Social Physics: Master Lexicon v1.0

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Purpose

This document establishes the precise, canonical notation for the Social Physics framework. It serves as the single source of truth for all mathematical symbols, ensuring consistency across theoretical papers, empirical studies, and computational implementations.

1 Core Notation Principles

- Scalars: italic lowercase (t, d, φ)
- Vectors: bold lowercase ($\mathbf{v}, \boldsymbol{\varphi}$)
- Matrices: bold uppercase ($\mathbf{J}, \mathbf{A}$)
- Random variables: uppercase (X, T)
- Functions and operators: text or mathrm (P-E-A, R_{fission})

2 Master Symbol Table

Symbol	Type	Definition and Constraints
t	Scalar	Time index. Usually discrete $t \in \mathbb{N}$ for empirical work, continuous $t \in \mathbb{R}^+$ for theory.
$\mathcal{U}$	Set	The set of all users/agents in the system. $\lvert\mathcal{U}\rvert = N$.
X	Label	A topic, event, or propositional content under analysis.

Table 1: Foundational symbols

Symbol	Type	Definition	First Ref
P-E-A(X, t)	Scalar function	**Perceived Exaggerated Amplification score** for topic X at time t. Computed as RMS of components. Range: $\mathbb{R}^+$.	Doc12, Eq.1
$A_p(X, t)$	Scalar function	**Prevalence Amplification**: $\frac{\hat{p}_{\text{platform}}(X,t)}{\hat{p}_{\text{baseline}}(X)}$ where $\hat{p}$ is exposure share.	Doc12, Sec.2.2
$A_e(X, t)$	Scalar function	**Emotional Intensity Amplification**: Ratio of average sentiment/arousal in feed vs. baseline about X.	Doc12, Sec.2.2
$A_b(X, t)$	Scalar function	**Bot/Coordination Amplification**: $1 + \gamma \cdot \text{relative\_bot\_activity}(X, t)$.	Doc12, Sec.2.2
$A_s(X, t)$	Scalar function	**Bias Skew Amplification**: Measures systematic slant of amplified sources.	Doc12, Sec.2.2
$\varphi_i(t)$	Vector	**Percepton**: Agent i's perceptual state (magnitude $\in [0, 1]$, direction $\in [-1, 1]$).	Doc15, Sec.3.1
d_i	Scalar	**Perceptual decay constant** for agent i. From $d\varphi_i/dt = -d_i\varphi_i + \dots$	Doc17, Eq.1
$R_{\text{fission}}(X, t)$	Scalar function	**Cascade reproduction metric**: Expected number of 2nd-generation events from one event at t, given system state.	Doc21, Sec.4.3
CDT(X)	Scalar	**Critical Distortion Threshold**: $\inf\{\alpha \geq 0 \mid R_{\text{fission}}(X, t) \geq 1$ when P-E-A$(X, t) = \alpha\}$.	Doc12, Eq.CDT

Table 2: Core Social Physics constructs

3 Core Social Physics Constructs

4 Core Social Physics Metrics: P-E-A and R_{fission}

This section defines the fundamental measurable quantities of the Social Physics framework. These definitions serve as the atomic components for all subsequent modeling, measurement, and analysis.

4.1 The P-E-A Score and Its Components

The Perceived Exaggerated Amplification (P-E-A) score quantifies the distortion between platform-mediated perception and a baseline reality. It decomposes into four amplification factors.

4.1.1 Prevalence Amplification (A_p)

For a topic X and time window t:

$$A_p(X,t) = \frac{\hat{p}_{\text{feed}}(X,t)}{\hat{p}_{\text{base}}(X)}$$

where:

- $\hat{p}_{\text{feed}}(X,t)$: Observed prevalence of X in user feeds (share of impressions).
- $\hat{p}_{\text{base}}(X)$: Baseline prevalence in a reference corpus (platform-wide, high-quality media, or offline polling). We assume $\hat{p}_{\text{base}}(X) > 0$ for any topic X under study.

Interpretation: $A_p > 1$ indicates over-representation; $A_p < 1$ indicates suppression.

4.1.2 Emotional Intensity Amplification (A_e)

$$A_e(X,t) = \frac{E_{\text{feed}}(X,t)}{E_{\text{base}}(X)}$$

where:

- $E_{\text{feed}}(X,t)$: Average emotional intensity (arousal) of content about X in feeds.
- $E_{\text{base}}(X)$: Baseline emotional intensity for X, assumed > 0.

Measurement: Emotional intensity $\text{emo}(c) \in [0,1]$ can be derived from sentiment analysis or transformer models.

4.1.3 Bot & Coordination Amplification (A_b)

$$A_b(X,t) = 1 + \gamma \cdot \left(\frac{B_{\text{feed}}(X,t)}{B_{\text{base}}} \right)$$

where:

- $B_{\text{feed}}(X,t)$: Bot-coordination footprint in feeds (product of bot probability and coordination score).
- B_{base}: Baseline bot-coordination level, assumed > 0.
- $\gamma \geq 0$: Sensitivity parameter to be calibrated empirically.

Note: $A_b \geq 1$, with equality when no abnormal bot/coordination activity is detected.

4.1.4 Bias Skew Amplification (A_s)

$$A_s(X,t) = 1 + \delta_1 \cdot |\mu_{\text{feed}}(X,t) - \mu_{\text{base}}| + \delta_2 \cdot \max(0, \sigma_{\text{base}} - \sigma_{\text{feed}}(X,t))$$

where:

- $\mu_{\text{feed}}, \mu_{\text{base}}$: Mean ideological slant of sources in feeds vs. baseline.
- $\sigma_{\text{feed}}, \sigma_{\text{base}}$: Standard deviation (diversity) of slants.
- $\delta_1, \delta_2 \geq 0$: Sensitivity parameters for directional skew and diversity loss, to be calibrated empirically.

4.1.5 Composite P-E-A Score

The overall P-E-A score is the root-mean-square of its components:

$$\text{P-E-A}(X,t) = \sqrt{\frac{A_p(X,t)^2 + A_e(X,t)^2 + A_b(X,t)^2 + A_s(X,t)^2}{4}}$$

Properties: Scale-invariant, emphasizes extremes, mathematically well-behaved. Range: P-E-A $\in [1, \infty)$ in practice.

4.2 Cascade Reproduction Metric (R_{fission})

$$R_{\text{fission}}(X,t) = \mathbb{E}\left[\frac{N_{\text{events}}(X, t+\Delta t)}{N_{\text{events}}(X,t)} \;\middle|\; \text{P-E-A}(X,t), \Theta\right]$$

where:

- $N_{\text{events}}(X,t)$: Count of coordination/hostile reaction events for topic X at time t.
- Δt: Typical generation time (e.g., 6–48 hours).
- Θ: Network structure and user susceptibility parameters.

Interpretation:

- $R_{\text{fission}} < 1$: **Subcritical** – cascades die out.
- $R_{\text{fission}} = 1$: **Critical threshold** – cascade size stable.
- $R_{\text{fission}} > 1$: **Supercritical** – runaway growth (Social Fission).

5 System Ontology: Fundamental Units

Social Physics models digital discourse as a dynamical system. These are its fundamental units.

5.1 Percepton (φ_i)

A **Percepton** is the atomic unit of perception for an agent i. It represents their current cognitive state regarding a topic.

$$\varphi_i(t) = (\, m_i(t),\, \mathbf{d}_i(t)\,)$$

where:

- $m_i(t) \in [0,1]$ is the **magnitude** (intensity) of the perception. 0 = unaware/neutral; 1 = maximum intensity (obsession, outrage).
- $\mathbf{d}_i(t)$ is the **direction** (stance/alignment) in a conceptual space. Often simplified to a scalar $d_i(t) \in [-1, +1]$, where -1 = strongly oppose, $+1$ = strongly support, 0 = neutral or conflicted.

Analogy: A Percepton is like a tiny magnet with a strength (m) and polarity (d). It can be influenced by other magnets (social influence).

5.2 Agent ($i \in \mathcal{U}$)

An **Agent** is a user, account, or node in the social network. Each agent i possesses a Percepton $\varphi_i(t)$ that evolves over time based on:

- Internal decay (forgetting).
- Social influence from connected agents.
- External shocks (news events, recommendations).

5.3 Interaction Edge ($\eta_{i \to j}$)

The **influence weight** from agent i to agent j. It determines how much i's Percepton affects j's update. It is a function of:

$$\eta_{i \to j} = f(\text{connection strength}, \text{algorithmic boost}, \text{credibility})$$

In your Lab Notes, this is the edge weight w_{ij} used in the toy model.

6 Dynamical Laws

6.1 The Perceptual Decay Law

In the absence of social reinforcement, a Percepton's intensity naturally fades (people forget, emotions cool). This is modeled as exponential decay:

$$\frac{dm_i(t)}{dt} = -\, d_i \cdot m_i(t)$$

where $d_i > 0$ is the **perceptual decay constant** for agent i. The solution is:

$$m_i(t) = m_i(0) \cdot e^{-d_i t}$$

Interpretation:

- **Large** d_i: Fast forgetter, quick to move on.
- **Small** d_i: Long memory, holds onto perceptions.
- The **half-life** of a perception is $t_{1/2} = \ln(2)/d_i$.

Why Exponential? It's the simplest, most natural law for a quantity that decays at a rate proportional to its current value (like radioactivity, or cooling).

6.2 The Social Influence Law

A Percepton is updated by combining decay with weighted influence from neighbors. To preserve interpretability, the updated magnitude is clipped to $[0, 1]$:

$$\tilde{m}_i(t+1) = m_i(t) \cdot (1 - d_i) + \sum_{j \in \mathcal{N}(i)} \eta_{j \to i} \cdot g(m_j(t), \mathbf{d}_j, \mathbf{d}_i) + \epsilon_i(t)$$

$$m_i(t+1) = \min(\max(\tilde{m}_i(t+1), 0), 1) \quad \text{(clipping)}$$

where:

- $\mathcal{N}(i)$: Neighbors of agent i in the social graph.
- $g(\cdot)$: A function modulating influence by alignment (e.g., $g = m_j \cdot (\mathbf{d}_j \cdot \mathbf{d}_i)$ for aligned influence).
- $\epsilon_i(t)$: External shock (news, platform recommendation).

This is the equation from which CDT and R_{fission} emerge as system-level properties.

7 Synthesis: From Micro-Laws to Macro-Cascades

This section connects the fundamental laws of agent-level perception to the system-level phenomena of cascades ($R_{\text{fission}} > 1$) and critical thresholds (CDT).

7.1 How P-E-A Modulates the Influence Law

Recall the Social Influence Law (Eq. 6.2):

$$\tilde{m}_i(t+1) = m_i(t) \cdot (1 - d_i) + \sum_{j \in \mathcal{N}(i)} \eta_{j \to i} \cdot g(m_j(t), \mathbf{d}_j, \mathbf{d}_i) + \epsilon_i(t).$$

The Perceived Exaggerated Amplification (P-E-A) score acts as a **systemic amplifier** on the components of this equation:

- **Prevalence (A_p)** increases the *probability* that influential neighbors j are posting about topic X (effectively enlarging the set $\mathcal{N}(i)$ for that topic).
- **Emotional Intensity (A_e)** increases the *magnitude* $m_j(t)$ of the influencing Perceptons.
- **Bot/Coordination (A_b)** and **Bias Skew (A_s)** systematically alter the *influence weights* $\eta_{j \to i}$ and the *alignment function* $g(\cdot)$, concentrating influence within coordinated or ideologically aligned clusters.

Thus, a high P-E-A(X, t) score **parametrically scales up** the social influence term in the dynamics, while decay (d_i) remains a fixed, individual property.

7.2 Deriving R_{fission} from the Linearized Dynamics

Consider a small perturbation (a single "infected" agent with heightened m_i) in a network near equilibrium. Linearizing the Influence Law around the baseline state (ignoring clipping for small perturbations) allows us to express the expected spread.

Let $\mathbf{m}(t)$ be the vector of perceptual magnitudes across all agents. The linearized update can be written as:

$$\mathbf{m}(t+1) = \mathbf{D} \cdot \mathbf{m}(t) + \mathbf{W} \cdot \mathbf{m}(t) = (\mathbf{D} + \mathbf{W}) \cdot \mathbf{m}(t)$$

where:

- $\mathbf{D} = \mathrm{diag}(1 - d_i)$ is the **decay matrix**.
- $\mathbf{W}$ is the **influence matrix**, with entries $W_{ij} = \eta_{j \to i} \cdot \frac{\partial g}{\partial m_j}$ evaluated at baseline.

The **spectral radius** $\rho(\mathbf{D}+\mathbf{W})$ (the largest absolute eigenvalue) determines the asymptotic growth rate of the perturbation. The cascade reproduction number is exactly:

$$R_{\text{fission}} = \rho(\mathbf{D} + \mathbf{W})$$

- If $\rho(\mathbf{D} + \mathbf{W}) < 1$, the perturbation decays ($R_{\text{fission}} < 1$).
- If $\rho(\mathbf{D} + \mathbf{W}) > 1$, the perturbation grows ($R_{\text{fission}} > 1$).

7.3 Defining the Critical Distortion Threshold (CDT) Formally

The influence matrix $\mathbf{W}$ is a function of the system's amplification state, which we measure by P-E-A. We can write $\mathbf{W} = \mathbf{W}(\text{P-E-A})$.

The **Critical Distortion Threshold** for topic X is therefore defined as the smallest P-E-A value for which the system's linearized dynamics become unstable:

$$\boxed{\mathrm{CDT}(X) := \inf \{ \, \alpha \geq 0 \mid \rho(\, \mathbf{D} + \mathbf{W}(\alpha) \,) \geq 1 \, \}}$$

Interpretation: The CDT is the **minimum amplification level** that tips the system's dominant mode of interaction from net decay ($\rho < 1$) to net growth ($\rho \geq 1$), enabling a single seed to trigger a runaway cascade.

7.4 The Complete Theoretical Pipeline

The causal logic of Social Physics is now fully specified:

1. **Platform mechanisms** (algorithmic ranking, network structure) produce a measurable **P-E-A score**.
2. A high P-E-A score parametrically **amplifies the influence matrix W** in the agent-based dynamics.
3. The amplified dynamics shift the system's **spectral radius** $\rho(\mathbf{D} + \mathbf{W})$.

4. When P-E-A $\geq$ CDT, we have $\rho(\mathbf{D} + \mathbf{W}) \geq 1$, meaning $R_{\text{fission}} \geq 1$.
5. This state of **Social Fission** leads to observable, large-scale cascades of coordinated/hostile behavior.

This pipeline transforms descriptive observation into a **predictive, testable theory**: by measuring P-E-A and estimating the network's decay parameters, one can forecast proximity to criticality.

Version History

- **v1.0** (December 18, 2025): Initial stable release.
  - Canonical notation for all core constructs established.
  - Formal definitions of P-E-A components (A_p, A_e, A_b, A_s) with baseline and parameter clarifications.
  - Ontology defined: Percepton (φ_i), Agent, Interaction Edge (η).
  - Dynamical Laws formalized: Perceptual Decay Law and Social Influence Law (with clipping).
  - Synthesis section added, deriving $R_{\text{fission}} = \rho(\mathbf{D} + \mathbf{W})$ and CDT from micro-dynamics.
  - Mathematical corrections applied: spectral radius, clipping, parameter calibration notes.
  - Document structure finalized with master symbol tables.
- **Next Version (Planned):** v1.1 – Integration with empirical case studies (Brexit) and computational lab notes.