

Python for finance and optimization
Homework 3: Option pricing

We consider a classical Black and Scholes model

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

and recall that the pricing PDE associated with a European payoff $g(S_T)$ is:

$$\partial_t P(t, S) - rP(t, S) + rS\partial_S P(t, S) + \frac{1}{2}\sigma^2 S^2 \partial_{SS}^2 P(t, S) = 0, \quad P(T, S) = g(S).$$

Propose and code an implicit scheme and a Crank-Nicolson scheme to price each of the following products:

- a call-spread – long leg: at-the-money call with 3-year maturity ; short leg: call of maturity equal to 3 years with moneyness 150%.
- a 10-year maturity put option with moneyness 80%.

We shall assume $S_0 = 10$, $r = 2\%$ and $\sigma = 25\%$.

Remark: In the Python notebook, please use Markdown cells to explain how you designed the schemes and the rationale for the boundary conditions you used.