LECTURE NOTES–UNIT II

SOLVING PROBLEMS BY SEARCHING

ECS302

ARTIFICIAL INTELLIGENCE

3/4 B.Tech [B3, B6]

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Module II Lecture Notes [*Problem solving*]

Syllabus

Problem Solving by Search and Exploration: Solving Problems by Searching:

Problem solving agents, example problems, searching for solutions, Uninformed search strategies, avoiding repeated states, searching with partial information; Informed Search and Exploration: Informed (heuristic) search strategies, heuristic functions, local search algorithms and optimization problems, local search in continuous spaces.

Problem Solving by Search:

An important aspect of intelligence is ***goal-based*** problem solving.

The solution of many problems can be described by finding a ***sequence of actions*** that lead to a desirable ***goal.*** Each action changes the ***state*** and the aim is to find the sequence of actions and states that lead from the initial (start) state to a final (goal) state.

A well-defined problem can be described by:

- ***Initial state***
- ***Operator or successor function*** - for any state x returns s(x), the set of states reachable from x with one action
- ***State space*** - all states reachable from initial by any sequence of actions
- ***Path*** - sequence through state space
- ***Path cost*** - function that assigns a cost to a path. Cost of a path is the sum of costs of individual actions along the path
- ***Goal test*** - test to determine if at goal state

What is Search?

- Search is one of the operational tasks that characterize AI programs best. Almost every AI program depends on a search procedure to perform its prescribed functions.
- Problems are typically defined in terms of state, and solution corresponds to goal states. Problem solving using search technique performs two sequence of steps:

(i) ***Define the problem*** - Given problem is identified with its required "*initial and goal state*".

(ii) ***Analyze the problem*** - The "*best search technique*" for the given problem is chosen from different AI search technique which derives one or more goal states in minimum number of states.

Types of problems:

In general the problem can be classified under anyone of the following four types which depends on two important properties. They are

(i) Amount of knowledge, of the agent on the state and action description.

(ii) How the agent is connected to its environment through its percepts and actions?

The four different types of problems are:

(i) Single state problem

(ii) Multiple state problems

(iii) Contingency problem

(iv) Exploration problem

1. *Problem Solving Agents:*

Problem solving agent is one kind of ***goal based agent***, where the agent decides what to do by finding sequence of actions that lead to desirable states. The complexity arises here is the knowledge about the formulation process, (from current state to outcome action) of the agent.

If the agent understood the definition of problem, it is relatively straight forward to construct a search process for finding solutions, which implies that problem solving agent should be an intelligent agent to maximize the performance measure. The sequence of steps done by the intelligent agent to maximize the performance measure:

i) ***Goal formulation*** - based on current situation is the first step in problem solving. Actions that result to a failure case can be rejected without further consideration.

ii) Problem formulation - is the process of deciding what actions and states to consider and follows goal formulation.

iii) Search - is the process of finding different possible sequence of actions that lead to state of known value, and choosing the best one from the states.

iv) Solution - a search algorithm takes a problem as input and returns a solution in the form of action sequence.

v) Execution phase - if the solution exists, the action it recommends can be carried out.

Thus, we have a simple "formulate, search, execute" design for the agent. This is shown in below figure 1.1.

```
function SIMPLE-PROELEM-SOLVING-AGENT (Percept) Returns an action
inputs: percept, a percept
static: seq, an action sequence, initially empty
state, some description of the current world state
goal, a goal, initially null
problem, a problem formulation
state ←UPDATE-STATE(State, Percept),
if seq is empty then do
goal ← FORMULATE-GOAL(State)
problem ←FORMULATE-PROBLEM(State, Goal),
seq ←SEARCH(problem)
action ←FIRST(Seq)
seq ← REST(seq)
return action
```

***Figure 1.1** A simple problem-solving agent. It first formulates a goal and a problem, searches for a sequence of actions that would solve the problem, and then executes the actions one at a time. When this is complete, it formulates another goal and starts over.*

Search - choosing the best one from the sequence of actions

Formulate-problem - sequence of actions and states that lead to goal state.

Update-state - initial state is forced to next state to reach the goal state

1.1. Well-defined problems and solutions:

A problem can be defined formally by four components:

1. *Initial state*
2. *Successor function*
3. *Goal test*
4. *Path cost*

Initial state - *is* that the agent starts in.

Successor function (S) - Given a particular state x, *S(x)* returns a set of ordered pair <Action, Successor>. Where each action is one of the legal actions in state x and each successor is a state that can be reached from x by applying an action.

Goal test **-** This determines whether a given state is a goal state. Sometimes there is an explicit set of possible goal states, and the test simply checks whether the given state is one of them.

A ***path cost*** function that assigns a numeric cost to each path. The problem-solving agent chooses a cost function that reflects its own performance measure.

[**Path (state space) -** The sequence of action leading from one state to another]

[**State space (or) state set space -** The set of all possible states reachable from the initial state by any sequence of actions.]

[**Step cost** – Taking action 'a' to go from state x to state y is denoted by c(x, a, y)]

A ***solution*** to a problem is a path from the initial state to a goal state. The effectiveness of a search can be measured using three factors. They are:

1 Solution is identified or not?

2. Is it a good solution? If yes, then path cost to be minimum.

3. Search cost of the problem that is associated with time and memory required to find a solution.

For Example: Imagine an agent in the city of Arad, Romania, enjoying a touring holiday. Now, suppose the agent has a nonrefundable ticket to fly out of Bucharest the following day. In that case, it makes sense for the agent to adopt the goal of getting to Bucharest. The agent's task is to

find out which sequence of actions will get it to a goal state.

This process of looking for such a sequence is called *"search"*.

A search algorithm takes a problem as input and returns a solution in the form of an action sequence. Once a solution is found, the actions it recommends can be carried out. This is called the execution phase.

1.2 Formulating problems:

Initial state: the initial state for our agent in Romania might be described as In(Arad)

Successor function: Given a particular state x, SUCCESSOR-FN(x) returns a set of (action, successor) ordered pairs, where each action is one of the legal actions in state x and each successor is a state that can be reached from x by applying the action. For example, from the state In(Arad), the successor function for the Romania problem would return

{(Go *(Sibzu), In (Sibiu)), (Go (Timisoara), In (Tzmisoara)), (Go (Zerznd), In (Zerind)))*

Goal test: The agent's goal in Romania is the singleton set {In (Bucharest)).

Path cost: The step cost of taking action '*a*' to go from state x to state y is denoted by *c(x, a,* y).

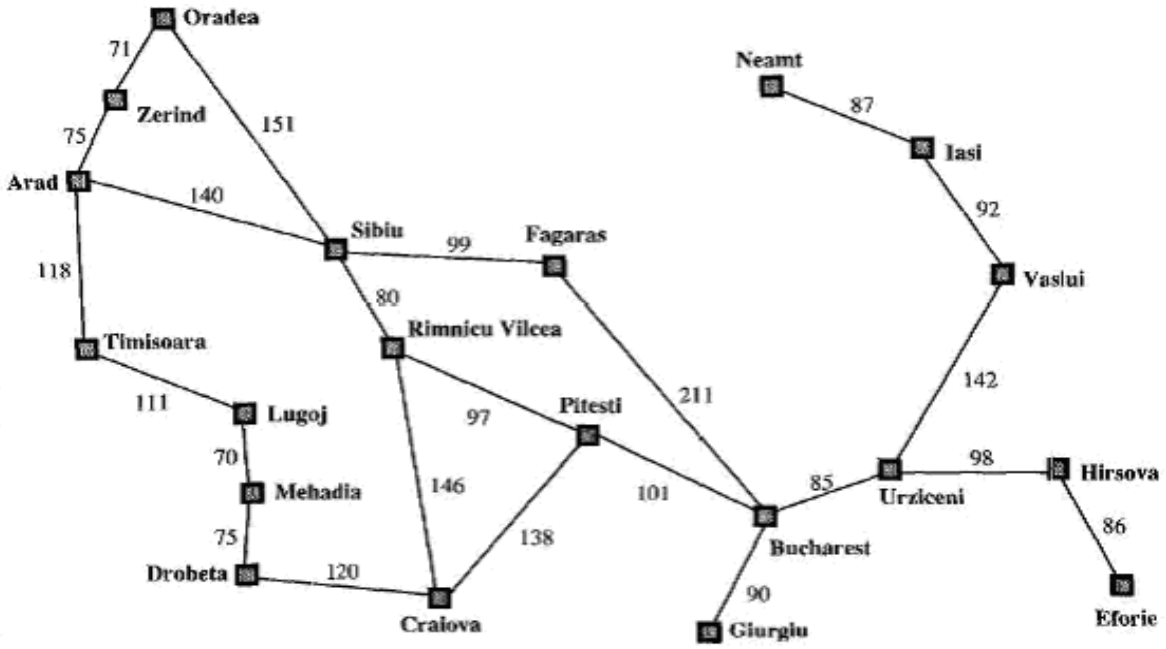


Figure 1.2: Simplified map of Romania

2. *Example Problems:*

The problem-solving approach has been applied to a vast array of task environments. A ***toy problem*** is intended to illustrate or exercise various problem-solving methods. It can be given a concise, exact description. It can be used easily by different researchers to compare the performance of algorithms. A ***real-world problem*** is one whose solutions people actually care about. Some list of best known *toy* and real-world problems.

2.1 Toy Problems

i) Vacuum world Problem States: The agent is in one of two locations, each of which might or might not contain dirt. Thus there are $2 * 2^2 = 8$ possible world states.

Initial state: Any state can be designated as the initial state.

Successor function: three actions (Left (L), *Right (R),* and *Suck (S)).*

Goal test: This checks whether all the squares are clean.

Path cost: Each step costs 1, so the path cost is the number of steps in the path.

This problem has discrete locations and the state is determined by both the location and the dirt cleaned. So, a larger environment with 'n' locations has **n x 2^n** states.

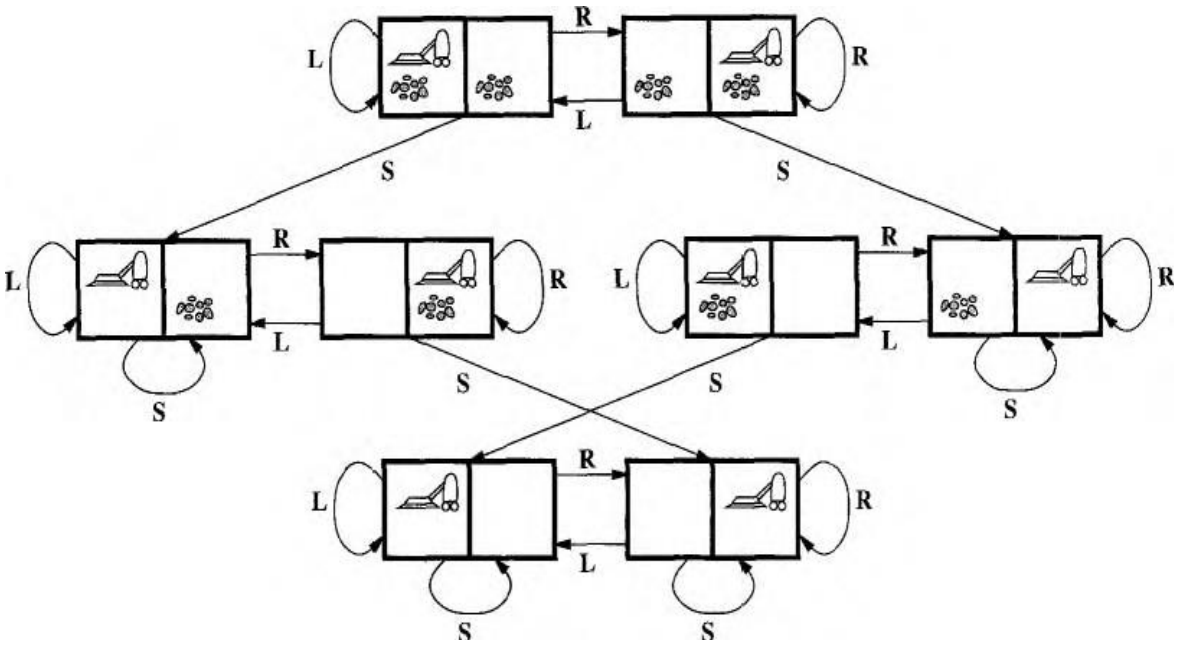
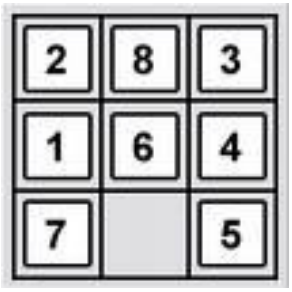
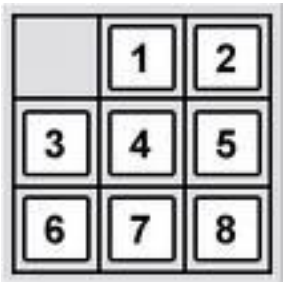


Figure: 2.1 complete state space for Vacuum World

***ii) 8-puzzle Problem*:** The 8-puzzle problem consists of a 3 x 3 board with eight numbered tiles and a blank space. A tile adjacent to the blank space can slide into the space. The object is to reach a specified goal state



Initial State



Goal State

States: A state description specifies the location of each of the eight tiles and the blank in one of the nine squares. Initial state: Any state can be designated as the initial state.

Successor function: This generates the legal states that result from trying the four actions (blank moves Left, Right, Up, or Down).

Goal test: This checks whether the state matches the goal configuration (Other goal configurations are possible.)

Path cost: Each step costs 1, so the path cost is the number of steps in the path.

This 8- Puzzle belongs to the family of sliding block puzzles which are used as test problems for new search algorithms in AI.

***iii) 8-queens problem*:** The goal of the *8-queens problem* is to place eight queens on a chessboard such that no queen attacks any other.

(Note: A queen attacks any piece in the same row, column or diagonal)

States: Any arrangement of 0 to 8 queens on the board is a state.

Initial state: No queens on the board.

Successor function: Add a queen to any empty square.

Goal test: 8 queens are on the board, none are attacked by others.

Path cost: Zero (search cost only exists)

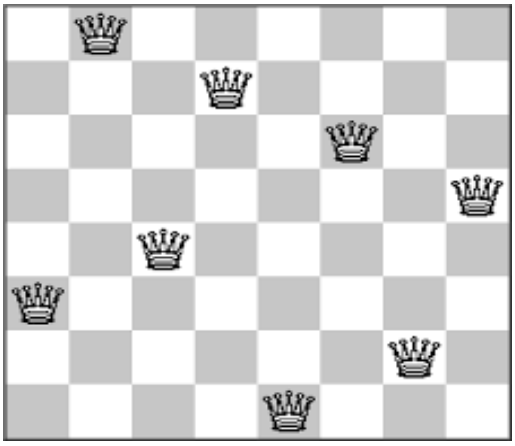


Figure 2.2: Solution to 8- queens

2.2 Real-world problems:

i) Route-finding problem: This problem can be defined in terms of specified locations and transitions along links between them. Route-finding algorithms are used in a variety of applications, such as routing in computer networks, military operations planning, and airline travel planning systems.

Let us consider the Airline travel problem;

Airline travel problem States:

States: Each is represented by a location (e.g., an airport) and the current time.

Initial state: This is specified by the problem.

Successor function: This returns the states resulting from taking any scheduled flight (perhaps further specified by seat class and location), leaving later than the current time plus the within-airport transit time, from the current airport to another.

Goal test: Are we at the destination by some pre specified time?

Path cost: This depends on monetary cost, waiting time, flight time, customs and immigration procedures, seat quality, time of day, type of airplane, frequent-flyer mileage awards, and so on.

ii) Touring Problems: Which are closely related to Route-finding problems, but it has an

important difference. For example: Consider the problem "***Visit every city at least once***" as shown in Romania Map fig: 1.2

- In Route finding the actions correspond to trips between adjacent cities
- But state space is quite different i.e., each state must be included (Not only the current location but also set of cities that the agent has visited)
- For example: The initial state would be "Arad " then visited{Arad}
- Suppose a typical intermediate state would be in "Lugoj" then visited{Arad, Timisoara, Lugoj}
- The goal test would check whether the agent is in Bucharest and all 20 cities have been visited.

iii) Travelling sales person Problem: [TSP]

It is a touring problem in which each city must be visited exactly once. The aim is to find the ***shortest*** tour. These problems are known as NP-hard, but an enormous amount of effort has been expended to improve the capacity of TSP algorithms.

These algorithms are also used in tasks such as planning movements of Automatic circuit board drills and Stocking machines on shop floors.

iv) VLSI layout

A VLSI layout problem requires positioning millions of components and connections on a chip to minimize area, minimize circuit delays, minimize stray capacitances, and maximize manufacturing yield. The layout problem comes after the logical design phase, and is usually split into two parts: ***cell layout*** and ***channel routing*****.**

- In cell layout, the primitive components of the circuit are grouped into cells, each of which performs some recognized function. Each cell has a fixed footprint (size and shape) and requires a certain number of connections to each of the other cells. The aim is to place the cells on the chip so that they do not overlap and so that there is room for connecting wires to be placed between the cells.
- Channel routing finds a specific route for each wire through the gaps between the cells.

v) Robot navigation

Robot navigation is a generalization of the route-finding problem described earlier. Rather than a discrete set of routes, a robot can move in a continuous space with (in principle) an infinite set of possible actions and states.

- For a circular robot moving on a flat surface, the space is essentially two-dimensional.
- When the robot has arms and legs or wheels that must also be controlled, the search space becomes many-dimensional. Advanced techniques are required just to make the search space finite. In addition to the complexity of the problem, real robots must also deal with errors in their sensor readings and motor controls.

vi) Automatic assembly sequencing

Automatic assembly sequencing of complex objects by a robot was first demonstrated by FREDDY (Michie, 1972). In assembly problems, the aim is to find an order in which to assemble the parts of some object.

- If the wrong order is chosen, there will be no way to add some part later in the sequence without undoing some of the work already done. Checking a step in the sequence for feasibility is a difficult geometrical search problem closely related to robot navigation.
- Another important problem is protein design. The goal is to find a sequence of Amino acids that will be fold in to a three- dimensional protein with right properties to cure some disease.

vii) Internet Searching

In recent years there is an increase of demand for *software robots* that performs Internet searching i.e., looking for answers to questions (or) for shopping deals. This is a good application for search technique i.e., internet is a graph of nodes connected by links.

3. Searching for Solutions

Search techniques use an explicit "*search tree*" that is generated by the initial state and the successor function that together define the state space. In general, we may have a "*search graph*" rather than a search tree, when the same state can be reached from multiple paths.

- Let us consider the example: Search tree for finding a route from *"Arad to Bucharest"* in Romania map.
- Here the Root of search tree is a "Search Node" corresponding to initial state In(Arad).
- The first step is to test whether this is a goal state.
- Apply the successor function to the current state, and generate a new set of states In this case, we get three new states: *In(Sibiu),In(Timisoara),* and *In(Zerind).* Now we must choose which of these three possibilities to consider further.
- Continue choosing, testing, and expanding until either a solution is found or there are no more states to be expanded.
- The choice of which state to expand is determined by the *"search strategy".*

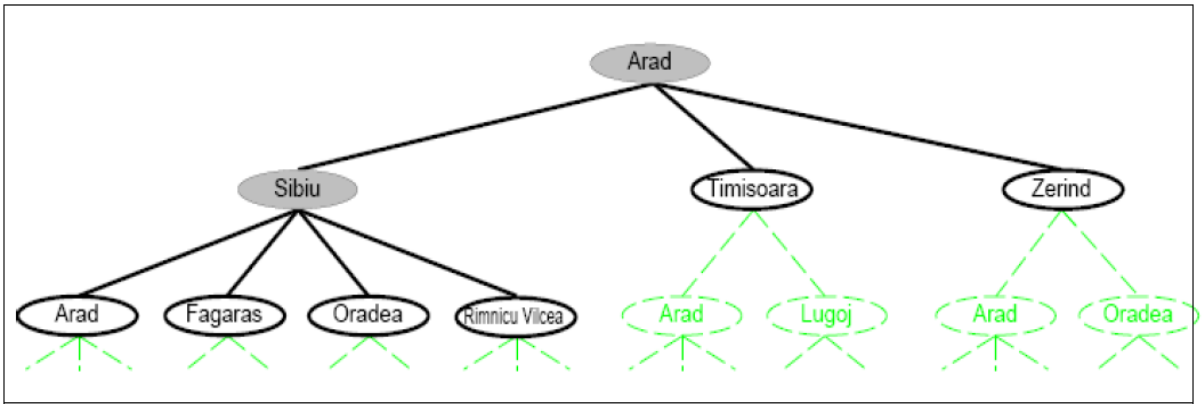
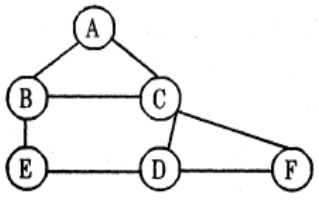


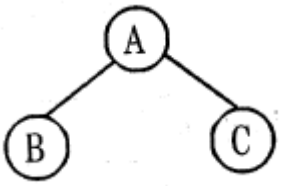
***Figure 3.1** Partial search trees for finding a route from Arad to Bucharest. Nodes that have been expanded are shaded. Nodes that have been generated but not yet expanded are outlined in bold. Nodes that have not yet been generated are shown in faint dashed line.*

Consider an example for tree Search algorithm: The task is to find a path to reach F from A

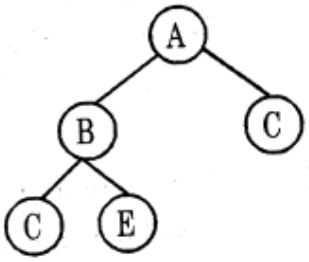


1. Start the sequence with the initial state and check whether it is a goal state or not. If it is a goal state return success. Otherwise perform the following sequence of steps from the initial state (current state) generate and expand the new set of states.
2. The collection of nodes that have been generated but not expanded is called *as fringe.*
3. Each element of the fringe is a *leaf node,* a node with no successors in the tree.
4. Sequence of steps to reach the goal state F from (A = A - C - F)

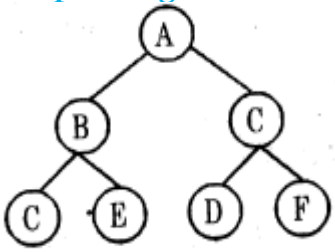
Expanding A



Expanding B



Expanding C



A General Tree-Search Algorithm:

function *TREE-SEARCH*(*problem. strategy*) **returns** a solution or failure

initialize the search tree using the initial state of *problem*

loop do

if there are no candidates for expansion *then return* failure

choose a leaf node for expansion according to *strategy*

if the node contains a goal state *then return* the corresponding solution

else expand the node and add the resulting nodes to the search tree

There are many ways to represent nodes, but we will assume that a node is a data structure with five components:

State: the state in the state space to which the node corresponds

Parent-node: the node in the search tree that generated this node;

Action (rule): the action that was applied to the parent to generate the node;

Path-cost: the cost, traditionally denoted by $g(n)$, of the path from the initial state to the node as indicated by parent pointers.

Depth: the number of steps along the path from the initial state. The collection of nodes represented in the search tree is defined using set or queue representation.

A node is a book keeping data structure used to represent the search tree. A state corresponds to configuration of the world.

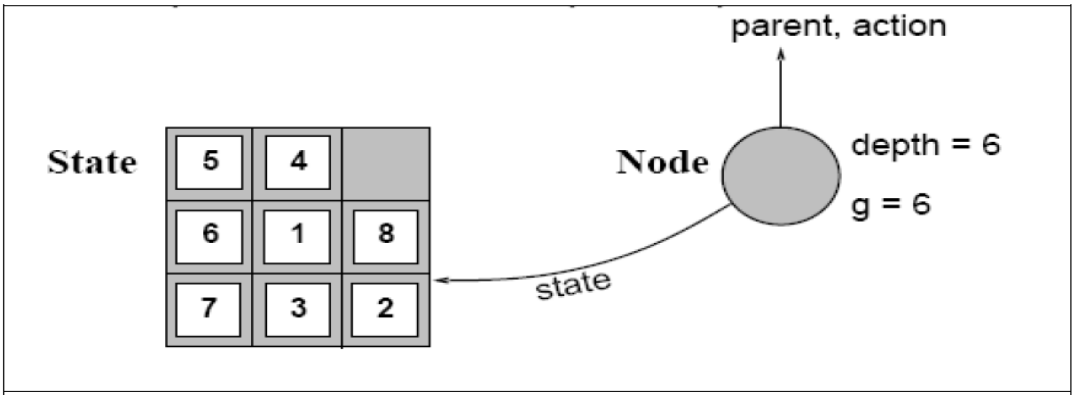


Figure 3.2 ***Nodes are data structures from which the search tree is constructed. Each has a parent, a state, Arrows point from child to parent.***

Fringe:

Fringe is a collection of nodes that have been generated but not yet been expanded. Each element of the fringe is a leaf node, that is, a node with no successors in the tree. The fringe of each tree consists of those nodes with bold outlines.

The collection of these nodes is implemented as a ***"Queue".***

The operations specified on a queue are as follows:

MAKE-QUEUE (element…) creates a queue with the given element(s).

EMPTY? (Queue) returns true only if there are no more elements in the queue.

FIRST (queue) returns FIRST (queue) and removes it from the queue.

INSERT (element, queue) inserts an element into the queue and returns the resulting queue.

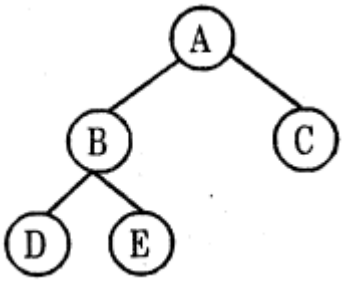
INSERT-ALL (elements, queue) inserts a set of elements into the queue and returns the resulting queue.

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
fringe <- INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
 loop do
 if EMPTY?(fringe) then return failure
node <- REMOVE-FIRST(fringe)
 if GOAL-TEST[problem]applied to STATE[node] succeeds then return SOLUTION(node)
fringe <- INSERT-ALL(EXPAND(node, problem),fringe)
```

```
function EXPAND(node, problem) returns a set of nodes
successors <- the empty set
for each <action, result> in SUCCESSOR-FN [problem](STATE[node])do
S <- a new NODE
STATE[s] <- result
PARENT-NODE[s] <- node
ACTION[s] <- action
PATH-COST[s] <- PATH-COST[node]+STEP-COST(node, action, s)
DEPTH[s] <- DEPTH[node] + 1
add s to successors
 return successors
```

***Figure: 3.3** General tree search algorithm with queue representation*

Example: Route finding problem



Find a path to reach E using Queuing function in general tree search algorithm.

Nodes	Queue
A	A
B C	C B
D E	E D C
C	E D
D	E
E	Goal state, Succeeded

Measuring problem solving performance:

The output of a problem- solving Algorithm is either failure or Solution. The search strategy algorithms are evaluated depends on four important criteria's. They are:

(i) Completeness: The strategy guaranteed to find a solution when there is one.

(ii) Time complexity: Time taken to run a solution.

(iii) Space complexity: Memory needed to perform the search.

(iv) Optimality: If more than one way exists to derive the solution then the best one is Selected

Definition of branching factor (b): The number of nodes which is connected to each of the node in the search tree. Branching factor is used to find space and time complexity of the search strategy. The total cost will be calculated as the combination of Search Cost and path cost to find a solution.

4. Solving Problems by Searching:

It divides the Algorithm in to two categories: **1) Uninformed Search Algorithms [Blind search]**

2) Informed Search Algorithms [Heuristic search]

Algorithms Comes Under Uninformed Search;

- Breadth First Search [BFS]
- Uniform-cost search
- Depth-first search [DFS]
- Depth-limited search [DLS]
- Iterative deepening depth-first search [IDDS / IDS]
- Bidirectional Search

i) Breadth-first search

It is a simple strategy in which the root node is expanded first, then all the successors of the root node are expanded next, then their successors, and so on. In general, all the nodes are expanded at a given depth in the search tree before any nodes at the next level are expanded.

Breadth-first search can be implemented by calling TREE-SEARCH with an empty fringe that is a first-in-first-out (FIFO) queue, assuring that the nodes that are visited first will be expanded first. In other words, calling *TREE-SEARCH (Problem, FIFO-QUEUE())* results in a breadth-first search. The FIFO queue puts all newly generated successors at the end of the queue, which means that shallow nodes are expanded before deeper nodes.

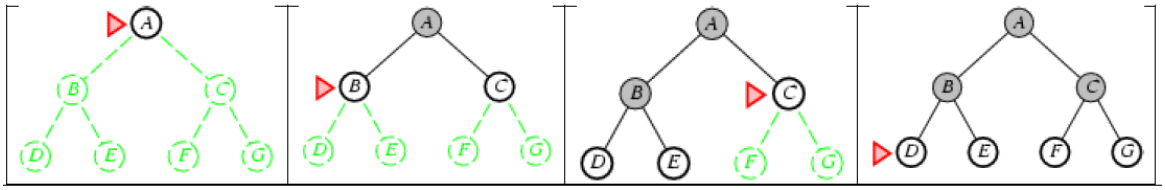


Figure 4.1: General expansion of Nodes

Time complexity for BFS:

Assume that every state has b successors. The root of the search tree generates b nodes at the first

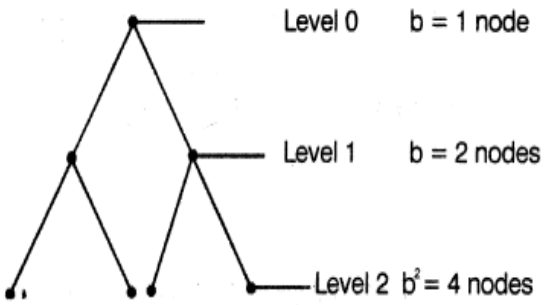
level, each of which generates b more nodes, for a total of b2 at the second level. Each of these generates b more nodes, yielding b3 nodes at the third level, and so on.

Now suppose that the solution is at depth d. In the worst case, we would expand all but the last node at level d, generating b^d+1 - b nodes at level d+1.

Then the total number of nodes generated is

b + b2 + b3 + …+ b ^d + (b^d+1 + b) = O(b^d).

Every node that is generated must remain in memory, because it is either part of the fringe or is an ancestor of a fringe node. The space complexity is, therefore, the same as the time complexity.



Advantages:

- BFS will never get trapped by exploring the useless path forever.
- If there is a solution then BFS will definitely find it out.
- If there is more than one solution then BFS can find the minimal that requires less number of steps.

Disadvantages:

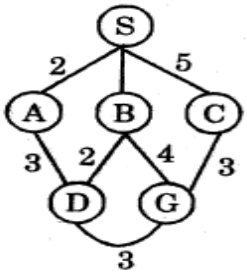
- The main problem of BFS is its memory requirement. Since each level of tree must be saved in order to generate the next level, and the amount of memory is proportional to the number of nodes stored.
- Suitable for only smallest instances problem (i.e.) (number of levels to be minimum (or) branching factor to be minimum)

Algorithm:

function ***BFS****{(problem)}* ***returns*** a solution or failure

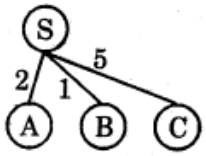
return ***TREE-SEARCH*** *(problem,* FIFO-QUEUE( ))

Example: Route finding problem [Find a , path from. S to G using BFS]

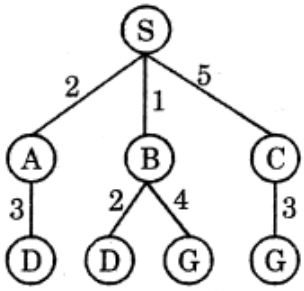


(i)

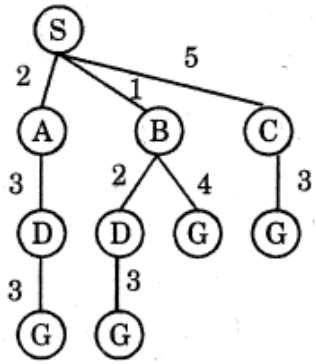
(ii)



(iii)



(iv)



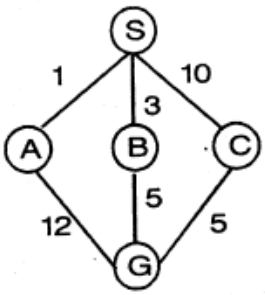
The path in the 2nd depth level is selected, (i.e.,) SBG {or) SCG.

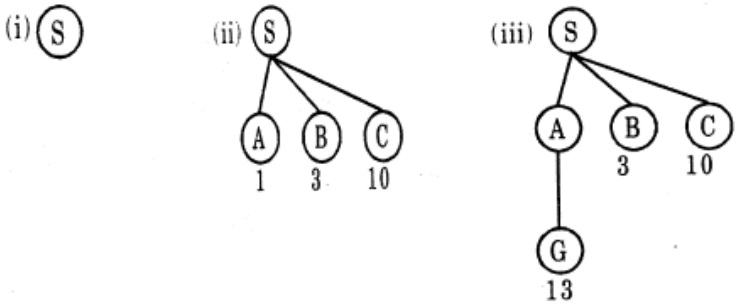
ii) Uniform-cost search

Breadth-first search is optimal when all step costs are equal, because it always expands the *shallowest* unexpanded node. By a simple extension, we can find an algorithm that is optimal with any step cost function. Instead of expanding the shallowest node, *uniform-cost search* expands the node n with the ***lowest path cost***. Note that if all step costs are equal, this is

identical to breadth-first search. Uniform-cost search does not care about the *number* of steps a path has, but only about their total cost.

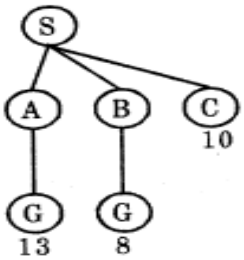
Example: Find a minimum path cost from S to G





Since the value of A is less it is expanded first, but it is not optimal.

B to be expanded next;



SBG is the path with minimum path cost. No need to expand the next path SC, because its path cost is high to reach C from S, as well as goal state is reached in the previous path with minimum cost.

Time and space complexity: Time complexity is same as breadth first search because instead of depth level the minimum path cost is considered.

Time complexity: $O(b^d)$ **Space complexity**: $O(b^d)$ ***Completeness:*** Yes ***Optimality:*** Yes

Advantage: Guaranteed to find the single solution at minimum path cost.

Disadvantage: Suitable for only smallest instances problem.

iii) Depth- First Search:

Depth-first search always expands the *deepest* node in the current fringe of the search tree. Here the search proceeds immediately to the deepest level of the search tree, where the nodes have no successors.

- As those nodes are expanded, they are dropped from the fringe, so then the search "backs up" to the next shallowest node that still has unexplored successors.
- This strategy can be implemented by TREE-SEARCH with a last-in-first-out (LIFO) queue, also known as a stack.

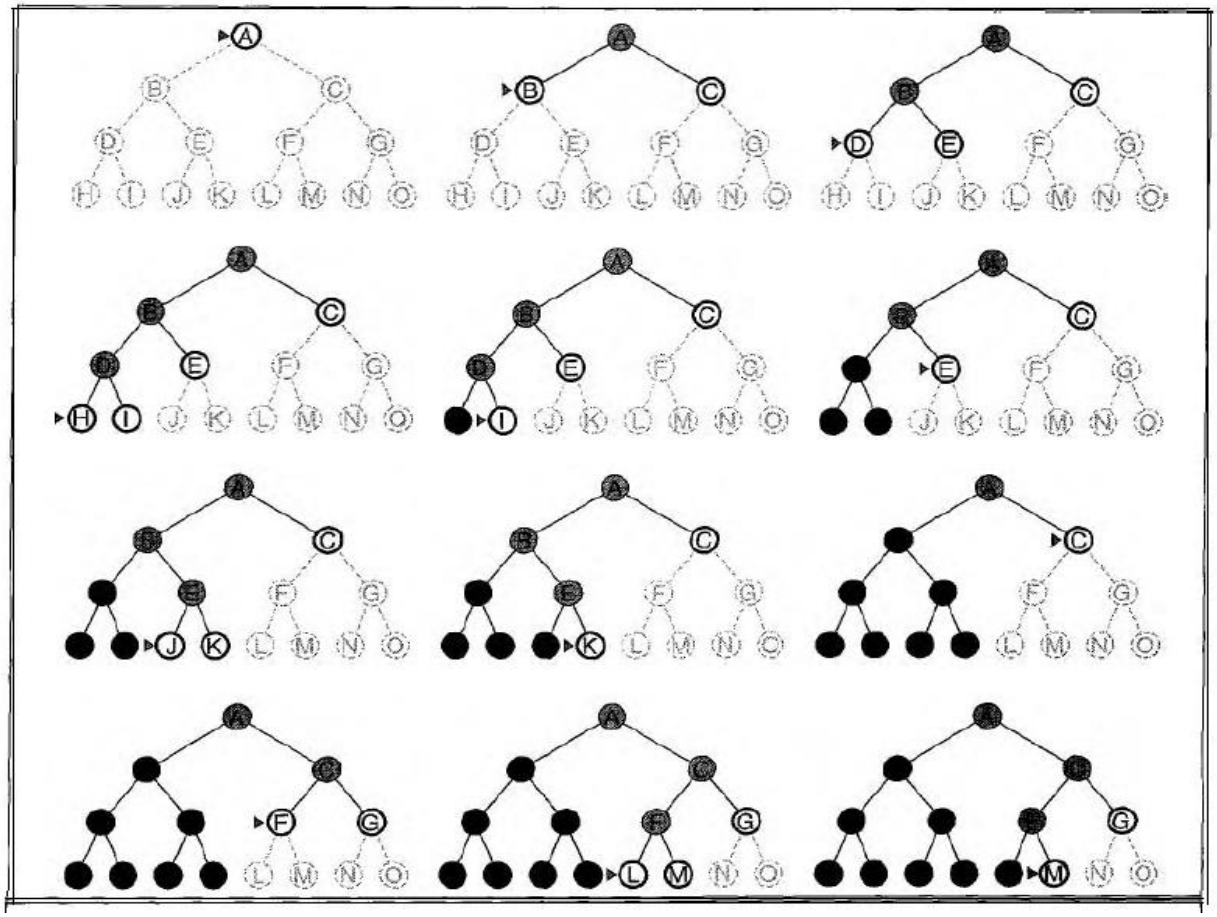
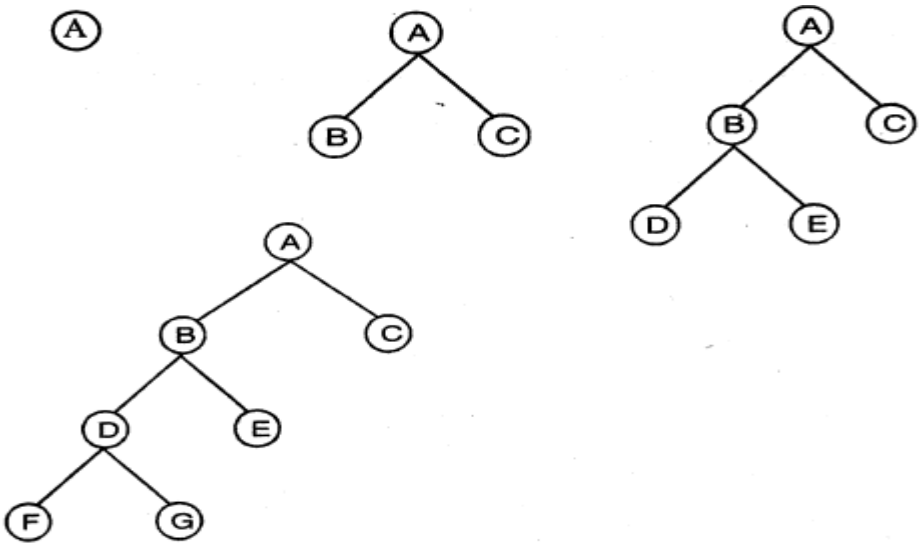


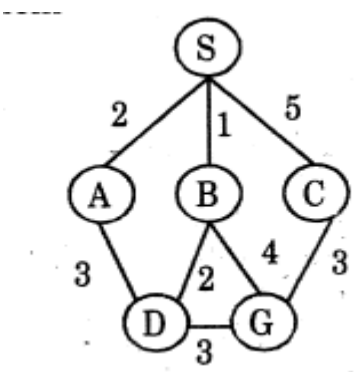
Figure 4.2 ***Depth****-first search on a binary tree. Nodes that have been expanded and have no descendants in the fringe can be removed from memory; these are shown in black. Nodes at depth 3 are assumed to have no successors and M is the only goal node.*

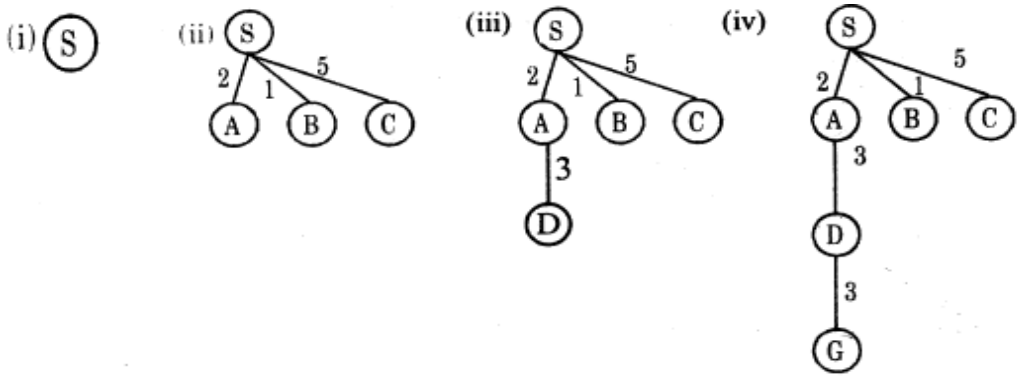
A variant of depth-first search called ***backtracking search*** uses still less memory. In backtracking, only one successor is generated at a time rather than all successors; each partially expanded node remembers which successor to generate next. In this way, only *O(m)* memory is needed rather than *O(bm)*. Backtracking search facilitates yet another memory saving (and time-saving) trick: the idea of generating a successor by ***modifying*** the current state description directly rather than copying it first. This reduces the memory requirements to just one state description and *O(m)* actions.

Depth first search tree with 3 level expansions.



Example: Find a path from S to G using DFS





The path in the 3rd depth level is selected. (i.e., S-A-D-G)

Algorithm:

function ***DFS****(problem)* ***return*** a solution or failure

TREE-SEARCH *(problem,* LIFO-QUEUE())

Time and space complexity:

For a state space with branching factor *b* and maximum depth *m,* depth-first search requires storage of only *bm* + 1 nodes. In the worst case depth first search has to expand all the nodes

Time complexity: *O(b^m).*

The nodes are expanded towards one particular direction requires memory for only that nodes.

Space complexity: O(*bm)*

b=2 m = 2 :. bm=4

Completeness: No

Optimality: No

Advantage: If more than one solution exists (or) number of levels is high then DFS is best because exploration is done only in a small portion of the whole space.

Disadvantage: It can make a wrong choice and get stuck going down a very long (or even infinite) path when a different choice would lead to a solution near the root of the search tree. For example, in Figure 4.2, depth-first search will explore the entire left sub tree even if node C is a goal node. If node J were also a goal node, then depth-first search would return it as a solution; hence, depth-first search is not optimal.

iv) Depth-limited search

To overcome the problem of DFS here we supply depth-first search with a predetermined depth limit ***l***. That is, nodes at depth *l* are treated as if they have no successors. This approach is called **depth-limited search.** The depth limit solves the infinite-path problem. Unfortunately, it also introduces an additional source of incompleteness if we choose ***l*** < d, that is, the shallowest goal is beyond the depth limit. (This is not unlikely when d is unknown.) Depth-limited search will also be non optimal if we choose ***l*** > d. Its ***time complexity*** is *O(b^* ***l****)* and its ***space complexity*** is *O(bl).*

Depth-first search can be viewed as a special case of depth-limited search with ***l=10***.

Sometimes, depth limits can be based on knowledge of the problem. For example, on the map of ***Romania*** there are 20 cities. Therefore, we know that if there is a solution, it must be of length 19 at the longest, so $l = 19$ is a possible choice. But in fact if we studied the map carefully, we would discover that any city can be reached from any other city in at most 9 steps. This number, known as the ***diameter*** of the state space, gives us a better depth limit, which leads to a more efficient depth-limited search. For most problems, however, we will not know a good depth limit until we have solved the problem.

Algorithm:

```
function Depth-Limited-Search( problem, limit) returns a solution/fail/cutoff
return Recursive-DLS(Make-Node(Initial-State[problem]), problem, limit)
function Recursive-DLS(node, problem, limit) returns solution/fail/cutoff
Cutoff - occurred? false
if Goal-Test(problem, State[node]) then return Solution(node)
else if Depth[node] = limit then return cutoff
else for each successor in Expand(node, problem) do
result Recursive-DLS(successor, problem, limit)
if result = cutoff then cutoff _ occurred? true
else if result not = failure then return result
if cutoff _ occurred? then return cutoff else return failure
```

Depth-limited search can terminate with two kinds of failure:

1) the standard failure value indicates no solution;
2) The *cutoff* value indicates no solution within the depth limit.

Time and space complexity:

The worst case time complexity is equivalent to BFS and worst case DFS.

Time complexity: $O(b^l)$ The nodes which is expanded in one particular direction above to be stored.

Space complexity: ***O(bl)***

Optimality*:* No, because not guaranteed to find the shortest solution first in the search technique. ***Completeness:*** Yes, guaranteed to find the solution if it exists.

Advantage: Cut off level is introduced in the DFS technique

Disadvantage: Not guaranteed to find the optimal solution.

v) Iterative deepening search (or iterative deepening depth-first search)

It is a general strategy, often used in combination with depth-first search that finds the best depth limit. It does this by gradually increasing the limit-first 0, then 1, then 2, and so on-until a goal is found. This will occur when the depth limit reaches d, the depth of the shallowest goal node. The algorithm is shown in Figure 3.14. Iterative deepening combines the benefits of depth-first and breadth-first search. Like depth-first search, its memory requirements are very modest: O(bd) to be precise. Like breadth-first search, it is complete when the branching factor is finite and optimal when the path cost is a non decreasing function of the depth of the node.

Algorithm:

function **ITERATIVE-DEEPENING-SEARCH** *(problem)* **returns** a solution, or failure

inputs : *problem*

for *depth* <- 0 to **do** *result* <-**DEPTH-LIMITED-SEARCH***(problem, depth)*

if *result not equal to cutoff* then return result .

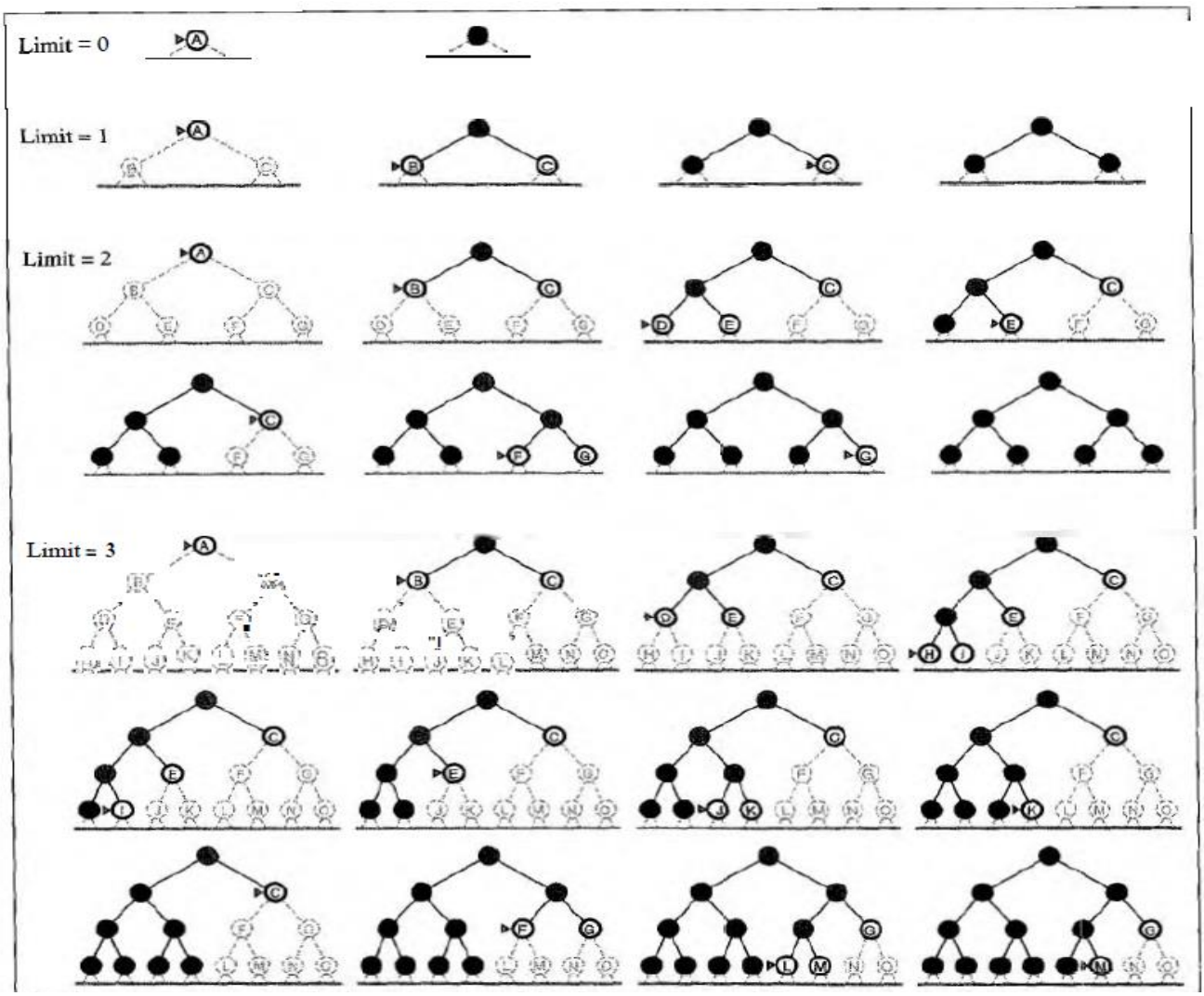
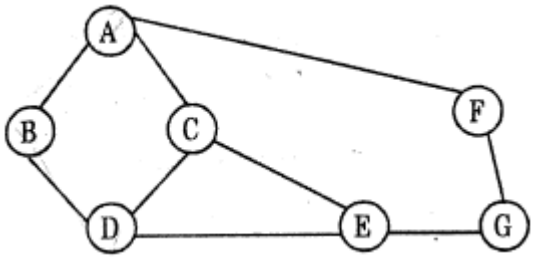


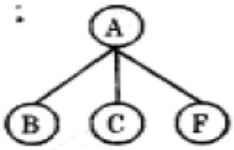
Figure 4.3 Four iterations of iterative deepening search on a binary tree.

Example: Find a path from A to G



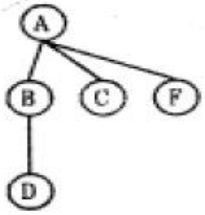
Limit = 0

Limit = 1

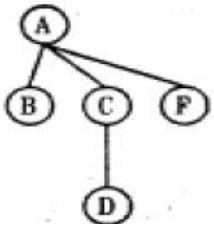


Limit = 2

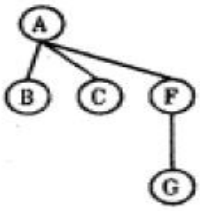
(i)



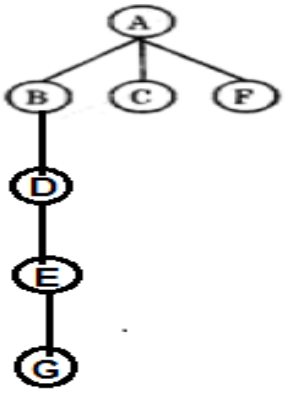
(ii)

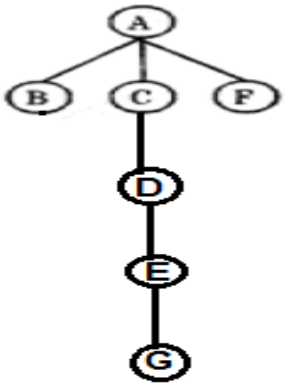


(iii)



Limit = 4





Solution: The goal state G can be reached from A in four ways. They are:

1. A – B – D - E – G ------- Limit 4
2. A - C - D - E - G ------- Limit 4
3. A - C - E - G ------- Limit 3
4. **A - F - G ------ Limit2**

Since it is a iterative deepening search it selects lowest depth limit (i.e.) **A-F-G** is selected as the solution path.

Time complexity:

- Iterative deepening search may seem wasteful, because states are generated multiple times. It turns out this is not very costly. The reason is that in a search tree with the same (or nearly the same) branching factor at each level, most of the nodes are in the bottom level, so it does not matter much that the upper levels are generated multiple times.
- In an iterative deepening search, the nodes on the bottom level (depth d) are generated once, those on the next to bottom level are generated twice, and so on, up to the children of the root, which are generated d times. So the total number of nodes generated is

N(1DS) = (d) b + (d - l)b2 + . . . + (1) bd ,

Which gives a time complexity of O(bd). We can compare this to the nodes generated by a breadth-first search:

N(BFS) = b + b2 + . . . + bd + (bdtl - b) .

For example, if b = 10 and d = 5, the numbers are

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

Advantage: This method is preferred for large state space and the depth of the search is not known.

Disadvantage: Many states can be expanded multiple times.

vi) Bidirectional search:

This is a search strategy that can simultaneously search both the directions (i.e.)

- Forward from the initial state and
- Backward from the goal, and stops when the two searches meet in the middle.

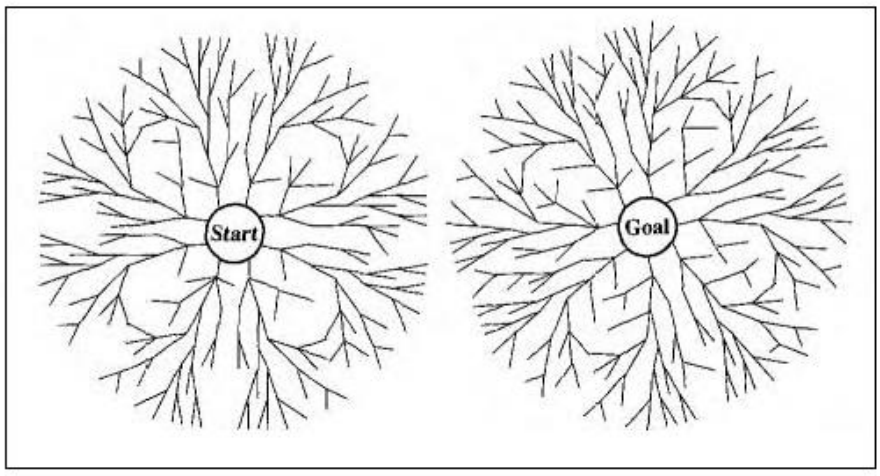
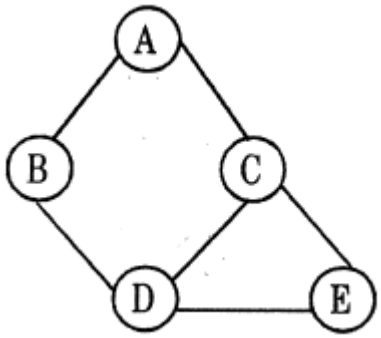
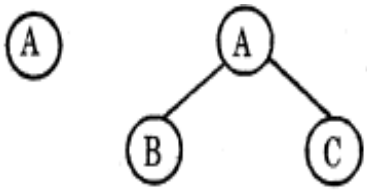


***Figure 4.4** A schematic view of a bidirectional search that is about to succeed, when a Branch from the Start node meets a Branch from the goal node.*

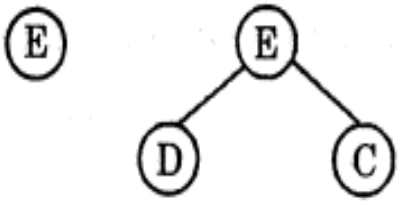
Example: Find a path from A to E.



Search from forward (A):



Search from backward (E):



Here backward and forward searches done at same time will lead to a solution i.e., **b^d/2+b^d/2= O (2b^d/2).**

This space requirement is most significant weakness of bidirectional search.

- This algorithm is complete and optimal if both searches are breadth-first.
- Here the reduction in time complexity makes bidirectional search as an attractive, but if we search backwards? This is not an easy one.
- Consider suppose, Let the ***predecessors*** of a state x, Pred(x), be all those states that have x as a successor. Bidirectional search requires that Pred (x) be efficiently computable.
- The easiest case is when all the actions in the state space are reversible, so that ***Pred (x) = Succ(x).***

Consider the question of what we mean by "the goal" in searching "backward from the goal." For the 8-puzzle and for finding a route in Romania, there is just one goal state, so the backward search is very much like the forward search. If there are several *explicitly listed* goal states-for example, the two dirt-free goal states, then we can construct a new dummy goal state whose immediate predecessors are all the actual goal states. Alternatively, some redundant node generations can be avoided by viewing the set of goal states as a single state, each of whose predecessors is also a set of states-specifically, the set of states having a corresponding successor in the set of goal states.

Advantage:

Time and space complexity is reduced

Disadvantage:

If two searches doesn't meet at all complexity arises in search technique. In backward search calculating predecessor is difficult task. If more than one goal state exists then explicit multiple state searches is required.

→More efficient search;

Example: b=10, d=6 then BFS will be examined as [b^d] i.e., [10^6] = 1,000,000 nodes

Where as Bidirectional search is [2b^d/2] i.e., [2x10^3] = 2,000 nodes

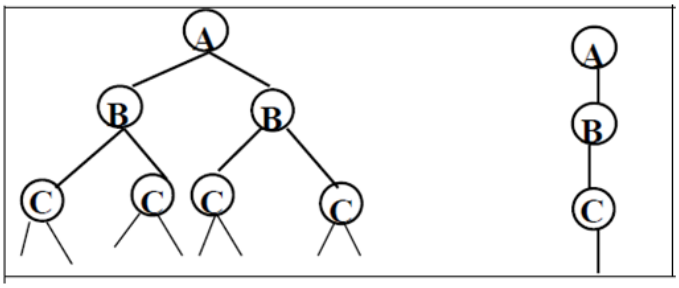
Comparing uninformed search strategies:

Criterion	Breadth First	Uniform Cost	Depth First	Depth Limited	Iterative Deepening	Bi direction
Complete	Yes	Yes	No	No	Yes	Yes
Time	$O(b^d)$	$O(b^d)$	O(bm)	$O(b^l)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^d)$	O(bm)	O(bl)	O(bd)	$O(b^{d/2})$
Optimal	Yes	Yes	No	No	Yes	Yes

5. *Avoiding Repeated States*

In searching, time is wasted by expanding states that have already been encountered and expanded before.

- For some problems repeated states are unavoidable. The search trees for these problems are infinite. If we prune some of the repeated states, we can cut the search tree down to finite size.
- Considering search tree up to a fixed depth, eliminating repeated states yields an exponential reduction in search cost. Repeated states can cause a solvable problem to become unsolvable if the algorithm does not detect them. Repeated states can be the source of great inefficiency: identical sub trees will be explored many times!



In the extreme case, a state space of size d + 1 (Figure 5.1(a)) becomes a tree with $2d$ leaves (Figure 5.1(b)). k more realistic example is the ***rectangular grid*** as illustrated in Figure 5.1(c). On a grid, each state has four successors, so the search tree including repeated states has 4^d leaves; but there are only about $2d$^2 distinct states within d steps of any given state. For d = *20,* this means about a trillion nodes but only about 800 distinct states. Repeated states, then, can cause a solvable problem to become unsolvable if the algorithm does not detect them. Detection usually means comparing the node about to be expanded to those that have been expanded

already; if a match is found, then the algorithm has discovered two paths to the same state and can discard one of them.

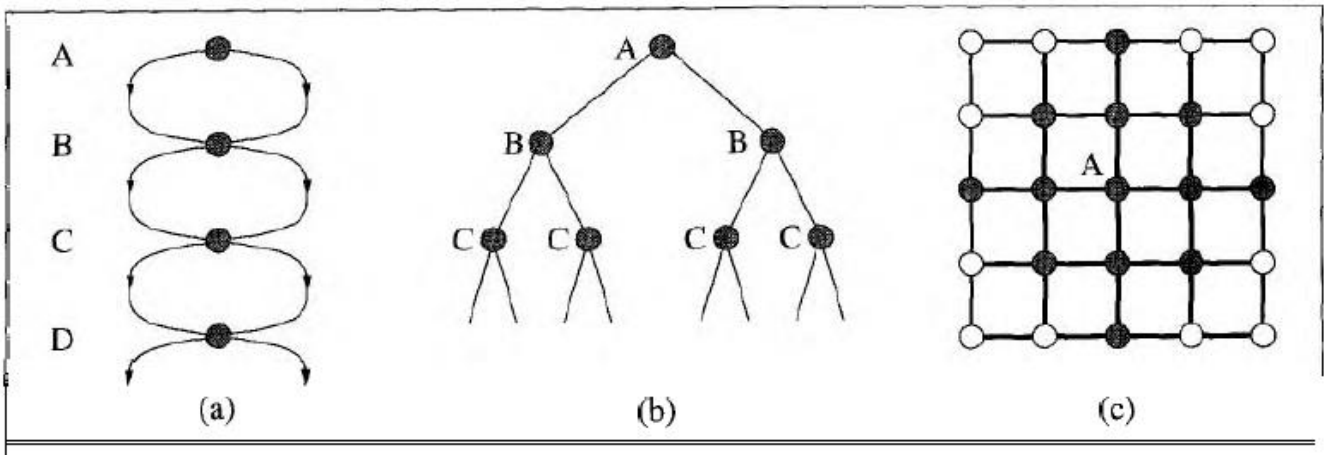


Figure 5.1 *State spaces that generate an exponentially larger search tree. (a) A state space in which there are two possible actions leading from A to B, two from B to C, and so on. The state space contains d + 1 states, where d is the maximum depth. (b) The corresponding search tree, which has 2d branches corresponding to the 2d paths through the space. (c) A rectangular grid space. States within 2 steps of the initial state (A) are shown in gray.*

The repeated states can be avoided using three different ways. They are:

1. Do not return to the state you just came from (i.e) avoid any successor that is the same state as the node's parent.
2. Do not create path with cycles (i.e) avoid any successor of a node that is the same as any of the node's ancestors.
3. Do not generate any state that was ever generated before.

The general TREE-SEARCH algorithm is modified with additional data structure, such as:

Closed list - This stores every expanded node.

Open list - fringe of unexpanded nodes.

If the current node matches a node on the closed list, then it is discarded and it is not considered for expansion. This is done with *GRAPH-SEARCH* algorithm. This algorithm is efficient for problems with many repeated states.

Algorithm:

```
function GRAPH-SEARCH (problem, fringe) returns a solution, or failure
 closed <- an empty set
fringe <- INSERT (MAKE-NODE(INITIAL-STATE[problem]), fringe)
 loop do
if EMPTY?(fringe) then return failure
node <- REMOVE-FIRST (fringe)
 if GOAL-TEST [problem ](STATE[node]) then return SOLUTION (node)
if STATE [node] is not in closed then add STATE [node] to closed
fringe <- INSERT-ALL(EXPAND(node, problem), fringe)
end
```

The worst-case time and space requirements are proportional to the size of the state space, this may be much smaller than $O(b^d)$.

6. *Searching with Partial Information*

When the knowledge of the states or actions is incomplete about the environment, then only partial information is known to the agent. This incompleteness lead to three distinct problem types. They are:

(i) ***Sensor less problems (conformant problems)*:** If the agent has no sensors at all, then it could be in one of several possible initial states, and each action might therefore lead to one of possible successor states.

(ii) ***Contingency problems*:** If the environment is partially observable or if actions are uncertain, then the agent's percepts provide new information after each action. A problem is called adversarial if the uncertainty is caused by the actions of another agent. To handle the situation of unknown circumstances the agent needs a contingency plan.

(iii) ***Exploration problem:*** It is an extreme case of contingency problems, where the states and actions of the environment are unknown and the agent must act to discover them.

Consider an example: "Vacuum World Agent"

Case 1: Where the agent know the eight possible states of vacuum world

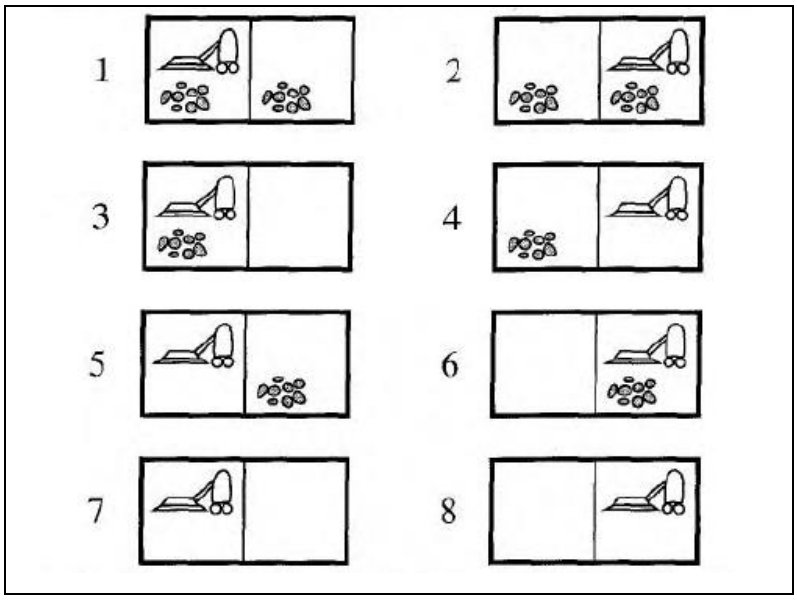


Figure 6.1 Eight possible states of vacuum world

There are three actions-Left, Right, and Suck-and the goal is to clean up all the dirt (states 7 and 8). If the environment is observable, deterministic, and completely known, then the problem is trivially solvable by any of the algorithms we have described. For example, if the initial state is 5, then the action sequence [Right, Suck] will reach a goal state, 8.

Case 2: This section deals with the sensor less and contingency versions of the problem.

Suppose that the vacuum agent knows all the effects of its actions, but has no sensors. Then it knows only that its initial state is one of the set {1,2,3,4,5,6,7,8). One might suppose that the agent's predicament is hopeless, but in fact it can do quite well. Because it knows what its actions do, it can, for example, calculate that the action Right will cause it to be in one of the states (2,4,6,8), and the action sequence [Right, Suck] will always end up in one of the states {4,8}. Finally, the sequence [Right, Suck, Left, Suck] is guaranteed to reach the goal state 7 no

matter what the start state. We say that the agent can coerce the world into state 7, even when it doesn't know where it started.

Belief State: when the world is not fully observable, the agent must reason about sets of states that it might get to, rather than single states. We call each such set of states a belief state, representing the agent's current belief about the possible physical states it might be;

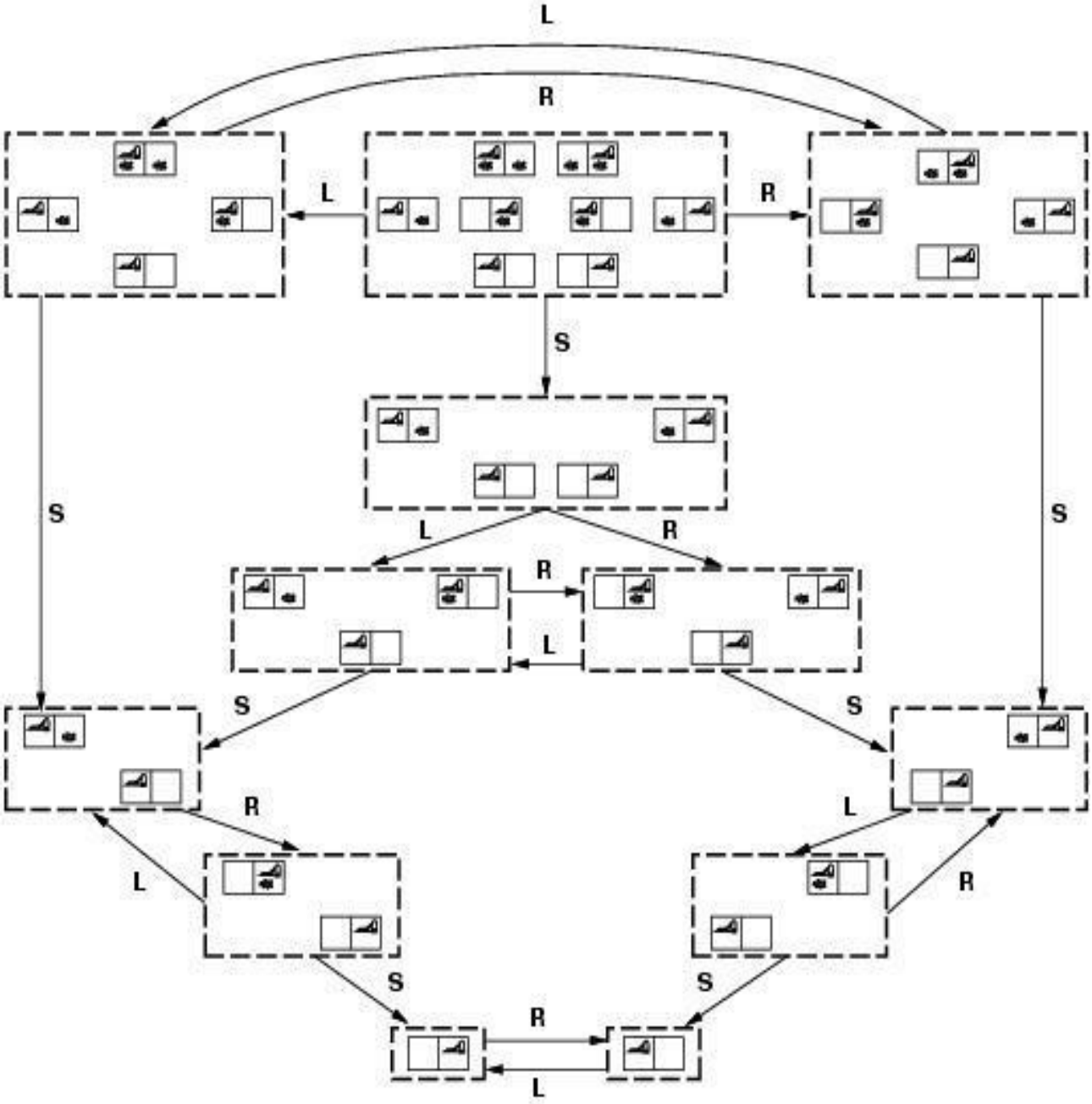


Figure 6.2: Sensor less vacuum world each box corresponds to single belief state

Contingency, start in {1,3}.

Murphy 's Law, Suck *can* dirty a clean carpet.

Local sensing: dirt, location only.

– Percept = [L, Dirty] ={1,3}

– [Suck] = {5,7}

– [Right] ={6,8}

– [Suck] in {6}={8} (Success)

– BUT [Suck] in {8} = failure

Solution?

– Belief-state: no fixed action sequence guarantees solution

Relax requirement:

– [*Suck*, *Right*, if *[R, dirty]* then *Suck*]

– Select actions based on contingencies arising during execution.

Time and space complexity are always considered with respect to some measure of the problem difficulty. In theoretical computer science, the typical measure is the size of the state space.

7. *Informed Search and Exploration*

Informed (Heuristic) Search Strategies:

An informed search strategy uses problem-specific knowledge beyond the definition of the problem itself and it can find solutions more efficiently than an uninformed strategy. The general approach is best first search that uses an evaluation function in *TREE-SEARCH or GRAPH-SEARCH*. Best-first search is an instance of the general TREE-SEARCH or GRAPH-SEARCH algorithm in which a node is selected for expansion based on an evaluation function, f(n) The node with the *lowest* evaluation is selected for expansion, because the evaluation measures distance to the goal. Best-first search can be implemented

within our general search framework via a priority queue, a data structure that will maintain the fringe in ascending order of f –values.

Heuristic function:

A heuristic function or simply a heuristic is a function that ranks alternatives in various search algorithms at each branching step basing on available information in order to make a decision which branch is to be followed during a search.

The *key component* of Best-first search algorithm is a heuristic function, denoted by h(n)

h(n) = estimated cost of the **cheapest path** from node n to a **goal node**.

For example, in Romania, one might estimate the cost of the cheapest path from Arad to Bucharest via a **straight-line distance** from Arad to Bucharest**.**

One constraint: if *n* is a goal node, then *h(n)* = 0

Heuristic functions are the most common form in which additional knowledge is imparted to the search algorithm.

The two types of evaluation functions are:

(i) Expand the node closest to the goal state using estimated cost as the evaluation is called **greedy best first search.**

(ii) Expand the node on the least cost solution path using estimated cost and actual cost as the evaluation function is called **A*search.**

i) Greedy best first search (Minimize estimated cost to reach a goal)

Greedy best-first search tries to expand the node that is closest to the goal, on the grounds that this is likely to a solution quickly. It evaluates the nodes by using the heuristic function

f(n) = h(n).

Taking the example of Route-finding problems in Romania, the goal is to reach Bucharest starting from the city Arad. We need to know the straight-line distances to Bucharest from various cities as shown in Figure 1.2. For example, the initial state is In(Arad) ,and the straight line distance heuristic hSLD(In(Arad)) is found to be 366.

Using the **straight-line distance** heuristic **hSLD ,** the goal state can be reached faster.

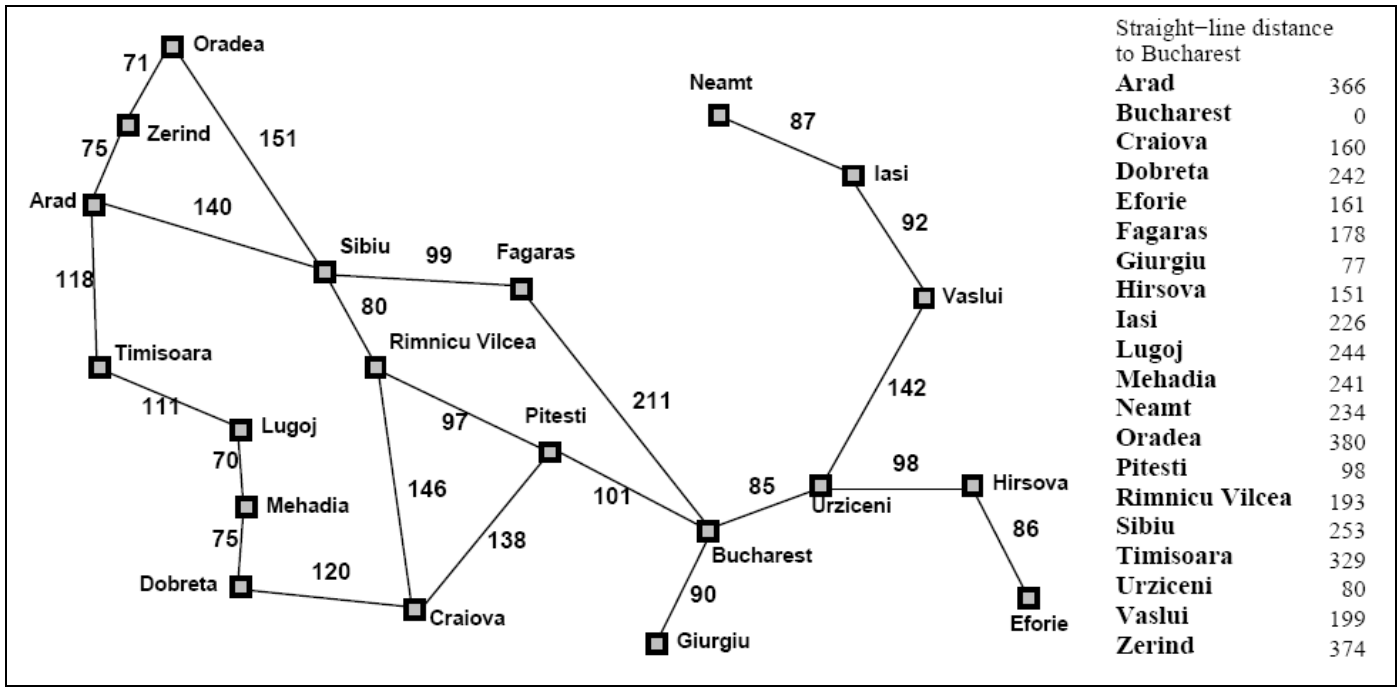
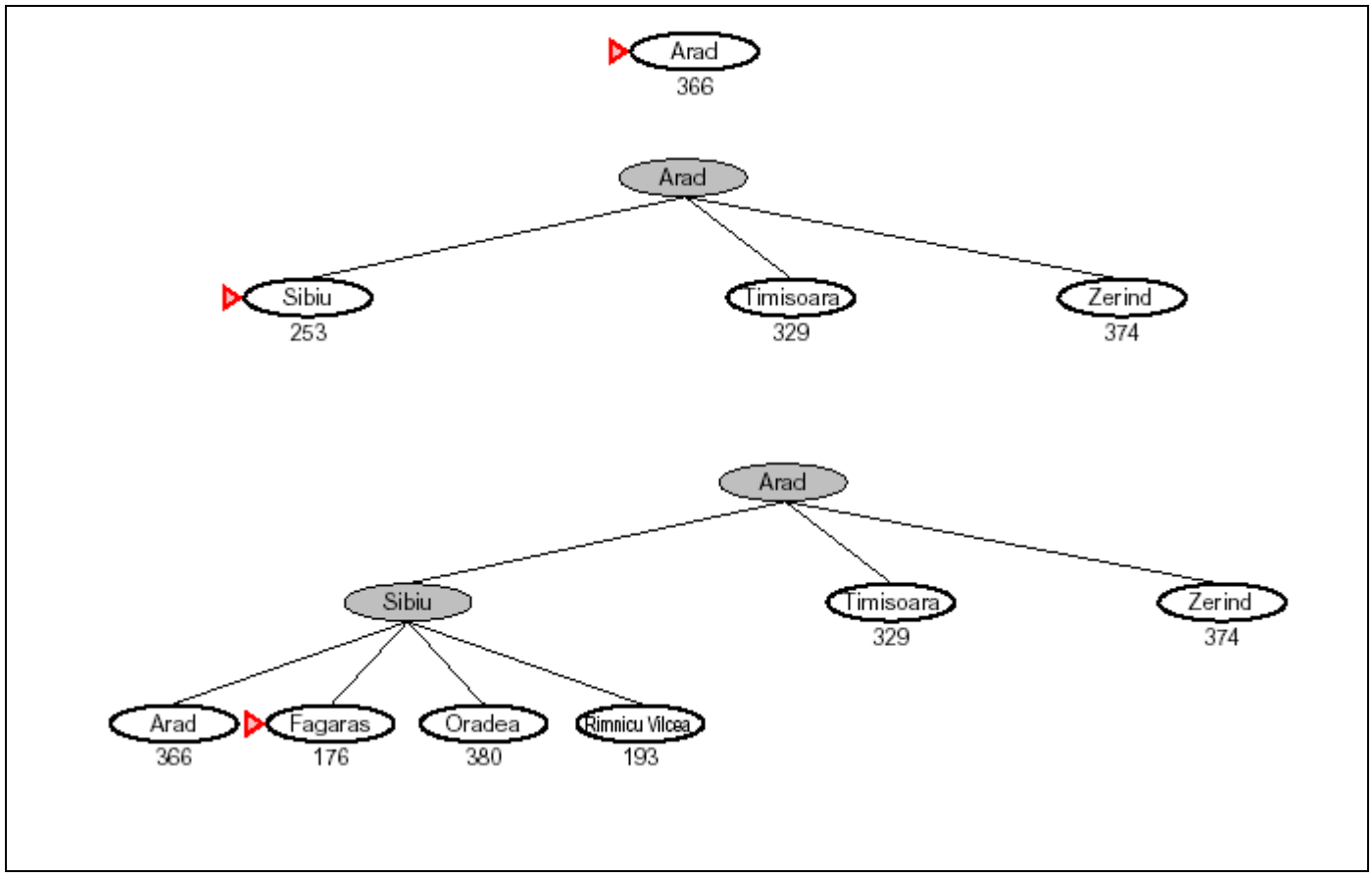


Figure 7.1 Values of hSLD - straight line distances to Bucharest



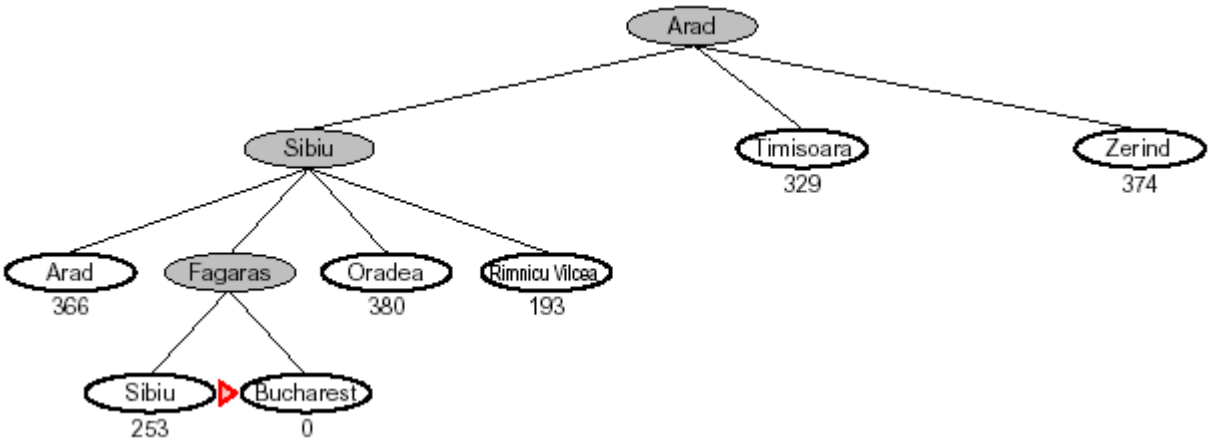


Figure 7.2 stages in greedy best-first search for Bucharest using straight-line distance heuristic hSLD.

Figure 7.2 shows the progress of greedy best-first search using h_{SLD} to find a path from Arad to Bucharest. The first node to be expanded from Arad will be Sibiu, because it is closer to Bucharest than either Zerind or Timisoara. The next node to be expanded will be Fagaras, because it is closest. Fagaras in turn generates Bucharest, which is the goal.

Complete? No–It can get stuck in loops, e.g., Iasi ! Neamt ! Iasi ! Neamt !

Complete in finite space with repeated-state checking

Time? O(bm), but a good heuristic can give dramatic improvement

Space? O(bm)—keeps all nodes in memory

Optimal? No

Greedy best-first search is not optimal, and it is incomplete. The worst-case time and space complexity is O(bm),where m is the maximum depth of the search space.

ii) A Search*

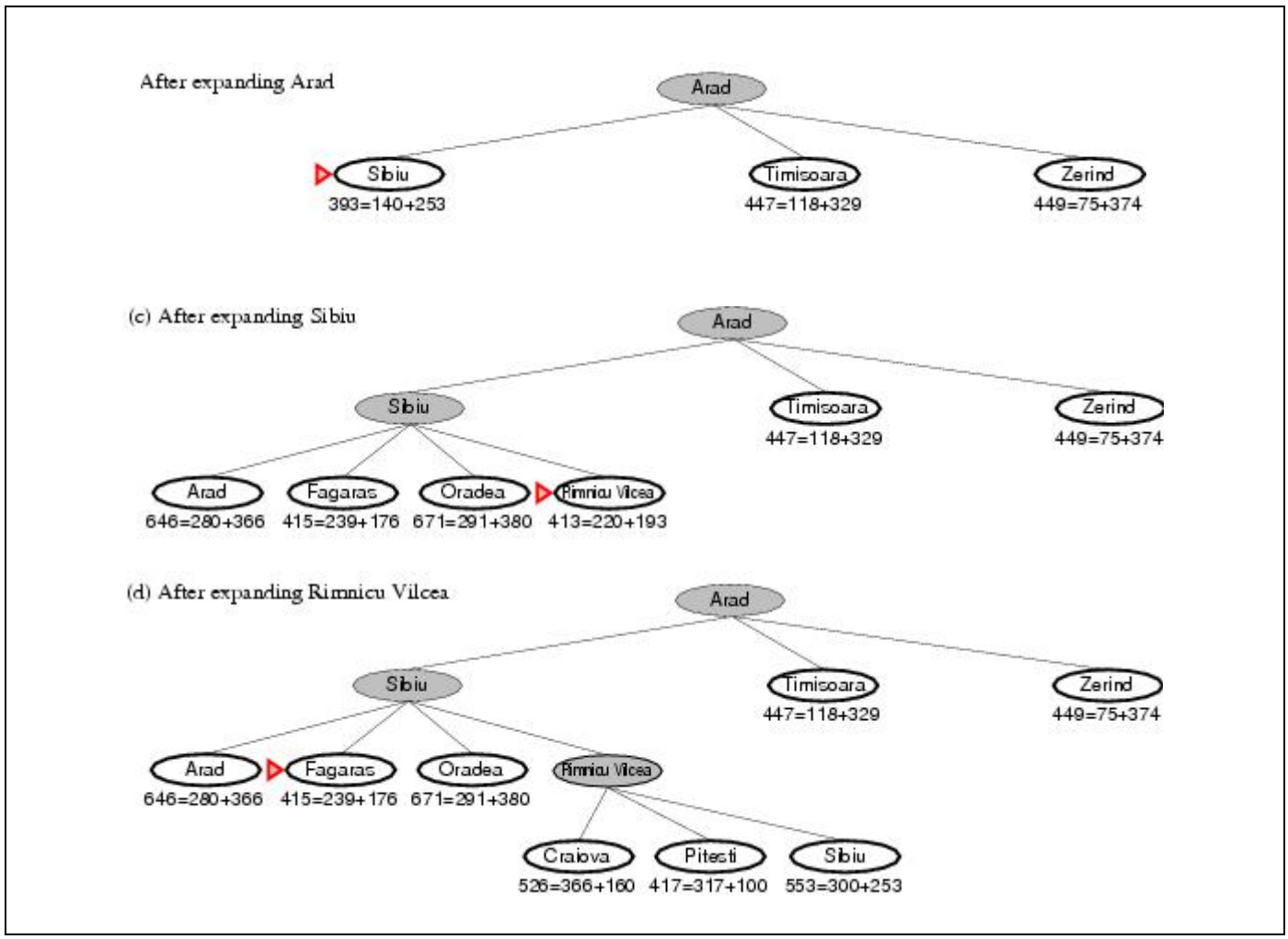
It is the most widely used form of best-first search. The evaluation function f(n) is obtained by combining

(1) **g(n) =** the cost to reach the node, and

(2) **h(n) =** the cost to get from the node to the **goal** : f(n) = g(n) + h(n).

A* Search is both optimal and complete. A* is optimal if h(n) is an *admissible heuristic*. The obvious example of admissible heuristic is the straight-line distance hSLD. It cannot be an overestimate. A* Search is optimal if h(n) is an admissible heuristic – *that is, provided that h(n) never overestimates the cost to reach the goal.*

An obvious example of an admissible heuristic is the straight-line distance hSLD that we used in getting to Bucharest. The values of 'g 'are computed from the step costs shown in the Romania map(figure 7.1). Also the values of hSLD are given in Figure 7.1.



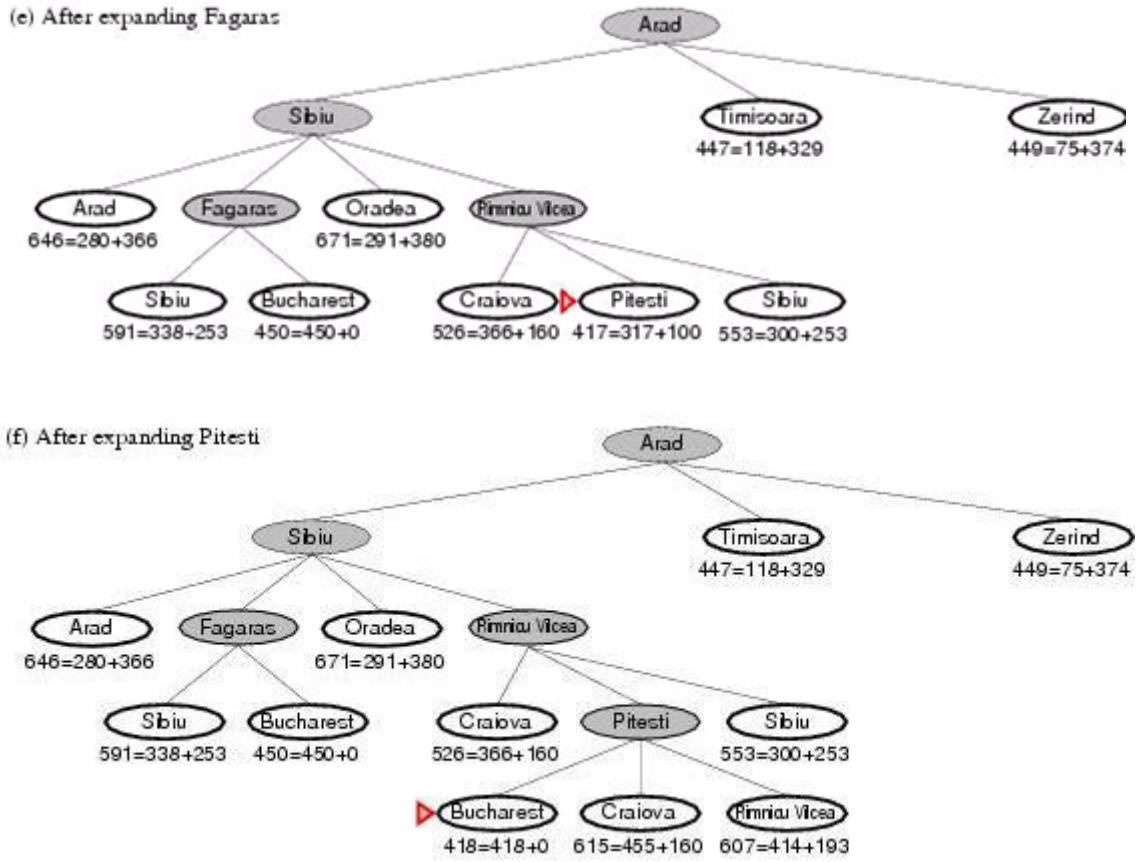


Figure 7.3 Stages in A Search for Bucharest. Nodes are labelled with f = g + h.*

The behavior of A search*

Monotonicity (Consistency) In search tree any path from the root, the f- cost never decreases. This condition is true for almost all admissible heuristics. A heuristic which satisfies this property is called *monotonicity.*

A heuristic *h(n)* is consistent if, for every node *n* and every successor *n'* of *n* generated by any action a, the estimated cost of reaching the goal from *n* is not greater than the step cost of getting to *n'* plus the estimated cost of reaching the goal from *n'*

Example for monotonic Let us consider two nodes n and n', where n is the parent of n'

For example g(n) = 3 and h(n) = 4. then f(n) = g(n) + h(n) = 7.

g(n′) = 54 and h(n′) = 3. then f(n′) = g(n′) + h(n′) = 8

Example for Non-monotonic

Let us consider two nodes n and n′, where n is the parent of n′. For example

g(n) = 3 and h(n) = 4. then f(n) = g(n) + h(n) = 7.

g(n′) = 4 and h(n′) = 2. then f(n′) = g(n′) + h(n′) = 6.

To reach the node *n* the cost value is 7, from there to reach the node *n'* the value of cost has to increase as per *monotonic* property. But the above example does not satisfy this property. So, it is called as non-monotonic heuristic.

How to avoid non-monotonic heuristic? We have to check each time when we generate anew node, to see if its f-cost is less that its parent's f-cost; if it is we have to use the parent's f- cost instead. Non-monotonic heuristic can be avoided using *path-max equation.*

$$f(n') = \max(f\{n), g(n') + h(n'))$$

Tree Search: Suppose, If suboptimal goal node G2 appears on the fringe, and c* be the cost of optimal solution and h(G2)=0

$$F(G2)= g(G2)+h(G2)=g(G2)> c^*$$

Now Consider a fringe node and if it is an optimal solution path like Pitesti i.e.,

$$f(n) = g(n)+h(n) <= c^*$$

Now, we have f(n)<=c*<f(G2) so G2 will not be considered and A* search selects an optimal solution.

Optimality: A* search is complete, optimal, and optimally efficient among all algorithms A* *using* GRAPH-SEARCH *is optimal if h(n) is consistent.*

Completeness: A* is complete on locally finite graphs (graphs with a finite branching factor) provided there is some constant *d* such that every operator costs at least *d.*

Drawback: A* usually runs out of space because it keeps all generated nodes in memory.

Graph Search:

Admissibility is not sufficient for graph search.

→ In Graph search the optimal path to a repeated state could be discarded if it is not the first one that was generated.

→ We can fix this problem by requiring consistency property for h(n).

→ A heuristic is consistent if for every successor m of a node n generated by any action a

$$h(n) <= c(n, a, m) + h(m)$$

Admissible heuristics are generally consistent. If h is consistent;

f(m) = g(m) + h(m)

= g(m) + c(n, a, m) + h(m)

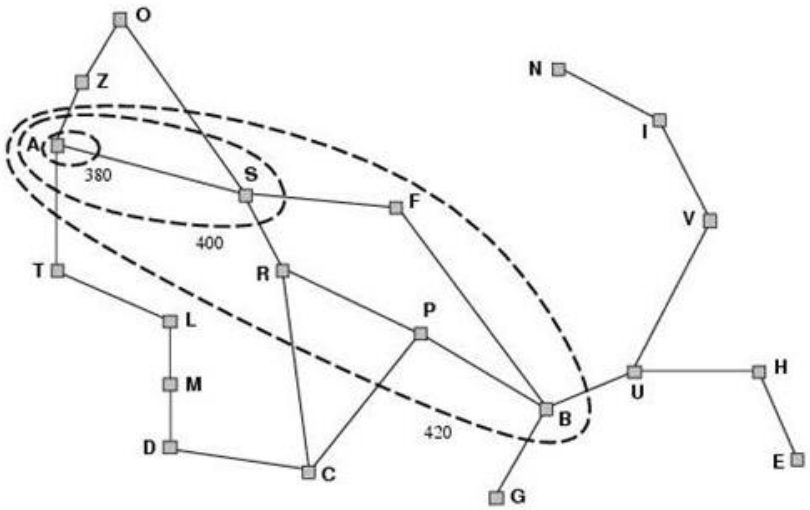
>= g(n) +h(n)

= f(n) is non decreasing along any path.

Contours of A:*

A* expands nodes in order of increasing f value and gradually adds f-contours of nodes

Contour I has all nodes with f=fi where fi<fi+1.



8. *Memory Bounded Heuristic Search:*

The simplest way to reduce memory requirements for A* is to adapt the idea of iterative deepening to the heuristic search context, resulting in the iterative-deepening A* (IDA*) algorithm. The memory requirements of A* is reduced by combining the heuristic function with iterative deepening resulting an IDA* algorithm.

Iterative Deepening A* search (IDA*) Depth first search is modified to use a*n f*-cost limit rather than a depth limit for IDA* algorithm. Each iteration in the search expands all the nodes inside the contour for the current f-cost and moves to the next contour with new *f* - cost.

Space complexity is proportional to the longest path of exploration that is *bd* is a good estimate of storage requirements

Time complexity depends on the number of different values that the heuristic function can take

Optimality: yes, because it implies A* search.

Completeness: yes, because it implies A* search.

Disadvantage: It will require more storage space in complex domains (i.e) each contour will include only one state with the previous contour. To avoid this, we increase the *f*-cost limit by a fixed amount on each iteration, so that the total number of iteration is proportional to $1/\epsilon$. Such an algorithm is called admissible.

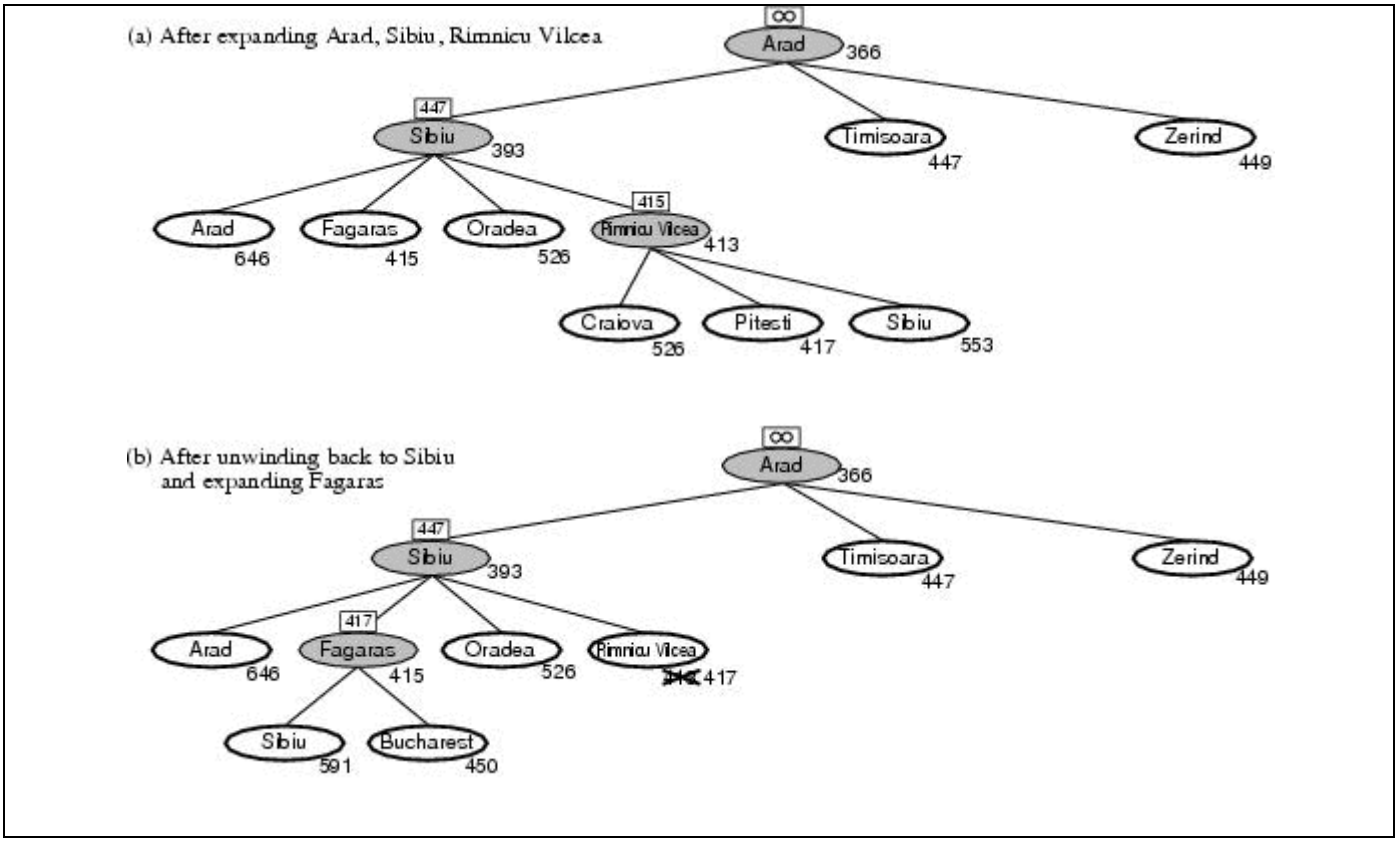
The two recent memory bounded algorithms are:

- *Recursive Best First Search (RBfS)*
- *Memory bounded A* search (MA*)*

Recursive Best First Search (RBFS) A recursive algorithm with best first search technique uses only linear space. It is similar to recursive depth first search with an inclusion (i.e.) keeps track of the f-value of the best alternative path available from any ancestor of the current node. If the current node exceeds this limit, the recursion unwinds back to the alternative path and replaces the f-value of each node along the path with the best f-value of its children. The main idea lies in to keep track of the second best alternate node (forgotten node) and decides whether it's worth to reexpand the subtree at some later time.

function RECURSIVE-BEST-FIRST-*SEARCH*(*problem*) returns a solution, or failure
RBFS(problem, MAKE-*NODE(INITIAL-STATE[problem]),* ∞)
function, *RBFS(problem, n*ode, f_*limit)* returns a solution, or failure and a new *f-cost* limit
if GOAL-*TEST[problem](state)* then return n*ode*
successors <- EXPAND(node, *problem)*
if *successors* is empty then *return failure,*
for each *s* in *successors* do
f[s]<-max(g(s) + *h(s),f[node])*
repeat
best <- the lowest f-value node in *successors*
if f[best] > f_*limit* then *return failure,f[best]*
alternative <- the second-lowest f-value among *successors*
result,f[best]<-RBFS(problem,best,min(f_limit,alternative))
if *result* failure then return result

Consider example of Romania Map;



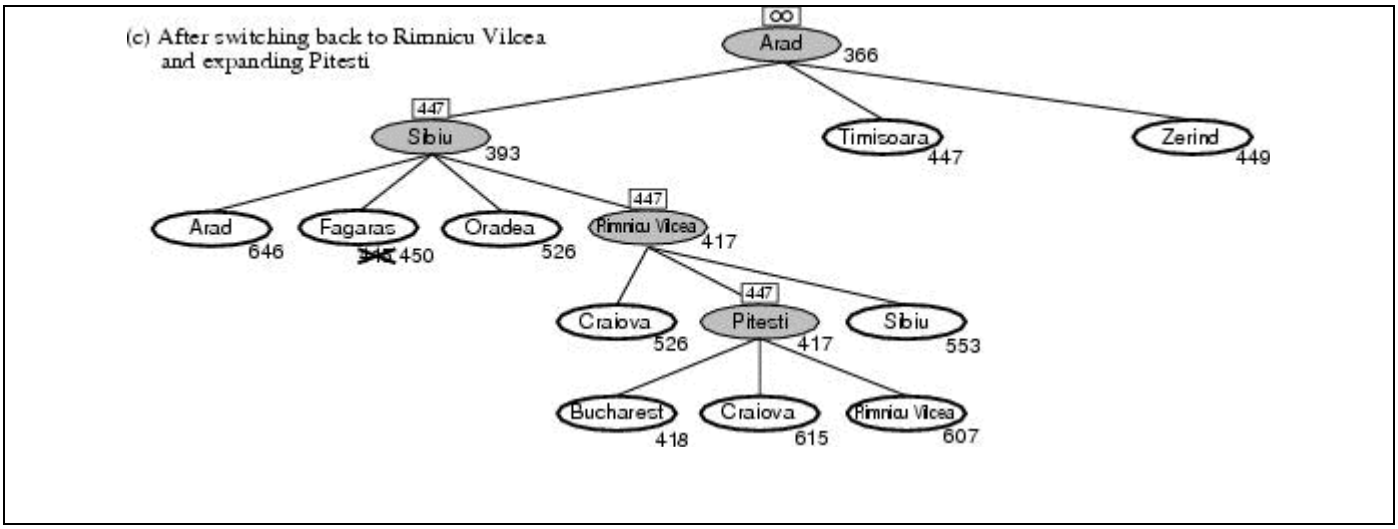


***Figure 8.1** Stages in an RBFS search for the shortest route to Bucharest.*

The f-limit value for each recursive call is shown on top of each current node.

(a) The path via Rimnicu Vilcea is followed until the current best leaf (Pitesti) has a value that is worse than the best alternative path (Fagaras).

(b) The recursion unwinds and the best leaf value of the forgotten subtree (417) is backed up to Rimnicu Vilcea; then Fagaras is expanded, revealing a best leaf value of 450.

(c) The recursion unwinds and the best leaf value of the forgotten subtree (450) is backed upto Fagaras; then Rimni Vicea is expanded. This time because the best alternative path (through Timisoara) costs atleast 447, the expansion continues to Bucharest.

***Time and space complexity*:** RBFS is an optimal algorithm if the heuristic function h(n) is admissible. Its time complexity depends both on the accuracy of the heuristic function and on how often the best path changes as nodes are expanded. Its space complexity is O(bd), even though more is available.

Advantage: RBFS is more efficient than IDA*. Like A* RBFS is an optimal algorithm if the heuristic function h(n) is admissible.

Drawback: It suffers from excessive node regeneration and uses too little memory even if more memory is available.

A search technique (algorithms) which uses all available memory are:

a. MA (Memory - bounded A*)*

b. SMA (Simplified MA*)*

Simplified Memory - bounded A* search (SMA*) SMA* algorithm can make use of all available memory to carry out the search.

Properties:

(a) It will utilize whatever memory is made available to it.

(b) It avoids repeated states as far as its memory allows.

It proceeds like A* by expanding the best leaf until memory is full. At this point I can't add a new node to the search tree without dropping the old one. So, it drops the worst leaf node with highest f-value, and it also back up the value of forgotten node to its parent.

It is ***complete*** if the available memory is sufficient to store the deepest solution path. It is *optimal* if enough memory is available to store the deepest solution path. Otherwise, it returns the best solution that can be reached with the available memory.

Advantage: SMA* uses only the available memory.

Disadvantage: If enough memory is not available it leads to un-optimal solution.

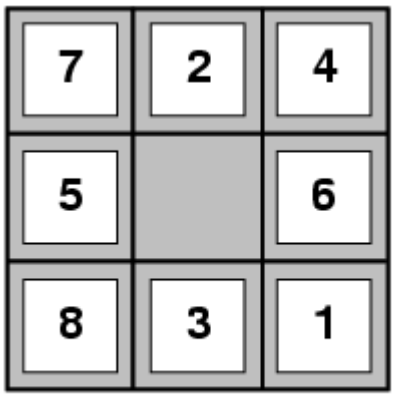
Space and Time complexity: depends on the available number of node.

9. Learning to Search Better

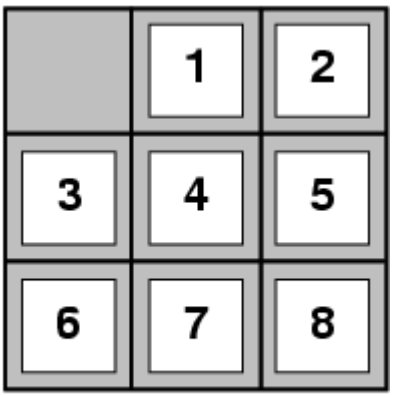
Here we need to observe that how the agent learns to search better on Meta level state space. Each state in a Meta level state space captures the internal state of program i.e., searching in an object level state space like Romania (or) expansion of Fagaras (or) RV. Here there is a misstep with Fagaras because this path is not helpful. So, Meta level learning Algorithm can learn from this experience to avoid unpromising sub trees→ minimize the total cost.

10. Heuristic Functions

A **heuristic function** or simply a heuristic is a function that ranks alternatives in various search algorithms at each branching step basing on available information in order to make a decision which branch is to be followed during a search.



Start State



Goal State

The 8-puzzle is an example of Heuristic search problem. The object of the puzzle is to slide the tiles horizontally or vertically into the empty space until the configuration matches the goal configuration.

The two commonly used heuristic functions are;

(1) h_1 = the number of misplaced tiles.

For above figure, all of the eight tiles are out of position, so the start state would have $h_1 = 8$. h_1 is an admissible heuristic.

(2) h_2 = the sum of the distances of the tiles from their goal positions.

This is called ***the city block distance*** or ***Manhattan distance*****.**

h_2 is admissible ,because all any move can do is move one tile one step closer to the goal.

Tiles 1 to 8 in start state give a Manhattan distance of

$h_2 = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18$.

Neither of these overestimates the true solution cost, which is 26.

Domination:

h2 always better than h1 if we see from definitions of two heuristics. We can say that for any node n,

h2(n) >=h1(n)

So, A* using h2 will never expand more nodes than h1.

The Effect of heuristic accuracy on performance:

One way to characterize the **quality of a heuristic** is the **effective branching factor b*.** If the total number of nodes generated by A* for a particular problem is **N**, and the **solution depth** is **d,** then b* is the branching factor that a uniform tree of depth d would have to have in order to contain N+1 nodes. Thus,

$N + 1 = 1 + b^* + (b^*)2+\ldots+(b^*)d$

Inventing admissible heuristic functions:

Is it possible for a computer to invent such a heuristic as above? YES

Relaxed problem:

A problem with fewer restrictions on the actions is called a ***relaxed problem***

1) The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem

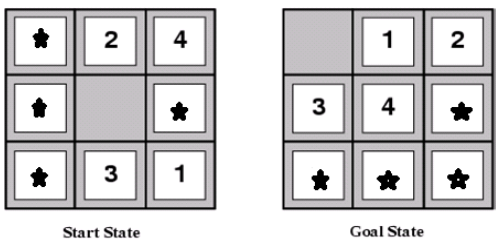
2) If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then *h1(n)* gives the shortest solution

3) If the rules are relaxed so that a tile can move to *any adjacent square,* then *h2(n)* gives the shortest solution

A program called ABSOLVER which can generate heuristics automatically from problem definition; using relaxed problem method and other various techniques this ABSOLVER generates new heuristics for 8- puzzle better than any other existing heuristic.

Sub problem:

Admissible heuristic also derived from the solution cost of a sub problem of a given problem.



Pattern Databases: Which are used to store the exact solution costs for every possible sub problem instance. So, we compute an admissible heuristic hDB for each possible state by looking up the corresponding sub problem in database.

11. Local search Algorithms and optimization Problems:

In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution. For example, in the 8-queens problem, what matters is the final configuration of queens, not the order in which they are added.

In such cases, we can **use local search algorithms.** They operate using a **single current state** (rather than multiple paths) and generally move only to neighbours of that state.

The important applications of these class of problems are (a) integrated-circuit design, (b)Factory-floor layout, (c) job-shop scheduling,(d)automatic programming, (e)telecommunications network optimization, (f)Vehicle routing etc…

They have two key advantages:

(1) They use very little memory-usually a constant amount;

(2) They can often find reasonable solutions in large or infinite (continuous) state spaces for which systematic algorithms are unsuitable.

Optimization Problems:

In addition to finding goals, local search algorithms are useful for solving pure **optimization problems**, in which the aim is to find the **best state** according to an **objective function**.

State Space Landscape: To understand local search, it is better explained using **state space landscape** as shown in following figure.

A landscape has both "**location**" (defined by the state) and "**elevation**"(defined by the value of the heuristic cost function or objective function).

If elevation corresponds to **cost**, then the aim is to find the **lowest valley** – a **global minimum**; if elevation corresponds to an **objective function**, then the aim is to find the **highest peak** – a **global maximum.**

Local search algorithms explore this landscape. A complete local search algorithm always finds a **goal** if one exists; an **optimal** algorithm always finds a **global minimum/maximum**.

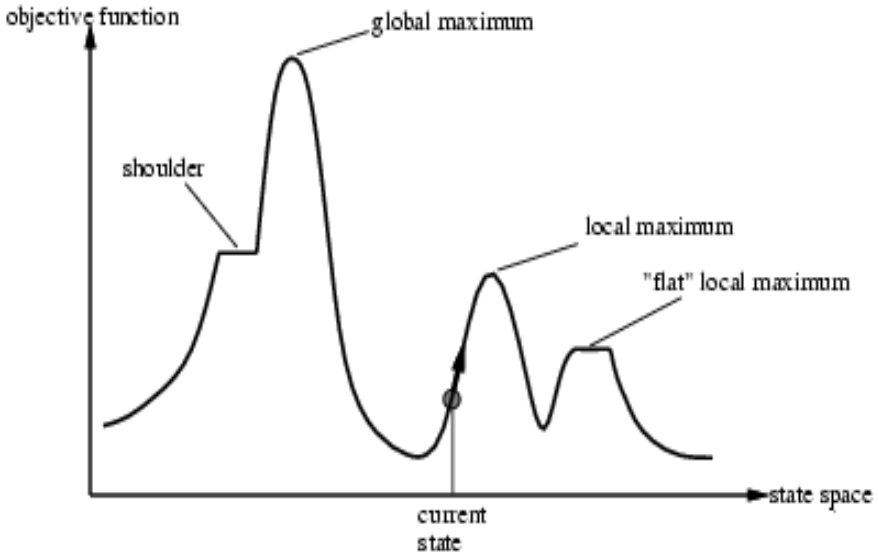


Figure 11.1 *A one dimensional **state space landscape** in which elevation corresponds to the **objective function**. The aim is to find the global maximum. Hill climbing search modifies the current state to try to improve it is as shown by the arrow.*

Some of the local search algorithms are:

1. Hill climbing search (Greedy Local Search)

2. Simulated annealing

3. Local beam search

4. Genetic Algorithm (GA)

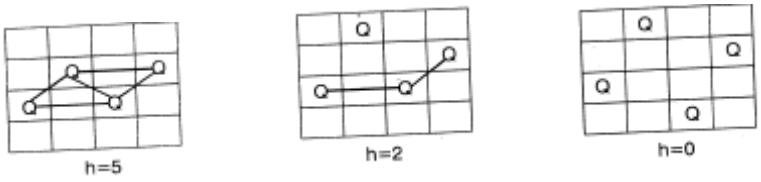
1. Hill Climbing Search (Greedy Local Search): The **hill-climbing** search algorithm is simply a loop that continually moves in the direction of increasing value. It terminates when it reaches a "peak" where no neighbor has a higher value. The algorithm does not maintain a search tree, so the current node data structure need only record the state and its objective function value. At each step the current node is replaced by the best neighbor;

Hill-**climbing** search algorithm

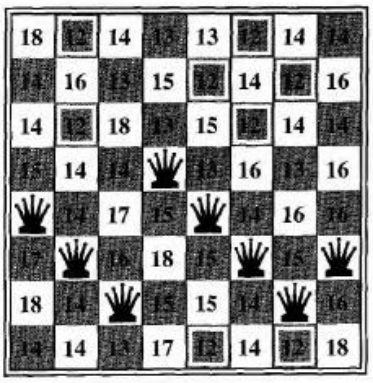
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
 inputs: problem, a problem
local variables: current, a node and neighbor, a node
current <- MAKE-NODE(INITIAL-STATE[problem])
 loop do
neighbor <- a highest-valued successor of current
if VALUE[neighbor] <= VALUE[current] then return STATE[current]
current <- neighbor
```

To illustrate hill-climbing, we will use the 8-queens, where each state has 8 queens on the board, one per column. The successor function returns all possible states generated by moving a single queen to another square in the same column (so each state has 8 x 7 = 56 successors). Hill-climbing algorithms typically choose randomly among the set of best successors, if there is more than one.

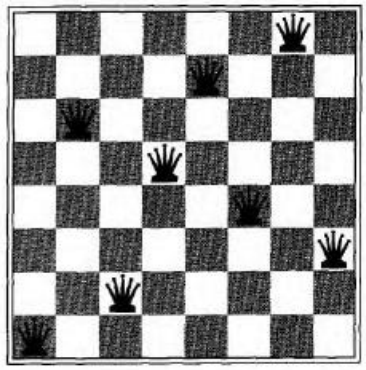
The heuristic cost function h is the number of pairs of queens that are attacking each other, either directly or indirectly. The global minimum of this function is zero, which occurs only at perfect solutions.



An 8-queens state with heuristic cost estimate h = 17, showing the value of h for each possible successor obtained by moving a queen within its column. The best moves are marked.

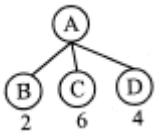


A local minimum in the 8-queens state space; the state has h = 1 but every successor has a higher cost.



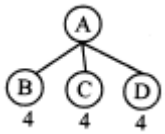
Hill climbing often gets stuck for the following reasons:

Local maxima or foot hills: A local maximum is a peak that is higher than each of its neighboring states, but lower than the global maximum



Example: The evaluation function value is maximum at C and from their there is no path exist for expansion. Therefore C is called as local maxima. To avoid this state, random node is selected using back tracking to the previous node.

Plateau or shoulder: a plateau is an area of the state space landscape where the evaluation function is flat. It can be a flat local maximum.



The evaluation function value of B C D are same, this is a state space of plateau. To avoid this state, random node is selected or skips the level (i.e) select the node in the next level.

Ridges: Ridges result in a sequence of local maxima that is very difficult for greedy algorithms to navigate. But the disadvantage is more calculations to be done.

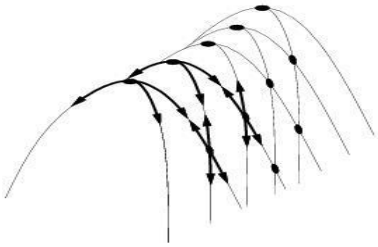


Illustration of why ridges cause difficulties for hill-climbing. The grid of states (dark circles) is superimposed on a ridge rising from left to right, creating a sequence of local maxima that are not directly connected to each other. From each local maximum, all the available options point Downhill.

Variants of hill-climbing:

Stochastic hill climbing - Stochastic hill climbing chooses at random from among the uphill moves; the probability of selection can vary with the steepness of the uphill move.

First-choice hill climbing - First-choice hill climbing implements stochastic hill climbing by generating successors randomly until one is generated that is better than the current state

Random-restart hill climbing - Random-restart hill climbing adopts the well known adage, "If at first you don't succeed, try, try again." It conducts a series of hill-climbing searches from randomly generated initial state, stopping when a goal is found.

2. Simulated annealing search: An algorithm which combines hill climbing with random walk to yield both efficiency and completeness In metallurgy, **annealing** is the process used to temper or harden metals and glass by heating them to a high temperature and then gradually cooling them When the search stops at the local maxima, we will allow the search to take some down Hill steps to escape the local maxima by allowing some "bad" moves but gradually decrease their size and frequency. The node is selected randomly and it checks whether it is a best move or not. If the move improves the situation, it is executed. E variable is introduced to calculate the probability of worsened. A Second parameter T is introduced to determine the probability.

Algorithm:

function SIMULATED-ANNEALING(problem, schedule) **returns** a solution state

inputs: problem, a problem schedule, a mapping from time to "temperature"

local variables: current, a node next, a node

T, a "variable" controlling the probability of downward steps

current <- **MAKE-NODE(INITIAL-STATE[problem])**

for t<- 1 to ∞ **do**

T <- schedule[t]

if T = 0 **then return** current

next <- a randomly selected successor of current

Δ E <- VALUE[next] - VALUE[current]

if Δ E > 0 **then** current <- next **else** current <- next only with probability e power ΔE/T

Property of simulated annealing search: T decreases slowly enough then simulated annealing search will find a global optimum with probability approaching one.

Applications VLSI layout, Airline scheduling.

3. Local beam search: Local beam search is a variation of beam search which is a path based algorithm. It uses K states and generates successors for K states in parallel instead of one state and its successors in sequence. The useful information is passed among the K parallel threads. The sequence of steps to perform local beam search is given below:

- Keep track of K states rather than just one.
- Start with K randomly generated states.
- At each iteration, all the successors of all K states are generated.
- If anyone is a goal state stop; else select the K best successors from the complete list and repeat.

This search will suffer from lack of diversity among K states. Therefore a variant named as stochastic beam search selects K successors at random, with the probability of choosing a given successor being an increasing function of its value.

Stochastic Beam Search:

Instead of choosing the best from the pool of candidate successors, this search chooses k successors randomly with a probability of choosing successor being an increasing factor of its value.

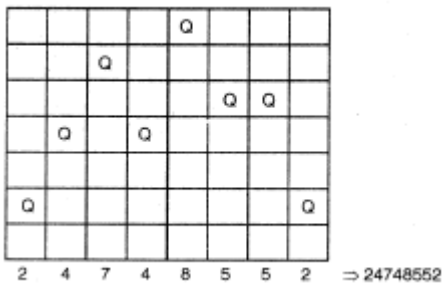
4. Genetic Algorithm:

A genetic algorithm (or **GA)** is a variant of stochastic beam search in which successor states are generated by combining *two* parent states, rather than by modifying a single state GA begins with a set of k randomly generated states, called the **population.** Each state, or **individual,** is represented as a string over a finite alphabet. For Example an 8 queen's state could be represented as 8 digits, each in the range from 1 to 8.

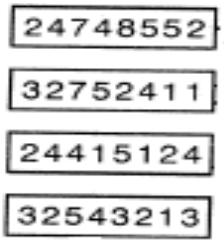
i) Initial population: K randomly generated states of 8 queen problem

Individual (or) state: Each string in the initial population is individual (or) state. In one state, the position of the queen of each column is represented.

Example: The state with the value 24748552 is derived as follows:



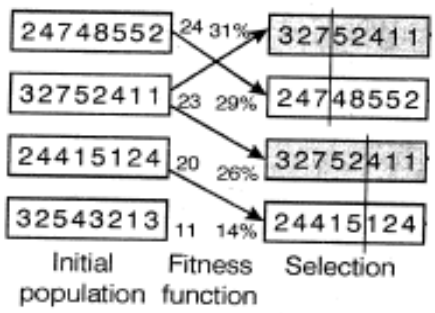
The Initial Population (Four randomly selected States) is:



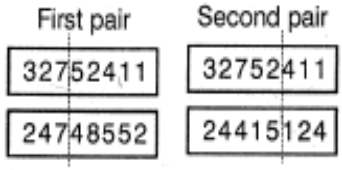
ii) Evaluation function (or) Fitness functions: A function that returns higher values for better State. For 8 queens problem the number of non attacking pairs of queens is defined as fitness function Minimum fitness value is 0 Maximum fitness value is : 8*7/2 = 28 for a solution The

values of the four states are 24, 23, 20, and 11. The probability of being chosen for reproduction is directly proportional to the fitness score, which is denoted as percentage. 24 / (24+23+20+11) = 31% 23 / (24+23+20+11) = 29% 20 / (24+23+20+11) = 26% 11 / (24+23+20+11) = 14%

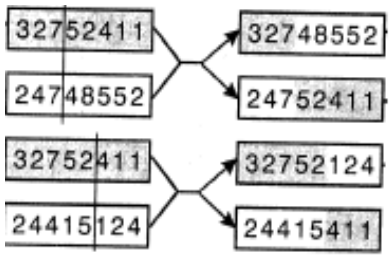
iii) Selection: A random choice of two pairs is selected for reproduction, by considering the probability of fitness function of each state. In the example one state is chosen twice (probability of 29%) and the another one state is not chosen (Probability of 14%)



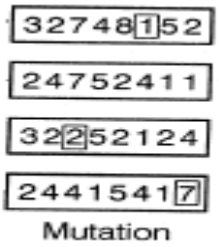
iv) Cross over: Each pair to be mated, a crossover point is randomly chosen. For the first pair the crossover point is chosen after 3 digits and after 5 digits for the second pair.



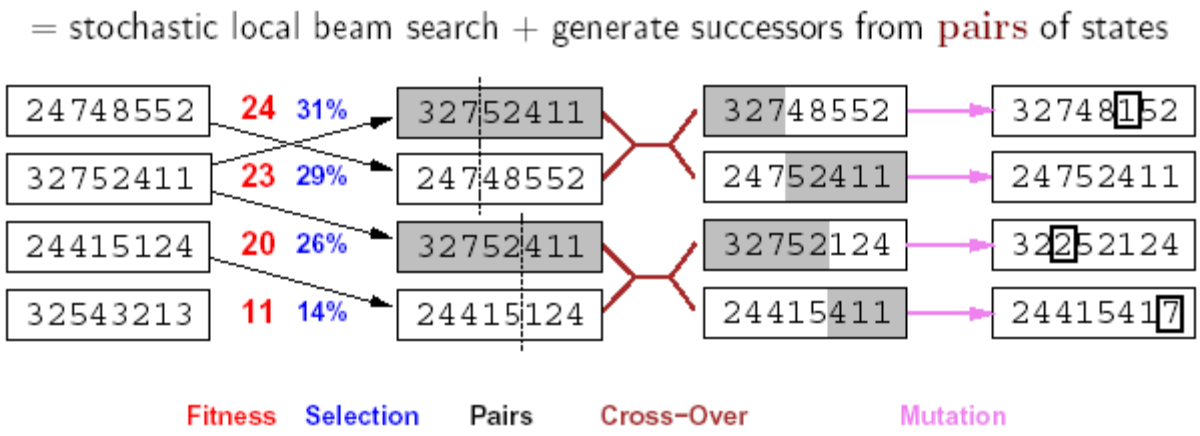
Offspring: Offspring is created by crossing over the parent strings in the crossover point. That is, the first child of the first pair gets the first 3 digits from the first parent and the remaining digits from the second parent. Similarly the second child of the first pair gets the first 3 digits from the second parent and the remaining digits from the first parent.



v) Mutation: Each location is subject to random **mutation** with a small independent probability. One digit was mutated in the first, third, and fourth offspring.



Production of Next Generation of States;

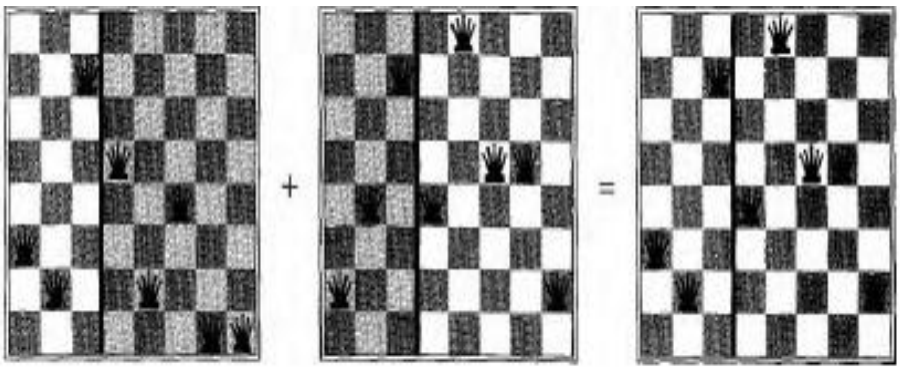


The 8-queens states corresponding to the first two parents;

Parent1: 32752411

\+

Parent2: 24748552



Algorithm:

```
function  GENETIC-ALGORITHM(population, FITNESS-FN)
returns an individual
inputs: population, a set of individuals
FITNESS-FN, a function that measures the fitness of an
individual
repeat
new-population <- empty set
loop for i from 1 to SIZE(population) do
x <- RANDOM-SELECTION(Population,FITNESS-FN)
y <- RANDOM-SELECTION(Population,FITNESS-FN)
child <- REPRODUCE(yX),
if (small random probability) then child  MUTATE(child)
add child to new-population
population <- new-population
until some individual is fit enough, or enough time has
elapsed
return the best individual in population, according to
FITNESS-FN
function REPRODUCE(x,y), returns an individual
inputs: x, y, parent individuals
n <- LENGTH(x)
c <- random number from 1 to n
return APPEND(SUBSTRING(x,1,c),SUBSTRING(y, c + 1, n ))
```

The sequence of steps to perform GA is summarized below:

• A successor state is generated by combining two parent states.

• Start with K randomly generated states population

• Each state or individual is represented as a string over a finite alphabet (often a string of O's and 1's)

• Evaluation function (fitness function) is applied to find better states with higher values.

• Produce the next generation of states by selection, crossover and mutation

12. Local Search In Continuous Spaces Local Search in continuous space is the one that deals with the real world problems.

• One way to solve continuous problems is to discretize the neighborhood of each state.

• Stochastic hill climbing and simulated annealing are applied directly in the continuous space

• Steepest - ascent hill climbing is applied by updating the formula of current state

x <- x + *f(x)* - small constant *f(x)* - magnitude & direction of the steepest slope. α ∇α∇

• Local search methods in continuous space may also lead to local maxima, ridges and plateau. This situation is avoided by using random restart method.