Lecture 8: Dictionaries and Hash Tables

Instructor: Saravanan Thirumuruganathan

Outline

1. Dictionaries
2. Hashing
3. Hash Tables
4. Briefly, DHTs and Bloom filters

In-Class Quizzes

- **URL:** `http://m.socrative.com/`
- **Room Name: 4f2bb99e**

Dictionary ADT

- Stores key-value pairs
- Required Operations:
  - Insert
  - Search (Membership check)
  - Delete

Motivation - I

Caller ID Implementation:

- **Objective:** Given phone number, output Caller's name
- Assume we need to worry about callers from Arlington only
- What is the universe/input space?

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- What is the universe/input space?
  - Ignore first three digits (why?)
  - Last 7 digits can input numbers between 0 to $10^7 - 1$
  - Number of phone numbers in Arlington way less than $10^7 - 1$

Motivation - II

Student ID Lookup:

- **Objective:** Given student id, retrieve student information
- Example: UTA graduate school, TA of this course
- What is the universe/input space?

Motivation - II

Student ID Lookup:

- **Objective:** Given student id, retrieve student information
- Example: UTA graduate school, TA of this course
- What is the universe/input space?
  - Ignore four digits (why?)
  - Last 6 digits can input numbers between 0 to $10^6 - 1$
  - Number of students in UTA/5311 is way less than 10^6

Potential Implementations

Possible Candidates:

Potential Implementations

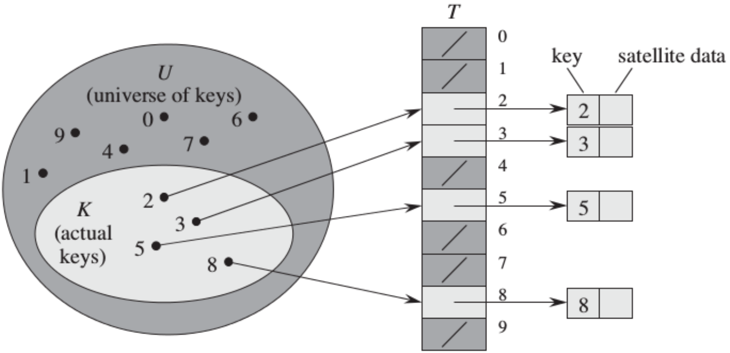
Possible Candidates:

- Linked List based
- Array based
- Balanced trees

Space Vs Time Tradeoff

- All our previous implementations optimized for time given linear storage cost
- What if time is more important than space?
- Think of companies like Google, Facebook, Amazon, AT&T etc

Direct Address Tables[1]



[1]CLRS Fig 11.1

Direct Address Tables

```
DAT-Search(T,k):
    return T[k]

DAT-Insert(T,x):
    T[x.key] = x

DAT-Delete(T,x):
    T[x.key] = NULL
```

Direct Address Tables

- Represent input in an array
- Each position/slot corresponds to a key in universe U
- Works well when U is small
- Pro:

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- Each position/slot corresponds to a key in universe U
- Works well when U is small
- Pro: Fast
- Con: Lot of space is wasted

Ideas to Improve DAT

- Let size of universe be N
- Let Space budget be m (for eg, $c \cdot \#$ max elements)

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- Let $\#$ elements inserted be n
- Caller ID Eg: $10^7 - 1$ vs $400K$ (size of Arlington)
- Student ID Eg: 10^6 Vs 8000
- 5311 Eg: 10^6 vs 50

Ideas to Improve DAT

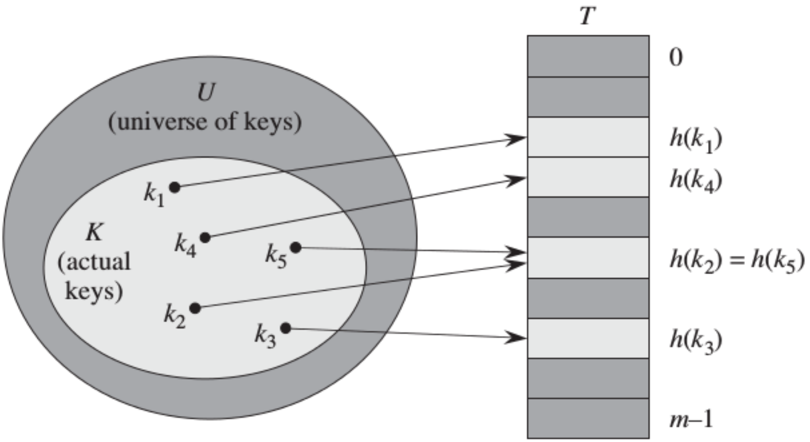
- Let size of universe be N
- Let Space budget be m (for eg, $c \cdot \#$ max elements)
- Let $\#$ elements inserted be n
- Caller ID Eg: $10^7 - 1$ vs $400K$ (size of Arlington)
- Student ID Eg: 10^6 Vs 8000
- 5311 Eg: 10^6 vs 50
- **Insight:** Try to have space proportional to m instead of N

Hash Tables

Hash Functions

- **Hash Function** h**:** Compute an array index from key value
- **Input:** $1..N$
- **Output:** $0..m-1$
- Formally, $h : U \rightarrow \{0, 1, \ldots, m-1\}$
- **Requirement:** (Ideal): Uniformly scramble elements across array
  - Efficient to compute (so peeking into array)
  - Each array position is uniformly likely

Hash Table[2]



[2] CLRS Fig 11.2

Hash Function Design: Student ID Example

- Space budget is $m = 100$ (array with 100 slots)
  - Objective: Design hash function $h(student\ id) \in \{0, 1, \ldots, 99\}$

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  - Last two digits of Student ID
  - Student ID be $h(1000 - 000 - 188) \Rightarrow 88$
  - Any two students with last two digits 88?
- Space budget is $m = 1000$ (array with 1000 slots)

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Hash Function Design: Student ID Example

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  - Objective: Design hash function $h(student\ id) \in \{0, 1, \ldots, 99\}$
  - Last two digits of Student ID
  - Student ID be $h(1000 - 000 - 188) \Rightarrow 88$
  - Any two students with last two digits 88?
- Space budget is $m = 1000$ (array with 1000 slots)
  - Objective: Design function $h(student\ id) \in \{0, 1, \ldots, 999\}$
  - Last three digits of Student ID
  - Student ID be $h(1000 - 000 - 188) \Rightarrow 188$
  - Any two students with last two digits 188?
- Tradeoff between Space and Collisions

Good and Bad Hash Functions

- 10-digit phone numbers
  - First three digits:

Good and Bad Hash Functions

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  - First three digits: Bad! (why?)
  - Last three digits:

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- 10-digit UTA student id
  - First three digits: Bad! (why?)
  - Last three digits: Better (why?)
- 9-digit SSN
  - First three digits: Bad! (why?)
  - Last three digits: Better (why?)

Hash Function Design

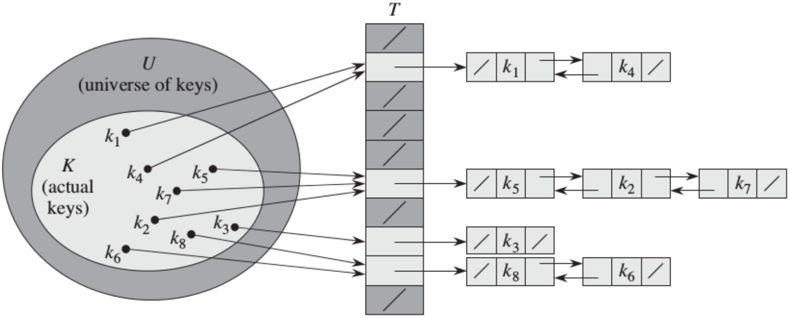
- **Division/Modular:** $h(k) = k \mod m$
  - Alternative: Mod by a prime P
  - Java Strings: $P = 31$
- **Multiplication:** $h(k) = \lfloor m(kA \mod 1) \rfloor$
  - $0 < A < 1$
  - Take the fractional part and multiply it by m
- Universal hashing

Collisions

- When two items are hashed to same slot $h(k_i) = h(k_j)$
- Collision Resolution Techniques
  - Separate Chaining
  - Open Addressing: Linear probing, Quadratic probing, Double Hashing

Separate Chaining[3]

- Idea: Place all elements that hash to same slot in a linked list



[3] CLRS Fig 11.3

Separate Chaining

```
Chained-Hash-Insert(T,x):
    Insert x at head of linked list T[h(x.key)]

Chained-Hash-Search(T,k):
    Search for element with key k in T[h(k)]

Chained-Hash-Delete(T,x):
    Delete x from linked list T[h(x.key)]
```

Open Addressing

- Separate Chaining used an external data structure to store all elements that collide
- Open Addressing
  - Do not use external storage (one element per slot)
  - Use hash table itself to store elements that collide
  - When a new key collides, find an empty slot and put it there
- Handling deletions is very messy - we will not discuss it here

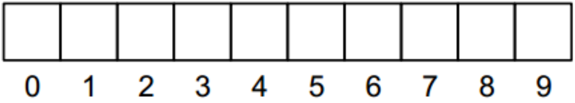
Linear Probing

Linear Probing:

- Using hash function, map key to an array index (say i)
- Put element at slot i if it is free
- If not try $i+1$, $i+2$, etc
- Roll around to start if needed

Linear Probing: Example[4]

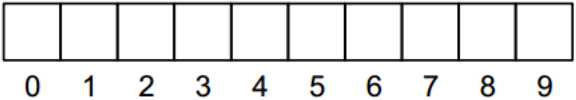
- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Resolve collisions via linear probing
- Hash function $h(k) = k\%10$ (i.e. take last digit)



[4] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[5]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$

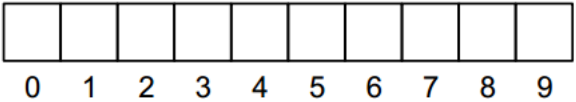


- First three elements have no collisions

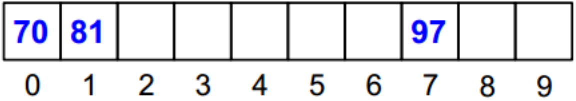
[5] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[5]

- Objective: Insert elements $\langle {\color{red}81, 70, 97}, 60, 51, 38, 89, 68, 24 \rangle$



- First three elements have no collisions



[5] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[6]

- Objective: Insert elements $\langle 81, 70, 97, \textcolor{red}{60}, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 60

70	**81**						**97**		
0	1	2	3	4	5	6	7	8	9

[6] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[6]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 60

70	**81**						**97**		
0	1	2	3	4	5	6	7	8	9

- Check slot 1 - it is full
- Check slot 2 - it is empty, so insert it

70	81	**60**					97		
0	1	2	3	4	5	6	7	8	9

[6] `https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf`

Linear Probing: Example[7]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 51

70	81	**60**					97		
0	1	2	3	4	5	6	7	8	9

[7] `https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf`

Linear Probing: Example[7]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 51

0	1	2	3	4	5	6	7	8	9
70	81	**60**					97		

- Check slot 2 - it is full
- Check slot 3 - it is empty, so insert it

0	1	2	3	4	5	6	7	8	9
70	81	60	**51**				97		

[7] `https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf`

Linear Probing: Example[8]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- No collisions when inserting 38 and 89

70	81	60	51				97	**38**	**89**
0	1	2	3	4	5	6	7	8	9

[8] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[9]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 68

70	81	60	51				97	**38**	**89**
0	1	2	3	4	5	6	7	8	9

[9] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[9]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 68

0	1	2	3	4	5	6	7	8	9
70	81	60	51				97	**38**	**89**

- Check slot 9 - it is full

[9] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[9]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 68

70	81	60	51				97	**38**	**89**
0	1	2	3	4	5	6	7	8	9

- Check slot 9 - it is full
- Wrap around: Check slots $0, 1, 2, 3$
- Insert 68 in slot 4

70	81	60	51	**68**			97	38	89
0	1	2	3	4	5	6	7	8	9

[9] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[10]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 68

0	1	2	3	4	5	6	7	8	9
70	81	60	51	68			97	38	89

[10] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[10]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 68

70	81	60	51	68			97	38	89
0	1	2	3	4	5	6	7	8	9

- Check slot 4 - it is full

[10] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[10]

- Objective: Insert elements $\langle 81, 70, 97, 60, 51, 38, 89, 68, 24 \rangle$
- Collision when inserting 68

70	81	60	51	**68**			97	38	89
0	1	2	3	4	5	6	7	8	9

- Check slot 4 - it is full
- Insert 24 in slot 5

70	81	60	51	68	**24**		97	38	89
0	1	2	3	4	5	6	7	8	9

[10] https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf

Linear Probing: Example[11]

Searching with Linear Probing:

70	81	60	51	68	24		97	38	89
0	1	2	3	4	5	6	7	8	9

- Easy Case: Search(T, 81)
- Harder Case I: Search(T, 60)
- Harder Case II: Search(T, 68)
- Harder Case III: Search(T, 80)

[11] `https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf`

Linear Probing Issues: Primary Clustering[12]

- Remember the scenario of inserting 68

70	81	60	51				97	38	89
0	1	2	3	4	5	6	7	8	9

- Had to travel long to find next empty slot
- Once collision happens, new keys are more likely to hash in middle of blocks
- So you have to spend more time to find an empty slot (extending the block size)
- You now increased the chance of a collision in the block!

[12] `https://ece.uwaterloo.ca/~cmoreno/ece250/2012-01-30--hash_tables.pdf`

Fixing Primary Clustering

- Idea: Look for empty slots increasingly further away from original slot
- Probe Sequence: The order in which successive slots are checked
- Linear Probing: $h(k, i) = h'(k) + i$
- Probe sequence for Linear Probing:

$$h'(k), h'(k) + 1, h'(k) + 2, \ldots$$

Fixing Primary Clustering

- Probe sequence for LP: $h'(k), h'(k)+1, h'(k)+2, \ldots$
- Quadratic probing: $h(k,i) = h'(k) + c_1 i + c_2 i^2$
  - Probe sequence when $c_1 = 0, c_2 = 1$,
    $$h'(k), h'(k) + 1^2, h'(k) + 2^2, \ldots$$
- Double Hashing
  - Choose two hash functions h_1 and h_2
  - Use h_1 first
  - If no collision, all is well
  - Else use the probing sequence
    $$h(k,i) = (h_1(k) + i \cdot h_2(k)) \mod m$$
- Search: Follow same procedure till you find the element or an empty slot

Hash Tables: Practical Advice I

- Load factor $\alpha = n/m$ (#elements / #table size)
- Low α: wasted space
- High α: long time for insert and search
- If you know n, pass it to Hash table (e.g. Java, Python)
- The data structure will be much faster
- For eg, most languages will set $m = \frac{4}{3}n$ (with $\alpha = \frac{3}{4}$)
- **Re-hashing:** Automatically adjusting number of budgets
- If α becomes too low or high, **re-hashing** happens (it is bad!)

Hash Tables: Practical Advice II

- Load factor $\alpha = n/m$ (#elements / #table size)
- Chaining can be used when $\alpha <$ 0.9
- Linear probing is used when table is sparse (α 0.5)
- Double hashing is used when $\alpha < 0.66$
- With **good** hash functions, Hash table outperforms BST, RBT etc
- Double hashing: $h_2(k)$ can never be 0 (else you get infinite loop)

Distributed Hash Tables (DHTs)

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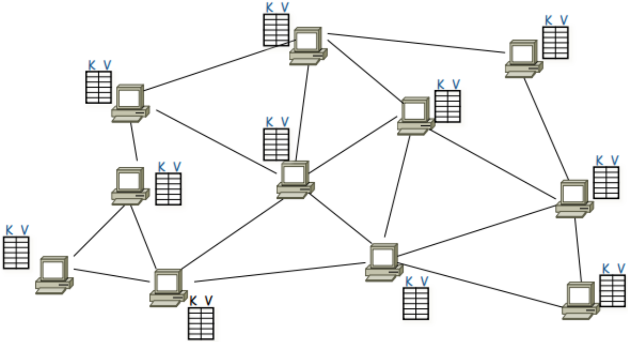
- **Idea:** Distribute the hash table content across many machines (typically a P2P network)
- **Motivation:**
  - Scalability: Eg. CDNs, NoSQL DBs
  - Fault Tolerance: Eg. Robust data archiving
  - Decentralization: Eg. BitTorrent
- **Issue:** We now have to determine which node to store data too!

Distributed Hash Tables (DHTs)

Applications:

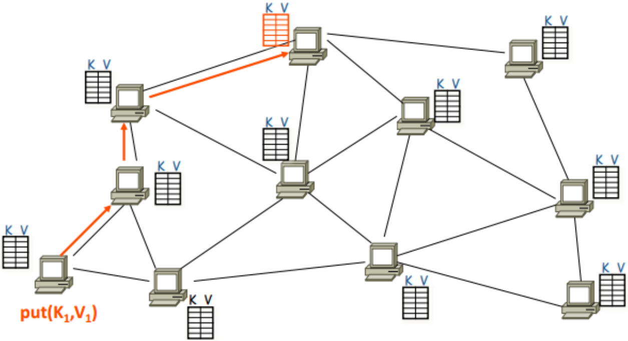
- Any internet scale application would have to use DHT
- Domain Name Service (hierarchical)
- File Sharing and Caching
- Archival/Retrieval of content (Eg. Dropbox: Deduplication)
- BitTorrent and other **trackerless** sharing sites
- Load balancing
- Anonymous web browsing
- Serverless email systems

DHTs: Visualization[13]



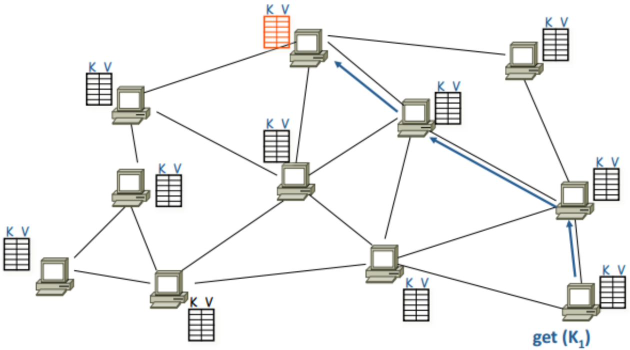
[13] http://www.cs.princeton.edu/courses/archive/spr09/cos461/docs/lec18-dhts.pdf

DHTs: Put[14]



[14] http://www.cs.princeton.edu/courses/archive/spr09/cos461/docs/lec18-dhts.pdf

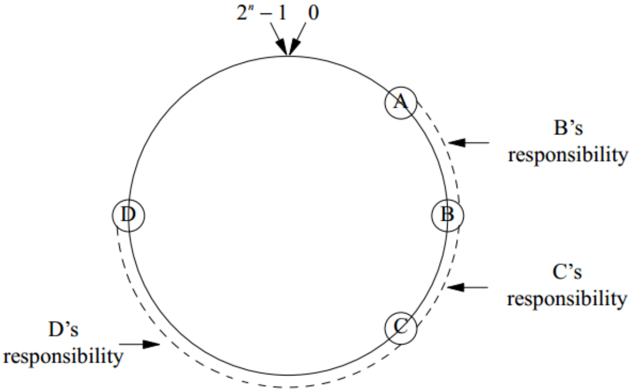
DHTs: Get[15]



15 http://www.cs.princeton.edu/courses/archive/spr09/cos461/docs/lec18-dhts.pdf

DHTs: Sample Implementation[16]

Chord Ring: Routing, Joining, Replication



[16] http://www.ietf.org/old/2009/proceedings/06mar/slides/plenaryt-2.pdf

DHTs: Consistent Hashing

- Consistent Hashing function: Special type of hashing function
- When m (#slots) or n (#item) is changed, at most $O(\lg n)$ items have to be moved
- Great idea for P2P systems with node arrival and removal

- GiG

neo@zionReloaded: ~/.config/google-chrome

```
neo@zionReloaded:~/.config/google-chrome$ ls -lh *Bloom*
-rw-rw-r-- 1 neo neo 5.5M Oct  3 01:43 Safe Browsing Bloom
-rw-rw-r-- 1 neo neo 1.3M Oct  3 01:43 Safe Browsing Bloom Prefix Set
neo@zionReloaded:~/.config/google-chrome$ 
```

- GiG

- GiG

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Summary

Major Concepts:

- Dictionary ADT
- Hash Tables
- Hashing, Collision Resolution
- DHTs, Bloom Filters