Stability and delay analysis of delay tolerant networks with random message arrivals

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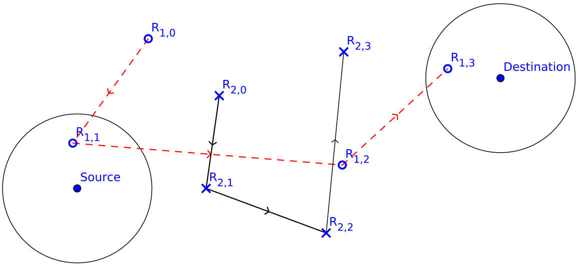
COMSNETS, 2017

Outline

1. System model
2. Stability analysis
3. Delay analysis
4. Future work

System model



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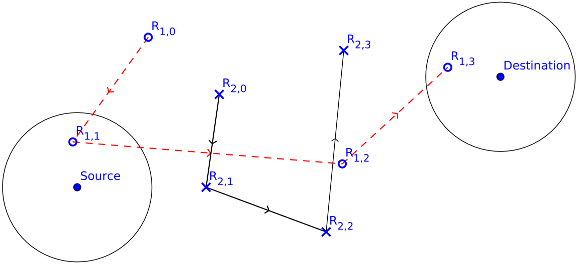

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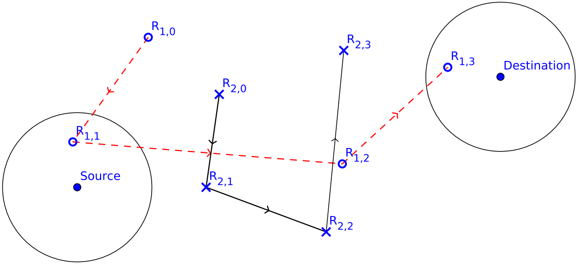

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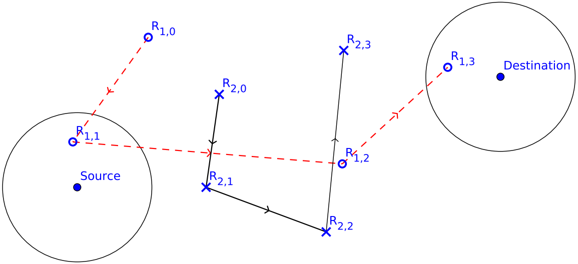

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- Mobility model: $\exp(\beta)$ intermeeting times between pairs of nodes, independent across node pair.
- Packets arrive to the source at the points of a Poisson point process of rate λ.
- Routing protocol: Two-hop relaying.

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- Assumptions:
  - A relay can hold at most one packet.
  - Source does not meet the destination.
  - No feedback mechanism for acknowledging message delivery.
- Goal:
  - What is the maximum arrival rate λ that the system can support?
  - How does end-to-end delay depend on the system parameters?

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- $\{(Q(t), R(t)), t \geq 0\}$ is a discrete-time Markov process on $\mathbb{Z}_+ \times \{0, 1, \cdots N\}$.
- Stability threshold:

$$\lambda^* = \sup\{\lambda : \{(Q(t), R(t)), t \geq 0\} \text{ is recurrent}\}.$$

Stability analysis

- Transition rates of the process $\{(Q(t), R(t)), t \geq 0\}$ for $i, j > 0$:
  - $$(i,j) \longrightarrow \begin{cases} (i+K, j), & \text{at rate } \lambda \\ (i, j-1), & \text{at rate } j\beta \\ (i-1, j+1), & \text{at rate } (N-j)\beta \end{cases}$$

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  - $(0,0) \to (K,0)$ at rate λ

Theorem

The stability threshold of the source queue is $\lambda^* = \frac{N\beta}{2K}$.

- Proof via verifying conditions for recurrence in [1]

[1] H. Kestan, Recurrent criteria for multidimensional Markov chains and multi-dimensional linear birth and death processes, Adv. App. Prob., 1976

Stability analysis: Two general cases

- Source-destination meetings are permitted.
- Transition rates:

$$(i,j) \longrightarrow \begin{cases} (i+K,j), & \text{at rate } \lambda \\ (i-1,j), & \text{at rate } \beta \\ (i,j-1), & \text{at rate } j\beta \\ (i-1,j+1), & \text{at rate } (N-j)\beta \end{cases}$$

Theorem

The stability threshold of the source queue is

$$\lambda^* = \frac{(N+2)\beta}{2K}.$$

Stability analysis: Two general cases

- Source-destination meetings are permitted.
- When a source-destination meeting occurs, all copies of the present message are discarded from the source buffer.
- Transition rates:

$$(i,j) \longrightarrow \begin{cases} (i+K,j), & \text{at rate } \lambda \\ (qK,j), & \text{at rate } \beta \\ (i,j-1), & \text{at rate } j\beta \\ (i-1,j+1), & \text{at rate } (N-j)\beta \end{cases}$$

where $i = qK + r$, $1 \leq r \leq K$, $q \in \mathbb{N}$.

Theorem

Let

$$\lambda_1 = \frac{(N+2)\beta}{2K}, \quad \text{and} \quad \lambda_2 = \frac{(N+2K)\beta}{2K}.$$

Then, the queue is stable for any rate λ *such that* $\lambda < \lambda_1$*, and the queue is unstable for any rate* λ *such that* $\lambda > \lambda_2$*.*

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- Stability threshold is linearly proportional to the number of relay nodes N, and inversely proportional to the redundancy parameter K.

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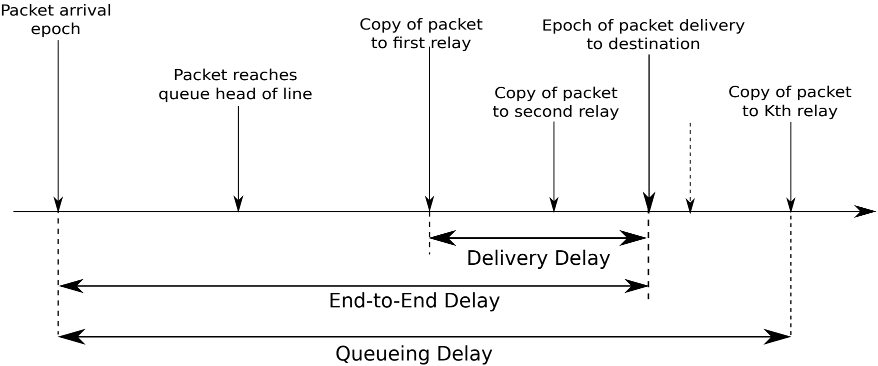
- For a fixed N, more number of relay nodes are infected with the same packet as we increase K, which reduces the stability threshold.
- For a fixed K, packets can be delivered quickly if we have more number of relay nodes which results in higher stability threshold.

Delay analysis

- Delay components: Delivery delay ($D_{d,i}$), end-to-end delay ($D_{e,i}$) and queueing delay ($D_{q,i}$).



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- $N = 1$. Source queue can be modelled by an $M/G/1$ queue with special service for the first customer.

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$$\text{source} \xrightarrow{E_{k,2}} \text{relay} \xrightarrow{E_{k,1}} \text{destination}$$

- During an idle period, service time is

$$X^* = \begin{cases} E_{1,2} + \sum_{k=2}^{K} (E_{k,1} + E_{k,2}), w.p. \frac{\beta}{\beta+\lambda}, \\ \sum_{k=1}^{K} (E_{k,1} + E_{k,2}), w.p. \frac{\lambda}{\beta+\lambda}. \end{cases}$$

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Theorem

The average queueing delay $\overline{d_q}$ is given by

$$\overline{d_q} = \frac{2\overline{X^*} + \lambda(\overline{(X^*)^2} - \overline{X^2})}{2(1 - \lambda\overline{X} + \lambda\overline{X^*})} + \frac{\overline{X^2}}{2(1 - \lambda\overline{X})}.$$

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- Assuming the service time of first customer to an empty queue to be distributed as X,

$$\overline{d_q} = \frac{\lambda\overline{X^2}}{2(1 - \lambda\overline{X})} + \overline{X}.$$

where $\overline{X} = \frac{2K}{\beta}$ and $\overline{X^2} = \frac{2K}{\beta^2} + \frac{4K^2}{\beta^2}$.

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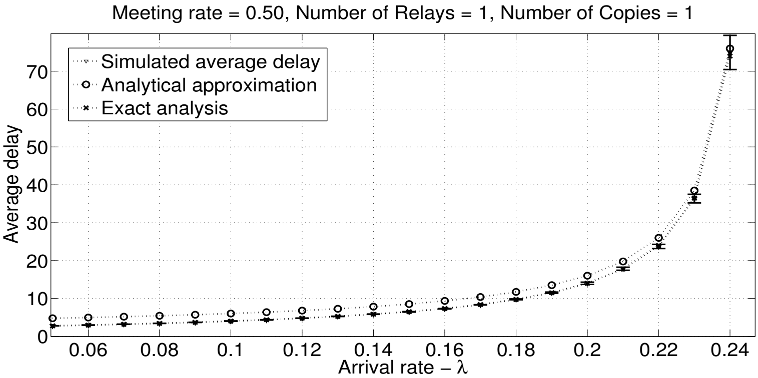
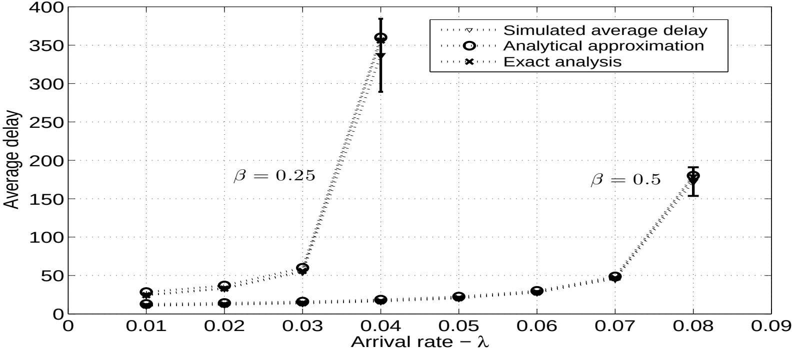
- Average delivery delay is $\frac{1}{\beta}$.
- $K = 1$ gives the minimum $\overline{d_q}$.

Simulation results: single relay case



- Stability threshold $\lambda^* = \frac{N\beta}{2K} = 0.25$

- $K = 3$



- $\lambda_1^* = 0.0416, \lambda_2^* = 0.083$

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- Approximate average queueing delay:

$$\overline{d_q} = \frac{\lambda \overline{(X^*)^2}}{2(1 - K\lambda\overline{X})} + K\overline{X},$$

where $\overline{X} = \frac{2}{N\beta}$, $\overline{(X^*)^2} = (\overline{X^2} - (\overline{X})^2)K + K^2(\overline{X})^2$, $\overline{X^2} = \frac{2}{N\beta^2} + \frac{4}{N^2\beta^2}$.

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Delay analysis: Multiple relays

- Simplification yields

$$\overline{d_q} = \frac{\lambda\left[\frac{2K}{N\beta^2} + \frac{4K^2}{N^2\beta^2}\right]}{2(1 - \frac{2K\lambda)}{N\beta}} + \frac{2K}{N\beta}.$$

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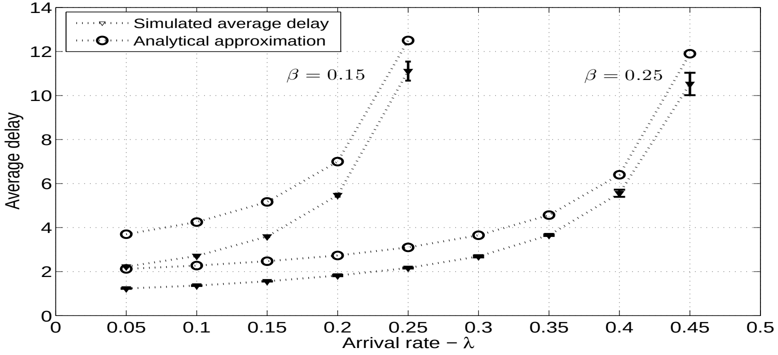
- Heuristic correction based on simulation results: $\frac{2K}{N^2\beta^2}$ instead of $\frac{2K}{N\beta^2}$.

$$\overline{d_q} \quad = \quad \frac{\lambda\left[\frac{2K}{N^2\beta^2}+\frac{4K^2}{N^2\beta^2}\right]}{2(1-\frac{2K\lambda}{N\beta})}+\frac{2K}{N\beta}.$$

Simulation results

- $N = 20, K = 5$



- $\lambda_1^* = 0.3, \lambda_2^* = 0.5$

Simulation results

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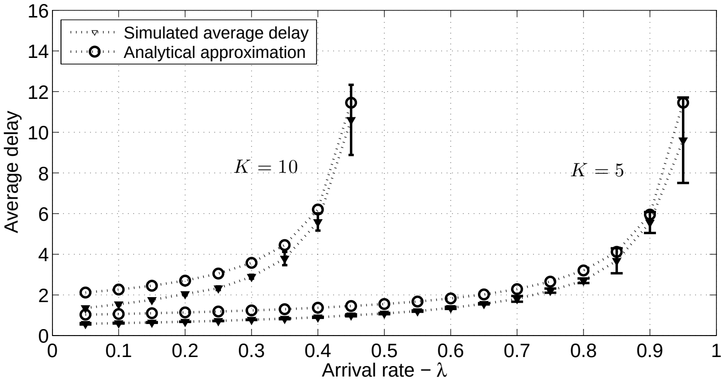
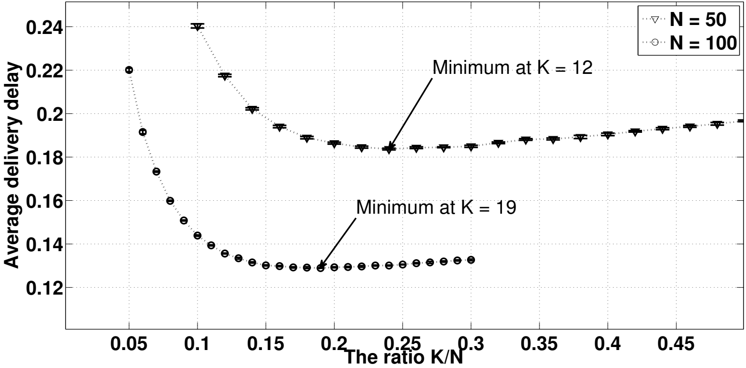


- $\lambda_1^* = 0.5, \lambda_2^* = 1$

Delivery delay vs. number of copies

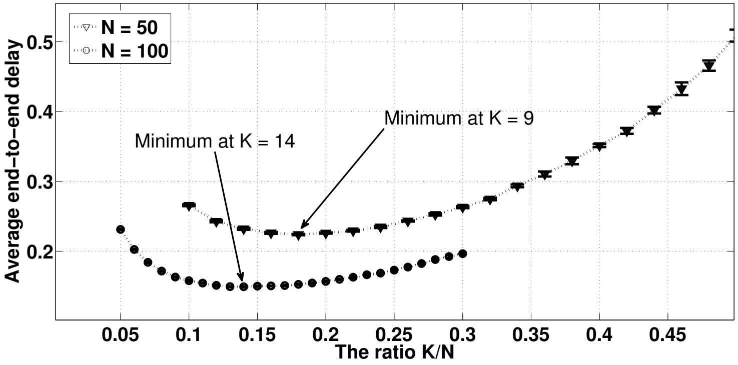
- $\lambda = 0.45, \beta = 1$



- An optimal value of K at with delivery delay is minimum.

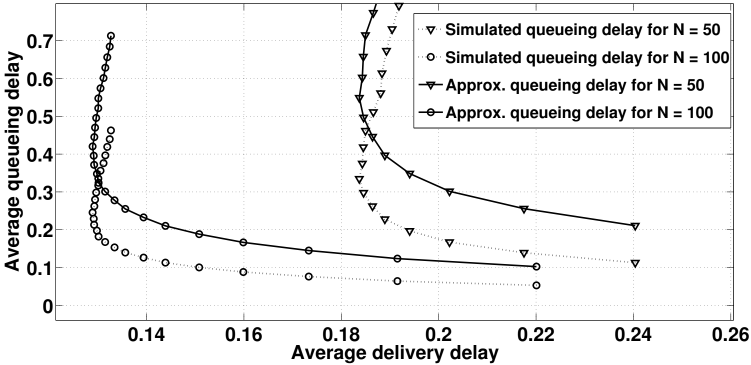
End-to-end delay vs. number of copies

- $\lambda = 0.45, \beta = 1$



Tradeoff: Queueing delay and delivery delay

- $\lambda = 0.45$, and $\beta = 1$



- Queueing delay $\overline{d_q}$ increases monotonically with K. Tradeoff between $\overline{d_q}$ and $\overline{d_d}$ for $K \in \{1, 2, \cdots, K^*\}$.

Future work

- Extension to multiple source-destination pairs.
- Relay nodes can carry more than one packet at a time.
- Behaviour for large number of relay nodes.

Thank you