Comparative Analysis of Chaotic Maps for Image Encryption

1. Introduction

Image encryption is a vital technique for securing visual information. Chaotic maps provide a promising approach due to their **sensitivity to initial conditions** and **pseudo-randomness**. In this study, we compare the encryption performance of three chaotic maps:

1. Henon Map
2. Tent Map
3. Lorenz–Rossler Map

Each method employs **confusion and diffusion** stages to secure the image.

2. Methodology

- **Confusion:** Rearranges pixel positions based on the sorted chaotic sequences.
- **Diffusion:** Alters pixel values using XOR operation with chaotic sequences.
- **Evaluation Metrics:**
    1. **Chi-Square Test** – measures uniformity of pixel intensity distribution. Lower values indicate better uniformity after encryption.
    2. **Correlation Analysis** – measures correlation between adjacent pixels. Lower absolute correlation indicates stronger encryption.
    3. **Timing Analysis** – measures computational efficiency of encryption.

3. Results

3.1 Histogram and Chi-Square Analysis

METHOD	CHANNEL	ORIGINAL IMAGE	ENCRYPTED IMAGE
HENON	Red	629925.93	241.88
	Green	660452.65	260.93
	Blue	616476.20	242.25
TENT	Red	629925.93	219.54
	Green	660452.65	215.62
	Blue	616476.20	230.74
LORENZ–ROSSLER	Red	629925.93	215.04
	Green	660452.65	256.06
	Blue	616476.20	266.21

Analysis:

- All three chaotic maps significantly reduce chi-square values compared to the original image.
- Tent and Lorenz–Rossler maps achieve slightly lower chi-square values than Henon, indicating a **more uniform pixel distribution**.

3.2 Correlation Analysis

METHOD	CHANNEL	HORIZONTAL	VERTICAL	DIAGONAL
ORIGINAL IMAGE	Red	0.9663	0.9568	0.9191
	Green	0.9670	0.9587	0.9221
	Blue	0.9688	0.9608	0.9263
HENON ENCRYPTED	Red	0.0010	0.1565	0.0045
	Green	-0.0052	0.1589	-0.0045
	Blue	-0.0052	0.1542	-0.0061
TENT ENCRYPTED	Red	0.0012	0.0111	0.0018
	Green	-0.0017	0.0060	-0.0010
	Blue	-0.0044	0.0101	-0.0023
LORENZ–ROSSLER ENCRYPTED	Red	-0.0021	0.0031	0.0035
	Green	0.0004	-0.0035	0.0045
	Blue	0.0049	-0.0031	0.0003

Analysis:

- All encrypted images show **near-zero correlation**, demonstrating strong encryption.
- Lorenz–Rossler achieves the lowest correlation values overall, especially in vertical and diagonal directions, indicating slightly better confusion/diffusion performance.
- Henon shows higher vertical correlation in some channels (~0.15), which is slightly weaker than Tent and Lorenz–Rossler.

3.3 Timing Analysis

METHOD	SINGLE RUN (S)	AVERAGE OVER 100 RUNS (S)
HENON	0.0710	0.0731
TENT	0.0768	0.0747
LORENZ–ROSSLER	0.3387	0.3483

Analysis:

- Henon and Tent maps are **fast and comparable** in execution time.
- Lorenz–Rossler is **significantly slower**, due to more complex differential equation calculations.
- If computational efficiency is critical, Henon or Tent may be preferable.

4. Overall Comparison

METRIC	HENON	TENT	LORENZ–ROSSLER
CHI-SQUARE	Moderate	Slightly lower	Slightly lower
CORRELATION	Low	Very low	Lowest overall
TIMING	Fast	Fast	Slow
SECURITY NOTES	Good balance	Efficient	Strongest security

Observations:

- **Security:** Lorenz–Rossler slightly outperforms others in correlation and uniformity, offering stronger encryption.
- **Efficiency:** Henon and Tent maps are much faster and suitable for real-time applications.
- **Trade-off:** There is a trade-off between **maximum security** (Lorenz–Rossler) and **computational efficiency** (Henon/Tent).

5. Hybrid Chaotic Encryption – Experimental Results

To overcome individual weaknesses of single chaotic maps, we propose an **Improved Hybrid Chaotic Image Encryption Scheme**. This method combines:

- **Henon Map**
- **Tent Map**
- **Logistic Map**
- **Sine Map**

Each map contributes unique dynamical behavior:

- Logistic → strong sensitivity
- Henon → 2D chaotic behavior
- Tent → uniform distribution
- Sine → high nonlinearity

These maps are **cascaded and whitened** to produce a highly random hybrid keystream. This keystream is used for **pixel scrambling (confusion)** and **pixel modification (diffusion)**, resulting in a significantly stronger and more secure encryption process.

5.1 Encryption Accuracy and Reversibility

Decryption fully recovers the original image, proving the method is:

- **Lossless**
- **Reversible**
- **Free from error amplification**

5.2 Histogram Uniformity and Chi-Square Analysis

Improved Hybrid – Chi-Square Values

CHANNEL	CHI-SQUARE
RED	244.8823
GREEN	257.2569
BLUE	298.8014

These values are **far lower than the original image**, indicating:

- Much higher randomness
- Better histogram flattening
- Strong resistance to statistical attacks

5.3 Correlation Analysis of Adjacent Pixels

Original Image Correlation

CHANNEL	HORIZONTAL	VERTICAL	DIAGONAL
RED	0.9663	0.9568	0.9191
GREEN	0.9670	0.9587	0.9221
BLUE	0.9688	0.9608	0.9263

Improved Hybrid Encrypted Image Correlation

CHANNEL	HORIZONTAL	VERTICAL	DIAGONAL
RED	0.0091	-0.0015	0.0016
GREEN	-0.0042	0.0021	0.0016
BLUE	0.0062	-0.0013	-0.0020

These values are extremely close to zero, showing:

- Excellent diffusion
- Strong decorrelation
- High resistance to statistical & differential attacks

5.4 Time Performance

Improved Hybrid Encryption Time:

- **Single-run (core operations): 0.015017 seconds**

This indicates:

- Real-time capability
- Very low computational load
- Suitability for resource-limited systems