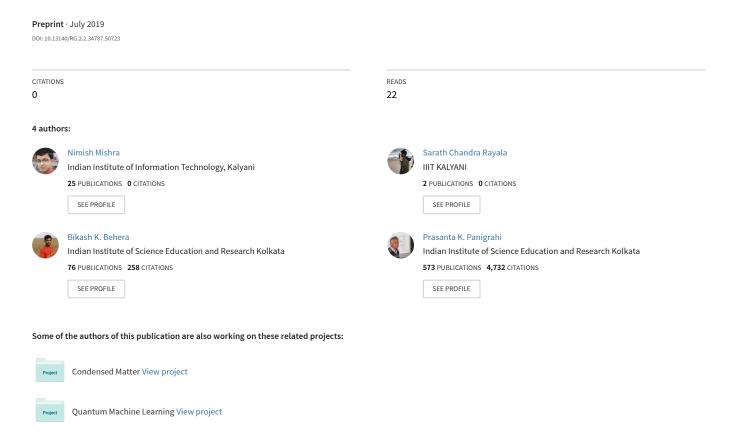
# Adding degrees of freedom to automated quantum Braitenberg vehicles



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Ever since practical quantum computation is possible, there have been attempts to apply the power of quantum mechanics to robotics and develop quantum equivalent of classical robots called quantum robots. Here, we discuss adding degrees of freedom to a quantum Braitenberg vehicle and then automating it. To achieve this, we draw parallelism between classical Boolean logic and quantum propositional logic of eigen-states. We find that it is possible to treat quantum propositional logic of a quantum robot as classical Boolean logic, and that we can create equivalent quantum circuits to realize such logic. We demonstrate adding a degree of freedom to a quantum Braitenberg vehicle previously having two degrees of freedom, and discuss how complex desired behaviour may be added. We demonstrate the ability of the memory of such a robot to introduce desired intrinsic behaviour. Our findings are a step towards our continued study on generalization of quantum logic for quantum robots.

## I. INTRODUCTION

The idea of quantum information was first suggested by Paul Benioff in 1980<sup>1</sup> amidst increasing inquiry into the physical nature of *classical* information first introduced by Rolf Landauer<sup>2</sup>. Fenyman furthered this idea by pointing to the intrinsic ability of a quantum computer (*if it could be constructed*) to handle matrix operations better than its classical counterpart<sup>3</sup>, while Manin wrote on similar lines in his book *Computable and Uncomputable*<sup>4</sup>. Quantum Turing machines were then conceptualized by Deutsch<sup>5</sup>.

The idea of a quantum robot was first introduced by Benioff<sup>6</sup> as a mobile quantum system coupled with a quantum computer and an ancilla system, that lets the robot take deterministic/non-deterministic action based on its measurement of the environment. A quantum robot is able to take advantage of purely quantum phenomena - superposition, entanglement, violation of Bell's inequality<sup>7</sup>, and non-deterministic nature of quantum computation and quantum algorithms. Thereafter, several inquiries into this field have been made. Dong et. al. <sup>8,9</sup> worked on learning algorithms for quantum robots. Zeno et. al. formalized the use of eigenlogic for introducing special behavioral possibilities to the robot <sup>10</sup>, and Zizzi<sup>11</sup> demonstrated a quantum metalanguage to control robots.

In this paper, we extend our previous work on non-automated quantum robot having three degrees of freedom in movement 12 and on automation of Braitenberg vehicles using finite automata Moore machines 13. We demonstrate an increase in degrees of freedom of movement in an automated quantum robot. We then explicate the usefulness of the automated quantum robot in a quantum game.

The paper is organized as follows. In Section II, we present an introduction to classical non-automated/automated Braitenberg vehicles and discuss previous works, including that of the authors, done on the

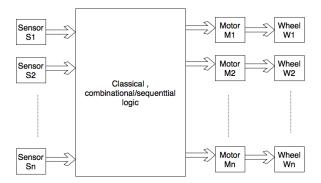


FIG. 1. Schematic of a classical Braitenberg vehicle. The sensors feed the inputs to the combinational/sequential logic. The motors receive the outputs and drive the wheels<sup>13</sup>.

subject. In Section III, we establish a theoretical reasoning on whether any arbitrary input-output relationship function may be realized for a quantum robot. In Section IV, we demonstrate building a quantum robot using three degrees of freedom and automating it. We discuss the flexibility of a quantum robot in allowing specific intrinsic behaviour to be added to it.

# II. AUTOMATED CLASSICAL BRAITENBERG VEHICLES

Braitenberg vehicles were first introduced by Braitenberg<sup>14</sup> as combinations of sensors and wheels attached to motors, all guided by electronic circuitry that activates certain motors based on sensor inputs (the sensors sense light as input). Different behaviours of the vehicles are manifested by different Boolean functions (sensor input- motor output relationships) implemented in the circuitry.

The above implementation in Fig. 1 has certain disadvantages: the need to continuously provide light sig-

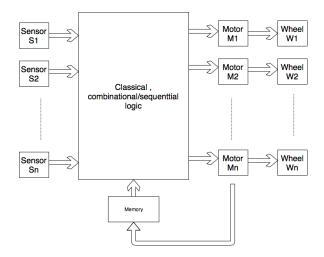


FIG. 2. Moore machine implementation of an automated Braitenberg vehicle. It can be observed here that how memory is a factor in the logic deciding the output states based on the inputs from sensors<sup>13</sup>.

nals as inputs, generating light signals of frequencies different from those already in the atmosphere, and preventing errors such as unintentional signals to a different sensor<sup>13</sup>. A plausible solution is to automate the vehicle using memory. Here, we keep the same procedure of automating as in<sup>13</sup> and Fig. 2.

We refer to Mishra et. al.  $^{13}$  for details on minimized Boolean function derivation of the circuitry and other details. The Braitenberg vehicle abstracted in Fig. 2 can be implemented as a quantum vehicle by involving quantum circuitry to compute the sensor input-motor output functions, as demonstrated in Mahanti et. al.  $^{12}$  and Raghuvanshi et. al.  $^{15}$ . It is also possible to introduce some desired intrinsic behaviour to the quantum robot, completely based on superposition and entanglement, and the flexibility of introducing external control over the operation of that intrinsic behaviour  $^{13}$ . Throughout the paper, we use  $|0\rangle$  and  $|1\rangle$  as indication of absence of light and presence of light respectively, in context of sensor inputs and as indication of no movement and forward movement respectively, in context of motors.

# III. DEGREES OF FREEDOM

The quantum version of propositional logic was first introduced by Birkhoff and Von Neumann<sup>16</sup>. In this version, to each property E is attached a quantity R such that measurement of R distinguishes the presence or absence of E. Therefore, any closed linear subspace corresponds to an elementary proposition (a true/false proposition). Co-measurability implies propositions can be measured by simultaneous measurements. Any two propositions, p and q are co-measurable iff there exists mutually orthogonal propositions a, b, c such that p = aVb and q = aVc, where xVy denotes all vectors

that are linear combination of x and y. The propositions we use in this paper correspond to the quantum states  $|0\rangle$  and  $|1\rangle$ , denoting the absence and presence of light respectively for sensors and no movement and movement for motors. We establish that these propositions are comeasurable since they are orthogonal to each other and thus physically distinguishable in measurement.

It is therefore possible to treat quantum logic in quantum robots like classical logic (for instance, distributive law is applicable in this case due to no violation of uncertainty principle of measurement of two observables). We further establish that any classical boolean function can be subjected to minimization techniques like Quine-Mccluskey method or the Karnaugh map method. Since quantum logic of quantum robots behaves as classical Boolean logic, the same minimization principles are applicable here. Classical Boolean logic may be sufficiently represented using basic gates - AND, OR, and NOT gates. We establish their equivalents in a quantum computer in Figures 3, 4, and 5. NOT operation is trivial.

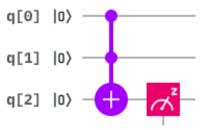


FIG. 3. Circuit for AND operation. The third qubit is the output qubit, resulting in eigenstate  $|1\rangle$  when both the inputs are  $|1\rangle$ .

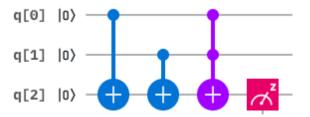


FIG. 4. Circuit for OR operation. The third qubit is the output qubit, resulting in eigenstate  $|1\rangle$  when either of the inputs is  $|1\rangle$ .

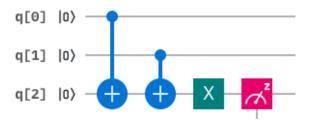


FIG. 5. Circuit for XOR operation. The third qubit is the output qubit, resulting in eigenstate  $|1\rangle$  when even number of the inputs are  $|1\rangle$ .

We conclude, henceforth, that, theoretically, desired number of degrees of freedom can be added to a quantum robot, since it possible to construct circuits representing the requisite functions. This flexibility allows complex behaviour to be added to the quantum robot.

#### IV. DEMONSTRATION OF ADDING A DEGREE OF FREEDOM

In our work in Mishra et. al. <sup>13</sup>, we constructed a Braitenberg vehicle with two degrees of freedom, i.e., two logic controlled motors. Here, we construct a Braitenberg vehicle using three degrees of freedom: turning left, turning right, and flying. We then automate the vehicle using finite automata Moore machine. It is possible to use Hadamard gate on one or more qubits to introduce a non-deterministic effect on the motors. We demonstrate the same in the circuit given in Fig. 6.



FIG. 6. Logic circuit for a quantum robot having three degrees of freedom. Circuit is derived after simplifying the underlying Boolean functions. q[0] and q[1] receive light inputs from sensors S1 and S2 and accordingly prepare in states  $|0\rangle$  and  $|0\rangle$ . On the output side, measurement of qubits in order from top to bottom yields states for M1, M2, and M3 respectively.

The circuit in Fig. 6 implements the following logic in Table I.

TABLE I. Logic implementation in a three degrees of freedom quantum robot. Here S1 and S2 denote the sensors, while M1, M2, and M3 denote motors that cause left turn, right turn, and flying action respectively. It can be observed that how S1 and S2 inputs are converted to outputs for M1 and M2 respectively, saving usage of 2 auxiliary qubits.

$\mathbf{S1}$	S2	M1 M2 M3				
0	0	110 or 100				
0	1	001 or 010				
1	0	110 or 100				
1	1	001 or 010				

It is henceforth observed how non-deterministic nature can be added to the robot. These non-deterministic natures depend on the implemented Boolean function. Since it is theoretically possible to construct a circuit for any Boolean function, it is possible to add any complex, desired behaviour to the robot. Practically though, quantum computer architecture, quantum gate application errors, and short coherence time might oppose huge circuitry and its proper functioning. In Fig. 7, an additional memory qubit is attached to the robot. The memory qubit participates in two situations: when based on memory, specific behaviour needs to be added to the robot, and when based on outputs, memory needs to be updated. Based on these specific Boolean functions, memory can be integrated into the logic of the robot, as presented in Table II.

TABLE II. Logic functions for the automated quantum robot. Mp denotes the memory from last iteration used as input in this iteration. Mn denotes the memory at the end of this iteration, to be fed as input in the next iteration.

S1	S2	$ \mathbf{Mp} $	M1	M2	M3	Mn		
0	0	0	1	1	0	0		
0	0	1	1000 or 1101					
0	1	0	0	1	0	1		
0	1	1	0100 or 0011					
1	0	0	1	0	0	1		
1	0	1	1000 or 1101					
1	1	0	0	0	1	0		
1	1	1	0100 or 0011					



FIG. 7. Logic circuit for automation of a quantum robot having three degrees of freedom as in Fig. 6. All symbols have their previous meaning as in Fig. 6 expect for an additional *memory* qubit q[3].

In this particular implementation, we have designed the circuit such that the memory achieves a couple of properties: when the memory is 0 in input, it causes the outputs to be deterministic and itself updates to 1 in turning configurations (i.e. when the robot turns right or left); it remains 0 in non-turning configurations (i.e. forward or flying movement) and when the memory is 1 as input, it causes the output to be non-deterministic and itself updates to 0 or 1 with around 50% probability for each.

#### V. CONCLUSION

To conclude, we have established here the possibility to add the desired number of degrees of freedom to a quantum Braitenberg vehicle since the logic used in such robots can be implemented in the same way as classical Boolean logic is implemented. This is possible due to comeasurability of the propositions involved, as discussed in Section III. We then established quantum implementation of basic classical Boolean logic gates and reasoned that any logic function can be implemented in a quantum

robot. We then demonstrated building a quantum robot having three degrees of freedom, and the flexibility we have in building such circuits. We introduced specific, desired behaviours to the robot completely using quantum phenomena. We then introduced memory to automate the robot; added specific behaviour of the memory in terms of how it updates the outputs and how it is itself updated.

The popularity of classical Braitenberg vehicles lies in the flexibility of implementable Boolean functions allowing complex behaviours in a robot. A quantum Braitenberg vehicle can, in addition, take advantage of quantum phenomena to exhibit non-deterministic nature. It also has the flexibility to accommodate more degrees of freedom, thus allowing the possibility of complex functions impossible for a classical Braitenberg vehicle. Robotics has been the center of academic and attraction since its inception. In the nascent field of quantum robotics, the ability to harness powers of quantum mechanics provides avenues yet to be explored. As quantum architecture increases in efficiency thereby increasing coherence time and the rate of errors in application of quantum gates decreases, we are set to witness astonishing progress in the field of quantum robotics.

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