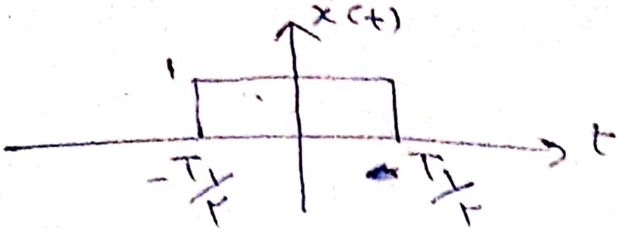
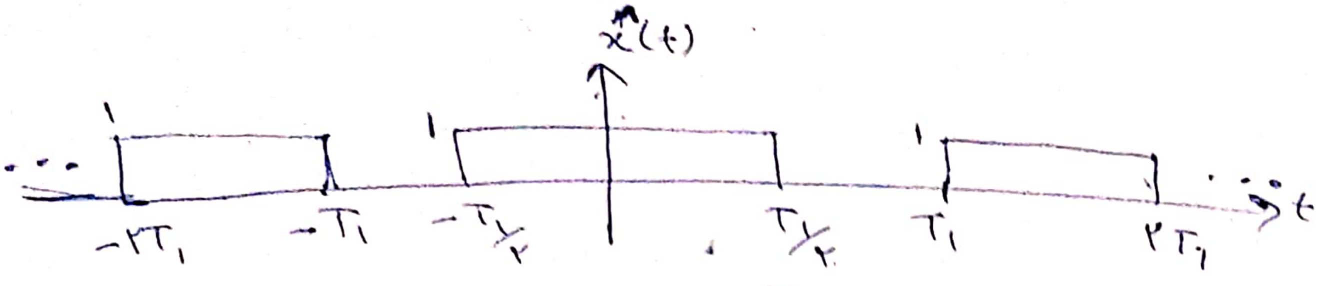
سوال ۱ - الف) :





سوال ۱ - ب) :

$$X(\omega)=\int_{-\infty}^{+\infty} x(t)\,e^{-j\omega t}\,dt=\int_{-T_1/2}^{T_1/2} 1\,e^{-j\omega t}\,dt=\frac{-1}{j\omega}e^{-j\omega t}\Big|_{-T_1/2}^{T_1/2}$$

$$=\frac{-1}{j\omega}\left[e^{-j\omega T_1/2}-e^{j\omega T_1/2}\right]=\frac{e^{j\omega T_1/2}-e^{-j\omega T_1/2}}{j\omega}=$$

$$\frac{2\sin\omega\frac{T_1}{2}}{\omega}\Rightarrow \mathrm{Sinc}\left(\frac{\omega T_1}{2\pi}\right)=\frac{\sin\frac{\omega T_1}{2\pi}\times\pi}{\frac{\omega T_1}{2\pi}\times\pi}=\frac{\sin\frac{\omega T_1}{2}}{\frac{\omega T_1}{2}}$$

$$=\frac{2\sin\frac{\omega T_1}{2}}{\omega\cancel{T_1}}\times\cancel{T_1}=\frac{2\sin\frac{\omega T_1}{2}}{\omega}\Rightarrow\frac{2\sin\frac{\omega T_1}{2}}{\omega}=T_1\,\mathrm{Sinc}\left(\frac{\omega T_1}{2\pi}\right)$$

$$\mathrm{Sinc}\left(\frac{\omega T_1}{2\pi}\right)=\frac{\sin\frac{\omega T_1}{2\pi}\times\pi}{\frac{\omega T_1}{2\pi}\times\pi}=\frac{\sin\frac{\omega T_1}{2}}{\frac{\omega T_1}{2}}=0\Rightarrow\frac{\omega T_1}{2}=\pi$$

$$\boxed{\omega=\frac{2\pi}{T_1}}$$

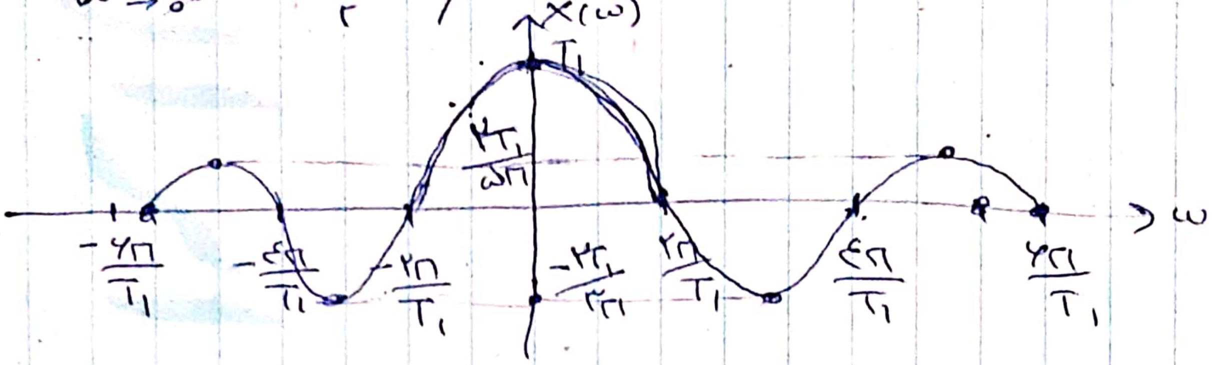
$$\frac{\omega T_1}{2}=2\pi\rightarrow\boxed{\omega=\frac{4\pi}{T_1},}$$

$$\rightarrow\frac{\omega T_1}{2}=3\pi\rightarrow\boxed{\omega=\frac{6\pi}{T_1}}$$

ادامه سوال ۱ - ب) :

$$\lim_{\omega\to 0} T_1 \operatorname{sinc}\left(\frac{\omega T_1}{2\pi}\right) = \lim_{\omega\to 0} T_1\left(\frac{\sin \frac{\omega T_1}{2\pi}\times\pi}{\frac{\omega T_1}{2\pi}\times\pi}\right) =$$

$$T_1 \lim_{\omega\to 0}\left(\frac{\sin\frac{\omega T_1}{2}}{\frac{\omega T_1}{2}}\right) = \boxed{T_1}$$

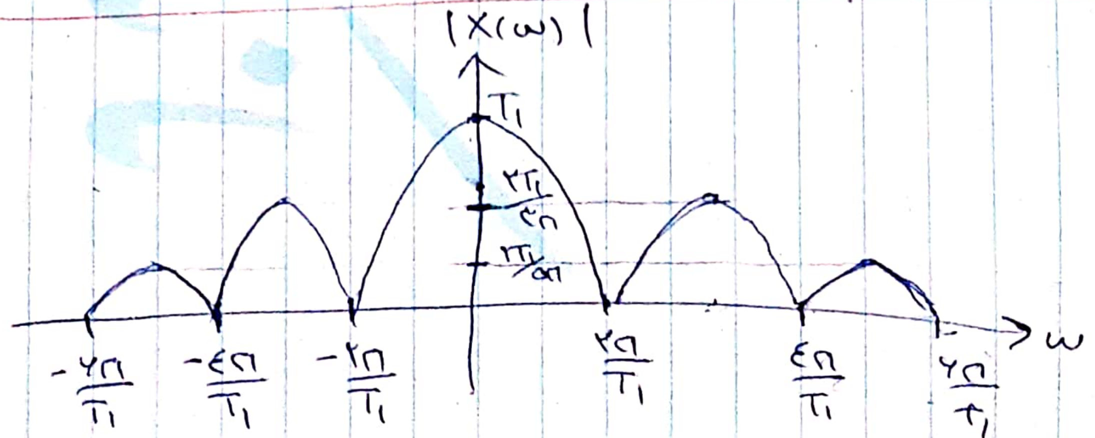


$$\omega = \frac{3\pi}{T_1} \to X\left(\frac{3\pi}{T_1}\right) = T_1 \operatorname{sinc}\left(\frac{\frac{3\pi}{T_1}\times T_1}{2\pi}\right) = T_1 \operatorname{sinc}\left(\frac{3}{2}\right)$$

$$= T_1 \frac{\sin\frac{3\pi}{2}}{\frac{3}{2}\pi} = \frac{-T_1}{\frac{3\pi}{2}} = \frac{-2T_1}{3\pi}$$

$$\omega = \frac{5\pi}{T_1} \to T_1 \operatorname{sinc}\left(\frac{\frac{5\pi}{T_1}\times T_1}{2\pi}\right) = T_1 \operatorname{sinc}\left(\frac{5}{2}\right) = T_1\frac{\sin\frac{5\pi}{2}}{\frac{5}{2}\pi}$$

$$= \frac{T_1}{\frac{5\pi}{2}} = \frac{2T_1}{5\pi}$$



سوال ۱ - ج)

$$a_k = \frac{1}{T}\int_T x(t)\,e^{-jk\omega_0 t}\,dt \rightarrow a_0 = \frac{1}{T}\int_{-T_1/2}^{T_1/2} 1\,dt = \frac{T_1}{T} = \frac{T_1}{\frac{3T_1}{2}} = \frac{2}{3}$$

$$\underset{\forall k\neq 0}{a_k} = \frac{1}{T}\int_{-T_1/2}^{T_1/2} 1\times e^{-jk\omega_0 t}\,dt = \frac{1}{T}\left[\frac{-1}{jk\omega_0}\,e^{-jk\omega_0 t}\Big|_{-T_1/2}^{T_1/2}\right] =$$

$$\frac{1}{T}\left\{\frac{-1}{jk\omega_0}\left(e^{-jk\omega_0 T_1/2} - e^{jk\omega_0 T_1/2}\right)\right\} = \frac{1}{Tjk\omega_0}\left[e^{jk\omega_0 T_1/2} - e^{-jk\omega_0 T_1/2}\right]$$

$$= \frac{2\sin k\omega_0 T_1/2}{Tk\omega_0}$$

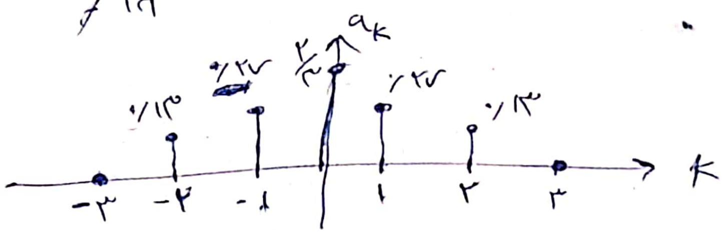
$$= \frac{2\sin k\times\frac{2\pi}{T}\times\frac{T_1}{2}}{T\times k\times\frac{2\pi}{T}} = \frac{2\sin k\pi\times\frac{2}{3}}{2k\pi} = \frac{\sin k\frac{2\pi}{3}}{k\pi} \quad \forall k\neq 0$$

$$a_k = \begin{cases} \frac{2}{3} & k=0 \\ \frac{\sin k\frac{2\pi}{3}}{k\pi} & k\neq 0 \end{cases}$$

$$|k|<4 \rightarrow -4<k<4 \rightarrow k=0,\pm1,\pm2,\pm3$$

$$a_{-1} = \frac{\sin -\frac{2\pi}{3}}{-\pi} = \frac{\sin\frac{2\pi}{3}}{\pi} = a_1 = 0/27$$

$$a_{-2} = \frac{\sin -\frac{4\pi}{3}}{-2\pi} = \frac{\sin\frac{4\pi}{3}}{2\pi} = 0/13 = a_2, \quad a_3 = a_{-3} = \frac{\sin 2\pi}{3\pi} = 0$$



سوال ۱-ت :

$$\frac{1}{T_0}X(\omega)=\frac{1}{\frac{3T_1}{2}}\times\frac{2\sin\omega\frac{T_1}{2}}{\omega}=\frac{2}{3T_1}\times\frac{2\sin\frac{2\pi k}{T_0}\times\frac{T_1}{2}}{\frac{2\pi k}{T_0}}=$$

$$\frac{1}{\cancel{T_0}}\times\frac{\cancel{2}\cancel{T_0}\sin\frac{\cancel{4}^{2}\pi k}{3\cancel{T_1}}\times\frac{\cancel{T_1}}{\cancel{2}}}{\cancel{2}\pi k}=\frac{\sin\frac{2\pi}{3}k}{\pi k}=a_k \qquad k\neq 0$$

$$k=0\Rightarrow\frac{1}{T_0}X(\omega)=\frac{1}{T_0}\underbrace{X(0)}_{T_1}=\frac{1}{\frac{3T_1}{2}}\times T_1=\frac{2}{3}=a_0$$

سوال ۲-

$$y(t)=x(t)*h(t)\Rightarrow F\{y(t)\}=Y(j\omega)=F\{x(t)*h(t)\}=X(j\omega)H(j\omega)$$

$$F\{x(3t)\}=\frac{1}{3}X\left(\frac{j\omega}{3}\right),\quad F\{h(3t)\}=\frac{1}{3}H\left(\frac{j\omega}{3}\right)$$

$$F\{g(t)\}=F\{x(3t)*h(3t)\}=G(j\omega)=F\{x(3t)\}F\{h(3t)\}=$$

$$\frac{1}{3}X\left(\frac{j\omega}{3}\right)\times\frac{1}{3}H\left(\frac{j\omega}{3}\right)=\frac{1}{9}\underbrace{X\left(\frac{j\omega}{3}\right)H\left(\frac{j\omega}{3}\right)}=G(j\omega)=\frac{1}{9}Y\left(\frac{j\omega}{3}\right)$$

از طرفی دیگر داریم: $Y(j\omega)=X(j\omega)H(j\omega)\Rightarrow \underbrace{Y\left(\frac{j\omega}{3}\right)=X\left(\frac{j\omega}{3}\right)H\left(\frac{j\omega}{3}\right)}$

$$g(t)=F^{-1}\{G(j\omega)\}=F^{-1}\left\{\frac{1}{9}Y\left(\frac{j\omega}{3}\right)\right\}=F^{-1}\left\{\frac{1}{3}\times\frac{1}{3}Y\left(\frac{j\omega}{3}\right)\right\}=$$

$$\frac{1}{3}\underbrace{\boxed{F^{-1}\left\{\frac{1}{3}Y\left(\frac{j\omega}{3}\right)\right\}}}_{y(3t)}=\frac{1}{3}y(3t)\longrightarrow A=\frac{1}{3},\quad B=3$$

سوال ۳ - الف)

$$y(t) = e^{-۳|t|} \longrightarrow x(t) = e^{-۳t}u(t) \longrightarrow y(t) = x(t) + x(-t),\ \forall t$$

$$Y(\omega) = X(\omega) + X(-\omega),\quad X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{+\infty} x(t)\, e^{-j\omega t}\, dt =$$

$$\int_{0}^{+\infty} e^{-۳t}\, e^{-j\omega t}\, dt = \int_{0}^{+\infty} e^{-(۳+j\omega)t}\, dt = \frac{-1}{۳+j\omega}\, e^{-(۳+j\omega)t}\Big|_{0}^{+\infty} =$$

$$\frac{-1}{۳+j\omega}\left[0 - 1\right] = \boxed{\frac{1}{۳+j\omega}} \qquad Y(\omega) = \frac{1}{۳+j\omega} + \frac{1}{۳-j\omega} = \frac{۲\times۳}{۹+\omega^۲} = \boxed{\frac{۶}{۹+\omega^۲}}$$

پریودیک است

$$z(t) = \sin ۲t = \frac{1}{۲j} e^{j۲t} - \frac{1}{۲j} e^{-j۲t} \longrightarrow Z(\omega) = ۲\pi \sum_{k} a_k\, \delta(\omega - k\omega_0)$$

$$Z(\omega) = ۲\pi\left[a_1\,\delta(\omega-\omega_0) + a_{-1}\,\delta(\omega+\omega_0)\right] = ۲\pi\left[\frac{1}{۲j}\delta(\omega-\omega_0) - \frac{1}{۲j}\delta(\omega+\omega_0)\right]$$

$$= \frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}\delta(\omega+\omega_0)$$

$$F\left\{e^{-۳|t|}\sin ۲t\right\} = F\left\{y(t)\, z(t)\right\} = Y(\omega) * Z(\omega) = \frac{۶}{۹+\omega^۲} * \left[\frac{\pi}{j}\delta(\omega-\omega_0)\right.$$

$$\left.- \frac{\pi}{j}\delta(\omega+\omega_0)\right] = \frac{\pi}{j}\left[\frac{۶}{۹+(\omega-\omega_0)^۲} - \frac{۶}{۹+(\omega+\omega_0)^۲}\right] =$$

$$\frac{۶\pi}{j}\left[\frac{1}{۹+(\omega-\omega_0)^۲} - \frac{1}{۹+(\omega+\omega_0)^۲}\right]$$

سوال ۳ - ب) :

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} \sum_{k=0}^{\infty} a^k \delta(t-kT)\, e^{-j\omega t}\, dt =$$

$$\sum_{k=0}^{+\infty} a^k \int_{-\infty}^{+\infty} \delta(t-kT)\, e^{-j\omega t}\, dt = \sum_{k=0}^{+\infty} a^k \underbrace{e^{-j\omega kT}}_{\text{sifting طبق خاصیت}} \;,\; \boxed{\int_{-\infty}^{+\infty} \delta(t-kT)\, dt}$$

$$= \sum_{k=0}^{+\infty} a^k e^{-j\omega kT} = \sum_{k=0}^{+\infty} \left(a e^{-j\omega T}\right)^k \frac{\text{سری هندسی با قدر نسبت}}{a e^{-j\omega T}} \;\; \frac{1}{1 - a e^{-j\omega T}}$$

سوال ۳ - ج) : طبق خاصیت $X(t) \xrightarrow{FT} 2\pi x(-\omega)$ می توان تبدیل فوریه $X(t)$ را به دست آورد و برعکس.

$$x(-\omega) = \frac{1}{\pi(\omega^2+1)} \;,\; 2\pi x(-\omega) = 2\pi \times \frac{1}{\pi(\omega^2+1)} = \boxed{\frac{2}{\omega^2+1}}$$

می دانیم که $F\{e^{-|t|}\} = \frac{2}{\omega^2+1}$ پس می توان از تبدیل فوریه $\frac{2}{\omega^2+1}$ تبدیل فوریه معکوس گرفت.

$$F^{-1}\left\{\frac{2}{\omega^2+1}\right\} = e^{-|t|} \longrightarrow X(t) = e^{-|t|}$$ پس $X(t)$ به دست آمد.

$$\hookrightarrow \boxed{X(\omega) = e^{-|\omega|}}$$

$$x(t) \xrightarrow{FT} X(\omega)$$
$$X(t) \xrightarrow{FT} 2\pi x(-\omega)$$

سوال ۴ - الف)

$$g(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}G(\omega)\,e^{j\omega t}\,d\omega=\frac{1}{2\pi}\int_{-2}^{2}1\,e^{j\omega t}\,d\omega=\frac{1}{2\pi}\left[\frac{1}{jt}\,e^{j\omega t}\Big|_{-2}^{2}\right]=$$

$$\frac{1}{2\pi jt}\left[e^{2jt}-e^{-2jt}\right]=\frac{\sin 2t}{\pi t}=\frac{\frac{\sin 2t}{\cancel{\pi}}\times 2\pi}{\frac{2t}{\cancel{\pi}}\times\pi}\times\frac{2\cancel{t}}{\pi\cancel{t}}=\frac{2}{\pi}\operatorname{sinc}\left(\frac{2t}{\pi}\right)$$

$$x(t)=\frac{g(t)}{\cos t}=\frac{\frac{2}{\pi}\operatorname{sinc}\left(\frac{2t}{\pi}\right)}{\cos t}$$

سوال ۴ - ب)

$$\underbrace{\cos\tfrac{2}{3}t}_{z(t)}=\frac{1}{2}e^{j\frac{2}{3}t}+\frac{1}{2}e^{-j\frac{2}{3}t}\Rightarrow Z(\omega)=2\pi\left[\frac{1}{2}\delta\left(\omega+\tfrac{2}{3}\right)+\frac{1}{2}\delta\left(\omega-\tfrac{2}{3}\right)\right]$$

$$=\pi\delta\left(\omega+\tfrac{2}{3}\right)+\pi\delta\left(\omega-\tfrac{2}{3}\right)$$

$$G(\omega)=\frac{1}{2\pi}\left[X(j\omega)*Z(\omega)\right]=\frac{1}{2\cancel{\pi}}\left[X_1(j\omega)*\left[\cancel{\pi}\delta\left(\omega+\tfrac{2}{3}\right)+\cancel{\pi}\delta\left(\omega-\tfrac{2}{3}\right)\right]\right]$$

$$=\frac{1}{2}\left[X_1\left(j\left(\omega+\tfrac{2}{3}\right)\right)+X_1\left(j\left(\omega-\tfrac{2}{3}\right)\right)\right]$$

از طرف دیگر داریم: $G(\omega)=\operatorname{rect}\left(\frac{\omega}{4}\right)$

سوال ۵ - الف)

$$X(t)=\frac{۲\sin[۳(t-۲\pi)]}{t-۲\pi}=\frac{۲\sin(۳t-۶\pi)}{t-۲\pi}=\frac{-۲\overbrace{\sin(۶\pi-۳t)}^{-\sin ۳t}}{t-۲\pi}$$

$$=\frac{۲\sin ۳t}{t-۲\pi}=۲\,\frac{\sin\frac{۳t}{\cancel{\pi}}\times\cancel{\pi}}{\frac{۳t}{\cancel{\pi}}\times\pi}\times\frac{۳t}{t-۲\pi}=\frac{۶t}{t-۲\pi}\,\text{sinc}\left(\frac{۳t}{\pi}\right)$$

از طرفی می‌دانیم: $\text{sinc}(t)\xrightarrow{FT}\text{rect}(f)=\text{rect}\left(\frac{\omega}{۲\pi}\right)$

scaling $\xrightarrow{\text{طبق خاصیت}}$ $\text{sinc}\left(\frac{۳}{\pi}t\right)\xrightarrow{F}\frac{1}{\frac{۳}{\pi}}\text{rect}\left(\frac{\omega}{۲\cancel{\pi}}\times\frac{\cancel{\pi}}{۳}\right)=\frac{\pi}{۳}\text{rect}\left(\frac{\omega}{۶}\right)$

$$x(t)\xrightarrow{FT}\frac{\overset{۲}{\cancel{۶}}t}{t-۲\pi}\times\frac{\pi}{\cancel{۳}}\text{rect}\left(\frac{\omega}{۶}\right)=\underbrace{\frac{۲t\pi}{t-۲\pi}\text{rect}\left(\frac{\omega}{۶}\right)}$$

می‌دانیم که برابر با $۲\pi\, x(-\omega)$ است

$$۲\cancel{\pi}\, x(-\omega)=\frac{۲t\cancel{\pi}}{t-۲\pi}\text{rect}\left(\frac{\omega}{۶}\right)\rightarrow x(-\omega)=\frac{t}{t-۲\pi}\text{rect}\left(\frac{\omega}{۶}\right)$$

$$\rightarrow x(\omega)=\frac{t}{t-۲\pi}\text{rect}\left(\frac{-\omega}{۶}\right)\rightarrow x(t)=\frac{t}{t-۲\pi}\text{rect}\left(\frac{-t}{۶}\right)$$

$x(t)\xrightarrow{FT}X(j\omega)$ از خاصیت دوگانی استفاده شد.

$X(t)\xrightarrow{FT}۲\pi\, x(-\omega)$

سوال ۵ - ب) : با توجه به خاصیت دوگانی $X(t) \xrightarrow{FT} 2\pi x(-\omega)$ پس باید تبدیل فوریه $x(t)$ را حساب کنیم. و $x(t) = \cos(4t + \frac{\pi}{3})$

$$z(t) = \cos 4t = \frac{1}{2} e^{j4t} + \frac{1}{2} e^{-j4t} \xrightarrow{FT} 2\pi \left[\frac{1}{2}\delta(\omega+4) + \frac{1}{2}\delta(\omega-4)\right]$$

$$= \pi\delta(\omega+4) + \pi\delta(\omega-4) = Z(\omega)$$

$$z'(t) = \cos\left(4t + \frac{\pi}{3}\right) \xrightarrow{FT} e^{-j\omega\frac{\pi}{3}} FT\{\cos 4t\} = e^{-j\omega\frac{\pi}{3}} \times Z(\omega) =$$

$$e^{-j\omega\frac{\pi}{3}} \left[\pi\delta(\omega+4) + \pi\delta(\omega-4)\right] = \cancel{\pi}\left[e^{-j(\omega+4)\frac{\pi}{3}} + e^{-j(\omega-4)\frac{\pi}{3}}\right]$$

$$= 2\cancel{\pi}\, x(-\omega) \Rightarrow x(-\omega) = \frac{1}{2}\left[e^{-j(\omega+4)\frac{\pi}{3}} + e^{-j(\omega-4)\frac{\pi}{3}}\right] =$$

$$\frac{1}{2}\left[\left(e^{-j\omega\frac{\pi}{3}} \times e^{-j\frac{4\pi}{3}}\right) + \left(e^{-j\omega\frac{\pi}{3}} \times e^{j\frac{4\pi}{3}}\right)\right] = \frac{e^{-j\omega\frac{\pi}{3}}}{2}\left[\underbrace{e^{-j\frac{4\pi}{3}} + e^{j\frac{4\pi}{3}}}_{=\cos\frac{4\pi}{3}}\right]$$

$$= \cos\frac{4\pi}{3} \times e^{-j\omega\frac{\pi}{3}} = -\frac{\sqrt{3}}{2} e^{-j\omega\frac{\pi}{3}} = x(-\omega)$$

$$\longrightarrow x(\omega) = -\frac{\sqrt{3}}{2} e^{j\omega\frac{\pi}{3}} \longrightarrow x(t) = -\frac{\sqrt{3}}{2} e^{j\frac{\pi}{3}t}$$

جواب نهایی

سوال ۲ - الف :

می‌دانیم : $2\sin s \cos t = \sin(s+t) + \sin(s-t)$

$\to 2\sin t \cos 2t = \sin(t+2t) + \sin(t-2t) = \sin 3t + \overbrace{\sin -t}^{-\sin t} \Rightarrow$

$h(t) = \dfrac{\sin 3t - \sin t}{\pi t} \to H(\omega) = F\left\{\dfrac{\sin 3t - \sin t}{\pi t}\right\} = F\left\{\dfrac{\sin 3t}{\pi t}\right\} - F\left\{\dfrac{\sin t}{\pi t}\right\}$

$= F\left\{\dfrac{3}{\pi}\,\mathrm{sinc}\left(\dfrac{3t}{\pi}\right)\right\} - F\left\{\dfrac{1}{\pi}\,\mathrm{sinc}\left(\dfrac{t}{\pi}\right)\right\} =$

$= \cancel{\dfrac{\pi}{3}} \times \cancel{\dfrac{3}{\pi}}\, \mathrm{rect}\left(\dfrac{\frac{\omega}{2\pi}}{\frac{3}{2\pi}}\right) - \dfrac{1}{\pi} \times \pi\, \mathrm{rect}\left(\dfrac{\omega}{2\pi} \times 2\pi\right) = \mathrm{rect}\left(\dfrac{\omega}{6}\right) - \mathrm{rect}\left(\dfrac{\omega}{2}\right)$

سوال ۲ - ب :

می‌دانیم : $1 - \cos mx = 2\sin^2 \dfrac{mx}{2} \to 1 - \cos \pi t = 2\sin^2 \dfrac{\pi}{2} t$

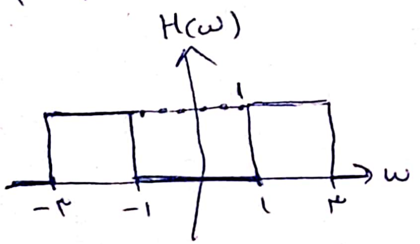
$\to \dfrac{1}{2} - \dfrac{1}{2}\cos \pi t = \sin^2 \dfrac{\pi}{2} t \to x(t) = 3 + \dfrac{1}{2} - \dfrac{1}{2}\cos \pi t = \dfrac{7}{2} - \dfrac{1}{2}\cos \pi t$

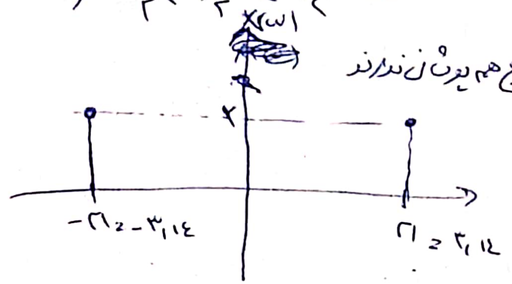
$\cos \pi t = \dfrac{1}{2} e^{j\pi t} + \dfrac{1}{2} e^{-j\pi t} \to FT\{\cos \pi t\} = 2\pi\left[\dfrac{1}{2}\delta(\omega+\pi) + \dfrac{1}{2}\delta(\omega-\pi)\right]$

$= \pi\delta(\omega+\pi) + \pi\delta(\omega-\pi) \to X(\omega) = \dfrac{7}{2} - \dfrac{\pi}{2}\delta(\omega+\pi) - \dfrac{\pi}{2}\delta(\omega-\pi)$

$Y(\omega) = X(\omega)H(\omega) = \left[\dfrac{7}{2} - \dfrac{\pi}{2}\delta(\omega+\pi) - \dfrac{\pi}{2}\delta(\omega-\pi)\right]\left[\mathrm{rect}\left(\dfrac{\omega}{6}\right) - \mathrm{rect}\left(\dfrac{\omega}{2}\right)\right]$

$-\dfrac{1}{2} \le \dfrac{\omega}{6} \le \dfrac{1}{2} \to -3 \le \omega \le 3$ و $-\dfrac{1}{2} \le \dfrac{\omega}{2} \le \dfrac{1}{2} \to -1 \le \omega \le 1$





با توجه به اینکه $X(\omega)$ و $H(\omega)$ هیچ هم‌پوشانی ندارند

پس $Y(\omega) = 0$ است

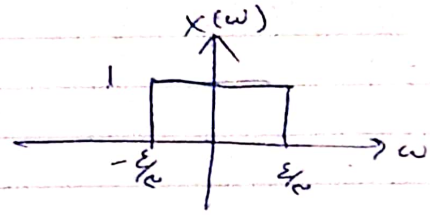
$y(t) = F^{-1}\{Y(\omega)\} = 0$

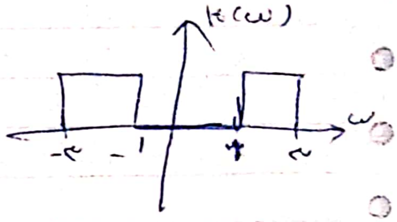
سوال ۶ - ج) :

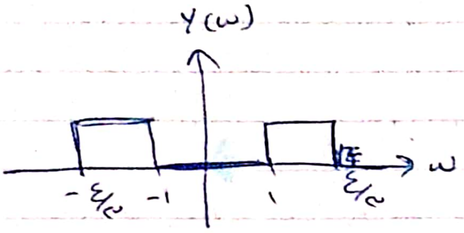
$$\frac{\sin \frac{4}{3} t}{\pi t} = \frac{4}{3\pi} \operatorname{sinc}\left(\frac{4t}{3\pi}\right)$$

$$F\left\{\frac{4}{3\pi} \operatorname{sinc}\left(\frac{4t}{3\pi}\right)\right\} = \cancel{\frac{4}{3\pi} \times \frac{3\pi}{4}}\, \operatorname{rect}\left(\frac{\omega}{2\pi} \times \frac{3\pi}{4}\right) = \operatorname{rect}\left(\frac{3\omega}{8}\right) = X(\omega)$$

$$-\frac{1}{2} \leq \frac{3\omega}{8} \leq \frac{1}{2} \longrightarrow -\frac{4}{3} \leq \omega \leq \frac{4}{3}$$







$$\rightarrow Y(\omega) = \begin{cases} 1 & -\frac{4}{3} \leq \omega \leq -1 \ \text{یا} \ 1 \leq \omega \leq \frac{4}{3} \\ 0 & \text{otherwise} \end{cases}$$

انرژی $x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-4/3}^{4/3} 1\, d\omega = \frac{4}{3\pi}$

انرژی $y(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |Y(j\omega)|^2 d\omega = \frac{1}{2\pi}\left[\int_{-4/3}^{-1} 1\, d\omega + \int_{1}^{4/3} 1\, d\omega\right] = \frac{1}{2\pi} \times \frac{2}{3} = \frac{1}{3\pi}$

سوال ۷ : $h_{\text{کل}} = \left(h_1(t) + (h_1(t) * h_2(t))\right) * h_3(t) * h_4(t)$

$$\rightarrow H(\omega) = \left(H_1(\omega) + (H_1(\omega) H_2(\omega))\right) H_3(\omega)\, H_4(\omega)$$

$$Y(\omega) = X(\omega)\, H(\omega) = \left(X(\omega) H_1(\omega) + X(\omega) H_1(\omega) H_2(\omega)\right) H_3(\omega)\, H_4(\omega)$$

$$= X(\omega) H_1(\omega) \left[1 + H_2(\omega)\right] \times H_3(\omega)\, H_4(\omega)$$

$$h_1(t) = \frac{\sin \omega_c t}{2\pi t} = \frac{\omega_c}{2\pi} \operatorname{sinc}\left(\frac{\omega_c t}{\pi}\right) *$$

ادامه سوال ۷:

$$F\{h_1(t)\} = H_1(\omega) = \frac{\omega_c}{2\pi} \times \frac{\pi}{\omega_c} \times \mathrm{rect}\left(\frac{\omega/2\pi}{\frac{\omega_c}{\pi}}\right) = \frac{1}{2}\,\mathrm{rect}\left(\frac{\omega}{2\omega_c}\right)$$

$$h_2(t) = \frac{\sin 2\omega_c t}{\pi t} = \frac{2\omega_c}{\pi}\,\mathrm{sinc}\left(\frac{2\omega_c t}{\pi}\right)$$

$$F\{h_2(t)\} = H_2(\omega) = \frac{2\omega_c}{\pi} \times \frac{\pi}{2\omega_c} \times \mathrm{rect}\left(\frac{\omega/\pi}{\frac{2\omega_c}{\pi}}\right) = \mathrm{rect}\left(\frac{\omega}{4\omega_c}\right)$$

$$H_3(\omega) = \int_{-\infty}^{+\infty} h_3(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} u(t)\, e^{-j\omega t}\, dt = \int_{0}^{+\infty} e^{-j\omega t}\, dt = \frac{-1}{j\omega} e^{-j\omega t}\Big|_0^{+\infty}$$

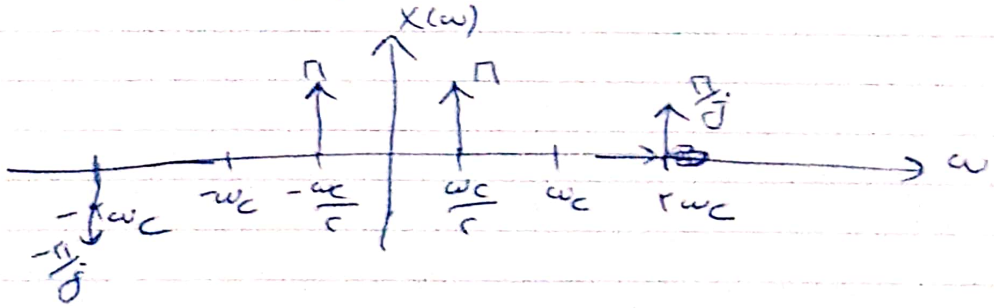
$$= \frac{1}{j\omega}$$

$$X(\omega) = FT\{x(t)\} = FT\{\sin 2\omega_c t + \cos \tfrac{\omega_c}{2} t\} = FT\{\sin 2\omega_c t\} +$$

$$FT\{\cos \tfrac{\omega_c}{2} t\} = 2\pi\left[\frac{1}{2j}\delta(\omega - 2\omega_c) - \frac{1}{2j}\delta(\omega + 2\omega_c)\right] +$$

$$2\pi\left[\frac{1}{2}\delta\left(\omega - \frac{\omega_c}{2}\right) + \frac{1}{2}\delta\left(\omega + \frac{\omega_c}{2}\right)\right] = \frac{\pi}{j}\delta(\omega - 2\omega_c) - \frac{\pi}{j}\delta(\omega + 2\omega_c)$$

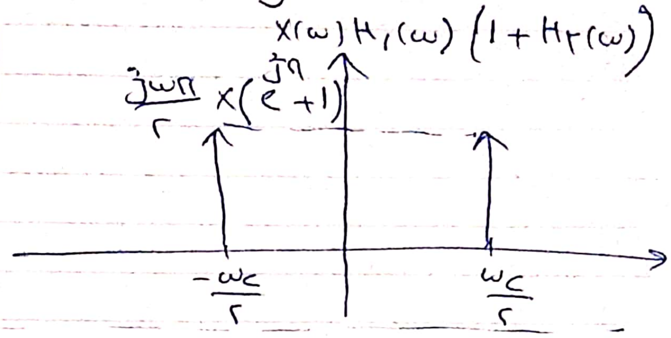
$$+ \pi\delta\left(\omega - \frac{\omega_c}{2}\right) + \pi\delta\left(\omega + \frac{\omega_c}{2}\right)$$



ادامه سوال ۷:

$$X(\omega)H_1(\omega) = \frac{j\omega}{2}\left[\pi\delta\left(\omega-\frac{\omega_c}{2}\right)+\pi\delta\left(\omega+\frac{\omega_c}{2}\right)\right]$$

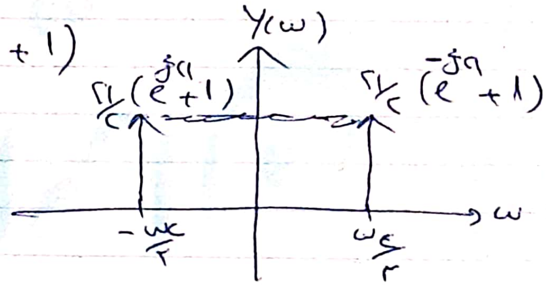
$$= \frac{j\omega\pi}{2}\left[\delta\left(\omega-\frac{\omega_c}{2}\right)+\delta\left(\omega+\frac{\omega_c}{2}\right)\right]$$



$$H_2\left(\frac{\omega_c}{2}\right) = e^{-j\pi}\ ,\quad H_2\left(\frac{-\omega_c}{2}\right) = e^{j\pi}$$

نقطه $-\frac{\omega_c}{2}$: $\frac{j\omega\pi}{2}\times(e^{j\pi}+1)\times\frac{1}{j\omega} = \frac{\pi}{2}(e^{j\pi}+1)$

نقطه $\frac{\omega_c}{2}$: $\frac{j\omega\pi}{2}\times(e^{-j\pi}+1)\times\frac{1}{j\omega} = \frac{\pi}{2}(e^{-j\pi}+1)$



$$\rightarrow Y(\omega) = \left[\frac{\pi}{2}(e^{j\pi}+1)\right]\delta\left(\omega+\frac{\omega_c}{2}\right)+\left[\frac{\pi}{2}(e^{-j\pi}+1)\right]\delta\left(\omega-\frac{\omega_c}{2}\right) = 0$$

$$\rightarrow y(t) = F^{-1}\{Y(\omega)\} = 0$$

سؤال ۸ ـ الف) از طرفین معادله تبدیل فوریه میگیریم:

$$(j\omega)^2 Y(j\omega) + 6(j\omega)Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\cancel{Y(j\omega)}}{\frac{(j\omega)^2 \cancel{Y(j\omega)} + 6(j\omega)\cancel{Y(j\omega)} + 8\cancel{Y(j\omega)}}{2}} = \frac{2}{\underbrace{(j\omega)}_{s}{}^2 + 6(j\omega) + 8}$$

$$= \frac{2}{s^2+6s+8} = \frac{2}{(s+2)(s+4)} = \frac{2}{(j\omega+2)(j\omega+4)} \overset{PFE}{=} \frac{A}{j\omega+2} + \frac{B}{j\omega+4}$$

$$A(j\omega+4) + B(j\omega+2) = Aj\omega + 4A + Bj\omega + 2B = (A+B)j\omega + 4A + 2B = 2$$

$$A+B=0 \rightarrow A=-B \quad , \quad 2A+B=1 \rightarrow -2B+B=-B=1 \rightarrow B=-1$$

$$A=1$$

$$H(j\omega) = \frac{1}{j\omega+2} + \frac{-1}{j\omega+4} \rightarrow h(t) = F^{-1}\{H(j\omega)\} = e^{-2t}u(t) - e^{-4t}u(t)$$

سؤال ۸ ـ ب) : از قبل میدانیم

$$FT\{t\,x(t)\} = j\frac{dX(\omega)}{d\omega}$$

$$FT\{t e^{-2t}u(t)\} = j\frac{d\left(\frac{1}{j\omega+2}\right)}{d\omega} = \frac{1}{(j\omega+2)^2} = X(j\omega)$$

$$Y(j\omega) = H(j\omega)X(j\omega) = \frac{1}{(j\omega+2)^2} \times \left[\frac{2}{(j\omega+2)(j\omega+4)}\right] = \frac{2}{(j\omega+2)^3(j\omega+4)} \overset{PFE}{=}$$

$$\frac{A}{j\omega+4} + \frac{B}{j\omega+2} + \frac{C}{(j\omega+2)^2} + \frac{D}{(j\omega+2)^3}$$

$$A(j\omega+2)^3 + B(j\omega+4)(j\omega+2)^2 + C(j\omega+4)(j\omega+2) + D(j\omega+4) = 2$$

$$(j\omega+2)\left[A(j\omega+2)^2 + B(j\omega+4)(j\omega+2) + C(j\omega+4)\right] + D(j\omega+4) = 2$$

(ادامه سوال ۸ ـ ب)

$$A(j\omega+2)^2 = A[-\omega^2+4j\omega+4] = -A\omega^2+4Aj\omega+4A$$

$$B(j\omega+4)(j\omega+2) = B(-\omega^2+6j\omega+8) = -B\omega^2+6Bj\omega+8B$$

$$[-A\omega^2 - B\omega^2 + 4Aj\omega + 6Bj\omega + 4A + 8B + Cj\omega + 4C](j\omega+2)$$

$$= [(A+B)j^2\omega^2 + (4A+6B+C)j\omega + 4(A+2B+C)](j\omega+2)$$

$$(A+B)j^3\omega^3 + (4A+6B+C)j^2\omega^2 + \overset{4A+8B+4C}{\cancel{4(A+2B+C)}}j\omega + \overset{2A+2B}{\cancel{2(A+B)}}j^2\omega^2$$

$$+ \overset{8A+12B+2C}{\cancel{2(4A+6B+C)}}j\omega + 8(A+2B+C) = (A+B)j^3\omega^3$$

$$+ (6A+8B+C)j^2\omega^2 + (12A+20B+6C+D)j\omega + 8A+16B+8C$$

$$+4D = 2 \Rightarrow \begin{cases} A+B=0 \longrightarrow A=-B \\ 6A+8B+C=0 \longrightarrow -6B+8B+C = 2B+C = 0 \quad (1) \\ 12A+20B+6C+D=0 \quad (2) \\ 8A+16B+8C+4D=2 \quad (3) \end{cases}$$

(1) $2B+C=0 \longrightarrow 2B=-C \longrightarrow C=-2B=2A$

(2) $12A-20A+12A+D=0 \longrightarrow 4A+D=0 \longrightarrow D=-4A$

(3) $8A-16A+\cancel{16A}-\cancel{16A}=2 \Rightarrow -8A=2 \longrightarrow A=\frac{-2}{8}=-\frac{1}{4}$

$B=\frac{1}{4}$, $C=-\frac{1}{2}$, $D=1$

$$Y(j\omega) = \frac{-\frac{1}{4}}{j\omega+4} + \frac{\frac{1}{4}}{j\omega+2} + \frac{-\frac{1}{2}}{(j\omega+2)^2} + \frac{1}{(j\omega+2)^3}$$

$$y(t) = -\frac{1}{4}e^{-4t}u(t) + \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t)$$

سوال ۵ - ج)

$$x(t) = F^{-1}\{X(j\omega)\} = F^{-1}\{X^*(\omega+3)\, e^{2j\omega}\} = 2\pi\, F^{-1}\{X^*(\omega+3)\} * F^{-1}\{e^{2j\omega}\}$$

از قبل می‌دانیم :
$$\begin{cases} x^*(t) \xrightarrow{FT} X^*(-\omega) \\ x^*(-t) \xrightarrow{FT} X^*(\omega) \end{cases}$$

$$F^{-1}\{X(\omega)\} = x(t) \rightarrow F^{-1}\{X^*(\omega)\} = x^*(-t) \rightarrow F^{-1}\{X^*(\omega+3)\} = x^*(-t)\, e^{-3jt}$$

طبق خاصیت که شیفت در حوزه فرکانس معادل تغییر در حوزه زمان است

$$F^{-1}\{e^{2j\omega}\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{2j\omega} e^{j\omega t}\, d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{j\omega(2+t)}\, d\omega = \frac{1}{2\pi} \times 2\pi\, \delta(t+2) = \delta(t+2)$$

طبق کم معادل است با

$$x(t) = 2\pi \big[\underbrace{x^*(-t)\, e^{-3jt} * \delta(t+2)}_{\text{sifting خاصیت}}\big] = 2\pi\, x^*(-t+2)\, e^{-3j(t+2)}$$