PERFORMANCE ANALYSIS OF THE BATCHES ALGORITHM FOR RANGE-DOPPLER MAP FORMATION IN PASSIVE BISTATIC RADAR

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Abstract

Passive Bistatic Radar (PBR), or Passive Coherent Location, is gaining interest in the radar community as it provides some advantages with respect to active radar. Although, passive radar does not aim to replace active radar, it provides a good complement to it. The computational effort that is required to implement the required signal processing is one of the drawbacks that affect passive radar. In this paper a computationally affordable algorithm is proposed and its performance tested by using a numerical simulation.

1 Introduction

Passive Bistatic Radar (PBR) is gradually making progress both in terms of signal processing and system development (demonstrators and prototypes). Several e.m. sources, i.e. Illuminator of Opportunity (IO), can be used to implement such a radar system. Ground-based broadcast transmitters such as FM and DAB radio, as well as analogue and digital TV (DVB-T) have been largely exploited to demonstrate the capability to perform air and coastal surveillance [1].

The standard signal processing adopted in PBR makes use of the two-dimensional Cross Ambiguity Function (CAF), which provides a Range-Doppler (RD) map. The CAF serves the following purposes:

1) to provide the necessary signal processing gain to allow detecting targets (matched filter);
2) to estimate target bistatic range and Doppler shifts.

The evaluation of the CAF can be computationally expensive if we consider that large 2D range-Doppler maps might be required depending on the desired surveillance area extent and the resolution, both along the range and the Doppler coordinates. Moreover, a long integration time is usually required in order to achieve a desirable signal processing gain to allow the detection of small and/or distant targets.

The main challenge faced in this work is the definition of a sub-optimum implementation of the CAF in order to achieve real time processing capabilities.

Several algorithms have been proposed in the literature to compute the CAF [2-4]. This paper will focus on one of such algorithms, namely the *like-FMCW algorithm* or *batches algorithm*, which was presented in [2]. Specifically, in [2], a particular example for FM waveform has been presented. This work proposes a new generalized formulation of the batches algorithm that is independent of the type of waveform. A comparative study between optimum and sub optimum algorithms is presented, both in terms of computational load, processing time and SNR loss. The paper is organized as follows. The theoretical definition of the cross correlation function and the main drawbacks of the optimum algorithms are analysed in Section 2. Section 3 deals with the newly proposed formulation of the sub-optimum "batches algorithm". In Section 4 some numerical results are presented followed by conclusions in Section 5.

2 CAF calculation

The theoretical CAF can be defined as [5]:

$$M(\tau,\nu) = \int_0^{T_{int}} x_{surv}(t)\, x_{ref}^*(t-\tau)\, e^{-j2\pi\nu t}\, dt \qquad (1)$$

$$0 \le \tau \le \tau_{max} \quad -\nu_{max} \le \nu \le \nu_{max}$$

where $M(\tau,\nu)$ represents the range-Doppler cross correlation function between the reference signal $x_{ref}(t)$ and the surveillance signal $x_{surv}(t)$, the variable τ denotes the time delay, corresponding to the bistatic time difference of arrival, τ_{max} is the maximum delay of interest and it is related to the maximum non ambiguous bistatic range, ν corresponds to the frequency Doppler shift of interest, ν_{max} is the maximum Doppler shift of interest and it is related to the maximum bistatic velocity of interest, T_{int} denotes the integration time or the so called Coherent Processing Interval (CPI). The integration time is typically chosen equal to $T_{int} = T_{obs} + \tau_{max}$, where T_{obs} is the length of the reference signal, in order to have no integration losses. The most obvious way to view the cross correlation processing would be to calculate the Fourier Transform of the signal $x_{surv}(t)x_{ref}^*(t-\tau)$, known in literature as the mixing product, for each bistatic time delay. The second way to view the cross correlation processing would be to calculate the cross correlation between $x_{surv}(t)$ and $x_{ref}(t)e^{j2\pi\nu t}$ for each Doppler shift. An efficient implementation of these approaches can be obtained by exploiting the well known FFT algorithm[2].

Although the FFT can reduce the computational load of the cross ambiguity function computation, several drawbacks of this approach should be underlined. First of all in the case of long integration time the FFT calculation for long sequences can be impractical. Secondly the FFT algorithm considers the Doppler frequencies with respect to the sampling frequency and typically the maximum Doppler shift is less than the sampling frequency (i.e. $\nu_{\max} << f_s$ where f_s is the sampling frequency). Therefore the majority of the FFT points are discarded.

3 Batches Algorithm

3.1 Theoretical analysis

The reference signal $x_{ref}(t)$ can be seen as the sum of n_B contiguous batches of length T_B and it can be written as

$$x_{ref}(t) = \sum_{i=0}^{n_B-1} x_i(t - iT_B) \tag{2}$$

where the number of batches n_B can be obtained by

$$\left\lfloor \frac{T_{obs}}{T_B} \right\rfloor \tag{3}$$

The signal belonged to each block is defined as

$$x_i(t) = x_{ref}(t + iT_B)\, q(t) \tag{4}$$

where $x_{ref}(t)$ is the transmitted signal of opportunity and $q(t)$ is defined as

$$q(t) = \begin{cases} 1 & t \in [0, T_B] \\ 0 & otherwise \end{cases} \tag{5}$$

Substituting Equation (2) into Equation (1), the CAF can be written as

$$M(\tau,\nu) = \sum_{i=0}^{n_B-1} \int_0^{T_{\text{int}}} x_{surv}(t) x_i^*(t - \tau - iT_B) e^{-j2\pi\nu t} dt \tag{6}$$

Starting from the following equation

$$x_i^*(t - \tau - iT_B) \neq 0 \quad \forall\, t \in [iT_B + \tau, iT_B + \tau + T_B] \tag{7}$$

we can modify the integral in Equation (6) as

$$M(\tau,\nu) = \sum_{i=0}^{n_B-1} \int_{iT_B+\tau}^{iT_B+\tau+T_B} x_{surv}(t) x_i^*(t - \tau - iT_B) e^{-j2\pi\nu t} dt \tag{8}$$

Then, with a change of variable $\alpha = t - iT_B$, it now follows that Equation (8) yields:

$$M(\tau,\nu) = \sum_{i=0}^{n_B-1} e^{-j2\pi\nu iT_B} \int_{\tau}^{\tau+T_B} x_{surv}(\alpha + iT_B) x_i^*(\alpha - \tau) e^{-j2\pi\nu\alpha} d\alpha \tag{9}$$

Because τ is in the range *0* to τ_{max}, the limits of integration, expressed in Equation (9), become *0* and $T_B+\tau_{max}$. In addition, Equation (8) can be reformulated as

$$M(\tau,\nu) = \sum_{i=0}^{n_B-1} e^{-j2\pi\nu iT_B} \int_0^{T_B+\tau_{\max}} x_i^{surv}(\alpha) x_i^*(\alpha - \tau) e^{-j2\pi\nu\alpha} d\alpha \tag{10}$$

where $x_i^{surv}(t)$ is the i-th block of the surveillance channel, $x_{surv}(t)$, and it is defined as

$$x_i^{surv}(t) = x_{surv}(t + iT_B)\, g(t) \tag{11}$$

where $g(t)$ is

$$g(t) = \begin{cases} 1 & t \in [0, T_B + \tau_{\max}] \\ 0 & otherwise \end{cases} \tag{12}$$

As it is possible to note from Equation (11) and Figure 1, the blocks on the surveillance channel are partially overlapped. By considering $x_i^{surv}(t)$ and $x_i(t)$, the CAF can be defined as

$$\chi_i(\tau,\nu) = \int_{-\infty}^{+\infty} x_i^{surv}(t) x_i^*(t - \tau) e^{-i2\pi\nu t} dt \tag{13}$$

Thus, substituting Equation (13) into Equation (10) we obtain

$$M(\tau,\nu) = \sum_{i=0}^{n_B-1} e^{-j2\pi\nu iT_B} \chi_i(\tau,\nu) \tag{14}$$

where $0 \le \tau \le \tau_{\max}$ and $-\nu_{\max} \le \nu \le \nu_{\max}$

The cross ambiguity function between $x_{surv}(t)$ and $x_{ref}(t)$ can be seen as a weighed sum of the ambiguity functions calculated within each batch.

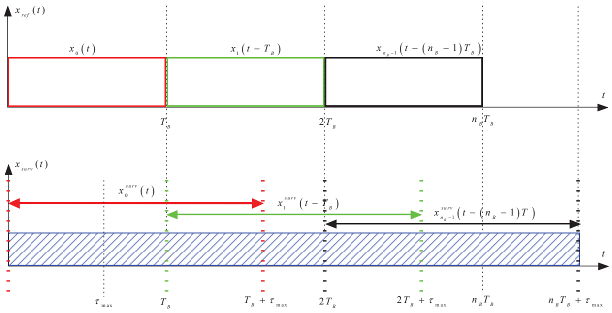

Figure 1: Reference and surveillance signals segmentation

In Equation (10), when the product between T_B and the maximum target Doppler $\nu_{\max}$ is small compared to unity, we can approximate the phase rotation within each block as a constant

$$e^{-j2\pi\nu\alpha} \approx e^{-j2\pi\nu T_B} \quad \forall\, \nu \in [-\nu_{\max}, \nu_{\max}] \tag{15}$$

and Equation (10) can be simplified as

$$M_b(\tau,\nu) = e^{-j2\pi\nu T_B} \sum_{i=0}^{n_B-1} e^{-j2\pi\nu iT_B} \int_0^{T_B+\tau_{\max}} x_i^{surv}(\alpha) x_i^*(\alpha - \tau) d\alpha \tag{16}$$

where the subscript “b” stands for “batches algorithm”.

Defining the cross correlation within the i-th block as

$$x_{cc}^i(\tau) = \int_0^{T_B+\tau_{\max}} x_i^{surv}(\alpha) x_i^*(\alpha - \tau) d\alpha \tag{17}$$

we may write the Equation (10) as

$$M_b(\tau,\nu) = e^{-j2\pi\nu T_B} \sum_{i=0}^{n_B-1} e^{-j2\pi\nu iT_B} x_{cc}^i(\tau) \tag{18}$$

The Doppler frequency compensation is neglected inside each block while it is considered with respect to consecutive blocks.

The main steps of this approach, schematically shown in Figure 2, can be summarized as:

- Select n_B consecutive batches of the reference channel $x_i(t)$ (see Equation (4)) , and the surveillance channel $x_i^{surv}(t)$ (see Equation (11))
- Iteratively, for i=0,…, $n_B - 1$:
  - Calculate the cross correlation, defined as $x_{cc}^{i}(\tau)$, between $x_i(t)$ and $x_i^{surv}(t)$ (see Equation (17))
- The Doppler dimension is then obtained by performing a FFT over the cross correlation values for each time delay, τ , or range bin

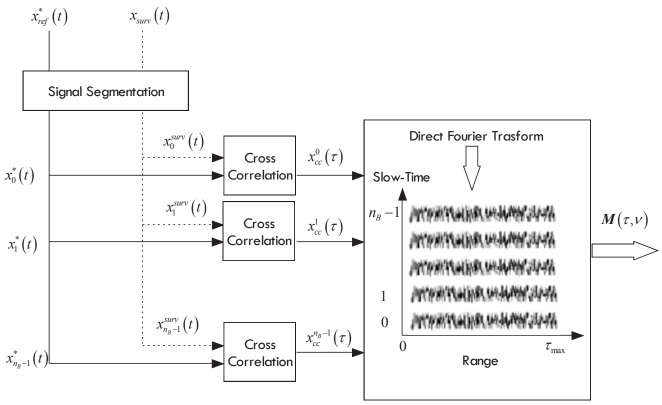

Figure 2: Reference and surveillance signals segmentation

3.2 Numerical implementation

The numerical version of Equation (16) can be written as

$$M_b[m,p] = \sum_{i=0}^{n_B-1} e^{-j2\pi\frac{m}{n_B}i} \sum_{n=0}^{L_B-1} x_i^{surv}[n]\, x_i^{*}[n-p] \qquad (19)$$
$$0 \le p \le N_{delay} - 1;\ 0 \le m \le N_{doppler} - 1$$

where $x_i^{surv}[n]$ and $x_i[n]$ are respectively the samples at the time $t = \frac{n}{f_s}$ of the surveillance and the reference signals belonged to the i-th block, L_B is the number of samples per batch corresponding to $\lfloor (T_B + \tau_{max}) f_s \rfloor$, p represents the range bin ($\tau = p / f_s$), m is the Doppler bin corresponding to $\nu = \frac{m}{N} f_s$, N_{delay} is the number of bistatic range bins corresponding to $\lfloor \tau_{max} f_s \rfloor$, $N_{Doppler}$ is the number of frequency Doppler bins corresponding to $\left\lfloor \frac{2\nu_{max} N}{f_s} \right\rfloor$.

In accordance with the small Doppler approximation, in Equation (19) we have neglected the exponential term in the inner summation

$$e^{-j2\pi\frac{p}{N}n} \approx 1 \qquad (20)$$

In order to guarantee that the exponential has a phase less than π/10 and to reduce the losses due the approximation given in Equation (16), we can impose:

$$2\pi\nu_{max} T_B \le \frac{\pi}{\gamma} \quad \Rightarrow \quad T_B \le \frac{1}{2\nu_{max}\gamma} \qquad (21)$$

where γ is an integer greater than 10. A detailed analysis of the losses has been conducted in section 4.

The batch length T_B is also related to the maximum unambiguous Doppler frequency by the relation

$$T_B < \frac{1}{2\nu_{max}} \qquad (22)$$

4 Numerical Results

In this section, we discuss the results that have been obtained by considering a 8K DVB-T transmitted signal. The surveillance channel signal has been modelled as

$$x_{surv}(t) = x_{ref}(t - \tau_T)\, e^{i2\pi\nu_T t} + w(t) \qquad (23)$$

where τ_T, ν_T are the actual time delay and Doppler shift of a single slowly fluctuating point target and $w(t)$ is a Gaussian white noise process. Typically, the radar performances are evaluated in terms of signal to noise ratio (SNR). However, the noise power at the output of the optimum and batch algorithm is the same. Therefore, to evaluate the losses of the batches algorithm with respect to the optimum algorithm, we define the following parameter

$$Loss(\tau_T,\nu_T) = 20\log_{10}\left(\left|\frac{M_b(\tau_T,\nu_T)}{M(\tau_T,\nu_T)}\right|\right) \qquad (24)$$

where $M(\tau_T,\nu_T)$ is the target peak calculated with optimum method and $M_b(\tau_T,\nu_T)$ is the target peak calculated with the sub-optimum method. Equation (24) can be evaluated by using Equation (10) and Equation (16) as

$$Loss(\tau_T,\nu_T) = 20 * \log_{10}\left(\left|\sum_{i=0}^{n_B-1}\chi_i(0,-\nu_T) \Big/ \sum_{i=0}^{n_B-1}\chi_i(0,0)\right|\right) \approx$$
$$\approx 20 * \log_{10}\left(\left|\frac{n_B\,\bar{\chi}_i(0,-\nu_T)}{E_i}\right|\right) \qquad (25)$$

We can observe that the losses depend only on the target Doppler frequency and on the shape of the ambiguity function.

A detailed analysis has been conducted by evaluating the losses with respect to the Direct-FFT method. The simulations were useful to find a length of the batch that is able to give a good compromise between losses and processing time.

A total of 48 different range Doppler target positions (τ_T,ν_T) have been considered. For each range Doppler position, 100 realizations have been done. The analysed CAF algorithm has been applied for five different batch lengths. The parameters used during the simulations can be summarized as:

- Sampling frequency, $f_s = 9.14\, MHz$

- $T_{obs} = 250\ ms$
- $\nu_{max} = 1500\ Hz$
- Target Doppler frequency:

$\nu_T = [0,100,200,300,400,500,600,700]\ Hz$

- Target time delay:

$\tau_T = [5.46,16.40,27.34,38.28,49.22,60.16]\ \mu s$

- Batch time:

$T_B = [15.64,31.28,109.38,218.76,333.29]\ \mu s$

Figure 3 shows the processing time for different range bins averaged on 800 realizations. The simulation time has been evaluated with a general purpose 4 core PC equipped with 8 GB of RAM. As it is possible to note, the processing time is strongly reduced (more than 96%) with respect to the Direct FFT method. Moreover the ratio between the target power calculated with both the Direct-FFT method and the Batch method has been shown in Figure 4 and Figure 5. The losses of the target power are negligible along the range (Figure 4). On the contrary, as long as the target Doppler frequency and the batch length increase, the target power decreases (Figure 5).

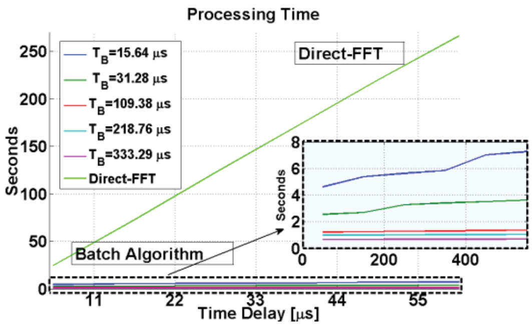

Figure 3: Time processing elapsed for Direct FFT method and batch algorithm

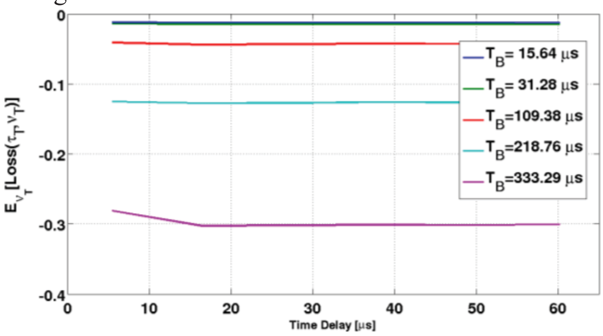

Figure 4: Losses of the target power averaged with respect to the Doppler frequencies

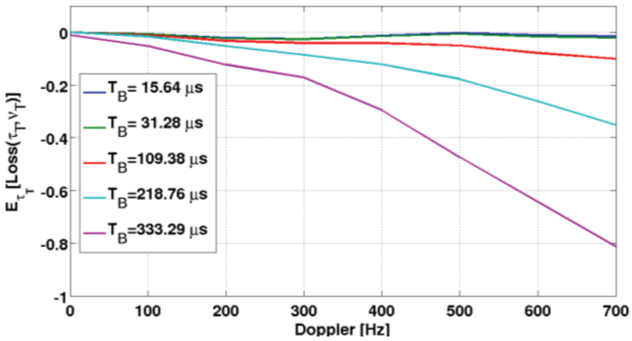

Figure 5: Losses of the target power averaged with respect to the range bin

5 Conclusion

In this paper, we have analysed the batches algorithm for the computation of the CAF, which provides an approximated result. We proposed some modifications to generalize the *like-FMCW approach* proposed in [2]. Simulation results have proven that the proposed algorithm is able to perform comparably with the complete calculation of the CAF whilst strongly reducing the computational effort.

Acknowledgements

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