Preparcial - Métodos comp.

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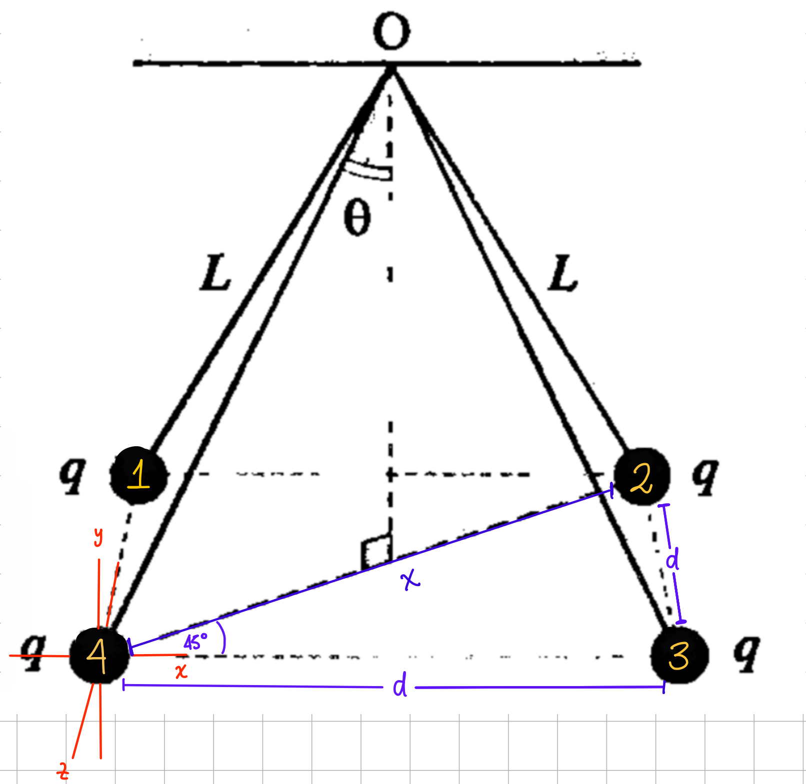
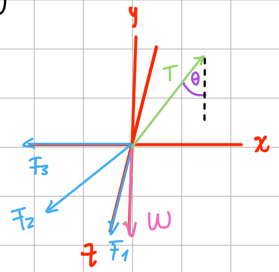
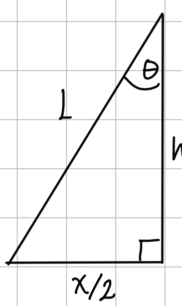


Carga # 4:



Hallar ecuación del ángulo para el cual las cargas están en reposo

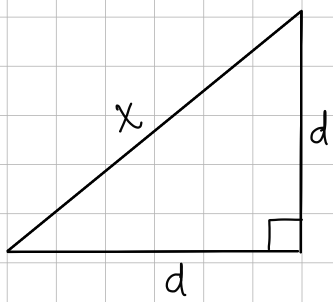
Hallo la distancias x y d que separan las cargas:



$$\sin\theta = \frac{x/2}{L}$$

$$\sin\theta = \frac{x}{2L}$$

$$x = 2L\sin\theta$$



$$d^2 + d^2 = x^2$$

$$2d^2 = x^2$$

$$d^2 = x^2/2$$

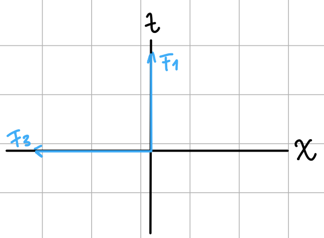
$$d = x/\sqrt{2}$$

$$d = \frac{2L\sin\theta}{\sqrt{2}}$$

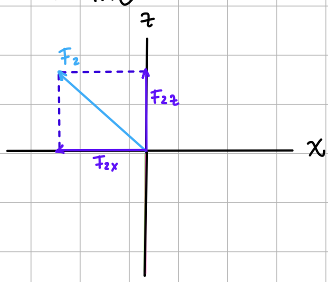
$$d = \sqrt{2}\,L\sin\theta$$

Dibujo los diagramas de cuerpo libre de la partícula 4.

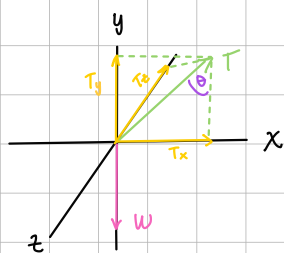
Plano xz



Plano xz



Plano xyz



La sumatoria de fuerzas debe dar 0 para que las cargas estén en reposo, por lo cual:

$$\Sigma F_x \Rightarrow -F_3 - F_{2x} + T_x = 0$$

$$\Sigma F_y \Rightarrow T_y - W = 0$$

$$\Sigma F_z \Rightarrow F_1 + F_{2z} - T_z = 0$$

Hallando las magnitudes de $\vec{F}_1, \vec{F}_2, \vec{F}_3$ (fuerzas entre cargas eléctricas): $F = \frac{K Q_1 Q_2}{r^2}$

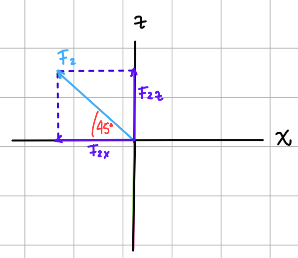
$$F_1 = \frac{K q^2}{d^2} \qquad F_2 = \frac{K q^2}{x^2} \qquad F_3 = \frac{k q^2}{d^2}$$

$$F_1 = \frac{k q^2}{2L^2 \sin^2\theta} \qquad F_2 = \frac{k q^2}{4L^2 \sin^2\theta} \qquad F_3 = \frac{k q^2}{2L^2 \sin^2\theta}$$

$$F_{2x} = F_2 \cos 45° \qquad F_{2z} = F_2 \sin 45$$

$$F_{2x} = \frac{F_2}{\sqrt{2}} \qquad F_{2z} = F_2 \cdot \frac{\sqrt{2}}{2}$$

$$F_{2x} = \frac{1}{\sqrt{2}} \cdot \frac{k q^2}{4L^2 \sin^2\theta} \qquad F_{2z} = \frac{\sqrt{2}}{2} \frac{k q^2}{4L^2 \sin^2\theta}$$

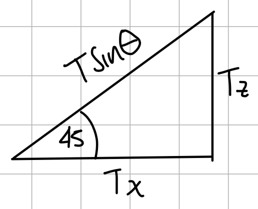


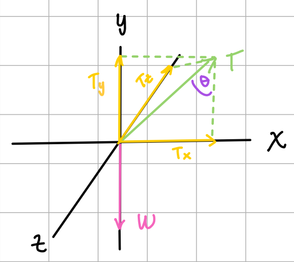
Hallando magnitud de T_x, T_y, T_z (tensión):

$$T_x = T \sin\theta \cos 45 = \frac{T \sin\theta}{\sqrt{2}}$$

$$T_y = T \cos\theta$$

$$T_z = T \sin\theta \sin 45 = \frac{\sqrt{2}\, T \sin\theta}{2}$$





Ahora, reemplazando en las sumatorias, obtenemos

$$\Sigma F_x \Rightarrow -F_3 - F_{2x} + T_x = 0$$

$$T_x = F_3 + F_{2x}$$

$$T_x = \frac{k q^2}{2L^2 \sin^2\theta} + \frac{1}{\sqrt{2}} \cdot \frac{k q^2}{4L^2 \sin^2\theta}$$

$$\frac{T \sin\theta}{\sqrt{2}} = \frac{k q^2}{2L^2 \sin^2\theta} + \frac{k q^2}{4\sqrt{2}\, L^2 \sin^2\theta}$$

$$\frac{T \sin\theta}{\sqrt{2}} = \frac{k q^2}{L^2 \sin^2\theta}\left(\frac{1}{2} + \frac{1}{4\sqrt{2}}\right) \Rightarrow T \sin\theta = \frac{k q^2}{L^2 \sin^2\theta}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4\sqrt{2}}\right)$$

$$(1) \quad T \sin\theta = \frac{k q^2}{L^2 \sin^2\theta}\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right)$$

$$\sum F_y \Rightarrow \quad T_y - w = 0$$

$$T_y = W$$

$$(2) \quad T\cos\theta = W$$

$$\sum F_z \Rightarrow \quad F_1 + F_{2z} - T_z = 0$$

$$T_z = F_1 + F_{2z}$$

$$T_z = \frac{kq^2}{2L^2\sin^2\theta} + \frac{\sqrt{2}}{2}\frac{kq^2}{4L^2\sin^2\theta}$$

$$\frac{\sqrt{2}\,T\sin\theta}{2} = \frac{kq^2}{2L^2\sin^2\theta} + \frac{\sqrt{2}}{2}\frac{kq^2}{4L^2\sin^2\theta}$$

$$\sqrt{2}\,T\sin\theta = \frac{kq^2}{L^2\sin^2\theta} + \frac{\sqrt{2}\,kq^2}{4L^2\sin^2\theta}$$

$$\sqrt{2}\,T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(1 + \frac{\sqrt{2}}{4}\right)$$

$$T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}\cdot 4}\right)$$

$$(3) \quad T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right)$$

Entonces, el sistema es:

$$\left.\begin{aligned} &(1) \quad T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right) \\ &(2) \quad T\cos\theta = W \\ &(3) \quad T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right) \end{aligned}\right\}$$

Vemos que (1) y (2) son la misma ecuación, por lo que el sistema se reduce a:

$$T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right)$$

$$T\cos\theta = W$$

$$(1)\quad T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)$$

$$(2)\quad T\cos\theta = W$$

Resolviendo, obtenemos:

$$\frac{(1)}{(2)} \Rightarrow \frac{T\sin\theta = \frac{kq^2}{L^2\sin^2\theta}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)}{T\cos\theta = W}$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{kq^2}{L^2\sin^2\theta\cdot W}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)$$

$$\frac{\sin^3\theta}{\cos\theta} = \frac{kq^2}{L^2 W}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)$$

$$\left(\frac{\sin^3\theta}{\cos\theta}\right)^2 = \left(\frac{kq^2}{L^2 W}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)\right)^2$$

$$\frac{\sin^6\theta}{\cos^2\theta} = \frac{k^2q^4}{L^4W^2}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)^2$$

$$\frac{\sin^6\theta}{1-\sin^2\theta} = \frac{k^2q^4}{L^4W^2}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)^2$$

$$\sin^6\theta = 1-\sin^2\theta\left(\frac{k^2q^2}{L^4W^2}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)^2\right)$$

$$\sin^6\theta = \frac{k^2q^4}{L^4W^2}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{4}\right)^2 - \sin^2\theta\left(\frac{k^2q^4}{L^4W^2}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)^2\right)$$

$$\sin^6\theta + \sin^2\theta\left(\frac{k^2q^4}{L^4W^2}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)^2\right) - \left[\frac{k^2q^4}{L^4W^2}\left(\frac{9}{16}+\frac{\sqrt{2}}{4}\right)^2\right] = 0$$

Si $\quad C = \frac{k^2q^4}{L^4W^2}\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)^2$, entonces:

$$\boxed{\sin^6\theta + C\sin^2\theta - C = 0}$$

Si sabemos que: $w = 114.6\,N$

$q = 3 \times 10^{-4}\,C$

$L = 5\,m$

Entonces, obtenemos:

$$\boxed{\sin^6\theta + C\sin^2\theta - C = 0}$$

Donde:

$$C = \frac{k^2 q^4}{L^4 w^2}\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right)^2$$

$$C = \frac{(9\times10^9)^2(3\times10^{-4})^4}{(5^4)(114.6)^2}\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right)^2$$

$$C = 0.07322$$