teórico Parcial 2

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25 Verificar cuadratura de Laguerre 2 puntos

a) hallar polinomio de Laguerre de orden 2 : $L_2(x) = \frac{1}{2}(x^2 - 4x + 2)$

Los polinomios de Laguerre están definidos por:

$$L_n(x) = \frac{e^x}{n!} \cdot \frac{d^n}{dx^n}(e^{-x}x^n)$$

Por lo que el polinomio 2 sería $L_2(x) = \frac{e^x}{2!} \cdot \frac{d^2}{dx^2}(e^{-x}x^2)$

Realizando la derivada $s \cdot p' + p s'$

$$\frac{d}{dx}(e^{-x}x^2) = -x^2e^{-x} + e^{-x} \cdot 2x$$

$$= -x^2e^{-x} + 2xe^{-x}$$

$$\frac{d^2}{dx^2}(e^{-x}x^2) = \frac{d}{dx}(-x^2e^{-x} + 2xe^{-x})$$

$$= (e^{-x}x^2 + 2xe^{-x} + (-e^{-x}2x + 2e^{-x})$$

multiplicando por $\frac{e^x}{2!}$

$$L_2(x) = \frac{\cancel{e^x}}{2!}(\cancel{e^{-x}}x^2 + 2x\cancel{e^{-x}} + (-\cancel{e^{-x}}2x + 2\cancel{e^{-x}})$$

$$\boxed{L_2(x) = \frac{1}{2}(x^2 + 4x + 2)}$$

b) hallando raíces del polinomio

igualando a cero:

$$\frac{1}{2}(x^2 + 4x + 2) = 0$$

$$x^2 + 4x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{16 - 4(1)(2)}}{2(1)}$$

$$x_1 = \frac{4 + \sqrt{8}}{2} = \boxed{2 + \sqrt{2}} \quad x_1$$

$$x_2 = \frac{4 - \sqrt{8}}{2} = \boxed{2 - \sqrt{2}} \quad x_2$$

c) hallar pesos de cuadratura

* Para w_1

$$w_1 = \int_0^{\infty} e^{-x}\left(\frac{x - x_2}{x_1 - x_2}\right) dx$$

Reemplazando raices x_1 y x_2

$$w_1 = \int_0^{\infty} e^{-x}\left(\frac{x - (2-\sqrt{2})}{(2+\sqrt{2}) - (2-\sqrt{2})}\right) dx$$

$$= \int_0^{\infty} e^{-x} \frac{x - 2 + \sqrt{2}}{\cancel{2} + \sqrt{2} - \cancel{2} + \sqrt{2}} dx$$

$$= \int_0^{\infty} e^{-x} \frac{x - 2 + \sqrt{2}}{2\sqrt{2}} dx$$

$$= \frac{1}{2\sqrt{2}} \int_0^{\infty} e^{-x} (x - 2 + \sqrt{2}) dx$$

$$= \frac{1}{2\sqrt{2}} \underbrace{\int_0^{\infty} e^{-x} x \, dx}_{\text{por partes}} - 2\int_0^{\infty} e^{-x} dx + \int_0^{\infty} \sqrt{2} e^{-x} dx$$

$$= \frac{1}{2\sqrt{2}} \cdot \left((x e^{-x}) - 2(e^{-x}) + \sqrt{2}(e^{-x})\right)\Big|_0^{\infty}$$

$$= \frac{e^{-x}}{2\sqrt{2}} \left(x - 2 + \sqrt{2}\right)\Big|_0^{\infty}$$

$$= \left(\frac{-e^{0}(0 - 2 + \sqrt{2})}{2\sqrt{2}}\right)$$

$$\boxed{\begin{aligned} w_1 &= \sqrt{2} - 1/2^{3/2} \\ w_1 &\approx 0.1464 \end{aligned}}$$

* Para W_2

$$W_2 = \int_0^{\infty} e^{-x}\left(\frac{x - x_1}{x_2 - x_1}\right)dx$$

Reemplazando raices x_1 y x_2

$$W_1 = \int_0^{\infty} e^{-x}\left(\frac{x-(2+\sqrt{2})}{(2-\sqrt{2})-(2+\sqrt{2})}\right)dx$$

$$= \int_0^{\infty} e^{-x}\left(\frac{x-2-\sqrt{2}}{\cancel{2}-\sqrt{2}-\cancel{2}-\sqrt{2}}\right)dx$$

$$= \int_0^{\infty} e^{-x}\left(\frac{x-2-\sqrt{2}}{-2\sqrt{2}}\right)dx$$

$$= \frac{1}{-2\sqrt{2}}\int_0^{\infty} e^{-x}(x-2-\sqrt{2})dx$$

$$= \frac{1}{-2\sqrt{2}}\int_0^{\infty} e^{-x}x\,dx - 2\int_0^{\infty} e^{-x}dx - \sqrt{2}\int_0^{\infty} e^{-x}dx$$

$$= \frac{1}{-2\sqrt{2}}\left(xe^{-x} - 2e^{-x} - \sqrt{2}e^{-x}\right)\Big|_0^{\infty}$$

$$= \frac{e^{-x}(x-1-\sqrt{2})}{2\sqrt{2}}\Big|_0^{\infty}$$

$$= \frac{e^{0}(0-1-\sqrt{2})}{2\sqrt{2}}$$

$$\boxed{W_2 = \frac{\sqrt{2}+1}{2^{3/2}} \qquad W_2 \approx 0.8535}$$

d) parte 1

$$\Gamma(x) = \int_0^\infty e^{-t} \cdot t^{x-1}\,dt$$

$$\Gamma(4) = \int_0^\infty e^{-t} \cdot t^{4-1}\,dt = \int_0^\infty e^{-t} \cdot t^{3}\,dt$$

$$= \int_0^\infty e^{-x} \cdot x^{3}\,dx$$

$$= (4-1)!$$

$$= 3!$$

$$= 6$$