2.1. Regla de Simpson de 3/8:

$$L_0 = \frac{(x-x_{i+1})(x-x_{i+2})(x-x_{i+3})}{(x_i-x_{i+1})(x_i-x_{i+2})(x_i-x_{i+3})}$$

$x_{i+1}-x_i = h$ $\quad$ $x - x_i = mh$

$x_{i+2}-x_i = 2h$ $\quad$ $x - x_{i+1} = mh - h$

$x_{i+3}-x_i = 3h$ $\quad$ $x - x_{i+2} = mh - 2h$

$x - x_{i+3} = mh - 3h$

$$L_0 = \frac{(mh-h)(mh-2h)(mh-3h)}{(-h)\cdot(-2h)(-3h)}$$

$$L_0 = \frac{h(m-1)\cdot h(m-2)\cdot h(m-3)}{-6h^3} = \frac{-1}{6}(m^2-3m+2)(m-3) = -\frac{1}{6}(m^3-3m^2+2m-3m^2+9m-6)$$

$$= -\frac{1}{6}(m^3-6m^2+11m-6)$$

$$\int_{x_i}^{x_{i+3}} L_0\, dx$$

$x = x_i \Leftrightarrow m = 0$ $\quad$ $x - x_i = mh$

$x = x_{i+3} \Leftrightarrow m = 3$ $\quad$ $x - x_{i+3} = mh - 3h$

$\frac{dx}{dm} = h \rightarrow dx = h\, dm$

$\hookrightarrow x = mh + x_i$

$$h\int_0^3 L_0\, dm = -\frac{h}{6}\int_0^3 (m^3-6m^2+11m-6)dm = -\frac{h}{6}\left[\frac{m^4}{4} - 2m^3 + \frac{11m^2}{2} - 6m\right]_0^3$$

$$= -\frac{h}{6}\left[\left(\frac{81}{4} - 54 + \frac{99}{2} - 18\right)\right] = \frac{3}{8}h$$

$$L_1 = \frac{(x-x_i)(x-x_{i+2})(x-x_{i+3})}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})(x_{i+1}-x_{i+3})} = \frac{(mh)(mh-2h)(mh-3h)}{h\cdot(-h)\cdot(-2h)} = \frac{m(m-2)(m-3)}{2}$$

$$= \frac{1}{2}(m^2-2m)(m-3) = \frac{1}{2}(m^3-3m^2-2m^2+6m) = \frac{1}{2}(m^3-5m^2+6m)$$

$$h\int_0^3 L_1\, dm = \frac{h}{2}\int_0^3 (m^3-5m^2+6m)\,dm = \frac{h}{2}\left[\frac{m^4}{4} - \frac{5}{3}m^3 + 3m^2\right]_0^3$$

$$= \frac{h}{2}\left[\frac{81}{4} - 45 + 27\right] = \frac{9}{8}h$$

$$L_2 = \frac{(x-x_i)(x-x_{i+1})(x-x_{i+3})}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})(x_{i+2}-x_{i+3})} = \frac{(mh)(mh-h)(mh-3h)}{(2h)(h)(-h)} = \frac{m(m-1)(m-3)}{-2}$$

$$= \frac{-1}{2}(m^2-m)(m-3) = -\frac{1}{2}(m^3-3m^2-m^2+3m) = -\frac{1}{2}(m^3-4m^2+3m)$$

$$h\int_0^3 L_2\, dm = -\frac{h}{2}\int_0^3 (m^3-4m^2+3m)\,dm = -\frac{h}{2}\left[\frac{m^4}{4} - \frac{4}{3}m^3 + \frac{3}{2}m^2\right]_0^3$$

$$= -\frac{h}{2}\left[\frac{81}{4} - 36 + \frac{27}{2}\right] = \frac{9}{8}h$$

$$L_3 = \frac{(x-x_i)(x-x_{i+1})(x-x_{i+2})}{(x_{i+3}-x_i)(x_{i+3}-x_{i+1})(x_{i+3}-x_{i+2})} = \frac{(mh)(mh-h)(mh-2h)}{3h\cdot 2h\cdot h} = \frac{m(m-1)(m-2)}{6}$$

$$= \frac{1}{6}(m^2-m)(m-2) = \frac{1}{6}(m^3-2m^2-m^2+2m) = \frac{1}{6}(m^3-3m^2+2m)$$

$$h\int_0^3 L_3\, dm = \frac{h}{6}\int_0^3 (m^3-3m^2+2m)\,dm = \frac{h}{6}\left[\frac{m^4}{4} - m^3 + m^2\right]_0^3$$

$$= \frac{h}{6}\left[\frac{81}{4} - 27 + 9\right] = \frac{3}{8}h$$

$$\int_{x_i}^{x_{i+3}} f(x)\,dx \approx \int_{x_i}^{x_{i+3}} f(x_i)L_0 + f(x_{i+1})L_1 + f(x_{i+2})L_2 + f(x_{i+3})L_3 \; dx$$

$$= f(x_i)\int_{x_i}^{x_{i+3}} L_0\,dx + f(x_{i+1})\int_{x_i}^{x_{i+3}} L_1\,dx + f(x_{i+2})\int_{x_i}^{x_{i+3}} L_2\,dx + f(x_{i+3})\int_{x_i}^{x_{i+3}} L_3\,dx$$

$$= f(x_i)\cdot\frac{3h}{8} + f(x_{i+1})\cdot\frac{9h}{8} + f(x_{i+2})\cdot\frac{9h}{8} + f(x_{i+3})\cdot\frac{3h}{8}$$

$$= \frac{3h}{8}\Big(f(x_i) + 3f(x_{i+1}) + 3f(x_{i+2}) + f(x_{i+3})\Big)$$