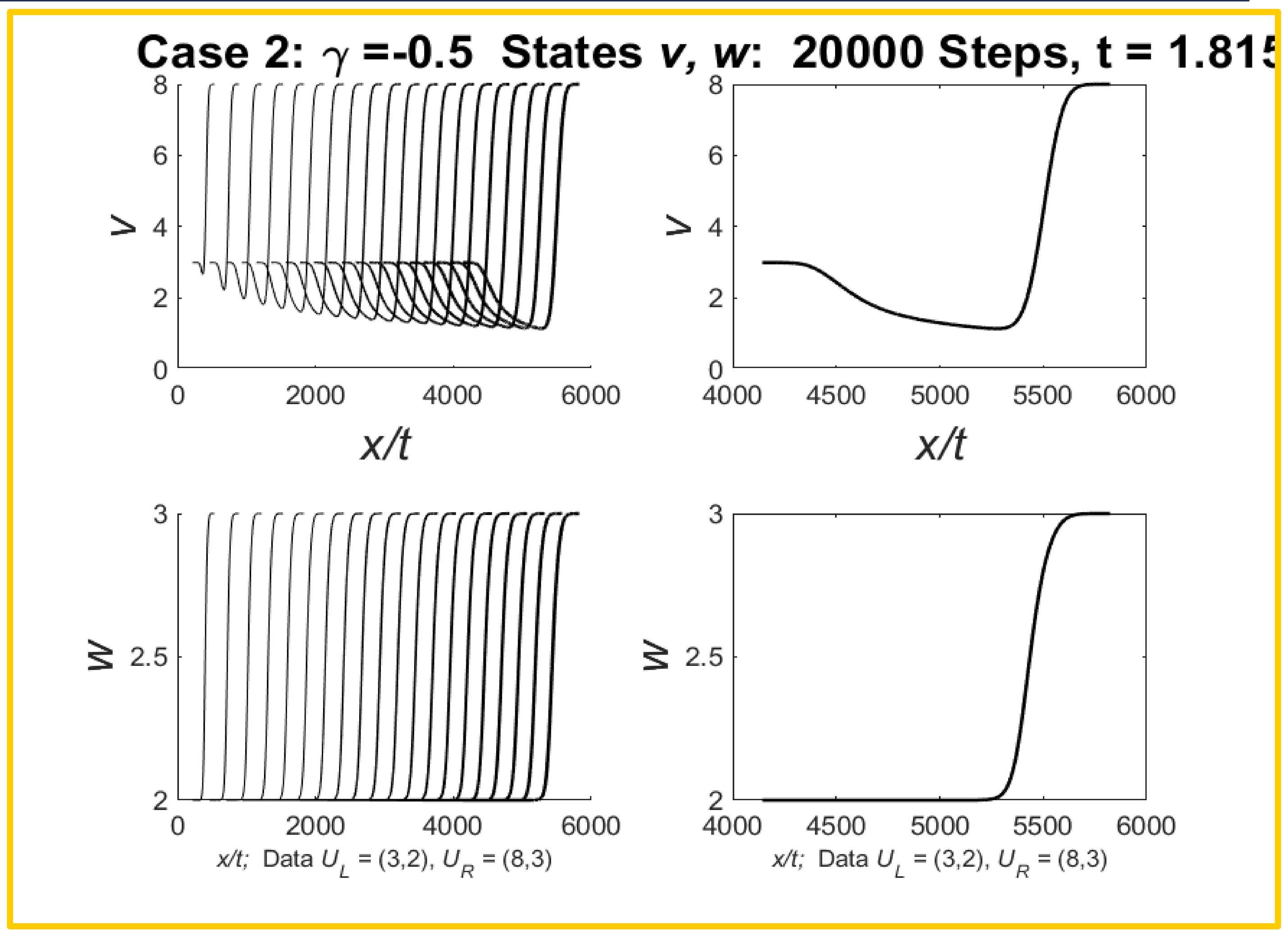
- Wave-speed calculated locally using eigenvalues
- Utilized change of variables for numerical stability

$$Y = VW + V\frac{\beta}{\eta - k}$$

- CFL condition imposed to ensure convergence to physically correct weak solution

## Results



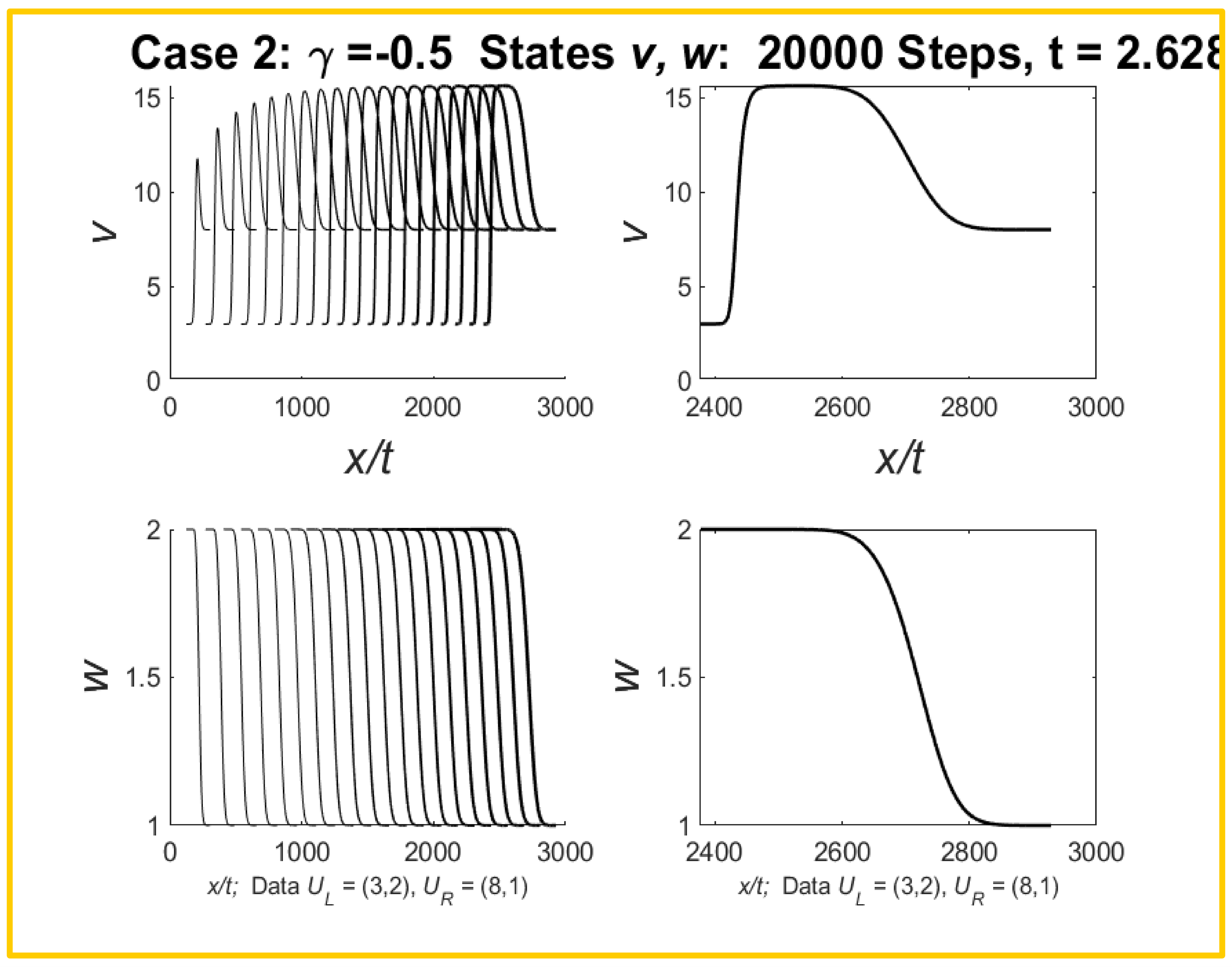

**Small Timescale on Left**

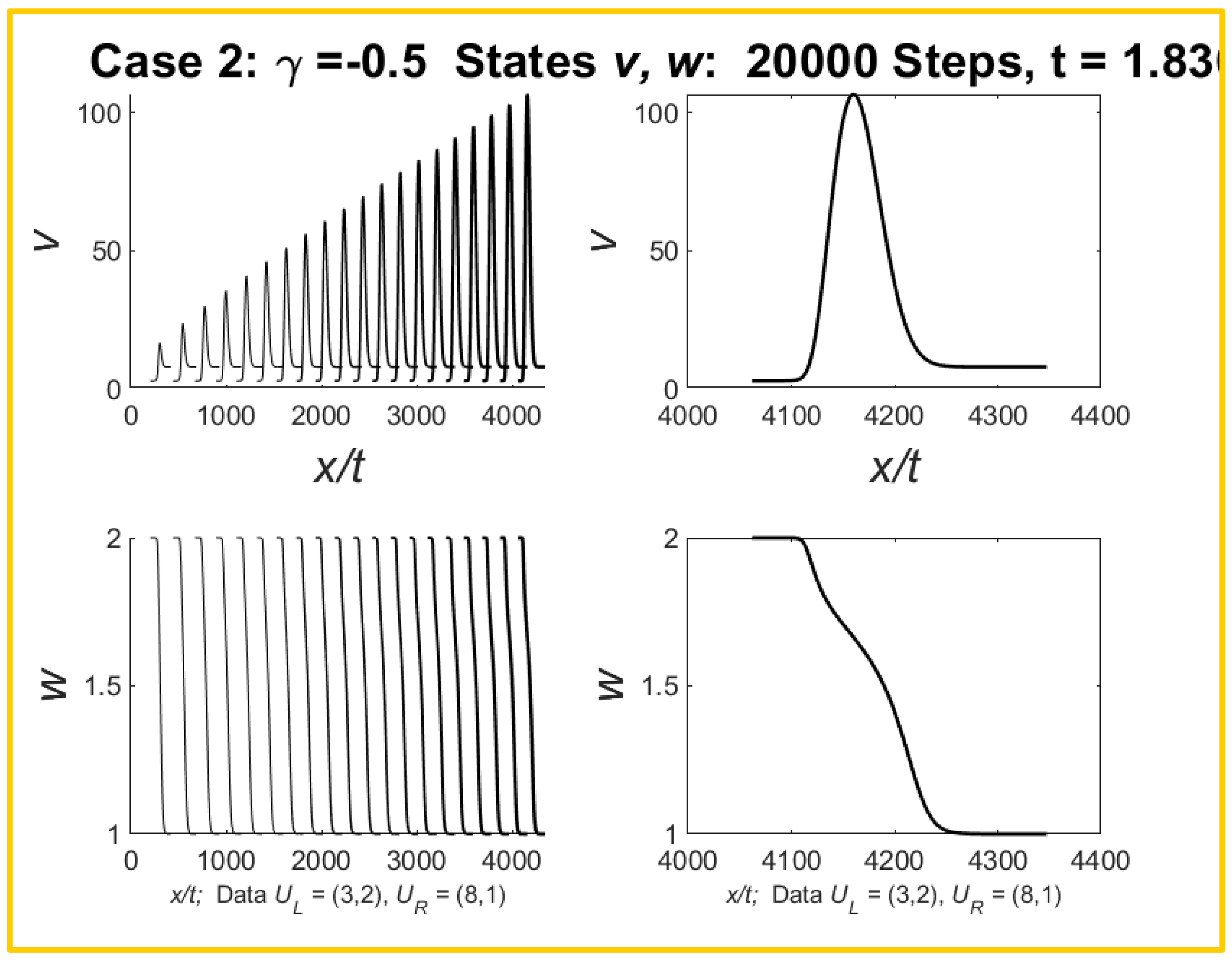
- Both points are initially $S_1$ $C_2$
- $C_2$ suffers heavily from diffusion
- Small time simulated with k = -0.01

**Large Timescale on Right**

- Point above $S_1$ shifts to $R_2$ $C_2$
- Movement in W is very small during $R_2$
- $R_2$ always appears to take priority over $C_2$

- Bottom point transforms into a delta-shock in large timescale
- Delta behavior appears only in V for all cases and all points in delta-shock regions




## Discussion and Future Work

- Time-dependent numerical methods can be used for time-dependent fluxes given parameter restrictions that slow convergence
- Future work involves less diffusive, higher order schemes

## Conclusion

- Successfully approximated physical weak solutions to the expanded varying Chapylgin gas system in time
- Validated analytical solutions derived by other members of our REU (see paired poster)
- Studying the aforementioned system offers insights into the mechanics of the universe that we reside in

## References and Acknowledgements


- Leveque. (2013). Numerical methods for conservation laws. Birkhäuser.
- Li, S. (2023). Delta-shocks for a 2 × 2 balance system of Keyfitz–Kranzer type with varying Chaplygin gas. Physics of Fluids, 35(7). https://doi.org/10.1063/5.0156662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