Analysis of Cross-Correlation Detector for Passive Radar Applications

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***Abstract*—For passive radar target detection, the cross-correlation (CC) based detector is a popular method, which cross-correlates the signal received in a reference channel (RC) and the signal in a surveillance channel (SC). The CC is simple to implement and resembles the optimum matched filter (MF) in idealistic conditions. However, there is limited understanding on its performance in realistic passive sensing environments with non-negligible noise in the RC and direct-path interference in the SC. This paper examines such effects on the detection performance of the CC detector. First, closed-form expressions for the probabilities of false alarm and detection of the CC detector are derived by using a central limit theory based approximation, which are verified with Monte Carlo simulations. Then, we show analytically to what extent the noise in the RC and the direct-path interference in the SC should be suppressed in order to achieve a desired performance loss of the CC detector with respect to the MF. These results are useful in designing practical CC solutions for passive radar sensing.**

I. INTRODUCTION

A passive radar system can detect and track a target of interest by exploiting non-cooperative illuminators of opportunity (IOs), which is of great interest in both civilian and military scenarios due to a number of advantages such as low cost, spatial diversity and availability of many existing IOs [1]–[8]. In passive radars, the locations and waveforms used by the IOs are no longer under control. As such, passive radar systems often require an additional separate channel, referred to as the reference channel (RC), to measure the transmitted signal from the IO to serve as a reference. One of the most popular detection strategies in passive radar is to conduct delay-Doppler cross-correlation (CC) between the data received in the RC and surveillance channel (SC) [1], [9]–[12], which mimics matched-filter (MF) processing in conventional active sensing systems where the transmitted signal is cross-correlated with the received signal. The principal advantages of the CC lie in its simplicity of implementation, and requirement of no prior knowledge of the transmitted waveform.

It is worth noting that under some ideal assumptions, the CC attains the detection performance of the optimum MF which maximizes the output signal-to-noise ratio (SNR). Specifically, the assumptions are (1) the RC is noiseless; and (2) the direct-path from the IO is absent from the SC. In practice, there inevitably exists noise in the RC. Moreover, commercial IOs such as radios and TV stations typically employ isotropic antennas to cover a wide area. Without any pre-processing, the direct-path signal seen in the SC is typically stronger than the target signal by several orders of magnitude [13]. It is therefore necessary to apply some direct-path signal cancellation techniques in the SC before target detection, e.g., by using an adaptive array with a spatial null formed in the IO source direction. Due to array size limitation, the null may not provide adequate direct-path cancellation. As a result, the SC may still see significant direct-path signal residual relative to the target signal strength. Apparently, the existence of the noise in the RC and the direct-path interference in the SC will deteriorate the CC detection performance. However, their impact on the CC detector has not been systematically studied in the open literature. It is unclear to what extent the noise in the RC and the direct-path interference in the SC should be suppressed in order to ensure an acceptable performance loss of the CC with respect to the optimal MF.

The goal of this work is to analyze the CC detector for passive sensing. Let SNR_r denote the SNR in the RC, while the INR_s denotes the direct-path interference-to-noise in the SC. Our main contribution here is to quantitatively analyze the effects of the SNR_r and INR_s on the detection performance of the CC detector. To this end, we first derive approximate expressions for the probability of false alarm (PFA) and probability of detection (PD) of the CC detector by taking into consideration the noise in the RC and the direct-path interference in the SC. Based on these theoretical results, we obtain simple expressions for the SNR_r and INR_s required by the CC detector to achieve a targeted performance loss with respect to the MF detector. Interestingly, it is found that there exists an upper bound for the INR_s above which it is impossible for the CC detector to achieve the targeted performance loss, no matter how clean the reference signal is. In addition, there exists a lower bound for the SNR_r, below which it is impossible to ensure the targeted performance of the CC detector. Monte Carlo (MC) simulations are provided to confirm the theoretical analysis.

II. SIGNAL MODEL

Consider a passive bistatic radar system as shown in Fig. 1. Denote by $x_\mathrm{s}(n)$ the signal received in the SC, which involves noise, a direct-path signal (i.e., interference) from the IO, and the echo of a target of interest, i.e.,

$$x_\mathrm{s}(n) = \gamma p(n) + \alpha p(n-\tau)\exp(j\Omega_d n) + w(n), \quad (1)$$

978-1-4799-8232-5/151$31.00@2015IEEE

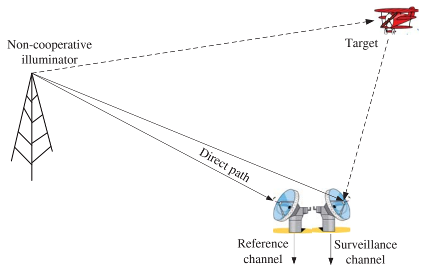

Fig. 1. Configuration of a passive radar system.

where $p(n)$ is the signal transmitted by the non-cooperative IO, γ is a scaling parameter accounting for the channel propagation effects of the direct path from the IO to the receive antenna in the SC, τ is the propagation delay of the target return relative to the direct path, α is a scaling parameter accounting for the target reflectivity as well as the channel propagation effects, Ω_d is a normalized Doppler frequency, and $w(n)$ denotes thermal noise modeled as identically and independently distributed (i.i.d.) circular complex Gaussian with zero mean and variance σ_w^2, i.e., $w(n) \sim \mathcal{CN}(0, \sigma_w^2)$. Unlike [9], [12], where the direct-path interference is assumed to be fully suppressed , we consider a more realistic scenario with direct-path residual due to imperfect interference mitigation.

The RC usually employs a directional antenna pointing toward the IO, and its received signal can be written as

$$x_\text{r}(n) = \beta p(n) + v(n), \tag{2}$$

where β is a scaling parameter accounting for the channel propagation effects from the IO to the receive antenna in the RC, and $v(n)$ is i.i.d. circular complex Gaussian thermal noise with zero mean and variance σ_v^2, i.e., $v(n) \sim \mathcal{CN}(0, \sigma_v^2)$. It is reasonable to assume that the thermal noise $v(n)$ and $w(n)$ are independent.

Let the null hypothesis (H_0) be such that the data in the SC is free of target echoes whereas the alternative hypothesis (H_1) be the opposite. Hence, the passive detection problem can be formulated in terms of the following binary hypothesis test

$$\begin{cases} H_0 : \begin{cases} x_\text{r}(n) = \beta p(n) + v(n), \\ x_\text{s}(n) = \gamma p(n) + w(n), \end{cases} \\ H_1 : \begin{cases} x_\text{r}(n) = \beta p(n) + v(n), \\ x_\text{s}(n) = \gamma p(n) + \alpha p(n-\tau) \exp(j\Omega_d n) + w(n). \end{cases} \end{cases} \tag{3}$$

III. Analysis of the CC Detector

A popular solution for the above passive detection problem is the CC detector given by

$$T_\text{CC} = |\bar{T}|^2 = \left| \sum_{n=0}^{N-1} T_n \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{4}$$

where $T_n = x_\text{s}^*(n) x_\text{r}(n-\tau) \exp(j\Omega_d n)$, N is integration time, λ is the detection threshold, $|\cdot|$ represents the modulus of a complex number, and the superscript $(\cdot)^*$ is the conjugate operation. In other words, the RC signal $x_\text{r}(n)$ is delay- and Doppler-compensated, before it is cross-correlated with the SC signal $x_\text{s}(n)$. This resembles the MF in active radar, except that the latter uses the noiseless waveform $p(n)$ instead of $x_\text{r}(n)$ for processing. The delay τ and Doppler Ω_d are generally unknown in practice. A standard approach for CC or MF implementation is to divide the uncertainty region of the target delay and Doppler frequency into small cells and the test is run on each cell with a given delay and Doppler frequency.

It is well-known that the MF is the optimum detector in active radar. The MF performance can be thought of as an upper bound for passive detection when the RC noise and SC direct-path interference vanish. An important question is, how far is the CC detector away from the MF bound in typical passive radar environments where the noise in the RC and the direct-path interference in the SC cannot be neglected? To the best of our knowledge, the problem has not be addressed in the open literature.

To answer the above question, we derive closed-form expressions for the PFA and PD of the CC detector by using a Gaussian distribution to approximate the distribution of $\bar{T}$ based on the central limit theorem (CLT). Although the Gaussian approximation is standard, it leads to a simple and accurate result which can be used to answer the previous question. In addition, it provides insights as to how clean the RC signal should be made relative to its noise level, as well as to what extent the direct-path interference in the SC should be mitigated, in order for the CC to be reasonably close to the MF bound. Such insights are useful to design practical CC solutions for passive radars.

Under H_1, we have

$$\begin{aligned} T_n \;=\;& \gamma^* \beta p^*(n) p(n-\tau) \exp(j\Omega_d n) \\ & + \gamma^* p^*(n) v(n-\tau) \exp(j\Omega_d n) \\ & + \alpha^* \beta |p(n-\tau)|^2 + \alpha^* p^*(n-\tau) v(n-\tau) \\ & + \beta p(n-\tau) w(n) \exp(j\Omega_d n) \\ & + w(n) v(n-\tau) \exp(j\Omega_d n). \end{aligned} \tag{5}$$

Assume that $p(n)$ are i.i.d. random variables with zero mean and unit variance[1]. Then, we have $\text{E}\{|p(n-\tau)|^2\} = 1$, and

$$\text{Var}\{|p(n-\tau)|^2\} = \text{E}\{|p(n-\tau)|^4\} - 1 \triangleq \phi, \tag{6}$$

where $\text{E}\{\cdot\}$ and $\text{Var}\{\cdot\}$ denote the mean and variance of a random variable, respectively. As a result, $\text{E}\{T_n\} = \alpha^* \beta$, and

$$\begin{aligned} \text{Var}\{T_n\} \;=\;& |\gamma|^2 |\beta|^2 + |\gamma|^2 \sigma_v^2 + \phi |\alpha|^2 |\beta|^2 + |\alpha|^2 \sigma_v^2 \\ & + |\beta|^2 \sigma_w^2 + \sigma_v^2 \sigma_w^2. \end{aligned} \tag{7}$$

The correlation between T_n can be neglected since the integration time N is usually large in passive radars [9]. Then, the mean μ_1 and variance σ_1^2 of $\bar{T}$ under H_1 are $\mu_1 = N\alpha^*\beta$

[1]Note that the actual variance of $p(n)$ can be absorbed into the foregoing scaling parameters.

and

$$\sigma_1^2 = N(|\gamma|^2|\beta|^2+|\gamma|^2\sigma_v^2+\phi|\alpha|^2|\beta|^2+|\alpha|^2\sigma_v^2+|\beta|^2\sigma_w^2+\sigma_v^2\sigma_w^2), \tag{8}$$

respectively. It is straightforward that the mean μ_0 and variance σ_0^2 of $\bar{T}$ under H_0 can be obtained by setting $\alpha = 0$ under H_1, i.e., $\mu_0 = 0$ and

$$\sigma_0^2 = N(|\gamma|^2|\beta|^2 + |\gamma|^2\sigma_v^2 + |\beta|^2\sigma_w^2 + \sigma_v^2\sigma_w^2). \tag{9}$$

Hence, the distributions of $\bar{T}$ under H_0 and H_1 can be approximated by $\mathcal{CN}(\mu_0, \sigma_0^2)$ and $\mathcal{CN}(\mu_1, \sigma_1^2)$ due to the CLT, respectively. It follows that the PFA of the CC detector is

$$P_{\text{FA}} = \exp\left(-\frac{\lambda}{\sigma_0^2}\right). \tag{10}$$

Accordingly, the detection threshold for a given P_{FA} is $\lambda = \sigma_0^2 \ln(P_{\text{FA}}^{-1})$. In addition, the PD of the CC detector is

$$P_{\text{D}} = Q_1\left(\sqrt{\frac{2|\mu_1|^2}{\sigma_1^2}}, \sqrt{\frac{2\sigma_0^2\ln(P_{\text{FA}}^{-1})}{\sigma_1^2}}\right), \tag{11}$$

where $Q_m(\cdot,\cdot)$ is the generalized Marcum Q-function of order m [14].

It is noted that the above analysis include the MF filter in active radars as a special case just by setting $\sigma_v^2 = 0$ and $\gamma = 0$. As noted earlier, the active MF performance can be thought of as an upper bound that cannot be achieved by any detectors in the passive detection set-up. Now, we investigate the performance loss of the passive CC detector with respect to the active MF detector. Let SNR_s be the ratio of the target returns' power to the noise power in the SC, which is defined by

$$\text{SNR}_\text{s} = 10\log_{10}\frac{|\alpha|^2}{\sigma_w^2}. \tag{12}$$

We also define

$$\text{SNR}_\text{r} = 10\log_{10}\frac{|\beta|^2}{\sigma_v^2}, \quad \text{INR}_\text{s} = 10\log_{10}\frac{|\gamma|^2}{\sigma_w^2}. \tag{13}$$

Theorem 3.1: To ensure a performance loss no more than δ dB in the CC detector with respect to the optimal MF detector for a given SNR_s, the INR_s in the SC must satisfy (in decibels) for a given SNR_r,

$$\text{INR}_\text{s} \leq 10\log_{10}\left[\frac{10^{\frac{\delta+\text{SNR}_\text{r}}{10}} - 10^{\frac{\delta+\text{SNR}_\text{s}}{10}} - 10^{\frac{\text{SNR}_\text{r}}{10}} - 1}{1 + 10^{\frac{\text{SNR}_\text{r}}{10}}}\right]. \tag{14}$$

Meanwhile, the SNR_r in the RC must satisfy (in decibels) for a given INR_s,

$$\text{SNR}_\text{r} \geq 10\log_{10}\left[\frac{10^{\frac{\text{INR}_\text{s}}{10}} + 10^{\frac{\delta+\text{SNR}_\text{s}}{10}} + 1}{10^{\frac{\delta}{10}} - 10^{\frac{\text{INR}_\text{s}}{10}} - 1}\right]. \tag{15}$$

Proof: For a given probability of false alarm P_{FA}, we can obtain from (11) that the PD of the MF detector for a given SNR_s is

$$P_\text{D}^{\text{MF}}(\text{SNR}_\text{s}) = Q_1\left(\sqrt{\eta_{\text{MF}}(\text{SNR}_\text{s})}, \sqrt{\zeta_{\text{MF}}(\text{SNR}_\text{s})}\right), \tag{16}$$

where

$$\eta_{\text{MF}}(\text{SNR}_\text{s}) = \frac{2N\cdot 10^{\frac{\text{SNR}_\text{s}}{10}}}{\phi\cdot 10^{\frac{\text{SNR}_\text{s}}{10}}+1}, \;\; \zeta_{\text{MF}}(\text{SNR}_\text{s}) = \frac{2\ln(P_{\text{FA}}^{-1})}{\phi\cdot 10^{\frac{\text{SNR}_\text{s}}{10}}+1}. \tag{17}$$

Similarly, for a given probability of false alarm P_{FA}, the PD of the CC detector is

$$P_\text{D}^{\text{CC}}(\text{SNR}_\text{s}) = Q_1\left(\sqrt{\eta_{\text{CC}}(\text{SNR}_\text{s})}, \sqrt{\zeta_{\text{CC}}(\text{SNR}_\text{s})}\right), \tag{18}$$

where $\eta_{\text{CC}}(\text{SNR}_\text{s})$ and $\zeta_{\text{CC}}(\text{SNR}_\text{s})$ are defined in (19) and (20), and are shown on the top of the next page, respectively.

In order to achieve a δ dB loss in the performance of the CC detector with respect to the MF detector, i.e., $P_\text{D}^{\text{CC}}(\text{SNR}_\text{s}+\delta) = P_\text{D}^{\text{MF}}(\text{SNR}_\text{s})$, it suffices that

$$\eta_{\text{CC}}(\text{SNR}_\text{s}+\delta) = \eta_{\text{MF}}(\text{SNR}_\text{s}), \tag{21}$$

which automatically means $\zeta_{\text{CC}}(\text{SNR}_\text{s}+\delta) = \zeta_{\text{MF}}(\text{SNR}_\text{s})$. After simplifying (21), we can obtain (14) and (15) in equalities. Obviously, the less the INR_s (and/or the more the SNR_r), the better the PD. Therefore, the inequalities in (14) and (15) follow to ensure a loss no larger than δ dB from the MF bound. The proof is completed. ■

Remarks: Eq. (14) gives the maximum INR_s to achieve a loss within δ dB for given SNR_r and SNR_s. In turn, Eq. (15) gives the minimum SNR_r to achieve a loss within δ dB for given INR_s and SNR_s.

Corollary 1: To ensure a performance loss no more than δ dB in the CC detector with respect to the optimal MF detector for a given SNR_s, the upper bound for the INR_s in the SC is

$$\text{INR}_\text{s} < 10\log_{10}\left(10^{\frac{\delta}{10}} - 1\right), \tag{22}$$

and the lower bound for the SNR_r in the RC is

$$\text{SNR}_\text{r} > 10\log_{10}\left(\frac{10^{\frac{\delta+\text{SNR}_\text{s}}{10}}+1}{10^{\frac{\delta}{10}}-1}\right). \tag{23}$$

Proof: Since the quantity in the bracket of the right-hand side of (15) must be positive, the condition (22) holds. Similarly, (23) must be satisfied in order to ensure the positiveness in the bracket of the right-hand side of (14). ■

Remarks: Eq. (23) specifies to the minimum how clean the reference signal is required to be. If (23) is not met, namely, the reference signal is not sufficiently clean, the performance loss of the CC detector with respect to the MF detector would exceed δ dB, however small the INR_s is in the SC. Meanwhile, (22) specifies to what extent the direct-path signal (i.e., the interference) in the SC should be suppressed at a minimum. If the INR_s in the SC does not satisfy (22), namely, the direct-path interference is not adequately cancelled, the loss would be greater than δ dB, no matter how large the SNR_r in the RC is. Finally, (22) also implies that the upper ceiling of the INR_s in the SC is determined only by the loss value δ. It is irrelevant to the sample number and the specific value of SNR_s.

$$\eta_{\mathrm{CC}}(\mathrm{SNR_s}) \triangleq \frac{2|\mu_1|^2}{\sigma_1^2} = \frac{2N \cdot 10^{\frac{\mathrm{SNR_s}+\mathrm{SNR_r}}{10}}}{10^{\frac{\mathrm{INR_s}+\mathrm{SNR_r}}{10}} + 10^{\frac{\mathrm{INR_s}}{10}} + \phi \cdot 10^{\frac{\mathrm{SNR_s}+\mathrm{SNR_r}}{10}} + 10^{\frac{\mathrm{SNR_s}}{10}} + 10^{\frac{\mathrm{SNR_r}}{10}} + 1}. \tag{19}$$

$$\zeta_{\mathrm{CC}}(\mathrm{SNR_s}) \triangleq \frac{2\sigma_0^2 \ln(P_{\mathrm{FA}}^{-1})}{\sigma_1^2} = \frac{2\ln(P_{\mathrm{FA}}^{-1})\left(10^{\frac{\mathrm{INR_s}+\mathrm{SNR_r}}{10}} + 10^{\frac{\mathrm{INR_s}}{10}} + 10^{\frac{\mathrm{SNR_r}}{10}} + 1\right)}{10^{\frac{\mathrm{INR_s}+\mathrm{SNR_r}}{10}} + 10^{\frac{\mathrm{INR_s}}{10}} + \phi \cdot 10^{\frac{\mathrm{SNR_s}+\mathrm{SNR_r}}{10}} + 10^{\frac{\mathrm{SNR_s}}{10}} + 10^{\frac{\mathrm{SNR_r}}{10}} + 1}. \tag{20}$$

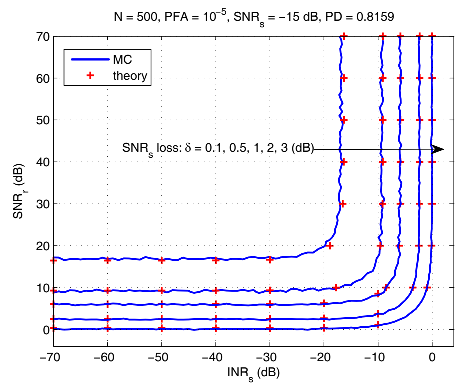

Fig. 2. Contour of the PD of the CC detector with different $\mathrm{SNR_s}$ loss with respect to the MF detector. The lines denotes MC simulation results, and the symbols "+" are the results obtained with the analytical expression in Theorem 3.1 (by making (14) or (15) an equality).

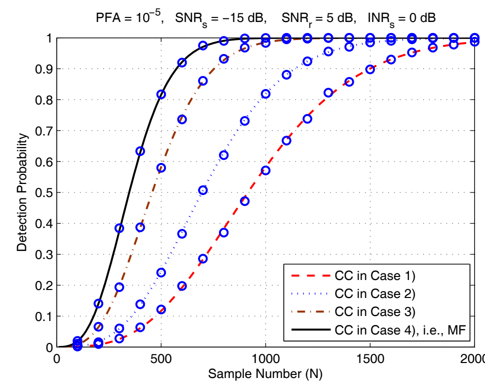

Fig. 3. Detection performance versus N. The symbols "○" denote the MC results, and the lines denote the results obtained using (11).

IV. Numerical Results

In this section, numerical simulations are conducted to confirm the validity of the above theoretical results. The PFA is set to be 10^{-5}. The transmitted signal $p(n)$ is sampled from the circular complex normal Gaussian distribution. In this case, it is easy to obtain that the parameter ϕ defined in (6) is 2. The combined effects of the noise in the RC and the direct-path interference in the SC on the performance of the CC detector with $N = 500$ is examined. The PD of the MF detector at the given $\mathrm{SNR_s} = -15$ dB is 0.8159. In order to achieve the same PD for the CC detector with a certain loss δ dB in the $\mathrm{SNR_s}$, i.e., $P_{\mathrm{D}}^{\mathrm{CC}}(\mathrm{SNR_s}+\delta) = P_{\mathrm{D}}^{\mathrm{MF}}(\mathrm{SNR_s})$, the requirements on $\mathrm{SNR_r}$ and $\mathrm{INR_s}$ are illustrated in Fig. 2. Inspections of these results highlight that there exists a lower floor for $\mathrm{SNR_r}$. For example, the $\mathrm{SNR_s}$ loss of the CC detector with respect to the MF detector would always be larger than 3 dB, if the value of $\mathrm{SNR_r}$ is below the lower floor (i.e., 0.2863 dB[2] in this example), no matter how small the $\mathrm{INR_s}$ in the SC is. Similarly, there exists a ceiling for the $\mathrm{INR_s}$ in the SC. For example, the loss exceeds 3 dB if the $\mathrm{INR_s}$ in the SC is greater than -0.0206 dB[3], no matter how high the $\mathrm{SNR_r}$ in the RC is. The above findings indicate that to ensure a targeted CC performance, we need to simultaneously clean up the reference signal in the RC and suppress the direct-path interference in the SC. Care to only one of the two requirements is not enough to guarantee good performance of the CC detector.

In Fig. 3, the detection performance versus the integration time N is examined for four cases: 1) with both noise in the RC and the direct-path interference in the SC; 2) no noise in the RC but with the direct-path interference in the SC; 3) no direct-path interference in the SC but with noise in the RC; 4) no noise in the RC, no direct-path interference in the SC; The dashed, dotted, dashed-dotted and solid lines denote the performance of the CC detector in Cases 1)–4), respectively. Note that the CC detector in Case 4) corresponds to the optimal MF detector whose performance serves as a benchmark. It can be seen that the existence of the noise in the RC and/or the direct-path interference in the SC leads to a notable loss in the detection performance. In particular, to achieve the same $\mathrm{PD} = 0.9$, the difference of the integration time required by the CC detector in Case 1) and the MF detector in Case 4) is as large as 950, which nearly doubles the integration time in this example. This is to say, by taking into consideration the noise in the RC and the direct-path interference in the SC, the CC detector needs approximately 3 times as much integration time as the MF detector in order to achieve the same PD. It signifies again the necessity of taking into account the noise in the RC and the direct-path interference in the SC during the design of passive detection systems.

V. Conclusion

In this letter, we derived approximate expressions for the PFA and PD of the CC detector in the presence of noise in the RC and the direct-path interference in the SC. The

[2]This value is predicted by computing the right-hand side of (23) with $\mathrm{SNR_s} = -15$ dB and $\delta = 3$ dB.

[3]This value is predicted by computing the right-hand side of (22) with $\delta = 3$ dB.

effects of noise in the RC and the direct-path interference in the SC on the performance of the CC detector have been evaluated analytically. These theoretical results enable designers to determine to what extent the noise in the RC and the direct-path interference in the SC should be mitigated in order to achieve a given performance loss of the CC detector with respect to the optimal MF detector. Our future work will focus on the development of new detection algorithms in passive radar by taking into account the noise in the RC and the direct-path interference in the SC, if they are non-negligible.

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