Advanced Computer Vision Methods
Homework 1

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Ex 1) *Take derivative of* $f(x) = (x + x^3)^4 - x$.

$$f'(x) = 4(x + x^3)^3 \cdot (1 + 3x^2) - 1$$

Ex 2) *Linearize the function* $f(x) = x + 2x^2$ *in* $x_0 = 3$.

By the definition:

$$f(x_0 + \delta) \approx f(x_0) + \frac{\partial f}{\partial x}|_{x_0} \cdot \delta$$

Now we calculate the derivative $\frac{\partial f}{\partial x}$:

$$\begin{aligned} \frac{\partial f}{\partial x} &= 1 + 4x \\ \Rightarrow \; f(x_0 + \delta) &\approx x_0 + 2x_0^2 + (1 + 4x_0)\delta \end{aligned}$$

If we denote $x_1 = x_0 = \delta$ it follows that $\delta = x_1 - x_0$, thus, considering $x_0 = 3$, $f(3) = 21$ and $\frac{\partial f}{\partial x}|_{x_0} = 13$

$$\begin{aligned} f(x_1) &\approx f(x_0) + \frac{\partial f}{\partial x}|_{x_0} \cdot (x_1 - x_0) \\ f(x_1) &\approx 21 + 13 \cdot (x_1 - 3) = 21 + 13x_1 - 39 \end{aligned}$$

Thus, linear approx can be written as a function $f(x) = 13x - 18$.

Another way to solve this task would be computing the "slope" coefficient directly from the derivative as $k = f'(3) = 13$ and then, considering that tangent linear function goes through the point $T_0(3, 21)$, compute $n = -18$, thus tangent line can be written as $y = 13x - 18$, which is the same as above.

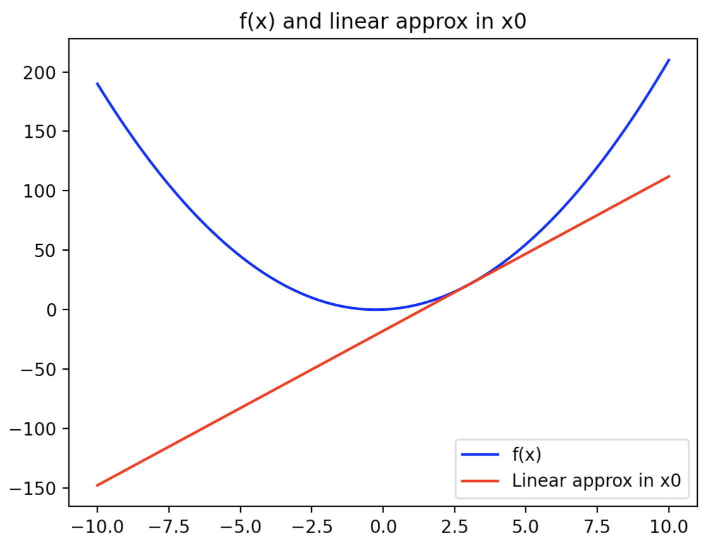

Figure 1: Function $f(x)$ and linear approximation in $x_0 = 3$.

Ex 3) *Linearize* $f(x_1(p), x_2(p)) = x_1^2(p) + x_2^5(p)$ *with parameters defined as* $x_1(p) = p_1, x_2(p) = p_1^2 + p_2^2$.

From the lectures we know

$$f(\vec{x}(p_0 + \delta)) \approx f(\vec{x}(p_0)) + \nabla f^T|_{p_0} \cdot J|_{p_0} \cdot \delta$$

and

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}, \quad p_0 = \begin{bmatrix} p_{0_1} \\ p_{0_2} \end{bmatrix}$$

We calculate

$$\nabla f^T = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right] = \left[2x_1(p), 5x_2^4(p)\right]$$

$$J = \begin{bmatrix} \frac{\partial x_1}{\partial p_1} & \frac{\partial x_1}{\partial p_2} \\ \frac{\partial x_2}{\partial p_1} & \frac{\partial x_2}{\partial p_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2p_1 & 2p_2 \end{bmatrix}$$

and then rewrite

$$\begin{aligned}
f(\vec{x}(p_0 + \delta)) &\approx f(\vec{x}(p_0)) + \left[2x_1(p), 5x_2^4(p)\right]|_{p_0} \begin{bmatrix} 1 & 0 \\ 2p_1 & 2p_2 \end{bmatrix} |_{p_0} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \\
&= x_1^2(p_0) + x_2^5(p_0) + \left[2x_1(p) + 10x_2^4(p) \cdot p_1 + 10x_2^4(p) \cdot p_2\right]|_{p_0} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \\
&= x_1^2(p_0) + x_2^5(p_0) + \left(2x_1(p)\delta_1 + 10x_2^4(p) \cdot p_1\delta_1 + 10x_2^4(p) \cdot p_2\delta_2\right)|_{p_0} \\
&= p_{0_1}^2 + (p_{0_1}^2 + p_{0_2}^2)^5 + 2p_{0_1}\delta_1 + 10(p_{0_1}^2 + p_{0_2}^2)^4 \cdot p_{0_1}\delta_1 + 10(p_{0_1}^2 + p_{0_2}^2)^4 \cdot p_{0_2}\delta_2
\end{aligned}$$

Ex 4) *Take* $f(x) = \vec{x}^T A\vec{x} \cdot 3$, *where* $\vec{x} \in \mathbb{R}^d$ *and* $A \in \mathbb{R}^{d\times d}$ *a symmetric positive definite matrix. Calculate* $\frac{\partial f}{\partial \vec{x}}$.

Following the hint and considering A is symmetric:

$$\frac{\partial f}{\partial \vec{x}} = 3(A + A^T)\vec{x} = 3 \cdot 2A\vec{x} = 6A\vec{x}$$