Maths assignment sarath

1. Find the Length of the curve $y=(1-x^2)^{1/2}$ for every $x \in [-1,1]$

Ans :- $L = \int_a^b \sqrt{1+(f(x))^2}\, dx$

here $a=1$ and $b=1$

$f(x) = (1-x^2)^{1/2}$

$f'(x) = \frac{1}{2}(1-x^2)^{-1/2}\, x - 2x$

$= \frac{-x}{(1-x^2)^{1/2}}$

Then the length of the curve y is,

$$L = \int_{-1}^{1} \sqrt{1+\frac{x^2}{1-x^2}}\, dx$$

$$= \int_{-1}^{1} \sqrt{\frac{1-\cancel{x^2}+\cancel{x^2}}{1-x^2}}\, dx$$

$$= \int_{-1}^{1} \sqrt{\frac{1}{1-x^2}}\, dx$$

$$= \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= \left[\sin^{-1} x \right]_{-1}^{1}$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= \sin^{-1}(-1) - (-\sin^{-1}(1))$$

$$= 2 \sin^{-1}(1) = \cancel{2}\pi/\cancel{2} = \pi$$

2. Determine the convergence and divergence of

i) $\int_{1}^{\infty} \frac{dx}{(4x-3)^{1/3}}$ ii) $\int_{0}^{1} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$

Ans: i) $\int_{1}^{\infty} \frac{dx}{(4x-3)^{1/3}}$

By the definition of integral

For $x \geq 1$, $4x \geq 1$ and hence we get

$$\frac{1}{(4x-3)^{1/3}} \geq \frac{1}{4x}$$

By the definition of integrals over unbounded intervals, we get

$$\int_1^\infty \frac{dx}{4x} = \lim_{b \to \infty} \frac{1}{4} \int_1^b \frac{dx}{x}$$

$$= \frac{1}{4} \lim_{b \to \infty} \int_1^b \frac{1}{x}\, dx$$

$$= \frac{1}{4} \lim_{b \to \infty} \left[\ln x \right]_1^b$$

$$= \frac{1}{4} \lim_{b \to \infty} \left[\ln b - 0 \right]$$

$$= \frac{1}{4} \ln \infty = \infty$$

Since the limit is infinite, the improper integrals on the left hand side of the above equation is divergent.

Then by comparison test $\int^\infty \frac{dx}{(4x-3)^{1/3}}$ is divergent.

ii) $\displaystyle\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

Since $\dfrac{e^{-\sqrt{x}}}{\sqrt{x}} \le \dfrac{1}{\sqrt{x}} \ \forall\ x \ge 0$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0} \int_0^1 \frac{1}{\sqrt{x}}\,dx$$

$$= \lim_{a\to 0} \left[2\sqrt{x}\right]_a^1$$

$$= \lim_{a\to 0} \left[2-a\right]$$

$$= 2$$

Since the limit is finite, the given improper integral is convergent and has the value $\pi/2$.

3. Show that $\displaystyle\int_{-\infty}^{\infty} \frac{|\sin x| + |\cos x|}{|x|+1}\,dx$ diverges

Ans:- since $|\sin x| + |\cos x| \ge \sin^2 x + \cos^2 x$

$$\sin^2 x + \cos^2 x = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{|x|+1}\,dx = \int_{-\infty}^{0} \frac{1}{|x|+1}\,dx + \int_0^{\infty} \frac{1}{|x|+1}\,dx$$

$$= \int_{\infty}^{1} \frac{du}{u} + \int_{-1}^{\infty} \frac{du}{u}$$

$$= [\ln u]_{-\infty}^{1} + [\ln u]_{1}^{\infty}$$

$$= -\ln|-\infty| + \ln\infty = 0$$

Since the limits on the right hand rule doesn't exist the given improper integral is divergent

hence the proof

4. use Newton's method to obtain a root correct to three decimal places for the equation $\sin x = 1 - x$

Ans:- $x_{n+1} = x_n - \frac{f(x_0)}{f'(x_0)}$

If $\lim_{n \to \infty} x_n = \bar{x}$ then $f(\bar{x}) = 0$

$f(x) = \sin x + x - 1$

$f'(x) = \cos x + 1$

Let us choose $x_0 = 1$ then the first approximation is computed by using Newton's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{\sin x + x - 1}{\cos x + 1}$$

$$= 1 - \frac{0.017}{1.999} = 0.991$$

$$x_2 = 0.991 - \frac{8.29 \times 10^{-3}}{1.999} = 0.986$$

$$x_3 = 0.986 - \frac{3.2 \times 10^{-3}}{1.999} = 0.984$$

$$x_4 = 0.984 - \frac{1.17 \times 10^{-3}}{1.999} = 0.983$$

$$x_5 = 0.983 - \frac{1.55 \times 10^{-4}}{1.999} = 0.982$$

$$x_6 = 0.982 - \frac{-8.61 \times 10^{4}}{1.999} = 0.982$$

Here the root of the given equation, lies b/w 0 and 1, correct to three decimal place is 0.982

5. Using the tabulated values of the integral evaluate $\int_0^1 x\sqrt{1-x^2}\,dx$ by Simpson's rule compare the result with exact value correct to 5 decimal places.

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1	—
$f(x)$	0	0.12402	0.254206	0.34763	0.43301	0.48784	0.49608	0.42361	0	—

Ans :- Hence $a=0$ and $b=1$ for $i=0,1,2,\ldots 10,$

let x_i denote given values of ~~f(x)~~ x and y_i denote the corresponding values of $f(x)$ ie, $y_i = f(x_i)$, for each $i=0,1,2,\ldots 10$. For convenience sake, let us make the following table of values of x_i & y_i:

x_i	0	0.125	.25	.375	.5	.625	.75	.875	1
y_i	0	0.12402	0.254206	.34763	.43301	.48784	.49608	.42361	0

Here the values of x are equally spaced and $h = \Delta x_i = .125$, then by Simpson's rule, we get,

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$$\int_0^1 f(x)\,dx = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + 2y_8\right]$$

$$= \frac{0.125}{3}\left[0 + (4 \times .12402) + (2 \times .24206) + (4 \times .34763) + (2 \times .43301) + (4 \times .48789) + (2 \times .49608) + (4 \times .42361) + 0\right]$$

$$= \frac{0.125}{3}\left[0.49608 + 0.48412 + 1.39052 + .86602 + 1.95156 + 0.99216 + 1.69444\right]$$

$$= \frac{0.125}{3}\left[7.8749\right]$$

$$= \frac{0.9843625}{3} = \underline{\underline{0.32812}}$$

The exacted value of the integral is

$$\int_0^1 x\sqrt{1-x^2}\,dx$$

$$= \int_1^0 -\frac{\sqrt{u}}{2}\,du$$

$$= \frac{-1}{2}\int_1^0 \sqrt{u}\,du$$

$$= \frac{-1}{2}\left[\frac{2u^{3/2}}{3}\right]_1^0 = \frac{-1}{2}\left[0 - \frac{2}{3}\right] = \frac{1}{3}$$

$$= 0.333$$

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