سوال ۱ ـ ۲ (الف) و

$$= \left(\frac{1}{2}\right)^{|n|}$$

$$x(n) = x'(n)\left(\frac{e^{jwn} + e^{-jwn}}{2}\right)$$

$$X(e^{jw}) = \frac{X'(e^{j(w-w_0)})}{2} + \frac{X'(e^{j(w+w_0)})}{2}$$

از طرفی می‌دانیم:

$$y(n) = \left(\frac{1}{2}\right)^{|n|} \xrightarrow{DTFT} = \frac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2 - 2\times\frac{1}{2}\cos w} =$$

$$\frac{\frac{3}{4} \times 4}{\frac{5}{4} - \cos w \times 4} = \frac{3}{5 - 4\cos w}$$

$$X(e^{jw}) = \frac{1}{2} \times \frac{3}{5-4\cos(w-w_0)} + \frac{1}{2} \times \frac{3}{5-4\cos(w+w_0)}$$

سوال ۱ - ب):

$$x(n) = (n-3)\left(\frac{1}{7}\right)^{|n|} = n\left(\frac{1}{7}\right)^{|n|} - 3\left(\frac{1}{7}\right)^{|n|}$$

$$\xrightarrow{\text{DTFT}} j\,\frac{d\left(\text{DTFT}\left(\frac{1}{7}\right)^{|n|}\right)}{d\omega} - 3\,\text{DTFT}\left(\left(\frac{1}{7}\right)^{|n|}\right)$$

$$= j\frac{d}{d\omega}\left(\frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2 - 2\times\frac{1}{7}\cos\omega}\right) - 3\times\frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2 - 2\times\frac{1}{7}\times\cos\omega}$$

$$= j\times\frac{-\left(-\frac{2}{7}(-\sin\omega)\right)\frac{48}{49}}{\left(\frac{50}{49} - \frac{2}{7}\cos\omega\right)^2} - 3\times\frac{\frac{48}{49}}{\frac{50}{49} - \frac{2}{7}\cos\omega}$$

$$= \frac{-j\,\frac{96}{343}\sin\omega}{\left(\frac{50}{49} - \frac{2}{7}\cos\omega\right)^2} - \frac{\frac{144}{49}}{\frac{50}{49} - \frac{2}{7}\cos\omega}$$

سوال ۱ـ ج) :

$-n-3 \triangleq m$

$$3^{-m-3} u(m) = \left(\frac{1}{3}\right)^{3} \left(\frac{1}{3}\right)^{m} u(m)$$

$$\xrightarrow{DTFT} \frac{1}{27} \times \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

سوال ۲ـ الف) :

$e^{-j\omega} \triangleq k$

$$X(e^{j\omega}) = \frac{1-\frac{1}{3}k}{1-\frac{1}{4}k-\frac{1}{4}k^{2}} = \frac{1-\frac{1}{3}k}{(\frac{1}{4}-\frac{1}{8}k)(k+4)}$$

$$= \frac{A}{\frac{1}{4}-\frac{1}{8}k} + \frac{B}{k+4}$$

$$\Rightarrow Ak + 4A + \frac{B}{4} - \frac{B}{8}k = 1 - \frac{1}{3}k$$

$$\Rightarrow \begin{cases} 4A + \frac{B}{4} = 1 \\ A - \frac{B}{8} = -\frac{1}{3} \end{cases} \qquad A = \frac{1}{18} \qquad B = \frac{28}{9}$$

① $X(e^{j\omega}) = \dfrac{\frac{1}{18}}{\frac{1}{4}(1-\frac{1}{2}e^{-j\omega})} + \dfrac{\frac{28}{9}}{4(1+\frac{1}{4}e^{-j\omega})}$

$\xrightarrow{DTFT^{-1}} \frac{2}{9}\left(\frac{1}{2}\right)^n u(n) + \frac{7}{9}\left(-\frac{1}{4}\right)^n u(n)$

سوال ۲ـ ج):

$X(e^{j\omega}) = \dfrac{1}{1-\frac{1}{3}e^{-j\omega}} - \left(\frac{1}{3}\right)^{n_0} e^{-jn_0\omega}\left(\dfrac{1}{1-\frac{1}{3}e^{-j\omega}}\right)$

$\xrightarrow{DTFT^{-1}} \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{3}\right)^{n_0} \times \left(\frac{1}{3}\right)^{n-n_0} u(n-n_0) =$

$\left(\frac{1}{3}\right)^n \left[u(n) - u(n-n_0)\right]$

سوال ۲ـ ب):

$X(e^{j\omega}) = \sum_k e^{jk\pi}\, \delta(\omega - \underbrace{\frac{2\pi}{5}k})$

در همه نقاط صفر است مگر $\omega = \frac{2\pi}{5}k$ یا حاصل $\frac{5\omega}{2\pi}$ عدد صحیح باشد.

$$= e^{j\frac{5\omega}{2}} \sum_{k} \delta\left(\omega - \frac{2\pi}{5}k\right) \xrightarrow{DTFT^{-1}}$$

$$= \delta\left(n + \frac{5}{2}\right) * \sum_{k} \delta(n - 5k)$$

سوال ۳ـ $x(n)$ حقیقی است $\Leftarrow x(n) = x_e(n) + x_o(n)$

$$x_o(n) \xrightarrow{DTFT} j\, Im\{X(e^{j\omega})\} = j(\sin\omega - \sin 2\omega)$$

$$= j\left(\frac{e^{j\omega} - e^{-j\omega}}{2j} - \frac{e^{2j\omega} - e^{-2j\omega}}{2j}\right) =$$

$$\frac{e^{j\omega} - e^{-j\omega}}{2} - \frac{e^{2j\omega} - e^{-2j\omega}}{2} \xrightarrow{DTFT^{-1}}$$

$$\frac{1}{2}\delta(n+1) - \frac{1}{2}\delta(n-1) - \frac{1}{2}\delta(n+2) + \frac{1}{2}\delta(n-2)$$

طبق شرط (الف): $\forall n<0 \quad x(n)=0$

$$x_o(1)+x_e(1)=0 \Rightarrow -\frac{1}{2}+x_e(1)=0$$

$$\to x_e(1)=\frac{1}{2}$$

$$x_o(2)+x_e(2)=0 \Rightarrow \frac{1}{2}+x_e(2)=0 \to x_e(2)=-\frac{1}{2}$$

$$x_o(n)+x_e(n)=0 \Rightarrow 0+x_e(n)=0 \Rightarrow x_e(n)=0 \quad \forall n\geq 3$$

$$x(0)=x_e(0)+x_o(0)=x_e(0)$$

طبق شرط (د):

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{j\omega})|^2\,d\omega=\sum_n |x(n)|^2$$

$x_e(n)$ که در قسمت قبل ها بدست آمد و طبق تعریف زوج است:

$$x_e(1)=\frac{1}{2} \to x_e(-1)=\frac{1}{2} \to x(-1)=\frac{1}{2}+\frac{1}{2}=1$$

$$x_e(2)=-\frac{1}{2} \to x_e(-2)=-\frac{1}{2} \to x(-2)=-\frac{1}{2}-\frac{1}{2}=-1$$

$$\forall n\geq 3 \quad x_e(n)=x_e(-n)=0$$

$$\Rightarrow \sum_n |x(n)|^2 = |x(0)|^2 + |x(-1)|^2 + |x(-2)|^2 = 3$$

$$\Rightarrow 1^2 + 1^2 + |x(0)|^2 = 3 \Rightarrow |x(0)| = 1$$

می‌دانیم $x(0)>0$ است $\Leftarrow$ $x(0) = 1$

$$x(n) = \delta(n) + \delta(n+1) - \delta(n+2)$$

سوال ۴ ـ الف) :

$$Y(e^{j\omega}) = \left(-H_1(e^{j\omega})X(e^{j\omega}) + X(e^{j\omega})\right)H_2(e^{j\omega})$$

$$H_1(e^{j\omega}) = e^{-2j\omega} = DTFT(\delta(n-2))$$

$$Y(e^{j\omega}) = \underbrace{X(e^{j\omega})(1-e^{-2j\omega})\Pi\left(\frac{\omega}{\pi}\right)}_{\text{پاسخ فرکانسی کل سیستم.}}$$

سوال ٤ - ب) :

$$X_1(e^{j\omega}) = DTFT\left(x_1(n) = 1\right) = 2\pi \sum_{k} \delta(\omega - 2k\pi)$$

$$Y_1(e^{j\omega}) = \left[2\pi \sum_{k} \delta(\omega - 2k\pi)\right]\left(1 - e^{-2j\omega}\right)\Pi\left(\frac{\omega}{\pi}\right)$$

$$X_2(e^{j\omega}) = DTFT\left(x_2(n) = (-1)^n = e^{jn\pi}\right) =$$

$$2\pi \sum_{k} \delta(\omega - \pi - 2k\pi)$$

$$Y_2(e^{j\omega}) = \left\{2\pi \sum_{k} \delta(\omega - \pi - 2k\pi)\right\}\left(1 - e^{-2j\omega}\right)$$

$$\Pi\left(\frac{\omega}{\pi}\right)$$

$$y_1(n) = DTFT^{-1}\left(Y_1(e^{j\omega})\right) = \cancel{\quad}$$

$$\underbrace{\left(x(n) - x(n-2)\right)}_{} * DTFT^{-1}\left(\Pi\left(\frac{\omega}{\pi}\right)\right) = 0$$

چون $x(n)$ ثابت است مساوی صفر می‌شود

(۱) $y_2(n) = DTFT^{-1}\{Y_2(e^{j\omega})\} =$

$$\underbrace{\left((-1)^n - (-1)^{n-2}\right)}_{=0} * DTFT^{-1}\left(\Pi\left(\frac{\omega}{\pi}\right)\right) = 0$$

سوال ک،

$$x(n) = (n+2)\left(\frac{1}{2}\right)^n u(n) = n\left(\frac{1}{2}\right)^n u(n) + 2\left(\frac{1}{2}\right)^n u(n)$$

$$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

از قبل داریم:

$$(n+1)\,a^n u(n) \quad |a|<1 \xrightarrow{DTFT} \frac{1}{(1-ae^{-j\omega})^2}$$

(۱)

$$n\,a^n u(n) + a^n u(n) \xrightarrow{DTFT} \frac{1}{(1-ae^{-j\omega})^2}$$

(۲)

$$a^n u(n) \xrightarrow{DTFT} \frac{1}{1-ae^{-j\omega}}$$

(۱) و (۲) $\Rightarrow n\,a^n u(n) \xrightarrow{DTFT} \frac{1}{(1-ae^{-j\omega})^2} - \frac{1}{1-ae^{-j\omega}}$

$$= \frac{a e^{-j\omega}}{(1-a e^{-j\omega})^{2}}$$

$$X(e^{j\omega}) = \frac{\frac{1}{2} e^{-j\omega}}{(1-\frac{1}{2} e^{-j\omega})^{2}} + \frac{2}{1-\frac{1}{2} e^{-j\omega}} \overset{*}{=}$$

$$\overset{*}{=} \frac{2-\frac{1}{2} e^{-j\omega}}{(1-\frac{1}{2} e^{-j\omega})^{2}}$$

معادله در سیستم LTI :

$$Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})$$

$$\rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$H(e^{j\omega}) = \frac{\frac{1}{1-\frac{1}{4} e^{-j\omega}}}{\frac{2-\frac{1}{2} e^{-j\omega}}{(1-\frac{1}{2} e^{-j\omega})^{2}}} = \frac{(1-\frac{1}{2} e^{-j\omega})^{2}}{2 - e^{-j\omega} + \frac{1}{8} e^{-2j\omega}}$$

$$y(n) = \delta(n) - \left(-\tfrac{1}{2}\right)^n u(n) = 1 - \frac{1}{1+\frac{1}{2}e^{-j\omega}} =$$

$$\frac{\frac{1}{2}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}} \longrightarrow X(e^{j\omega}) = \frac{Y(e^{j\omega})}{H(e^{j\omega})} =$$

$$\frac{\frac{1}{2}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}} \times \frac{2 - e^{-j\omega} + \frac{1}{8}e^{-2j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}$$

$$X(e^{j\omega}) = e^{-j\omega} \times \frac{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{16}e^{-2j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2\left(1+\frac{1}{2}e^{-j\omega}\right)} =$$

$$= e^{-j\omega} \times \left[\frac{A}{1+\frac{1}{2}e^{-j\omega}} + \frac{B}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2} + \frac{C}{1-\frac{1}{2}e^{-j\omega}}\right]$$

صورت کسر $= A + \frac{A}{4}e^{-2j\omega} - Ae^{-j\omega} + B + \frac{B}{2}e^{-j\omega} + C$

$$- \frac{C}{4}e^{-2j\omega}$$

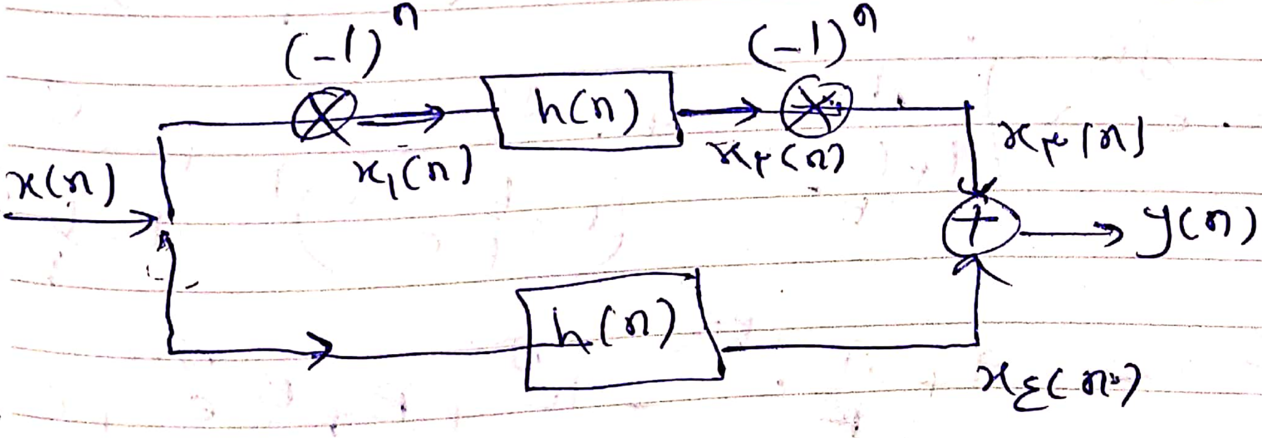
$$\left.\begin{array}{l} A+B+C=1 \\ \frac{A}{4}-\frac{C}{4}=\frac{1}{16} \\ -A+\frac{B}{2}=-\frac{1}{2} \end{array}\right\} \longrightarrow C=\frac{5}{16},\ A=\frac{9}{16},\ B=\frac{1}{8}$$

$$X(e^{j\omega})=\frac{\frac{9}{16}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}+\frac{\frac{1}{8}e^{-j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}+\frac{\frac{5}{16}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}$$

$$\xrightarrow{DTFT^{-1}} \frac{9}{16}\left(-\frac{1}{2}\right)^{n-1}u(n-1)+\frac{1}{4}n\left(\frac{1}{2}\right)^n u(n)$$

$$+\frac{5}{16}\left(\frac{1}{2}\right)^{n-1}u(n-1)$$

سوال ۶ :



(۱) $Y(e^{j\omega}) = DTFT(y(n)) = X_3(e^{j\omega}) + X_4(e^{j\omega})$

$X_1(e^{j\omega}) = DTFT(x(n)\, e^{jn\pi}) = X(e^{j(\omega-\pi)})$

$X_2(e^{j\omega}) = X_1(e^{j\omega}) H(e^{j\omega}) = X(e^{j(\omega-\pi)}) H(e^{j\omega})$

$X_3(e^{j\omega}) = X_2(e^{j(\omega-\pi)}) = X(e^{j(\omega-2\pi)}) H(e^{j(\omega-\pi)})$

$X_4(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$

$Y(e^{j\omega}) = X(e^{j(\omega-2\pi)}) H(e^{j(\omega-\pi)}) +$

$X(e^{j\omega}) H(e^{j\omega})$

چون $X(e^{j\omega})$ نسبت به ω با دوره تناوب 2π متناوب است پس:

$Y(e^{j\omega}) = X(e^{j\omega})\left(H(e^{j(\omega-\pi)}) + H(e^{j\omega})\right)$

این فیلتر باند فرکانسی $\frac{\pi}{4}$ تا $\frac{3\pi}{4}$
و $\frac{3\pi}{4}$ تا $\frac{5\pi}{4}$ را صفر و بقیه
باندهای فرکانسی را
حذف می‌کند.