Automated Production Scheduling for Artificial Teeth Manufacturing

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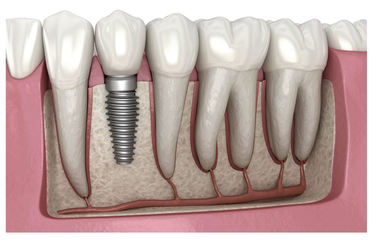
Problem Statement

Artficial Teeth Manufacturing



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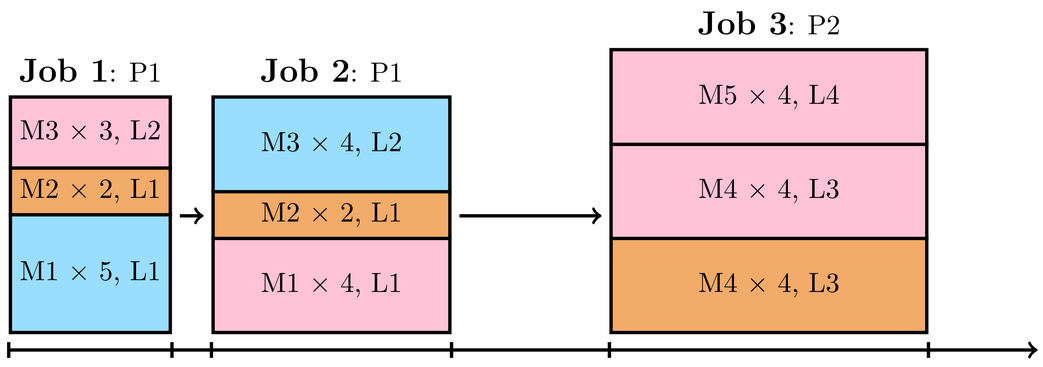
- **Task:** Create efficient single machine schedules
- **Aim**: Automated solutions method to assist human planners

Contribution

- We introduce a novel realistic scheduling problem
- Real-life benchmark instances publicly available
- Exact constraint modeling approach
- Construction Heuristic
- Local Search Approach using Simulated Annealing

The Artificial Teeth Scheduling Problem

Example Schedule



- **Moulds:** M1, M2, M3
- **Colors:** Orange, Pink, Blue
- **Product Lines**: L1, L2, L3, L4
- **Production Program**: P1, P2

Constraints

- **Number of moulds per job** $=$ number of program slots
- **Mould availability** must be considered
- The **number of colors/lines per job** must be $\leq$ the allowed maximum
- Job **moulds must be compatible** with the job's program
- A job cannot assign **incompatible colors** at the same time
- All **product demands** need to be fulfilled

Minimization Objectives

- Makespan
- Tardiness
- Waste

Constraint Modeling and Solution Approaches

Constraint Model

We modeled the problem with the MiniZinc language:

- High-level Constraint Programming (CP) Model
- We can use the model with state-of-the-art CP and integer programming (IP) solvers:
- We propose a linear (L) and a bilinear (B) variant

Constraints (Examples)

- The amount of available moulds must not be exceeded:

$$\sum_{c \in C} jm_{i,m,c} \leq a_m \quad \forall i \in J, m \in M$$

- The number of product types must not be larger than the allowed maximum:

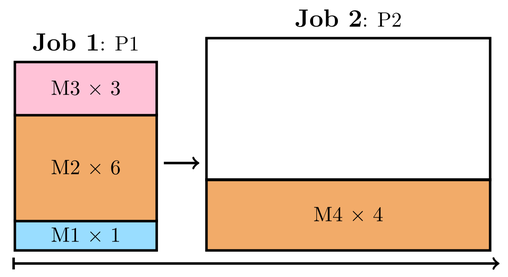
$$\sum_{c \in C} \sum_{l \in L} \left[\sum_{m \in M} ([l_m = l] jm_{i,m,c}) > 0 \right] \leq w \; \forall i \in J$$

- Moulds have to be compatible with the job's program:

$$(jp_i \neq p_m) \Rightarrow (jm_{i,m,c} = 0) \quad \forall i \in J, m \in M, c \in C$$

Construction Heuristic

Order demands by due date and greedily create jobs
Example: $D1 = (1 \times M1, Blue), D2 = (8 \times M4, Orange), \ldots$



Local Search Approach

7 neighborhood moves:

- Increase job length
- Reduce job length
- Change mould
- Delete job
- Add new job
- Swap mould
- Swap two jobs

Experimental Evaluation

Benchmark Instances

- 6 real-life instances
- 6 generated smaller instances
- Runtime Limit of 30 min per instance

Constraint Modeling Approach

- Evaluated solvers: Gecode (Gce), Chuffed (Chu), Gurobi (Grb), CPLEX (Cpx)

Local Search Approach

- Simulated annealing to escape local optima
- 10 repeated runs for each instance

Computational Results

- Best exact results (Exact) with IP solvers and bilinear model
- Optimal results for 5 instances
- Lower bounds (LB) for all instances
- Best result are shown in bold face
- Local search (LS) reaches optimal results for 4 instances and could solve all real-life instances

Inst.	Cpx L	Cpx B	Grb L	Grb B
I 1	2.96	4.66	2.54	**2.53**
I 2	2.01	3.24	**1.96**	2.01
I 3	**2.23**	**2.23**	**2.23**	**2.23**
I 4	**2.54**	**2.54**	**2.54**	**2.54**
I 5	2.11	2.12	2.11	**2.10**
I 6	**3.00**	**3.00**	**3.00**	**3.00**

Inst.	Gce L	Gce B	Chu L	Chu B
I 1	-	37.53	579.59	-
I 2	-	-	19.31	272.87
I 3	69.69	3.00	3.00	1123.83
I 4	37.87	37.87	99.09	619.50
I 5	43.85	45.11	10.32	-
I 6	**3.00**	**3.00**	**3.00**	**3.00**

Inst.	LB	Exact	LS
I 1	2.08	**2.53**	**2.53**
I 2	1.25	1.96	**1.94**
I 3	2.23	**2.23**	**2.23**
I 4	2.54	**2.54**	**2.54**
I 5	1.63	**2.10**	2.13
I 6	3.00	**3.00**	**3.00**
I 7	0.50	-	**2.98**
I 8	0.15	-	**2.39**
I 9	0.59	-	**2.97**
I 10	0.53	-	**2.78**
I 11	0.34	-	**2.76**
I 12	1.02	-	**2.89**

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