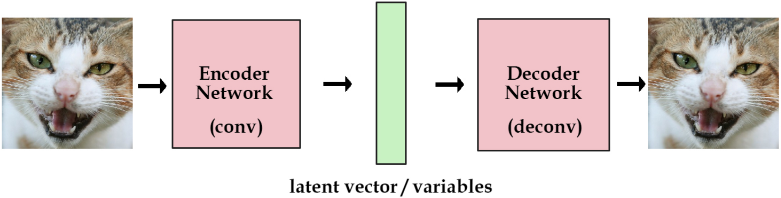
Variational Autoencoder

Introduction - Autoencoders

- 

- Attempt to learn identity function
- Constrained in some way (e.g., small latent vector representation)
- Can generate new images by giving different latent vectors to trained network
- Variational: use probabilistic latent encoding

Deep Learning Perspective

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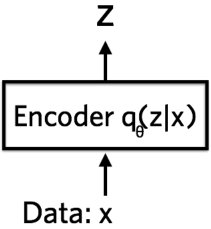
- 

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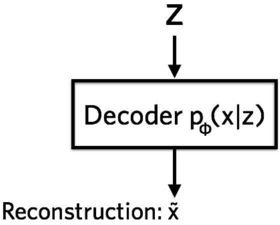
- 

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- Measures how effectively the decoder has learned to reconstruct x given the latent representation z

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- Loss for datapoint x_i:

$$l_i(\theta, \phi) = -\mathbb{E}_{z \sim q_\theta(z|x_i)}\big[\log p_\phi(x_i|z)\big] + KL\big(q_\theta(z|x_i)||p(z)\big)$$

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- Expectation taken over distribution of latent representations

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- KL Divergence as regularizer:

$$KL\big(q_\theta(z|x_i)||p(z)\big) = \mathbb{E}_{z\sim q_\theta(z|x_i)}\big[\log q_\theta(z|x_i) - \log p(z)\big]$$

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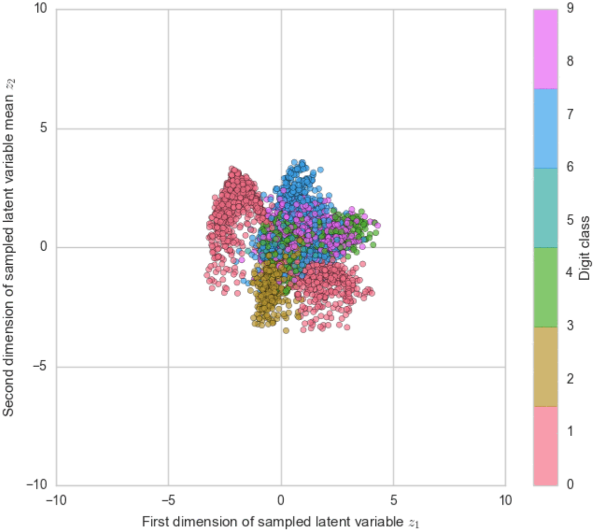
- Representation for images of the same digit are close together in latent space
- Otherwise could "memorize" the data and map each observed datapoint to a distinct region of space

MNIST latent variable space



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- Output of $q_\theta(z|x)$ is vector of μ's and vector of σ's

Summary

- Deep Learning objective is to minimize the loss function:

$$L(\theta, \phi) = \sum_{i=1}^{N} \Big(-\mathbb{E}_{z \sim q_\theta(z|x_i)} \big[\log p_\phi(x_i|z) \big] + KL\big(q_\theta(z|x_i) || p(z)\big) \Big)$$