Adaptive Smoothing for Path Integral Control

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Summary

✓ We propose a model-free algorithm called ASPIC that smoothes the cost function by applying an **inf-convolution** aiming to speedup convergence of policy optimization

✓ ASPIC bridges two well-known policy optimization methods: **direct cost optimization** and the **cross-entropy method**

✓ We show that **intermediate levels of smoothing are optimal**

Path Integral Control

The optimization objective: **entropy-regularized expected cost**

$$C(p_u) = \langle V(\tau) \rangle_{p_u} + \gamma KL(p_u||p_0) \qquad \text{with } V(\tau) = \int_0^T V(x_t, t)dt$$

$V(\tau)$: state cost of trajectory $\tau = \{x_t | \forall t : 0 < t \leq T\}$

$p_0(\tau)$: **uncontrolled dynamics**

$p_u(\tau)$: induced probability over **controlled state trajectories**

Parameter γ controls the strength of the regularization $KL(p_u||p_0)$

(Approach I) → Direct-Cost Optimization

Gradient descent over parametrized path probabilities p_{u_θ}

$$\nabla_\theta C(p_{u_\theta}) = \left\langle S^\gamma_{p_{u_\theta}}(\tau) \nabla_\theta \log p_{u_\theta}(\tau) \right\rangle_{p_{u_\theta}}$$

with the stochastic cost $S^\gamma_{p_{u_\theta}}(\tau) := V(\tau) + \gamma \log \frac{p_{u_\theta}(\tau)}{p_0(\tau)}$

(Approach II) → Cross-Entropy Method

KL divergence minimization has analytical solution

$$p^* := \arg\min_p C(p) = \frac{1}{Z}\, p_0(\tau) \exp\left(-\frac{1}{\gamma} V(\tau)\right), \quad \text{with } Z = \left\langle \exp\left(-\frac{1}{\gamma} V(\tau)\right) \right\rangle_{p_0}$$

The Cross-Entropy method results in the gradient

$$\nabla_\theta KL\left(p^* || p_{u_\theta}\right) = \frac{1}{Z_{p_{u_\theta}}} \left\langle \exp\left(-\frac{1}{\gamma} S^\gamma_{p_{u_\theta}}(\tau)\right) \nabla_\theta \log p_{u_\theta}(\tau) \right\rangle_{p_{u_\theta}}$$

Approach I and Approach II share the same global minima
Which approach is better?

Smoothing Stochastic Control Problems

Inf-Convolution applied the original cost

✓ The smoothed cost $J^\alpha(\theta)$ (also known as **Moreau envelope** function)

$$J^\alpha(\theta) = \inf_{\theta'} \alpha KL(p_{u_{\theta'}}||p_{u_\theta}) + \underbrace{\gamma KL(p_{u_{\theta'}}||p_0) + \langle V(\tau) \rangle_{p_{u_{\theta'}}}}_{C(p_{u_{\theta'}})}.$$

– Parameter α penalizes the deviation of $p_{u_{\theta'}}$ from p_{u_θ}

– Parameter γ penalizes the deviation of $p_{u_{\theta'}}$ from the uncontrolled dynamics p_0

✓ **Preserves global minima**

Smoothing bridges Direct-Cost Optimization and the Cross-Entropy Method

Smoothing has analytical solution for path integral control

The smoothed cost becomes

$$J^\alpha(\theta) = -(\gamma + \alpha) \log \left\langle \exp\left(-\frac{1}{\gamma + \alpha} S^\gamma_{p_{u_\theta}}(\tau)\right) \right\rangle_{p_{u_\theta}}$$

and its gradient

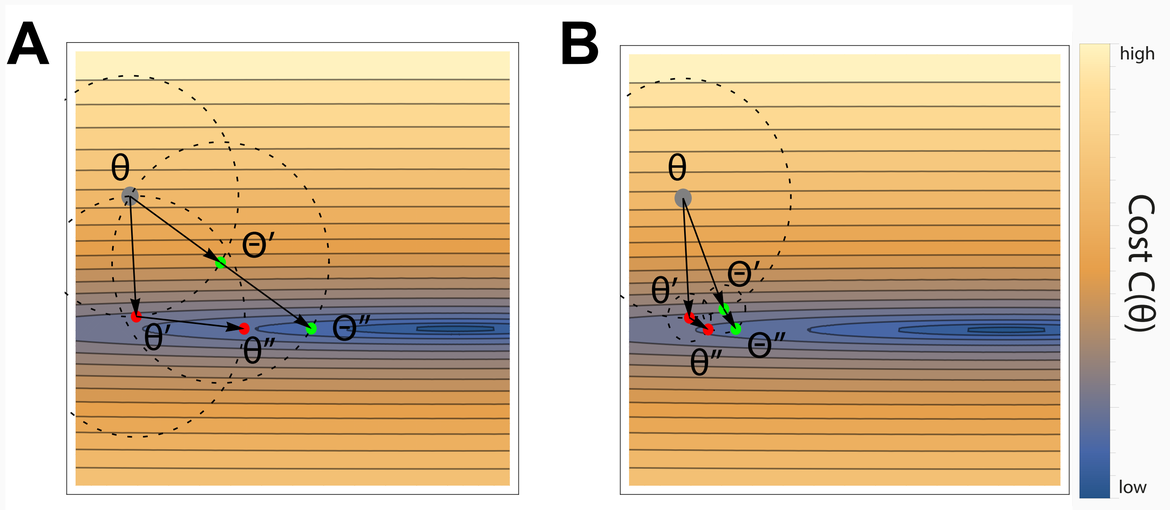
$$\nabla_\theta J^\alpha(\theta) = -\frac{\alpha}{Z^\alpha_{p_{u_\theta}}} \left\langle \exp\left(-\frac{1}{\gamma + \alpha} S^\gamma_{p_{u_\theta}}(\tau)\right) \nabla_\theta \log p_{u_\theta}(\tau) \right\rangle_{p_{u_\theta}}$$

$\alpha \to \infty$ (**weak** smoothing) recovers **Direct-Cost Optimization**
$\alpha \to 0$ (**strong** smoothing) recovers **Cross-Entropy Method**

Smoothing Accelerates Policy Optimization

A two-step task with 2D cost landscape $C(\theta)$ parametrized by θ

Theorem: a smoothed update $\Theta' = \Theta^{J^\alpha}_{\mathcal{E}}(\theta)$ with stepsize $\mathcal{E}$ followed by a direct update $\Theta'' = \Theta^{C}_{\mathcal{E}'}(\Theta')$ with stepsize $\mathcal{E}'$ is optimal, for all $\mathcal{E}$, α.



✓ (B) When the size of the second step becomes small $\mathcal{E}' \ll \mathcal{E}$, the smoothed update $\theta \to \Theta'$ becomes more similar to the direct update $\theta \to \theta'$.

The ASPIC Algorithm

▷ **Adapts the smoothing parmeter** α to keep the variance of the gradient estimator at a predefined level, independently of the number of samples N per iteration

▷ **Performs natural gradient updates** $\theta_{n+1} = \theta_n - \beta^{-1} F^{-1} \nabla_{\theta'} J^\alpha(\theta')|_{\theta'=\theta_n}$

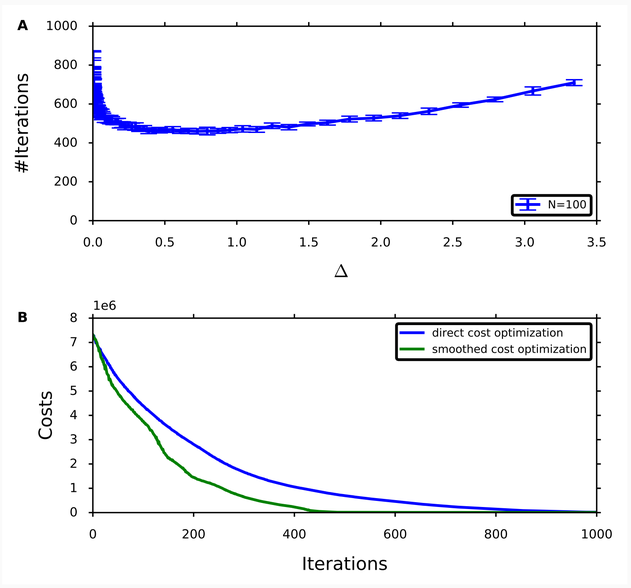
✓ Two hyper-parameters:

– The smoothing strength Δ (a bound on the variance)

– The size of the trust regions $\mathcal{E}$

Experiments

▷ Linear-Quadratic control problems ("exact" policies)

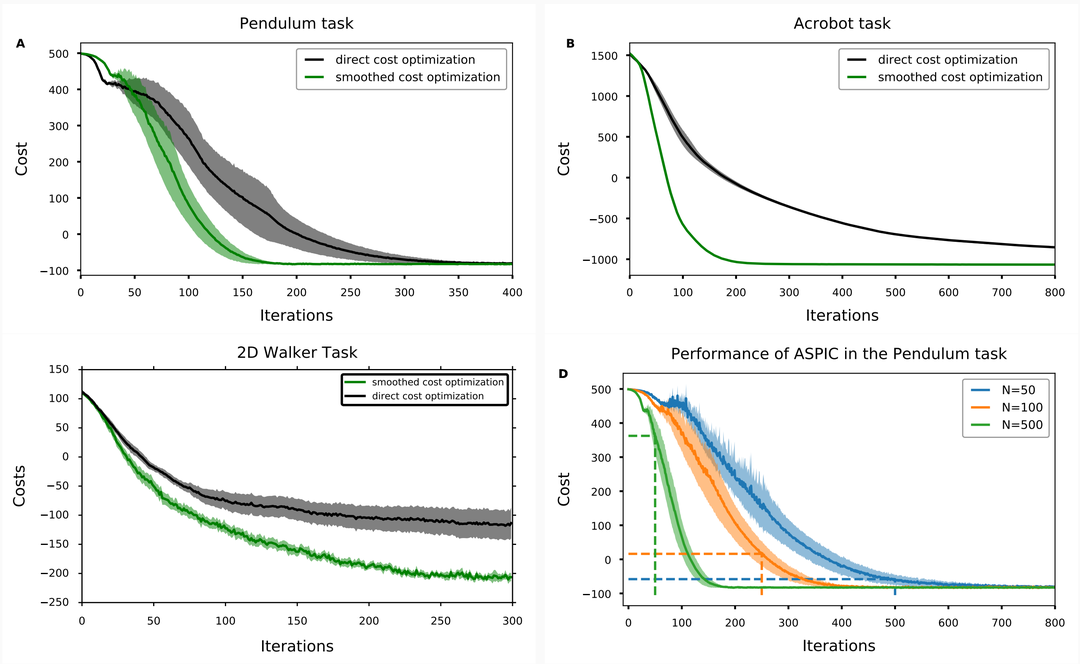


✓ Smoothing ($\Delta > 0$) accelerates

✓ For too large Δ : gradient estimation is not reliable

✓ Optimal smoothing at intermediate values of $\Delta \approx 0.4$

▷ Nonlinear Control Problems ("approximated" policies)



✓ A strong performance boost can also be achieved

✓ Scales well to complex problems using neural network policies

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