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Exam: Mathematical Optimization

Analysis and results reproduction of:

Dynamic Wireless Charging Lanes location model in urban networks considering route choices

Cong Quoc Tran, Mehdi Keyvan-Ekbatani, Dong Ngoduy, David Watling

Transportation Research Part C
2022

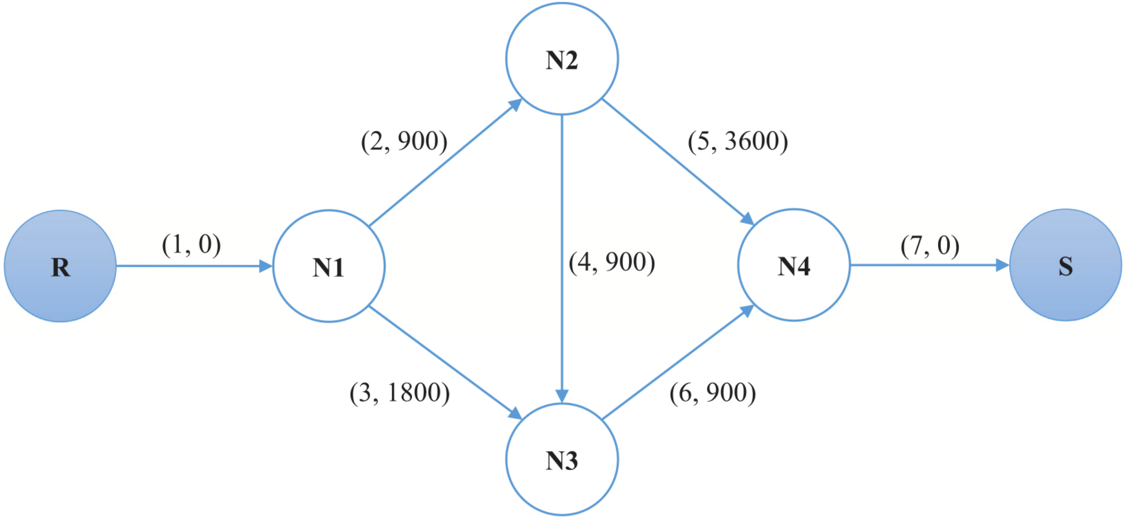
To determine the optimal location for dynamic Wireless Charging Lanes

To support the system planner in deploying the WCL, the paper takes into consideration traffic dynamics and congestion under multiple vehicle classes (EV and ICV).

The problem combines a mixed-integer linear program with dynamic routing behaviour into the charging location problem.

The objective is to maximise network performance while providing insights into traffic propagation patterns over the network.

Network design problem → limited to single Origin-Destination networks



Upper level: minimise the system cost with specific demands

Lower level: Traffic Assignment Problem

Traffic Assignment Problem

Determine which routes will be used and how much traffic can be expected on each route.

Requirements:
- Estimated n.vehicles
- Available routes and travel times on each route
- Decision criteria for route choice (route choice behaviours and path feasibility constraints)

Route choice decisions are a function of travel times

Travel times are determined by traffic flow

Traffic flow is a product of route choice decisions

Traffic Assignment Problem

User Equilibrium:

Users choose the route that minimizes their own travel time.

✅ REALISTIC SCENARIO

❌ EFFICIENT IN THE BI-LEVEL FRAMEWORK

System Optimal:

Users cooperatively achieve the minimum system-wide travel time.

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Traffic Assignment Problem with Dynamic System Optimal

User Equilibrium:

Users choose the route that minimizes their own travel time.

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- ❌ EFFICIENT IN THE BI-LEVEL FRAMEWORK

System Optimal:

Users cooperatively achieve the minimum system-wide travel time.

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↓

Problem reformulated as single-level

Assumptions

- All EVs have homogeneous battery size.
- The energy recharge rate is underestimated proportionally to travel time at free-flow speed.
- The energy consumption rate of an EV is assumed proportional only to the travelled distance and homogeneous for all EVs.
- Network must be cleared at time horizon.

General formulation

Variables:

$x_a \in \{0,1\}$ if link a has WCL

$y_p \in \{0,1\}$ if path p is feasible

$B_{a,p}$ = state of energy of an EV after traversing link a on path p

$n_a^m(i)$ = number of vehicles class m on link a at time i

$u_a^m(i)$ = incoming traffic flow of vehicles class m to link a at time i

$v_a^m(i)$ = outgoing traffic flow of vehicles class m from link a at time i

$f_{ab}^m(i)$ = upstream traffic of vehicles class m from link a to link b at time i

Parameters:

I = budget

D^m = demand rate of vehicles of class m (vehicles/timestep)

V_a = free-flow speed (m/min)

W_a = backward speed (m/min)

K_a = jam density (vehicles/m)

$Q_a = \frac{K_a V_a W_a}{V_a + W_a}$ = maximum flow capacity speed (vehicles/min)

α_a^m = aggregate link-based share factor of vehicles of class m

Constraints

Recharge:
ω = energy received
$t_a^0 = \frac{l_a}{V_a}$ travel time at velocity V_a

Consumption:
ϵ = consumption rate
l_a = length of link a

$$\iota \sum_{a \in \mathcal{E}_A} l_a x_a \leq I \quad \text{budget} \tag{3}$$

$$B_{a,p} = B_0 \quad \text{initial state of energy} \qquad \forall p \in \mathcal{P}, a \in \mathcal{E}_R | a \in p \tag{4}$$

$$B_{a,p} = \min\{B_{max}, B_{b,p} - \epsilon l_a + \omega t_a^0 x_a\} \qquad \forall p \in \mathcal{P}, a \in \mathcal{E}_A, b \in Y_a^- | (b,a) \in p \tag{5}$$

$$M(y_p^{EV} - 1) \leq B_{a,p} \quad \text{path feasibility} \qquad \forall p \in \mathcal{P}, a \in \mathcal{E}_A | a \in p \tag{6}$$

$$\mathbf{x}, \mathbf{y} : \textbf{binary} \tag{7}$$

$$\mathbf{B} : \textbf{unrestricted} \tag{8}$$

Sets:
$\mathcal{P}$ = paths
$\mathcal{E}$ = links, including source links ($\mathcal{E}_R$), sink links ($\mathcal{E}_S$) and normal links ($\mathcal{E}_A$)
$Y_a^{\pm}$ = incoming/outgoing links to/from link a

Constraints

$$n_a^m(i) - \sum_{k=0}^{i}[u_a^m(k) - v_a^m(k)] = 0 \qquad \text{vehicles conservation} \qquad \forall a \in \mathscr{E}, m \in \mathscr{M}, i \in \mathscr{T} \tag{12}$$

$$\sum_{k=i-\frac{l_a}{V_a}+1}^{i} u_a^m(k) - n_a^m(i) \leq 0 \qquad \text{incoming flow} \leq \text{vehicles on } a \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{13}$$

$$n_a^m(i) + \sum_{k=i-\frac{l_a}{W_a}+1}^{i} v_a^m(k) \leq K_a l_a \hat{\alpha}_a^m \qquad \text{capacity per class} \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{14}$$

$$u_a^m(i) = D^m(i) \qquad \text{demand for incoming vehicles at origin} \qquad \forall a \in \mathscr{E}_R, m \in \mathscr{M}, i \in \mathscr{T} \tag{15}$$

$$\left.\begin{aligned} & u_a^m(i) - \sum_{b \in Y_a^-} f_{ba}^m(i) = 0 && \forall a \in \mathscr{E}/\mathscr{E}_R, m \in \mathscr{M}, i \in \mathscr{T} \quad (16) \\ & v_a^m(i) - \sum_{b \in Y_a^+} f_{ab}^m(i) = 0 && \forall a \in \mathscr{E}/\mathscr{E}_S, m \in \mathscr{M}, i \in \mathscr{T} \quad (17) \end{aligned}\right\} \text{flow conservation}$$

$$v_a^m(i) = 0 \qquad \text{network cleared at destination} \qquad \forall a \in \mathscr{E}_S, m \in \mathscr{M}, i \in \mathscr{T} \tag{18}$$

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It can be used to give priority to one vehicle class over the other

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Not coherent with assumption of network cleared at time horizon

$$\left.\begin{aligned} & u_a^m(i) - \sum_{b \in Y_a^-} f_{ba}^m(i) = 0 && (16) \\ & v_a^m(i) - \sum_{b \in Y_a^+} f_{ab}^m(i) = 0 && (17) \end{aligned}\right\} \text{flow conservation} \qquad \forall a \in \mathcal{E}/\mathcal{E}_S, m \in \mathcal{M}, i \in \mathcal{T}$$

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Constraints

$$\sum_{b \in Y_a^-} f_{ba}^m(i) \leq Q_a \sum_{p \in \mathscr{P}} \delta_{a,p} y_p^m \qquad \text{different route choices} \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{19}$$

$$S_a(i) = min\left\{ Q_a; K_a l_a + \sum_{m \in \mathscr{M}} \sum_{k \leq i - \frac{l_a}{W_a}} v_a^m(k) - \sum_{m \in \mathscr{M}} \sum_{k \leq i-1} u_a^m(k) \right\} \qquad \text{supply} \qquad \forall a \in \mathscr{E}_A, i \in \mathscr{T} \tag{20}$$

$$D_a(i) = min\left\{ Q_a; \sum_{m \in \mathscr{M}} \sum_{k \leq i - \frac{l_a}{V_a}} u_a^m(k) - \sum_{m \in \mathscr{M}} \sum_{k \leq i-1} v_a^m(k) \right\} \qquad \text{demand} \qquad \forall a \in \mathscr{E}_A, i \in \mathscr{T} \tag{21}$$

$$\left.\begin{aligned} &\sum_{m \in \mathscr{M}} u_a^m(i) \leq S_a(i) && \forall a \in \mathscr{E}_A, i \in \mathscr{T} \quad (22) \\ &\sum_{m \in \mathscr{M}} v_a^m(i) \leq D_a(i) && \forall a \in \mathscr{E}_A, i \in \mathscr{T} \quad (23) \end{aligned}\right\} \text{true capacity}$$

$$\mathbf{n, u, v, f \ : \ non - negative} \tag{24}$$

Constraints

$$\sum_{b \in \Upsilon_a^-} f_{ba}^m(i) \le Q_a \sum_{p \in \mathscr{P}} \delta_{a,p} y_p^m \qquad \text{different route choices} \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \qquad (19)$$

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$$\mathbf{n, u, v, f \ : \ non - negative} \qquad (24)$$

$$\boxed{Q_a \text{ (veh/min)} \longrightarrow Q_a \tau \text{ (veh/timestep)}}$$

Objective function

$$\max OF = \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{T}} \left\{ \sum_{a \in \mathcal{E}_S} \sum_{b \in Y_a^-} (|\mathcal{T}| + 1 - i) f_{ba}^m(i) + o \sum_{a \in \mathcal{E}/\mathcal{E}_S} \sum_{b \in Y_a^-} (|\mathcal{T}| + 1 - i) f_{ba}^m(i) \right\}$$

Set of discrete time steps

Penalty cost to incentivise travellers to move along instead of waiting

Maximising total traffic outflow is mathematically equivalent to minimising total travel times.

Minimising constraints

$$\boldsymbol{B_{a,p}} = \min\{\boldsymbol{B_{max}}, \boldsymbol{B_{b,p}} - \epsilon l_a + \omega t_a^0 x_a\}$$

$$B_{a,p} \leq B_{max}$$

$$B_{a,p} \leq B_{b,p} - \epsilon l_a + \omega t_a^0 x_a + M(1 - y_p)$$

$$B_{a,p} \geq B_{b,p} - \epsilon l_a + \omega t_a^0 x_a - M(1 - y_p)$$

$$S_a(i) = min\left\{Q_a; K_a l_a + \sum_{m \in \mathcal{M}} \sum_{k \leq i - \frac{l_a}{W_a}} v_a^m(k) - \sum_{m \in \mathcal{M}} \sum_{k \leq i-1} u_a^m(k)\right\}$$

$$D_a(i) = min\left\{Q_a; \sum_{m \in \mathcal{M}} \sum_{k \leq i - \frac{l_a}{V_a}} u_a^m(k) - \sum_{m \in \mathcal{M}} \sum_{k \leq i-1} v_a^m(k)\right\}$$

$$\sum_{m \in \mathcal{M}} u_a^m(i) \leq S_a(i)$$

$$\sum_{m \in \mathcal{M}} v_a^m(i) \leq D_a(i)$$

Modified demand rate at origin

$$u_a^m(i) = D^m(i) \qquad \forall a \in \mathscr{E}_R, m \in \mathscr{M}, i \in \mathscr{T}$$

$$u_a^m(i) = D^m(i) \qquad \forall a \in \mathscr{E}_R, m \in \mathscr{M}, \; i \leq T_{stop}$$

$$u_a^m(i) = 0 \qquad \forall a \in \mathscr{E}_R, m \in \mathscr{M}, \; i > T_{stop}$$

Clearance time in worst case scenario:

$$T_{stop} = \frac{N}{2D_a^m \big/ Q_a \tau + 1}$$

n.vehicles entering each timestep τ

Exit capacity of sink link per timestep τ

Last time step to clear network

Extra constraint

To avoid conflict with constraint 19 $\left(\sum_{b \in Y_a^-} f_{ba}^m(i) \le Q_a \sum_{p \in \mathscr{P}} \delta_{a,p} y_p^m \right)$

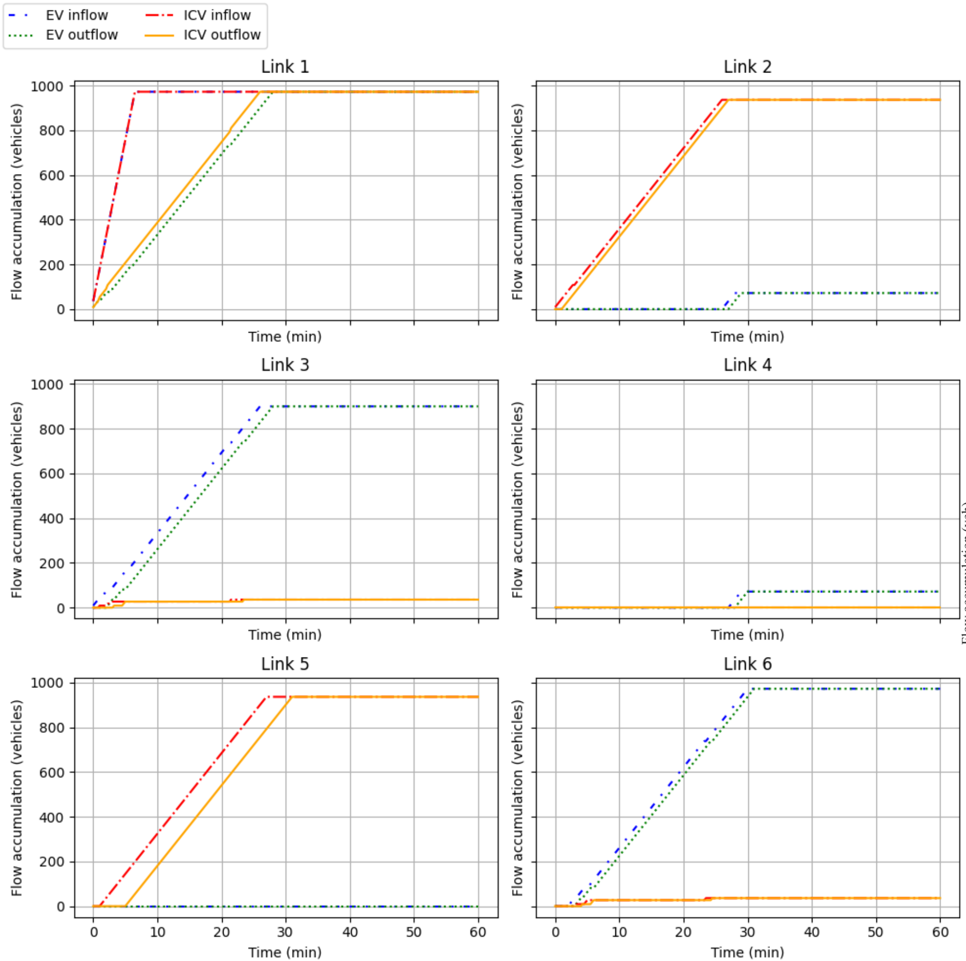
impose a minimum upstream traffic for feasible paths.

Otherwise if there is too much congestion, constraint 19 makes $y_p = 0$ even if feasible.

$$\sum_{p \in P} \sum_{a \in p} \sum_{b \in Y_a^-} \sum_{i}^{N} f_{ab}^{EV}(i) \ge \lambda y_p$$

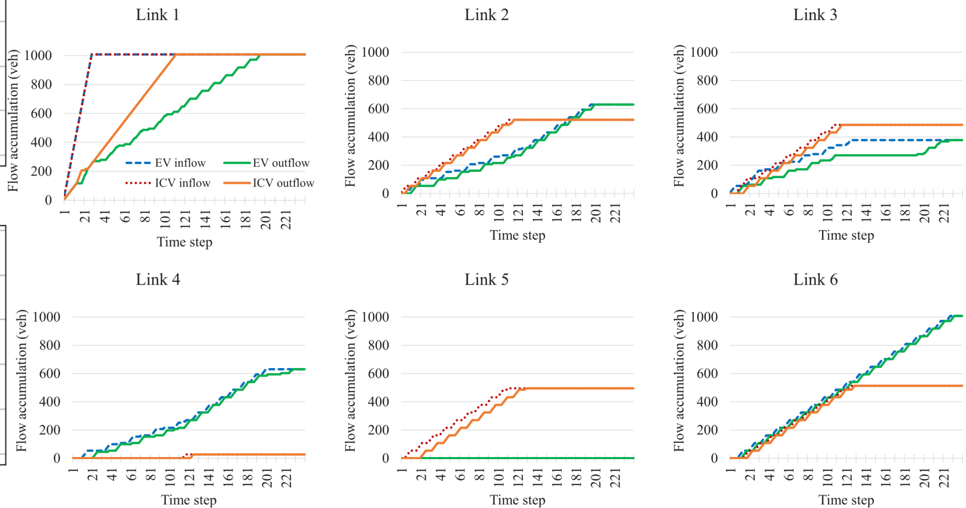
Arbitrary small

Inflow-outflow profiles of Braess network

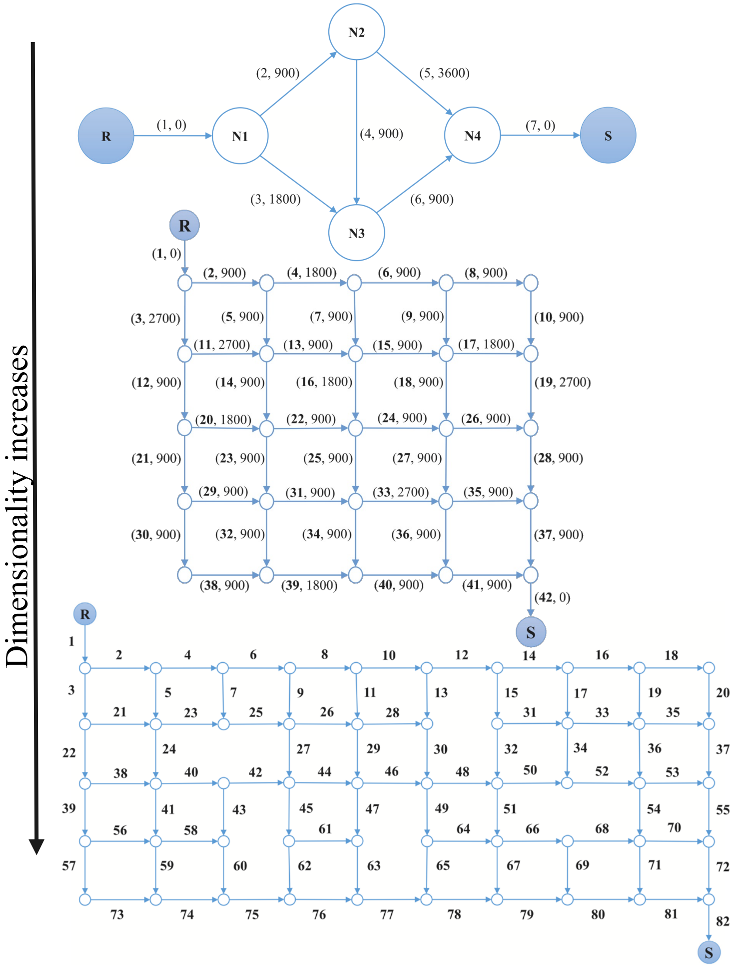


Slightly different implementation → different plots

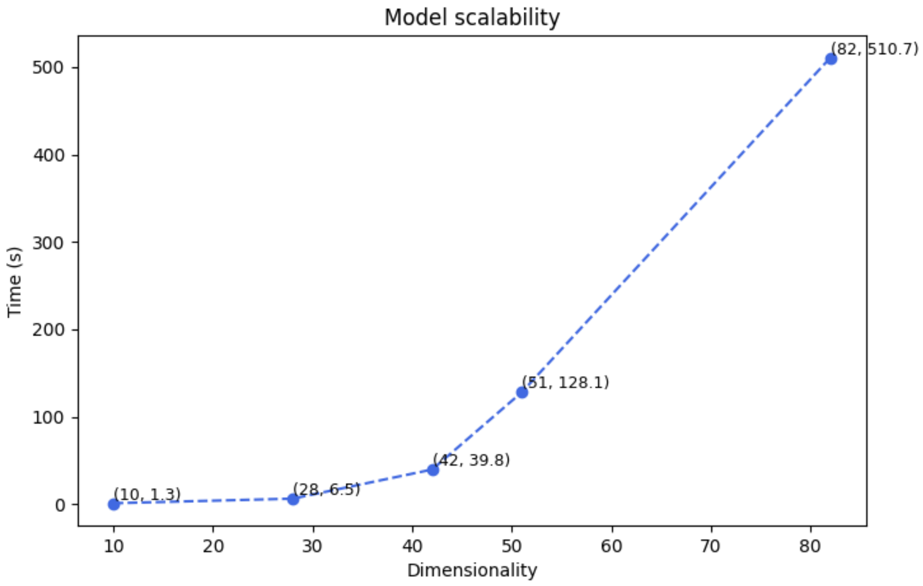
But overall a similar representation



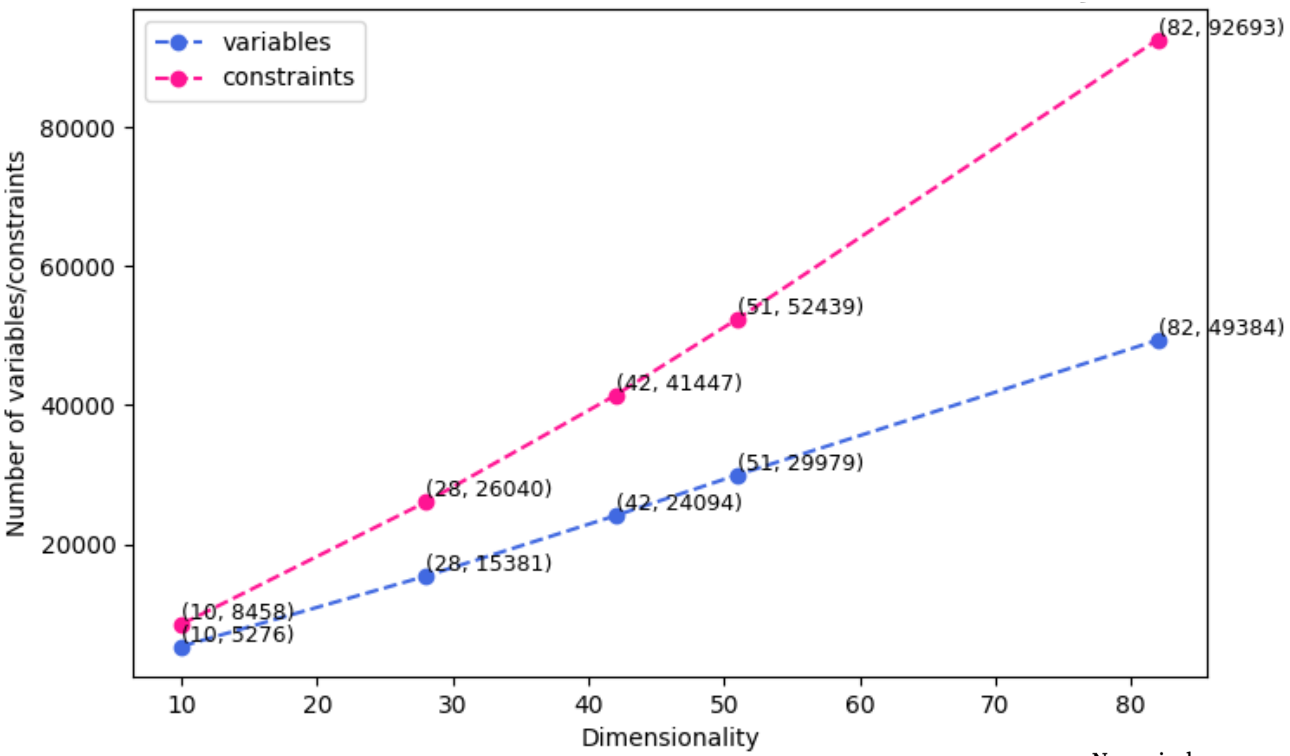
Gurobi optimization: increasing dimensionality



Boosting the dimensionality of the problem, the time of execution highly increases.



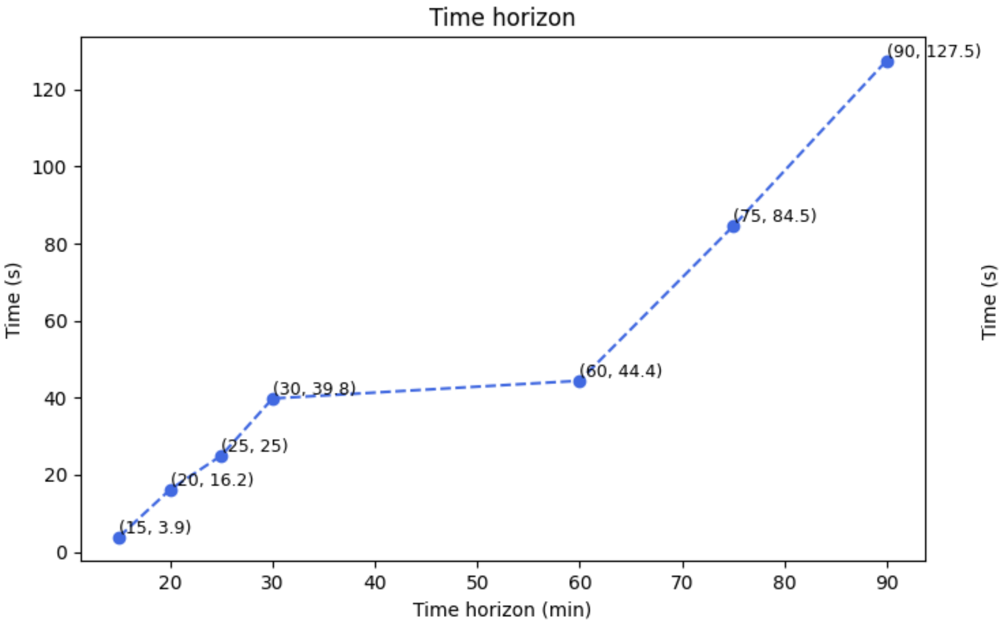
Number of variables and constraints VS dimensionality

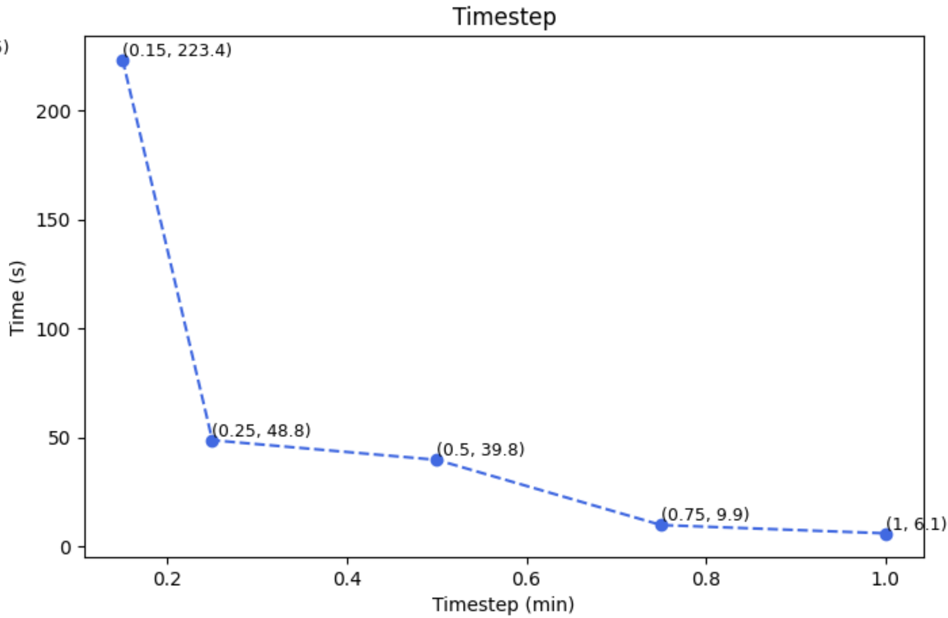


Tran et al.'s computational performance adopting PYOMO with the CPLEX solver

Numerical test	No. variables (binary)	No. linear constraints	Run-time (s)
Grid-42	31,067 (180)	46,432	283.96
Grid-82	63,465 (640)	116,282	28,823.83

Parameters value's effect on execution time

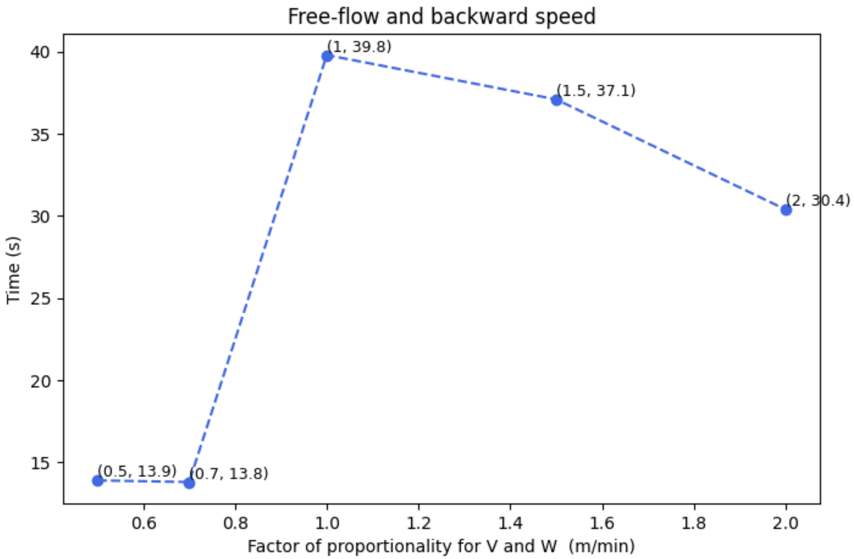


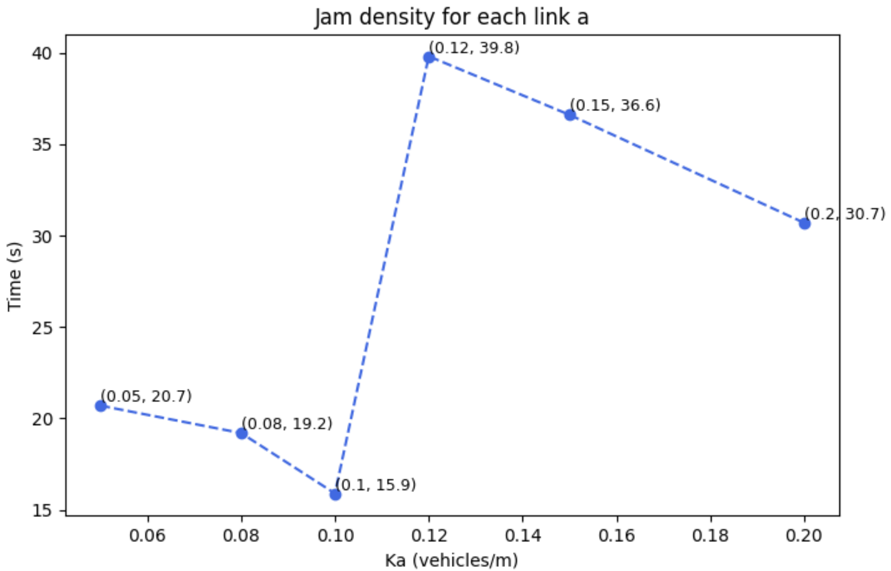


Increasing time horizon increases execution time

With increasing timestep, less vehicles enter the network, lowering the execution time

Parameters value's effect on execution time





Direct proportionality between Q_a and K_a, V_a, W_a,
highlighting a peak congestion at given values.

$$\boxed{Q_a = \frac{K_a V_a W_a}{V_a + W_a}}$$

Thank you for the attention

Arriva l'asfalto magico per auto elettriche, la ricarica a induzione è pronta.

Operaio della Serenissima che dovrà rifare tutto da capo:

Sitography

- Implemented paper: «Dynamic wireless charging lanes location model in urban networks considering route choices», Tran et al., 2022, https://www.sciencedirect.com/science/article/pii/S0968090X2200095X
- Cited paper for α: «Multiclass dynamic system optimum solution for mixed traffic of human-driven and automated vehicles considering physical queues», Ngoduy et al., 202,1 https://www.sciencedirect.com/science/article/pii/S0191261521000011
- Traffic Assignment Problem: lecture 4 of «Principles of Transportation Engineering» at the Iowa State University, https://www.youtube.com/watch?v=qjSLM3-ENxU