Lecture 2: Divide&Conquer Paradigm, Merge sort and Quicksort

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Outline

1. Divide and Conquer
2. Merge sort
3. Quick sort

In-Class Quizzes

- **URL:** `http://m.socrative.com/`
- **Room Name: 4f2bb99e**

Divide And Conquer Paradigm

- D&C is a popular algorithmic technique
- Lots of applications
- Consists of three steps:
  1. **Divide** the problem into a number of sub-problems
  2. **Conquer** the sub-problems by solving them *recursively*
  3. **Combine** the solutions to sub-problems into solution for original problem

Divide And Conquer Paradigm

When can you use it?

- The sub-problems are easier to solve than original problem
- The number of sub-problems is **small**
- Solution to original problem can be obtained easily, once the sub-problems are solved

Recursion and Recurrences

- Typically, D&C algorithms use **recursion** as it makes coding simpler
- Non-recursive variants can be designed, but are often slower
- If all sub-problems are of equal size, can be analyzed by the recurrence equation

$$T(n) = aT(\frac{n}{b}) + D(n) + C(n)$$

- a: number of sub-problems to solve
- b: how fast the problem size shrinks
- $D(n)$: time complexity for the divide step
- $C(n)$: time complexity for the combine step

D&C Approach to Sorting

How to use D&C in Sorting?

- Partition the array into sub-groups
- Sort each sub-group recursively
- Combine sorted sub-groups if needed

Why study Merge Sort?

- One of the simplest and efficient sorting algorithms
- Time complexity is $\Theta(n \log n)$ (vast improvement over Bubble, Selection and Insertion sorts)
- Transparent application of D&C paradigm
- Good showcase for time complexity analysis

Merge Sort

High Level Idea :

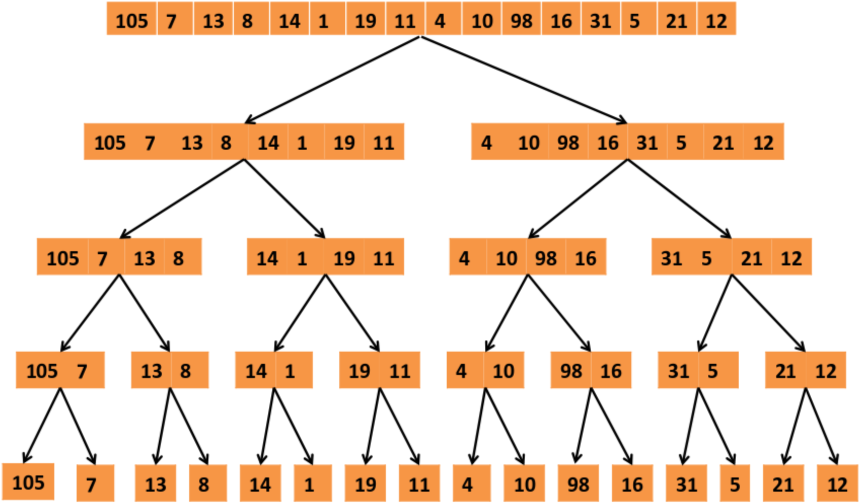
- Divide the array into two **equal** partitions - left and right (ignore edge cases for now)
- Sort left partition recursively
- Sort right partition recursively
- Merge the two sorted partitions into the output array

Merge Sort Pseudocode

Pseudocode:

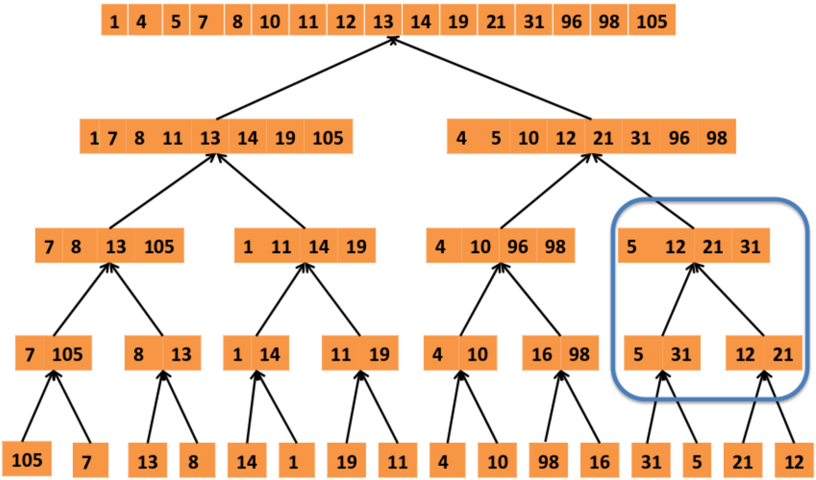
```
MergeSort(A, p, r):
    if p < r:
        q = (p+r)/2
        Mergesort(A, p , q)
        Mergesort(A, q+1, r)
        Merge(A, p, q, r)
```

Merge Sort - Divide[1]



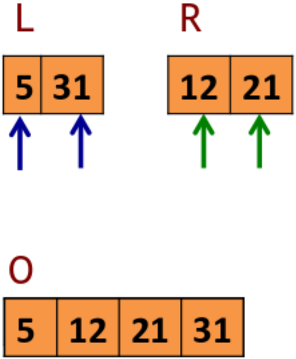
[1] http://web.stanford.edu/class/cs161/slides/0623_mergesort.pdf

Merge Sort - Combine[2]



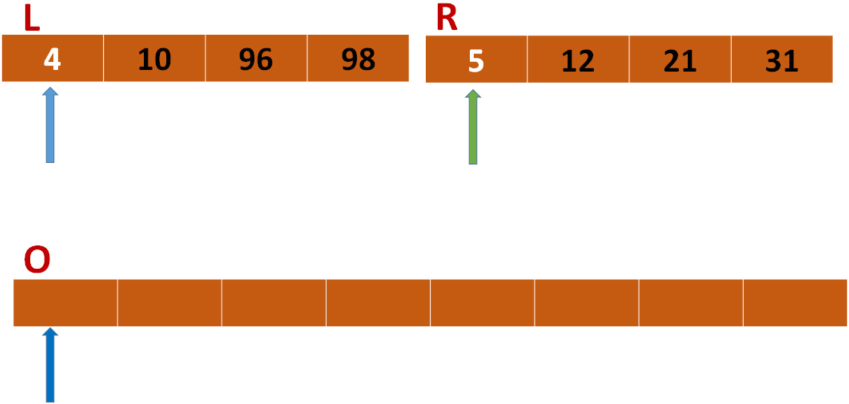
[2] http://web.stanford.edu/class/cs161/slides/0623_mergesort.pdf

Merging Two Sorted Lists[3]

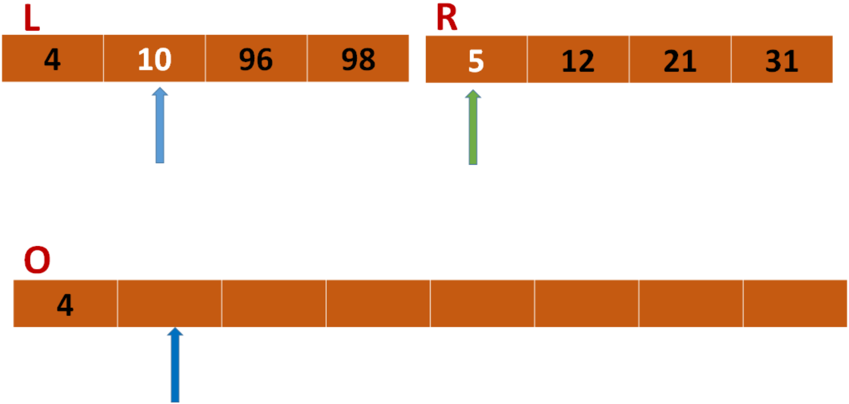


[3] http://web.stanford.edu/class/cs161/slides/0623_mergesort.pdf

Merging Two Sorted Lists



Merging Two Sorted Lists



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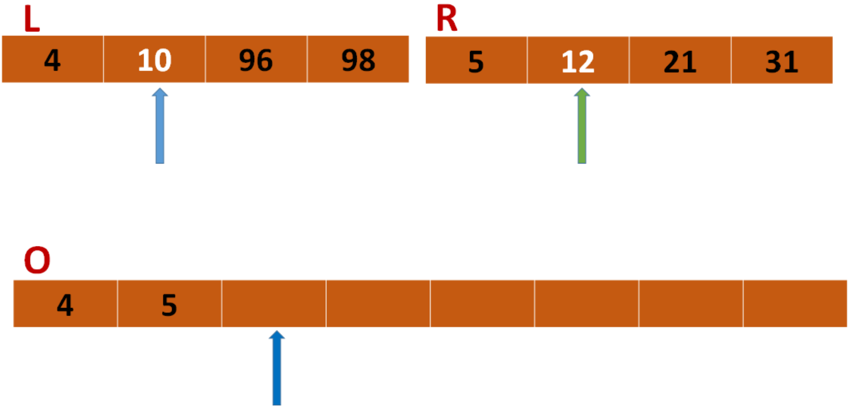

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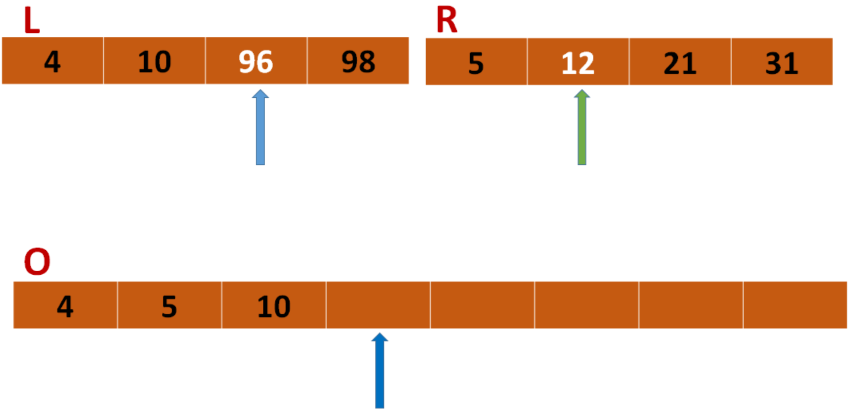

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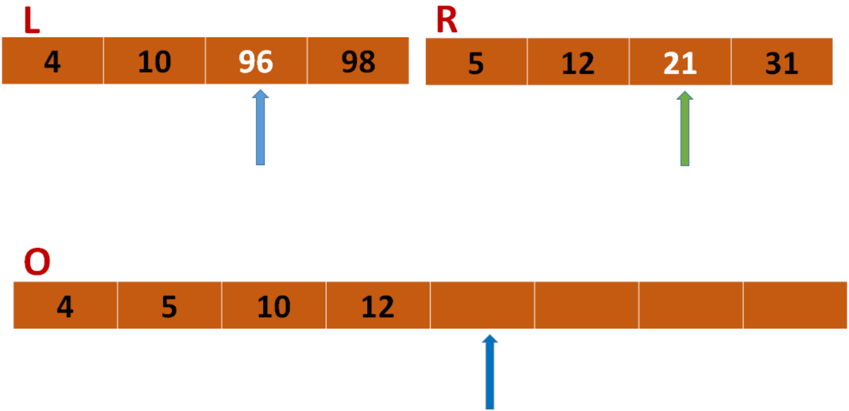

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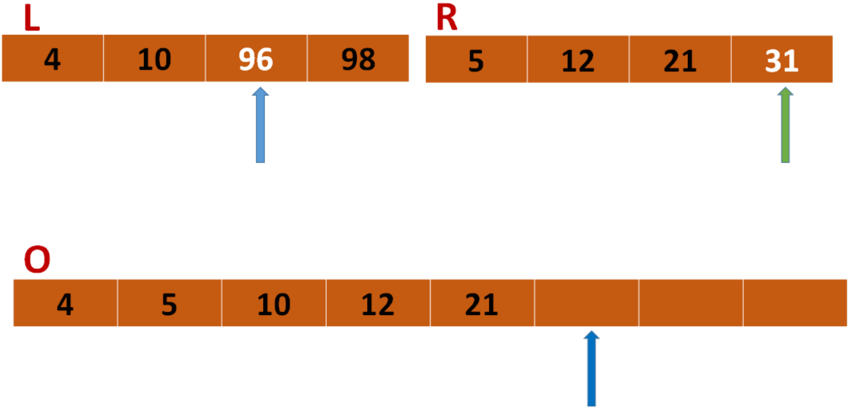

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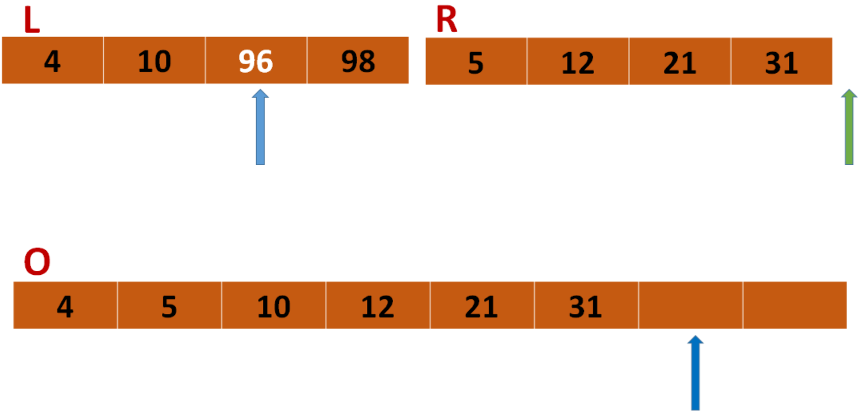

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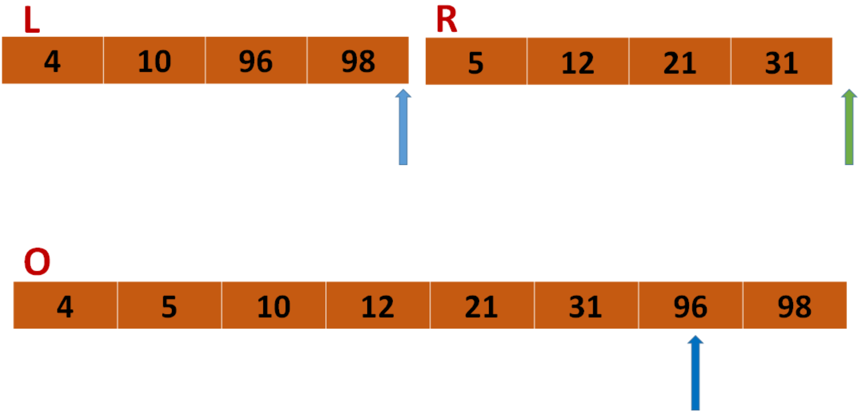


Merging Two Sorted Lists

Merge Pseudocode:

```
Merge(A,B,C):
    i = j = 1
    for k = 1 to n:
        if A[i] < B[j]:
            C[k] = A[i]
            i = i + 1
        else: (A[i] > B[j])
            C[k] = B[j]
            j = j + 1
```

Analyzing Merge Sort: Master Method

Quiz!

- General recurrence formula for D&C is $T(n) = aT(\frac{n}{b}) + D(n) + C(n)$
- What is a?
- What is b?
- What is $D(n)$?
- What is $C(n)$?

Quiz!

- General recurrence formula for D&C is

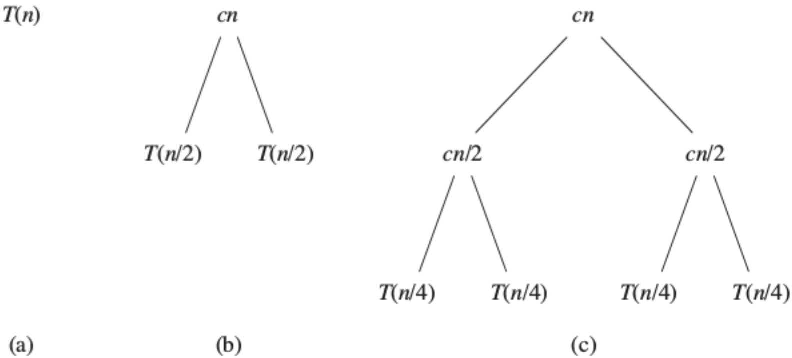
$$T(n) = aT(\frac{n}{b}) + D(n) + C(n)$$

- $a = 2,\ b = 2$
- $D(n) = O(1)$
- $C(n) = O(n)$
- Combining, we get:

$$T(n) = 2T(\frac{n}{2}) + O(n)$$

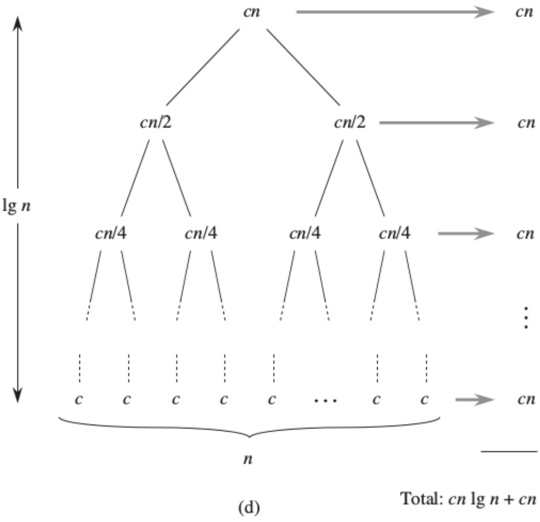
- Using Master method, we get $T(n) = O(n \log n)$
- If you are picky, $T(n) = T(\lceil \frac{n}{2} \rceil) + T(\lfloor \frac{n}{2} \rfloor) + O(n)$

Analyzing Merge Sort: Recursion Tree[4]



[4] CLRS Book

Analyzing Merge Sort: Recursion Tree[5]



[5]CLRS Book

Merge Sort Vs Insertion Sort

- Merge Sort is very fast in general ($O(n \log n)$) than Insertion sort ($O(n^2)$)
- For "nearly" sorted arrays, Insertion sort is faster
- Merge sort has $\Theta(n \log n)$ (i.e. both best and worst case complexity is $n \log n$
- Overhead: recursive calls and extra space for copying
- Insertion sort is in-place and adaptive
- Merge sort is easily parallizable