

Probabilistic Model Checking with PRISM

Production Line

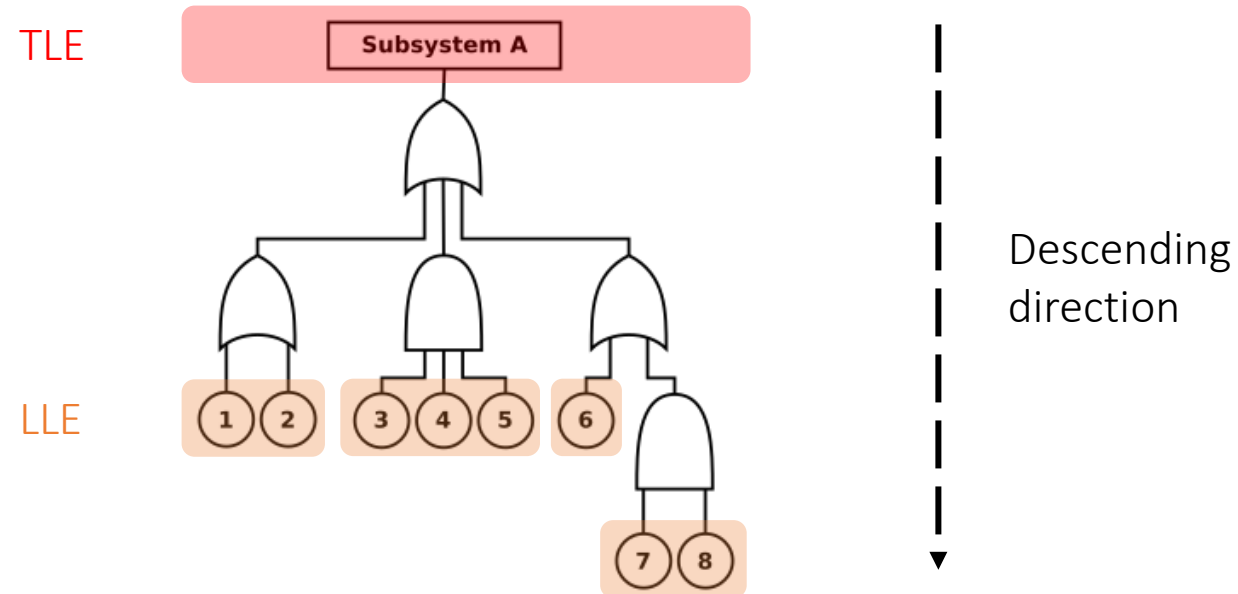
Patrick SARDINHA

Fault Trees



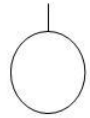
Fault Trees (FT) graphically represent possible combinations of events (Low Levels Events) leading to a predefined undesirable event (Top Level Event)

Representation:

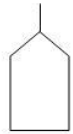


Graphic symbols

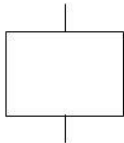
Events:



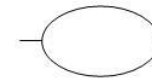
Basic event: failure in a system component



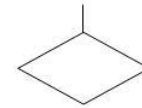
External event: expected to occur



Intermediate event: events occurring at the exit of a door



Conditioning event: an event with conditions



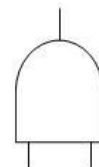
Undeveloped event: an event with insufficient information

Gates:

Describe the relationship between input and output events.



OR gate: the output occurs if any input occurs



AND gate: the output occurs only if all inputs occur

Fault Tree Analysis (FTA)

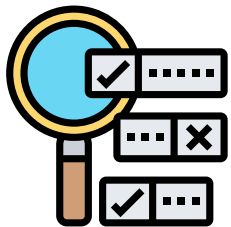


Method used to evaluate the risks of a system and allows to:

Understand how a system can fail

Know how to reduce the risks

Visualize the event rates of an accident



FTA is often performed by transforming the FT into a Boolean function which is used for simulation ...



... but this methodology has a lot of constraints (time/resources)

A new formal Probabilistic FTA methodology



Efficient Probabilistic Fault Tree Analysis of Safety
Critical Systems via Probabilistic Model Checking



Marwan Ammar, Ghaith Bany Hamad, Otmane Ait Mohamed, Yvon Savaria

A new formal Probabilistic FTA methodology

The idea is as follows:

1. Model the system (composed of components) and specify event parameters
2. Synthesize the system fault tree
3. Model the behavior of each FT gate as a probabilistic automaton (PA)
4. Generate a formal MDP(?) model of the fault tree with the parallel composition of the PA (PRISM)
5. Analyze the MDP(?) model to evaluate the maximum probability of Top Level Event (TLE)

System description

We have a **production chain** made up of:



Machines that extract resources

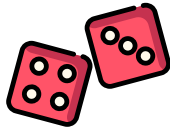


Machines that transform resources

We have different types of **disruptive primary events**:



Technical failures on machines with a certain probability




Non-deterministic quantities of extracted resources


...

And others

Resource extraction


We have different kinds of machine:


	Burner mining drill
Inputs	Raw minerals (from a source)
Outputs	Minerals
Basic event	<ul style="list-style-type: none">- Can break down- May be affected by an external event- The input quantity may vary

	Offshore pump
Inputs	Water (from a source)
Outputs	Water
Basic event	<ul style="list-style-type: none">- Can break down- May be affected by an external event- The input quantity may vary


Resource transformation


We have different kinds of machine:

	Boiler
Inputs	Fuel & water
Outputs	Steam
Basic event	<ul style="list-style-type: none">- Can break down- May be affected by an external event- Fuel not supplied- Water not supplied

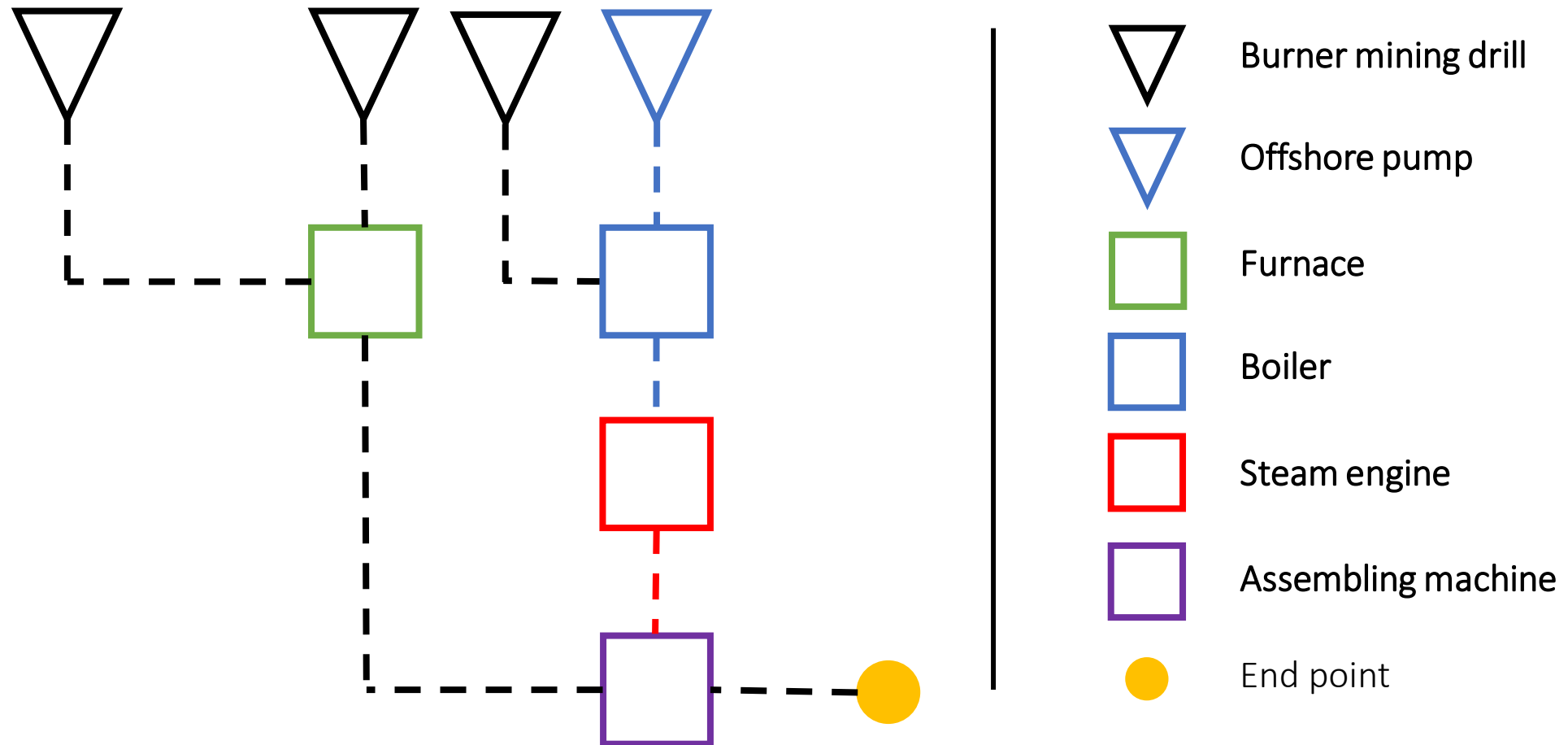
	Steam engine
Inputs	Steam
Outputs	Electricity
Basic event	<ul style="list-style-type: none">- Can break down- May be affected by an external event- Steam not supplied

Resource transformation

	Furnace
Inputs	Fuel & minerals
Outputs	Processed minerals
Basic event	<ul style="list-style-type: none">- Can break down- May be affected by an external event- Fuel not supplied- Minerals not supplied

	Assembling machine
Inputs	Electricity & Processed minerals
Outputs	Final product
Basic event	<ul style="list-style-type: none">- Can break down- May be affected by an external event- Electricity not supplied- Processed minerals not supplied

Production line: Example



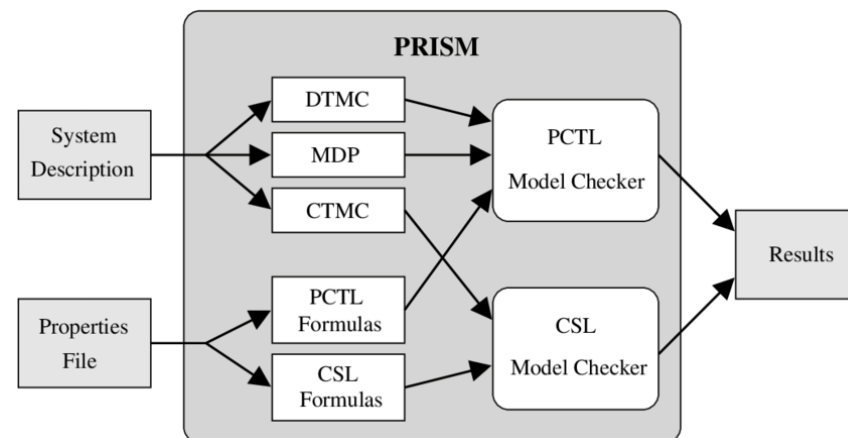
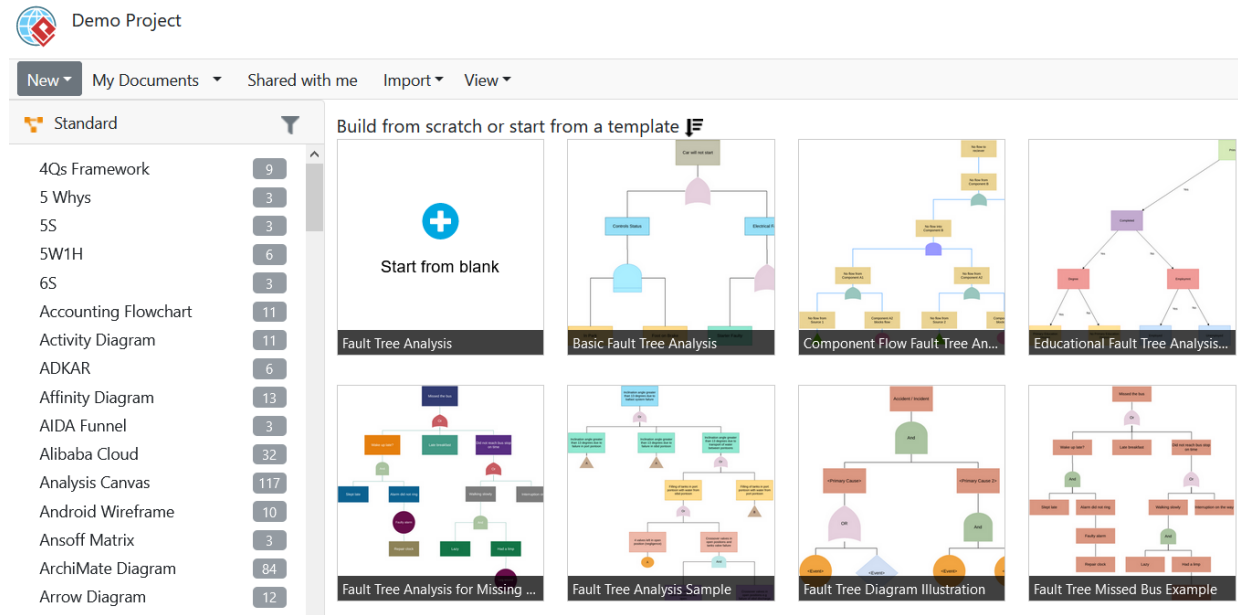
Some tools



VisualParadigm Online



PRISM



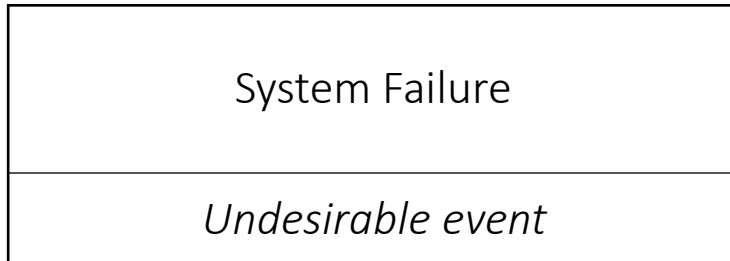
Interesting property to check



Estimate and compare the probability that faults from different low-level events cause a system failure



Production of Assembling Machine is zero



Top Level Event (TLE)

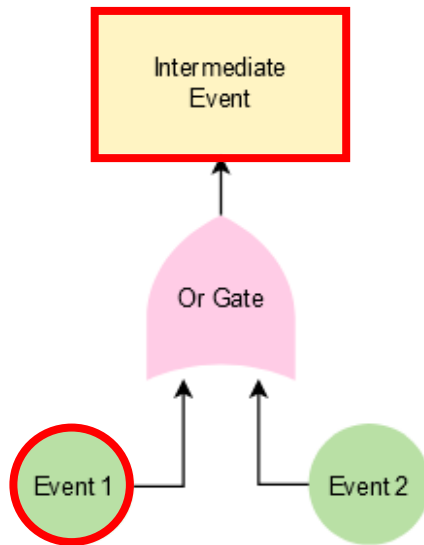


This, allows us to identify the most critical component of the system and we can then apply redundancy (TMR) on this element for example

Some mechanisms

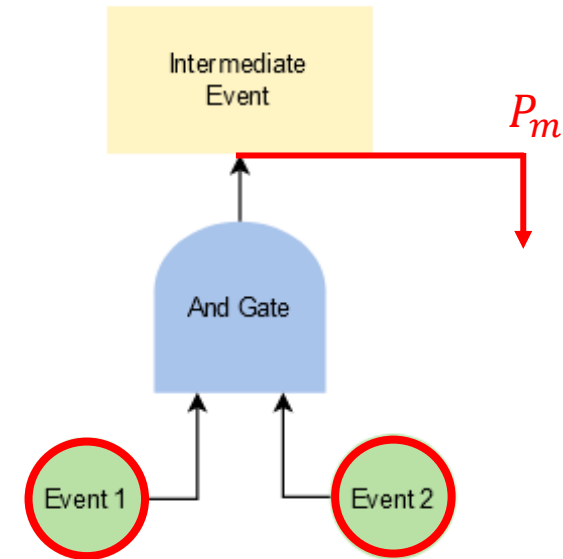
Fault propagation:

An error in a node at a lower level of the tree can propagate to a higher level

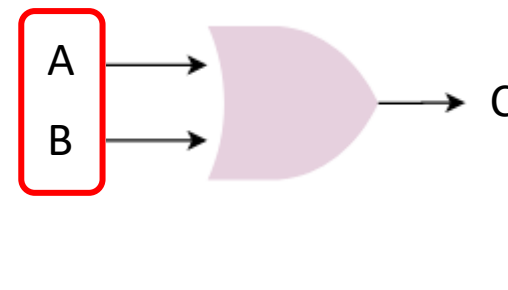
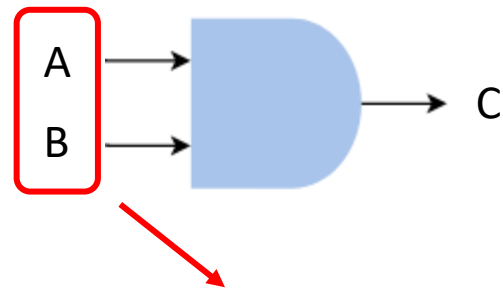


Fault masking:

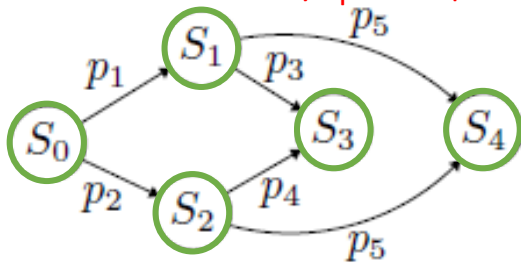
Adds a probability of fault mitigation inside the gates



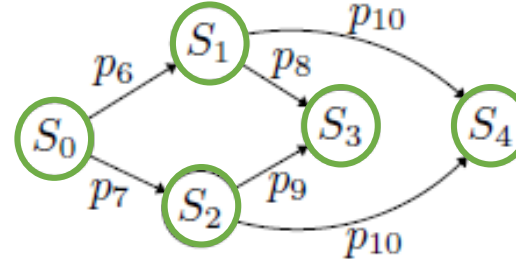
FT gates



All door entries. Only probability with the probability of being triggered
There is a probability that the gate is masked

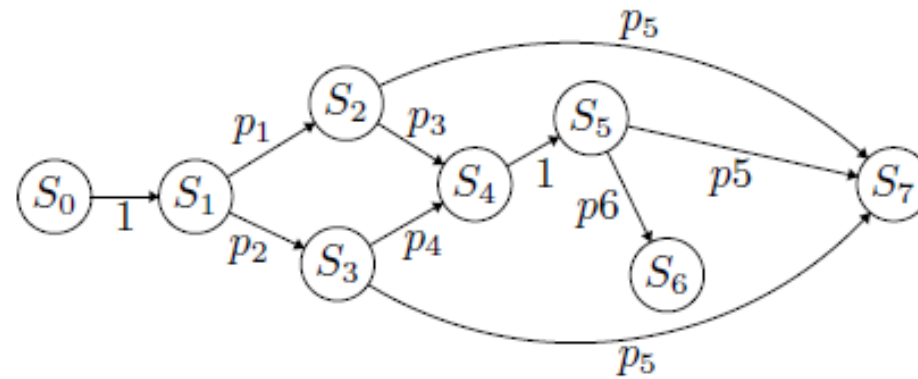
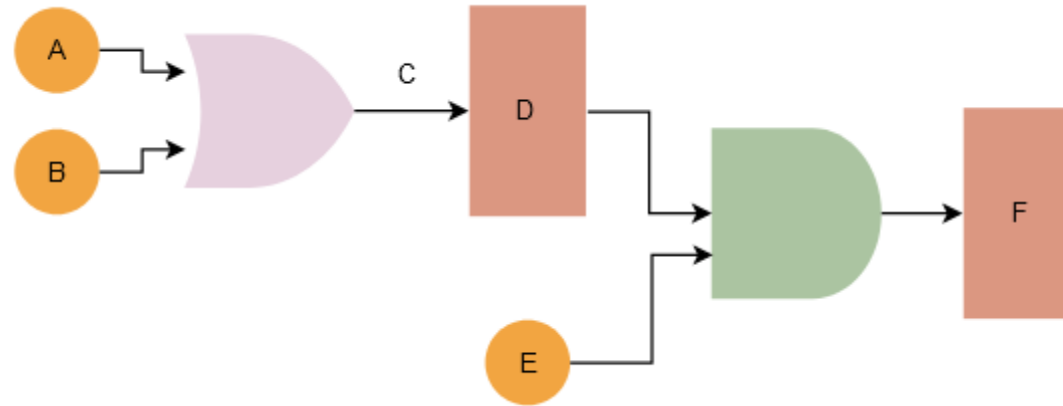


$$S_q(A = 0, B = 0, M = 11)$$



$$S_q(A = 0, B = 0, M = 11)$$

Example: Door combination



FT gates with PRISM

AND gate

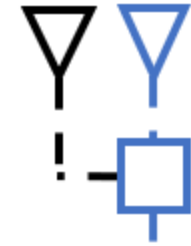
```
module and_gate
[] (and=1) & (X=0) & (Y=0) & (M=0) & (Z=0) ->p1: (X'=1) & (and'=2)
                                     +p2: (Y'=1) & (and'=2);
[] (X=1) & (Y=0) & (M=0) ->p5: (M'=1) & (X'=0) +p3: (Y'=1) & (Z'=1);
[] (Y=1) & (X=0) & (M=0) ->p5: (M'=1) & (Y'=0) +p4: (X'=1) & (Z'=1);
endmodule
```

OR gate

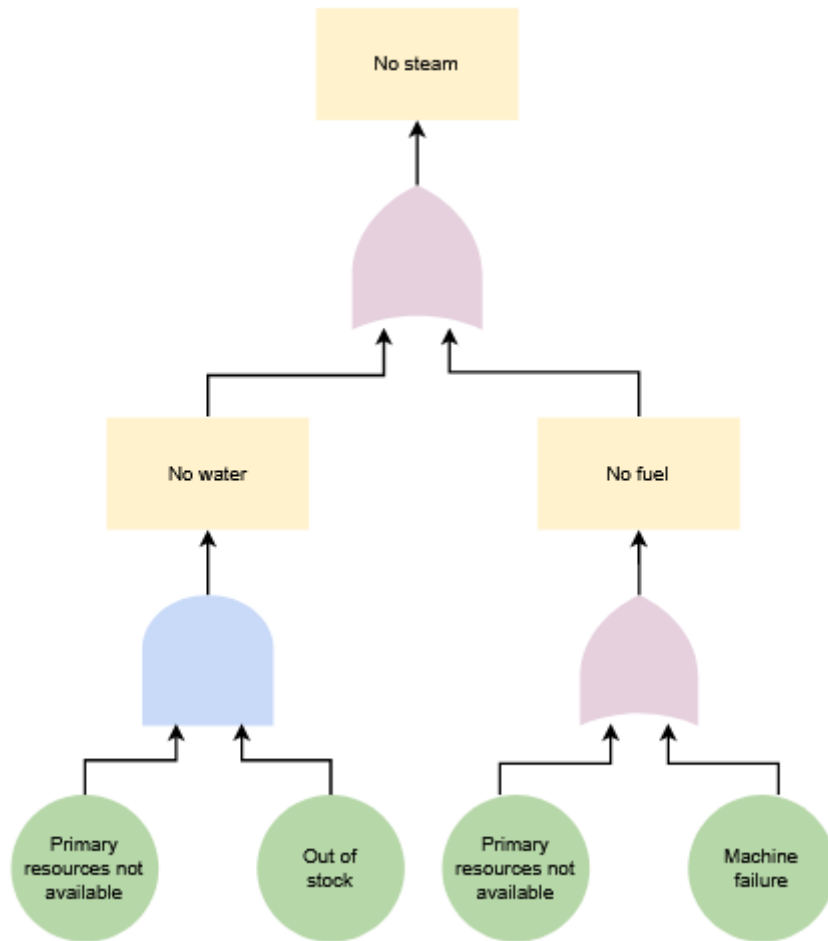
```
module or_gate
[] (or=1) & (A=0) & (B=0) & (M=0) & (C=0) ->p1: (A'=1) & (or'=2)
                                     +p2: (B'=1) & (or'=2);
[] (A=1) & (C=0) & (M=0) ->p5: (M'=1) & (A'=0) +p3: (A'=0) & (C'=1);
[] (B=1) & (C=0) & (M=0) ->p5: (M'=1) & (B'=0) +p4: (B'=0) & (C'=1);
endmodule
```

```
module twogate
[] or=0 -> (or'=1);
[] c=1 -> (c'=0) & (d'=1);
endmodule
module or_gate
[] (or=1) & (a=0) & (b=0) & (m=0) & (c=0) ->p1: (a'=1) & (or'=2)
                                     +p2: (b'=1) & (or'=2);
[] (a=1) & (c=0) & (m=0) ->p5: (m'=1) & (a'=0) +p3: (a'=0) & (c'=1);
[] (b=1) & (c=0) & (m=0) ->p5: (m'=1) & (b'=0) +p4: (b'=0) & (c'=1);
endmodule
module and_gate
[] (and=1) & (d=0) & (e=0) & (m=0) & (f=0) ->p6: (d'=1) & (and'=2)
                                     +p7: (e'=1) & (and'=2);
[] (d=1) & (e=0) & (m=0) ->p5: (m'=1) & (d'=0) +p8: (e'=1) & (f'=1);
[] (e=1) & (d=0) & (m=0) ->p5: (m'=1) & (e'=0) +p9: (d'=1) & (f'=1);
endmodule
```

Combination OR / AND



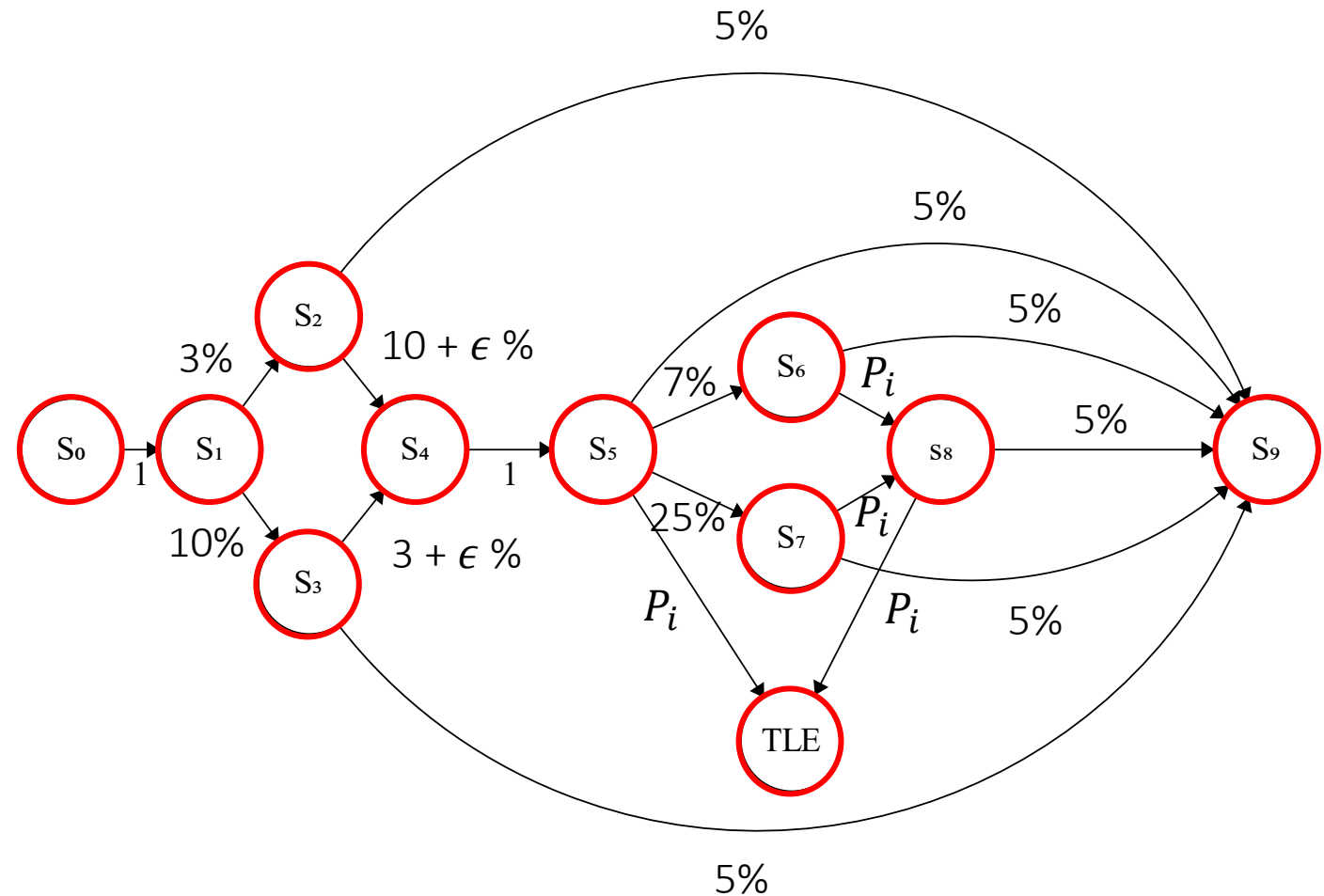
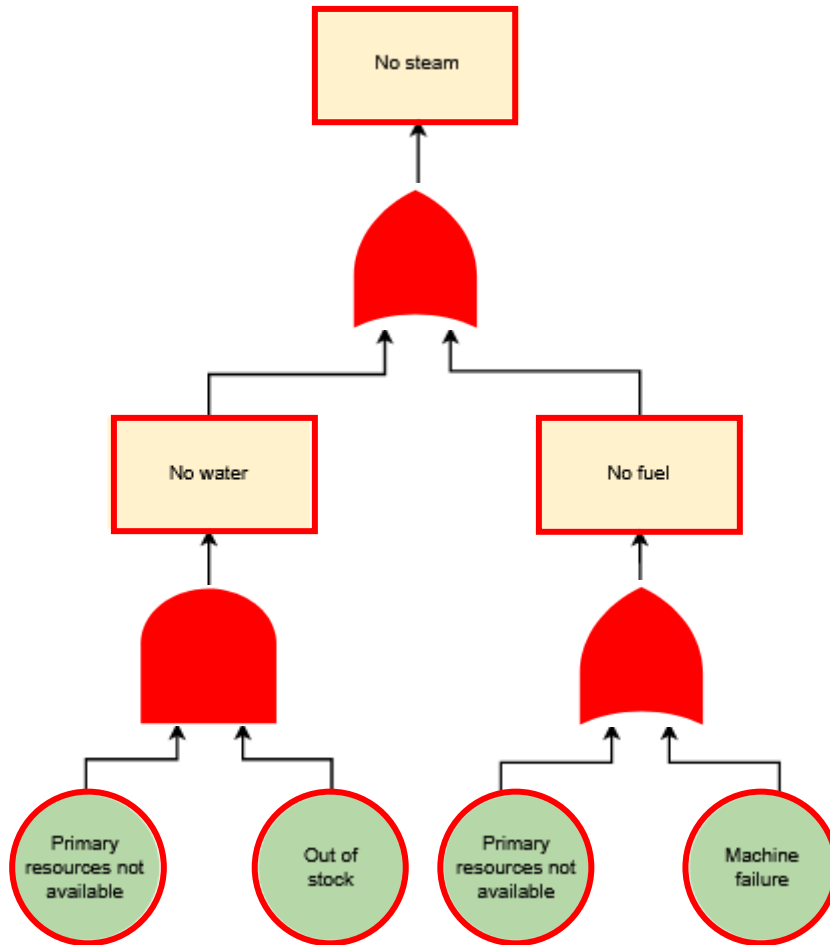
Application example: Boiler



Bottom Event	Probability
Primary resources not available	3%
Out of stock	10%
Primary resources not available	7%
Machine failure	25%

Fault masking prob: 5%

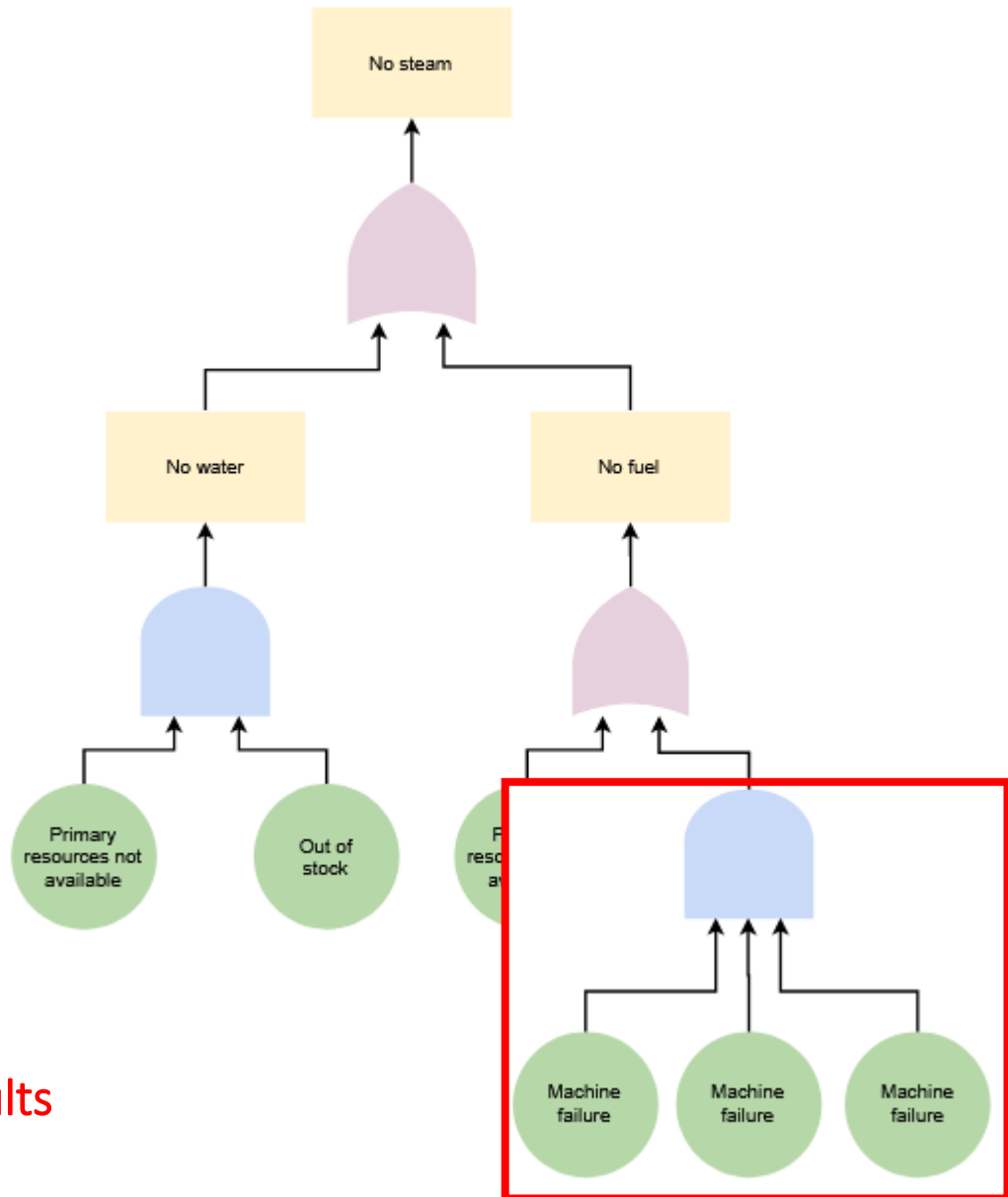
Application example: Boiler



Add redundancy

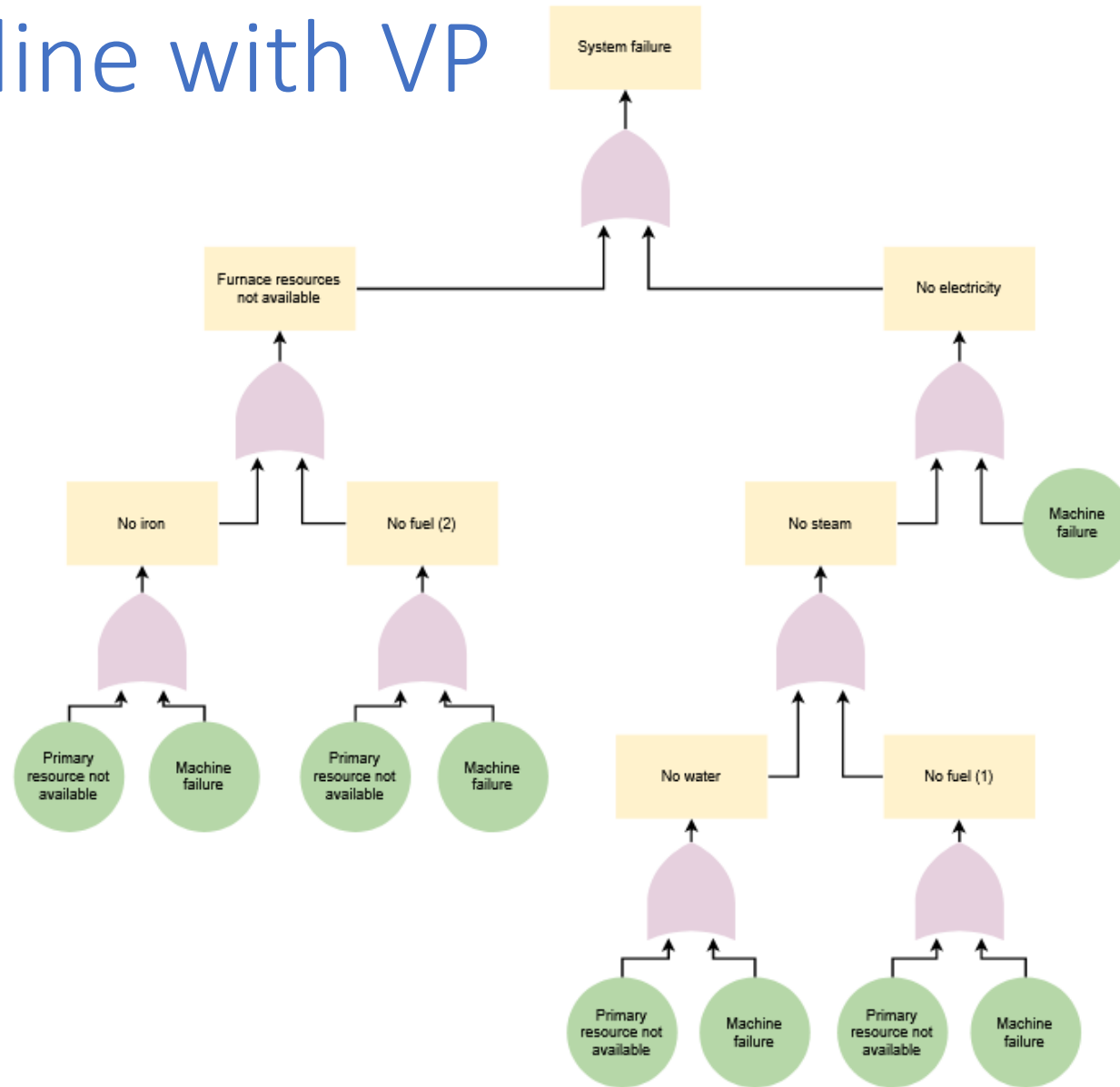
Example of TMR
(Triple Modular Redundancy)

It's a way to improve a production system
with the use of thresholds ...



→ We can abstract this method by **masking faults**

Prod. line with VP



Production line fault tree

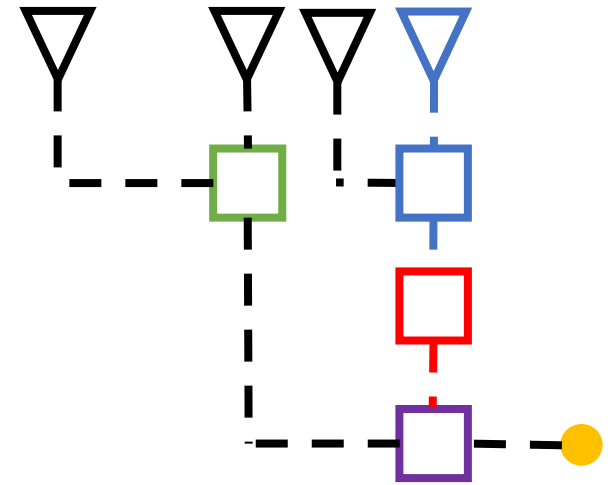


Diagram of the production line

Check property with PRISM

To estimate the probability that a low-level event leads to system failure with PRISM

For a node connected to an **OR** gate:

$$P_{max} = ? [(F X_i = 1) \& (F TLE = 1)]$$



The maximum probability that event X_i will trigger and the fault is propagated to the TLE ?

For a node connected to an **AND** gate:

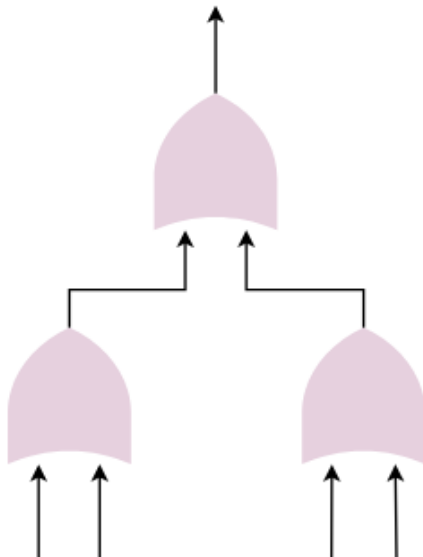
$$P_{max} = ? [(F X_i = 1) \& (F X_j = 1) \& (F TLE = 1)]$$



The maximum probability that event X_i will trigger and event X_j will trigger and the fault is propagated to the TLE ?

Gates' order

Depending on the order in which the gates are evaluated, the PA of the TF will be different but **intuitively** the probability that a low-level event leads to the system failure **will not change**



- Events on different gates have no connection
- When evaluating a door, we consider all the inputs

Idea

However, depending on the gate, only one or more inputs are needed

~~Only one entry required :~~

All entries required

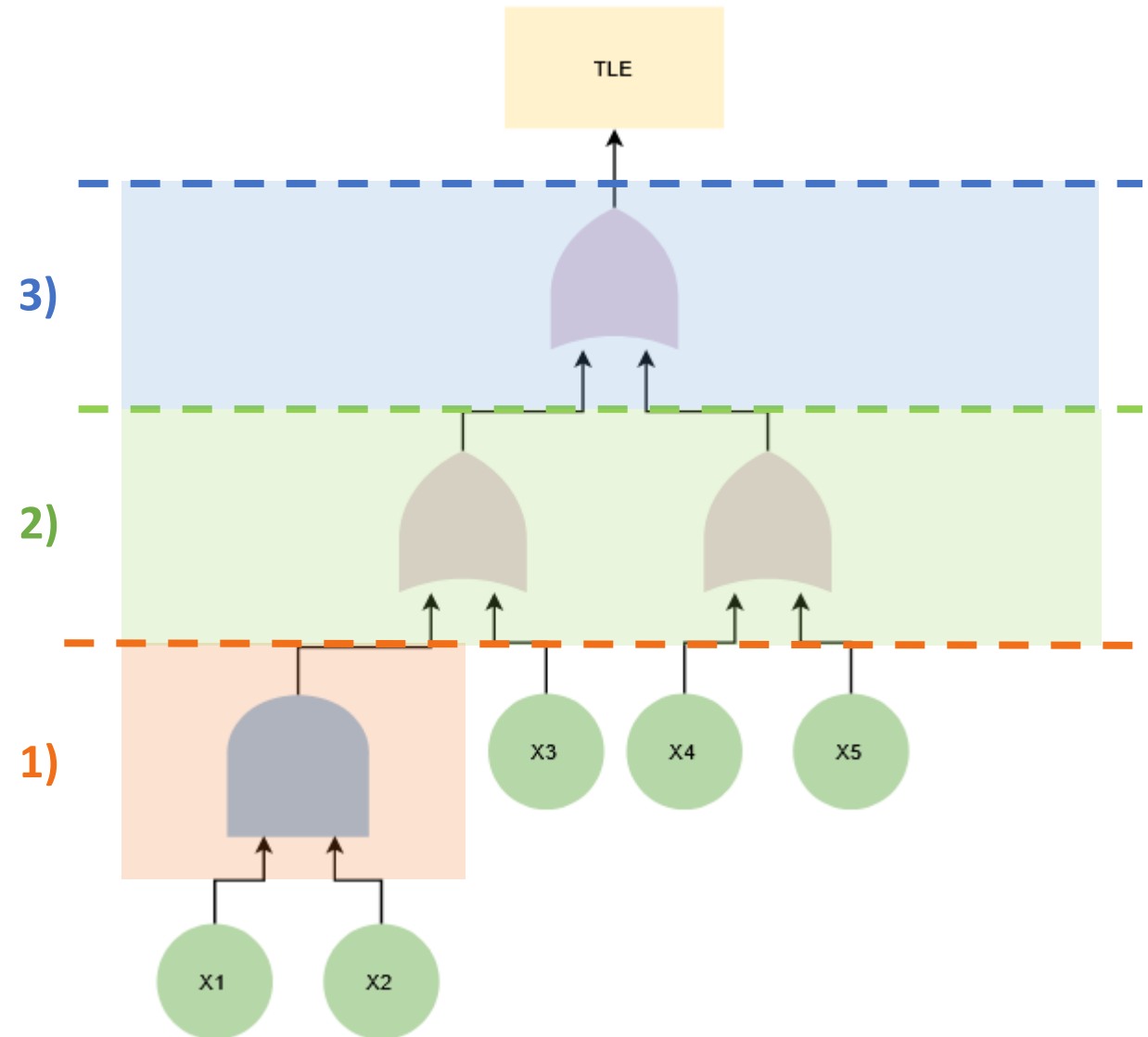


: All entries required

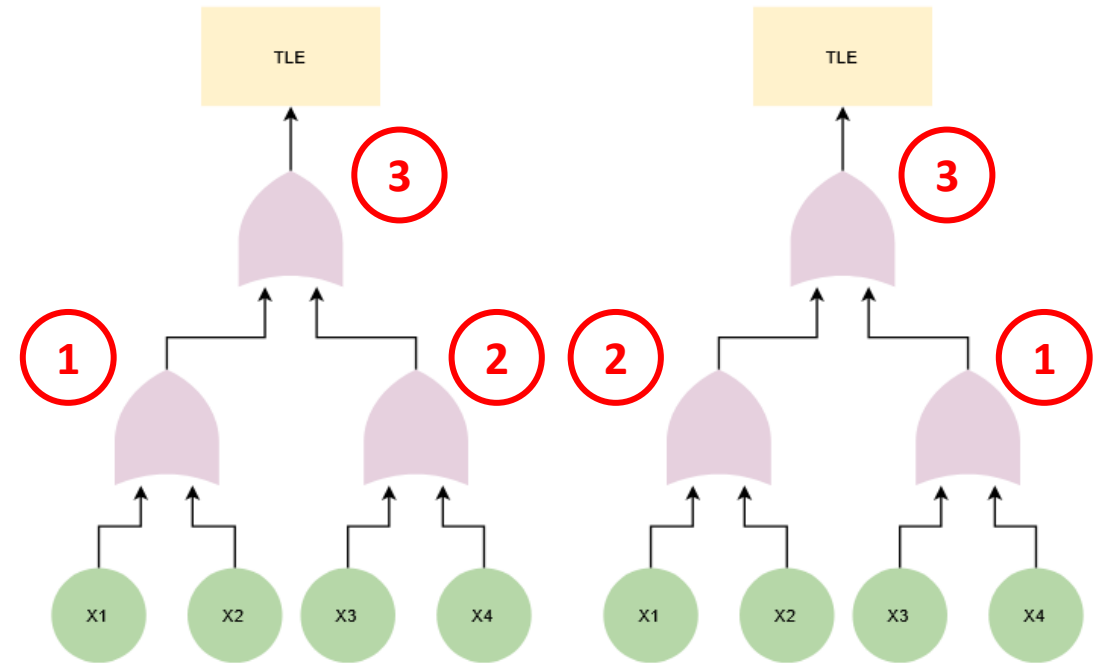
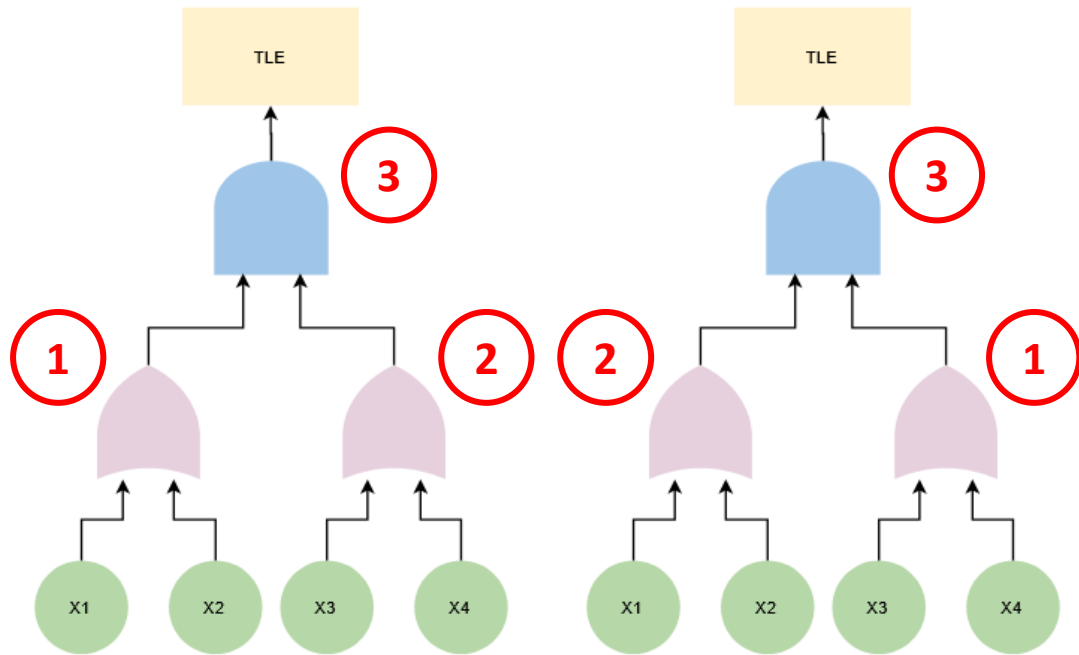
Idea

The idea is therefore to first evaluate all the doors of a layer ...

.. and then move to a higher layer

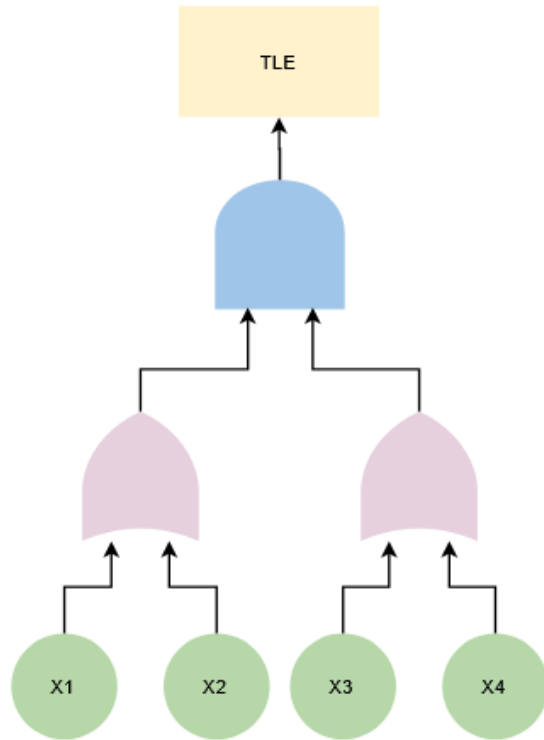


Example



The result of property valuations are equal

Using PRISM



Simulation parameters:

Confidence: 0.01
Number samples: 1000
Max path length: 10000

Bottom Event	Probability trigger
X_1	5%
X_2	32%
X_3	15%
X_4	20%

Fault masking prob: 10%

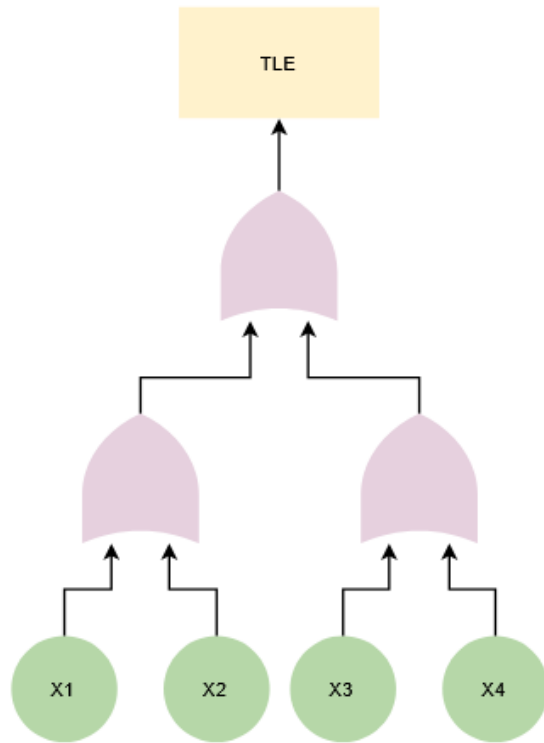
$$P = ? [(F X_1 = 1) \& (F TLE = 1)] = 0.102$$

$$P = ? [(F X_2 = 1) \& (F TLE = 1)] = 0.623$$

$$P = ? [(F X_3 = 1) \& (F TLE = 1)] = 0.319$$

$$P = ? [(F X_4 = 1) \& (F TLE = 1)] = 0.406$$

Using PRISM



Simulation parameters:

Confidence: 0.01
Number samples: 1000
Max path length: 10000

Bottom Event	Probability trigger
X_1	5%
X_2	32%
X_3	15%
X_4	20%

Fault masking prob: 10%

$$P = ? [(F X_1 = 1) \& (F TLE = 1)] = 0.104$$

$$P = ? [(F X_2 = 1) \& (F TLE = 1)] = 0.709$$

$$P = ? [(F X_3 = 1) \& (F TLE = 1)] = 0.350$$

$$P = ? [(F X_4 = 1) \& (F TLE = 1)] = 0.461$$

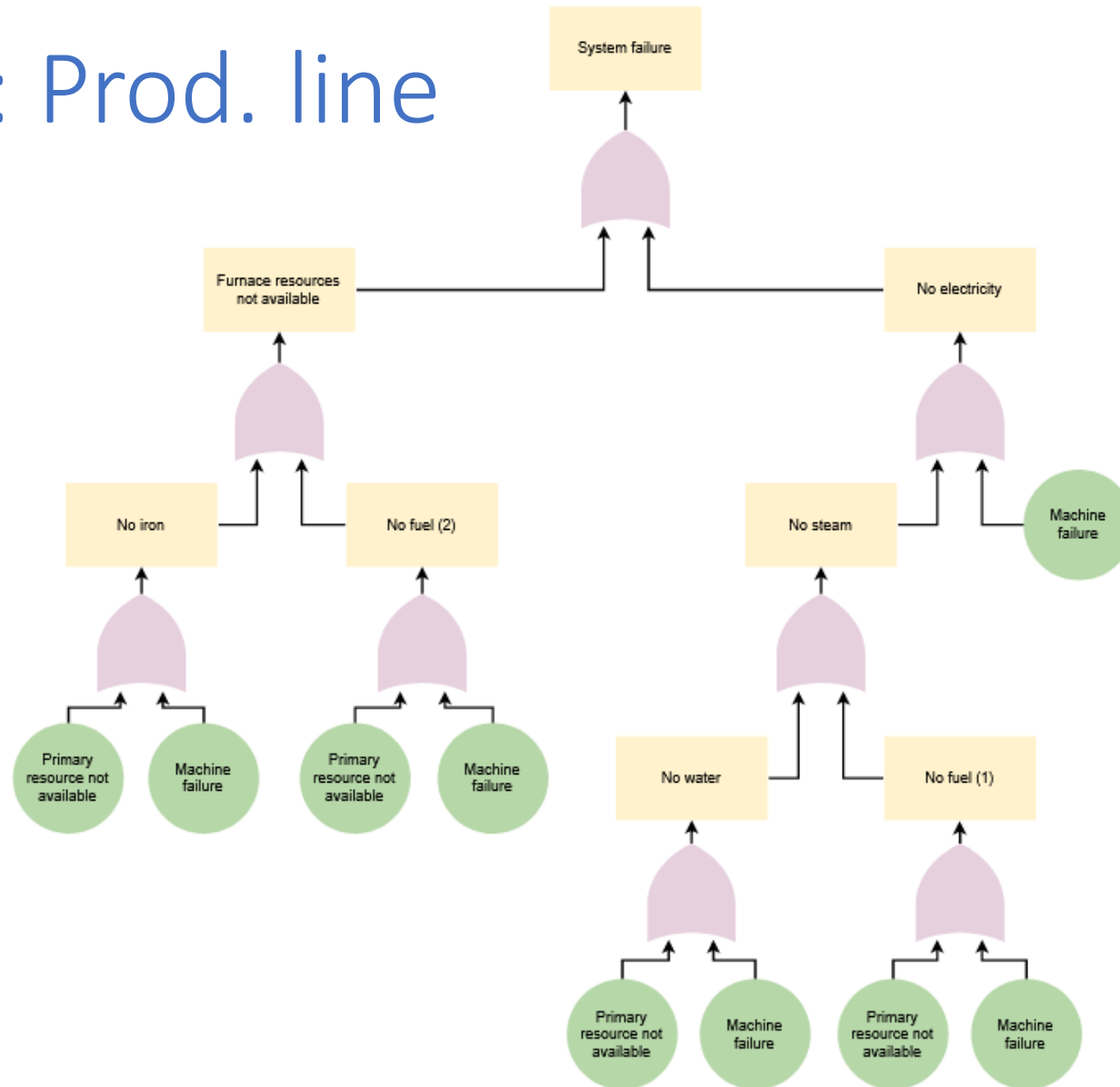
DTMC vs MDP

The main difference between DTMC & MDP:

DTMC	→	In each state, the successor state is determined by <u>a discrete probability distribution</u> (Known & predictable environment)
MDP	→	In each state, the successor state is determined by <u>a nondeterministic choice between several discrete probability distributions</u> (Unknown environment)

We will test both and observe the differences

Recall: Prod. line



Production line fault tree

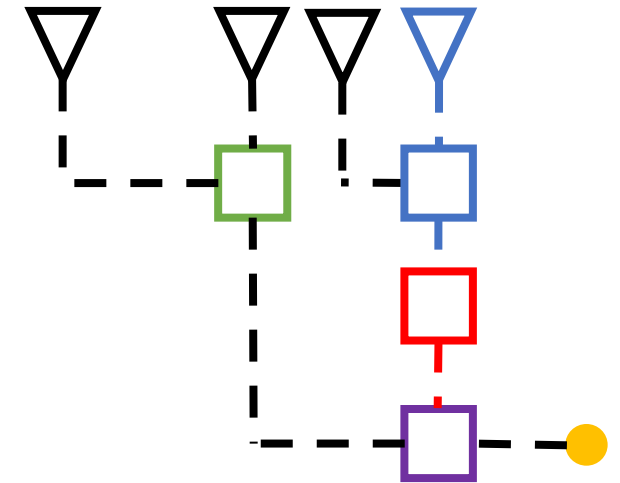
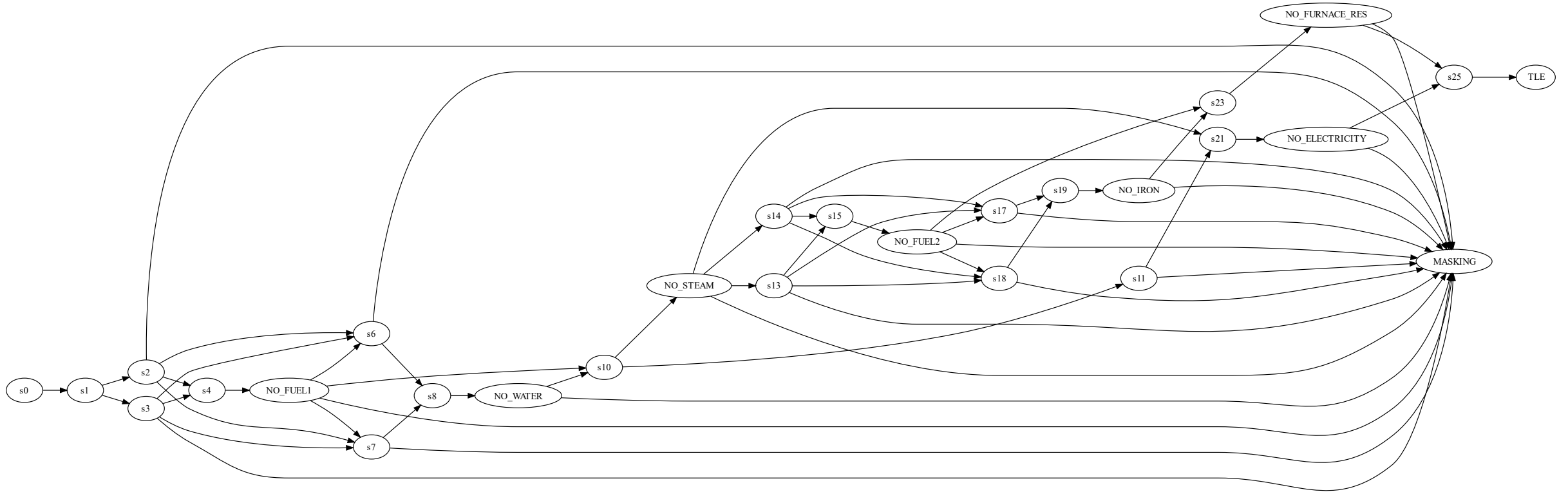


Diagram of the production line

Corresponding PA



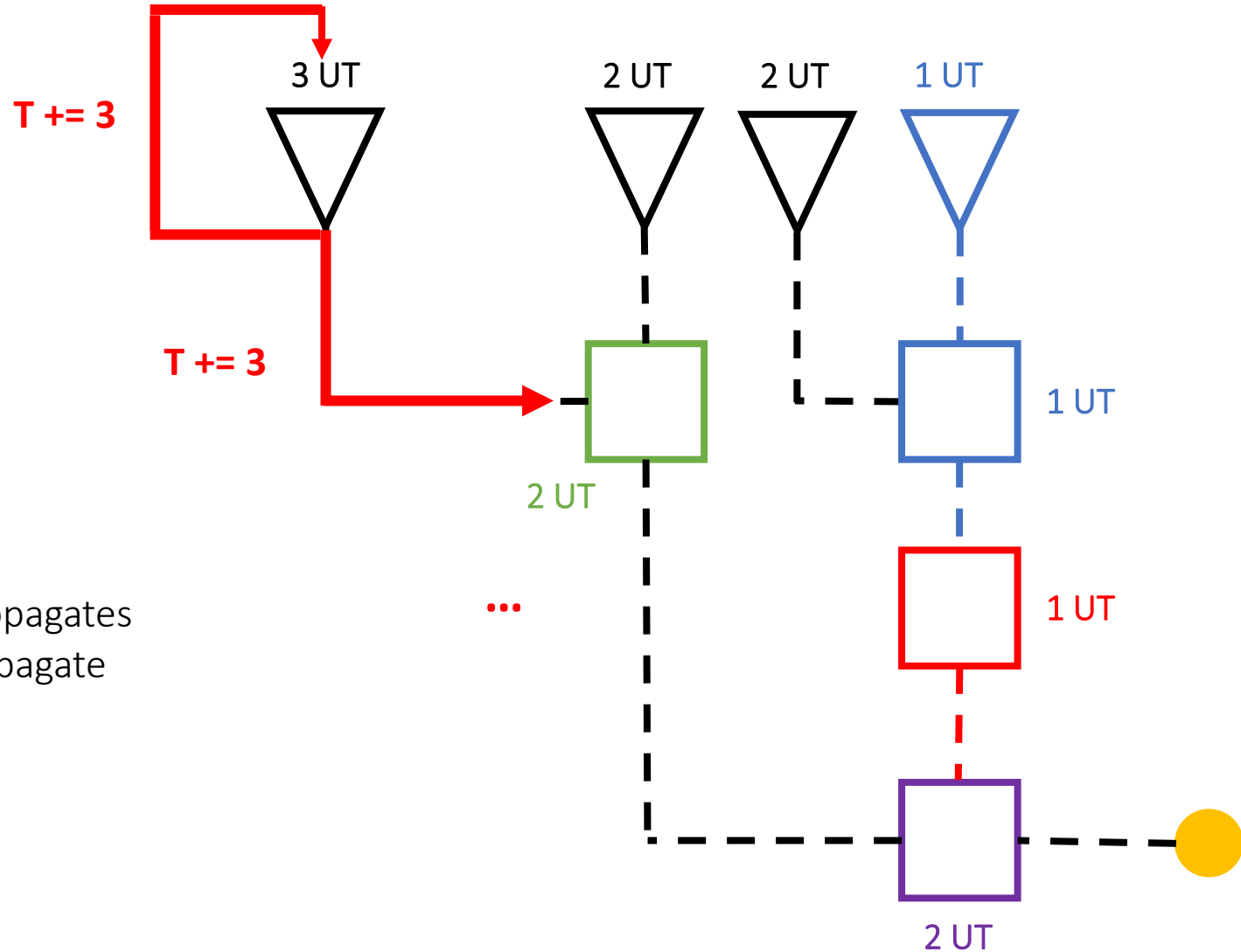
Properties to check with our propagation model



Estimate and compare the probability that a fault in a low-level events cause a system failure **before X_t time units**

$$P_{max} = ? [(F X_i = 1) \& (F TLE = 1) \& (F T < X_t)]$$

Add time value to each machine



Time increment when a fault propagates through a gate or does not propagate

Using PRISM

Bottom Event	Probability trigger
Primary res. (X_1)	0.32
Machine failure (X_2)	0.05
Primary res. (X_3)	0.15
Machine failure (X_4)	0.20
Primary res. (X_5)	0.32
Machine failure (X_6)	0.05
Primary res. (X_7)	0.10
Machine failure (X_8)	0.05
Machine failure (X_9)	0.25

Fault propagation prob: 90%

Fault masking prob: 10%

$$P = ? [(F X_1 = 1) \& (F TLE = 1) \& (T < 20)] = 0.73147581$$

$$P = ? [(F X_2 = 1) \& (F TLE = 1) \& (T < 20)] = 0.13139391$$

$$P = ? [(F X_3 = 1) \& (F TLE = 1) \& (T < 20)] = 0.34049867$$

$$P = ? [(F X_4 = 1) \& (F TLE = 1) \& (T < 20)] = 0.45398480$$

$$P = ? [(F X_5 = 1) \& (F TLE = 1) \& (T < 20)] = 0.73571484$$

$$P = ? [(F X_6 = 1) \& (F TLE = 1) \& (T < 20)] = 0.14989343$$

$$P = ? [(F X_7 = 1) \& (F TLE = 1) \& (T < 20)] = 0.51207102$$

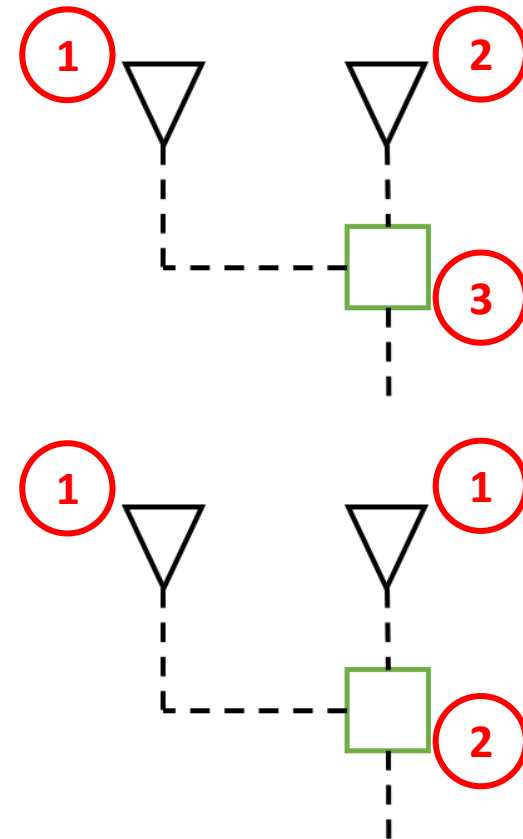
$$P = ? [(F X_8 = 1) \& (F TLE = 1) \& (T < 20)] = 0.25612831$$

$$P = ? [(F X_9 = 1) \& (F TLE = 1) \& (T < 20)] = 0.06466131$$

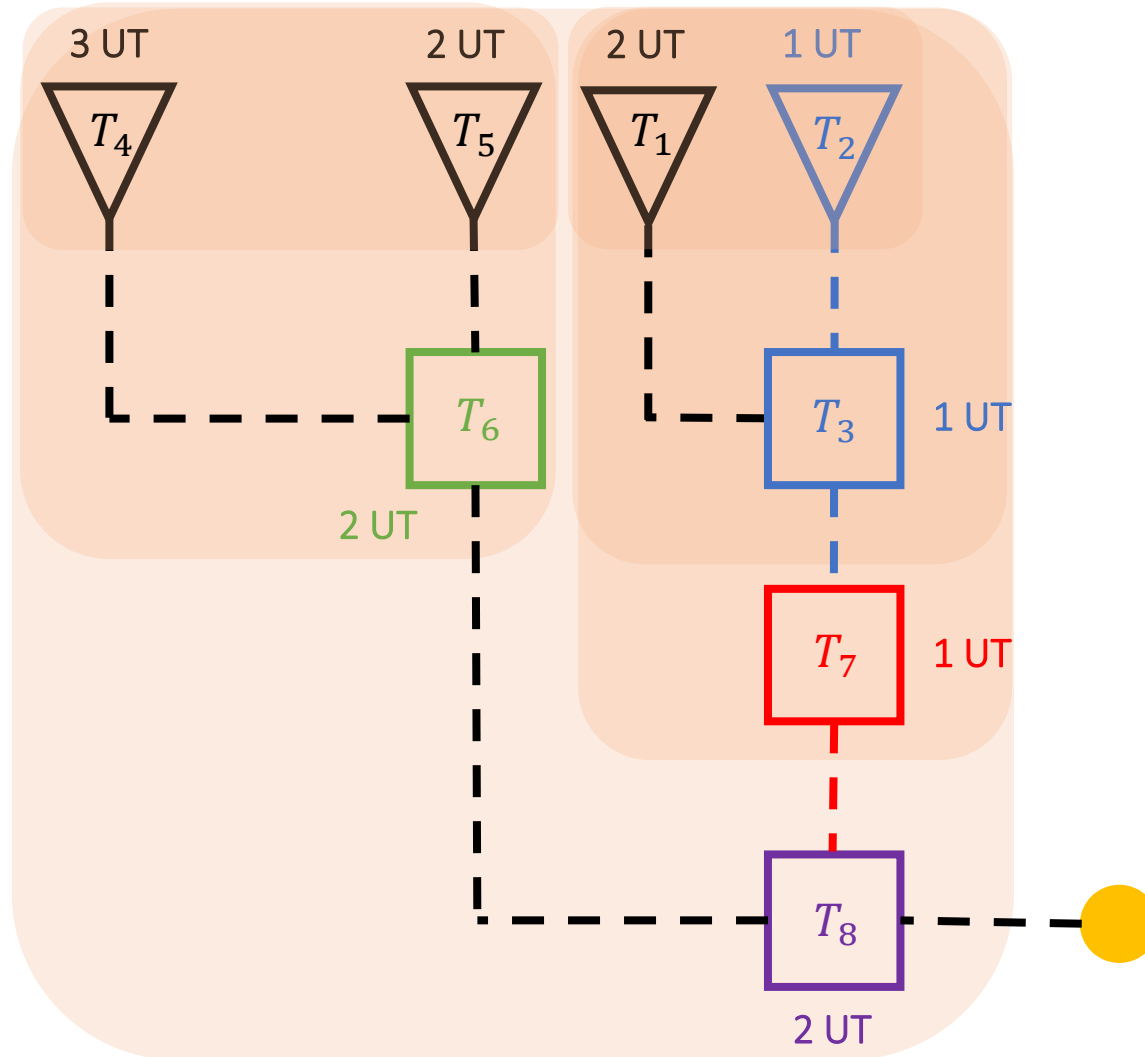
Parallelism between machines

We go through the gates of the tree sequentially for the propagation of faults ...

... but for the temporal notion we must consider the parallelism of certain machines



Parallelism: How it works



$$\begin{aligned}
 T_1 &= \begin{cases} T_1 + 2 * v_1 & v_1 < \max_M_1 \end{cases} \\
 T_2 &= \begin{cases} T_2 + 1 * v_2 & v_2 < \max_M_2 \end{cases} \\
 T_3 &= \begin{cases} \max(T_1, T_2) + 1 \\ T_1, T_2 = \max(T_1, T_2) + 1 \end{cases} \\
 T_4 &= \begin{cases} T_4 + 3 * v_4 & v_4 < \max_M_4 \end{cases} \\
 T_5 &= \begin{cases} T_5 + 2 * v_5 & v_5 < \max_M_5 \end{cases} \\
 T_6 &= \begin{cases} \max(T_4, T_5) + 2 \\ T_4, T_5 = \max(T_4, T_5) + 2 \end{cases} \\
 T_7 &= \begin{cases} T_3 + 1 \\ T_1, T_2 = T_3 + 1 \end{cases} \\
 T_f &= \begin{cases} \max(T_6, T_7) + 2 \\ T_1, T_2, T_4, T_5 = \max(T_6, T_7) + 2 \end{cases}
 \end{aligned}$$

Results: $T_{f1} < 10$

Bottom Event	Probability trigger
Primary res. (X_1)	0.32
Machine failure (X_2)	0.05
Primary res. (X_3)	0.15
Machine failure (X_4)	0.20
Primary res. (X_5)	0.32
Machine failure (X_6)	0.05
Primary res. (X_7)	0.10
Machine failure (X_8)	0.05
Machine failure (X_9)	0.25

Fault propagation prob: 90%

Fault masking prob: 10%

$$P = ? [(F X_1 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.54066383$$

$$P = ? [(F X_2 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.08447872$$

$$P = ? [(F X_3 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.26791824$$

$$P = ? [(F X_4 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.35722432$$

$$P = ? [(F X_5 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.54066383$$

$$P = ? [(F X_6 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.08447872$$

$$P = ? [(F X_7 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.41676170$$

$$P = ? [(F X_8 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.20838085$$

$$P = ? [(F X_9 = 1) \& (F TLE = 1) \& (T_{f1} < 10)] = 0.12502851$$

Results: $T_{f2} < 20$

Bottom Event	Probability trigger
Primary res. (X_1)	0.32
Machine failure (X_2)	0.05
Primary res. (X_3)	0.15
Machine failure (X_4)	0.20
Primary res. (X_5)	0.32
Machine failure (X_6)	0.05
Primary res. (X_7)	0.10
Machine failure (X_8)	0.05
Machine failure (X_9)	0.25

Fault propagation prob: 90%

Fault masking prob: 10%

$$P = ? [(F X_1 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.83347989$$

$$P = ? [(F X_2 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.13023123$$

$$P = ? [(F X_3 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.41301905$$

$$P = ? [(F X_4 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.55069207$$

$$P = ? [(F X_5 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.83347989$$

$$P = ? [(F X_6 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.13023123$$

$$P = ? [(F X_7 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.64247408$$

$$P = ? [(F X_8 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.32123704$$

$$P = ? [(F X_9 = 1) \& (F TLE = 1) \& (T_{f2} < 20)] = 0.19274222$$

Results analysis

Estimate and compare the probability that a fault in a low-level events cause a system failure **before X_t time units**

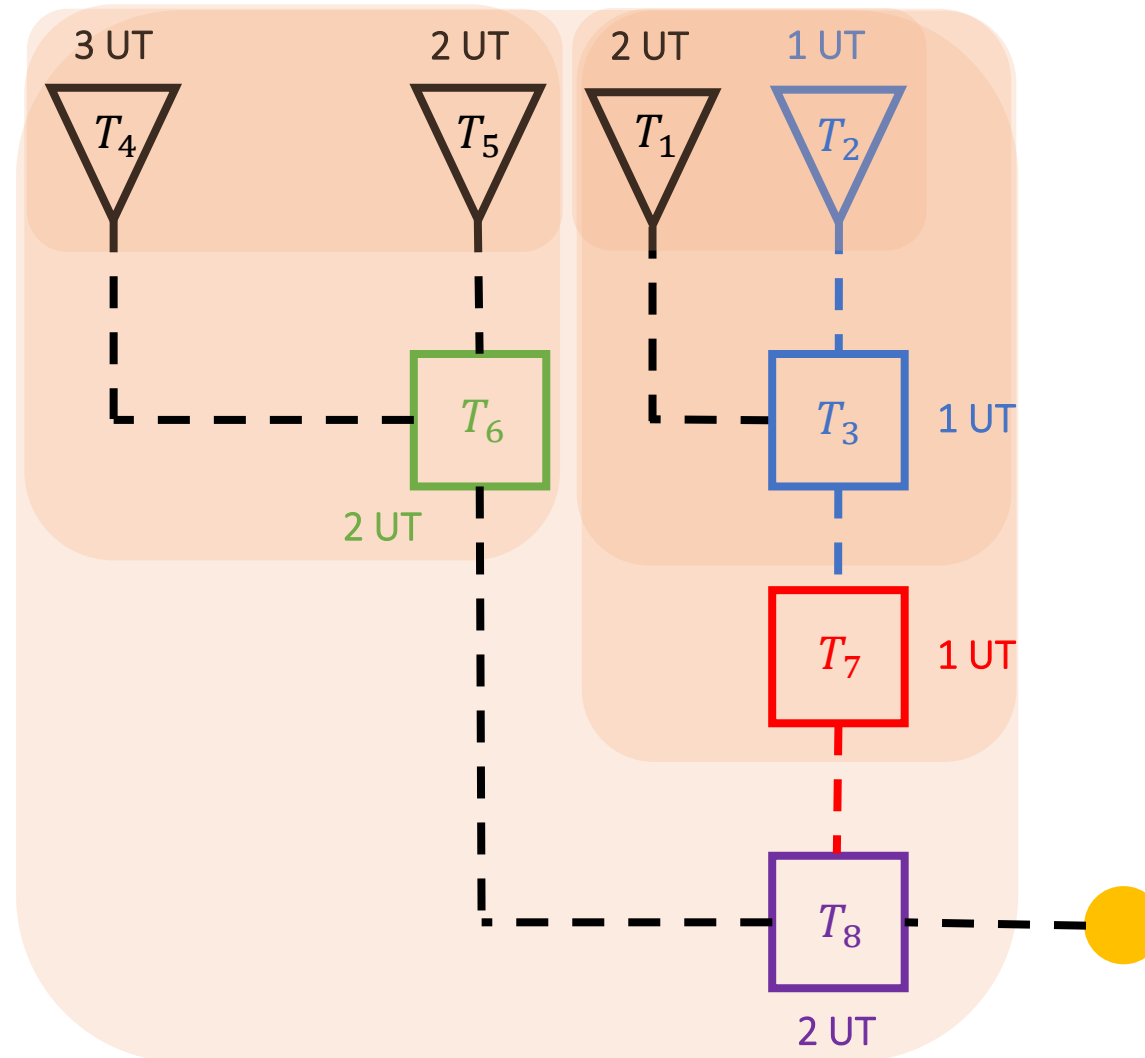
We can observe that the probability that a fault propagates and reaches the top of the tree is much higher for $T_2 > T_1$
which implies that the longer the production time, the more likely a technical failure will occur

The same machines with the same properties at the same level (of the FT) **give similar results**

The **number of time units** associated with each machine will influence the results

A particular event will be **more impacted** by events that are **more or less directly linked to it**
(same door, sub-part of the tree, etc.)

Parallelism: How it works



Disadvantages

Our model is a bit complex and is made up of many states ...

Built Model
States: 897646
Initial states: 1
Transitions: 1562080

$$T_f < 10$$

Built Model
States: 147876427
Initial states: 1
Transitions: 275801627

$$T_f < 20$$

... that's why we must limit the maximum number of loops concerning fault masking

Advancement



Try to reduce complexity (code)

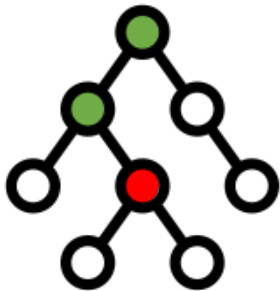


Write the report

Modifications

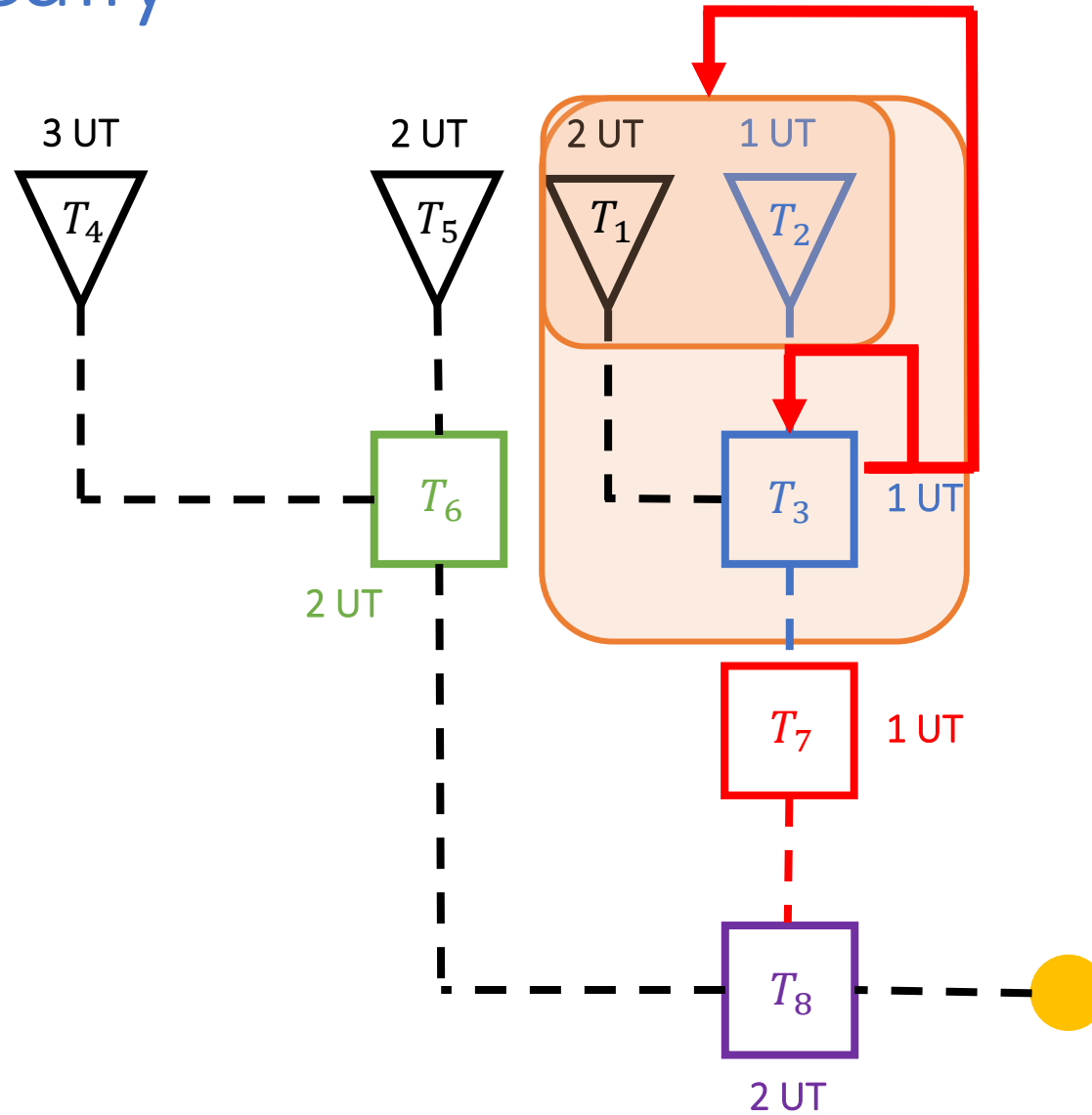


To reduce the complexity of the code, the idea is to recover the intermediate results of the subsystems



This prevents when a fault is masked in a machine that the whole branch of the tree is reassessed

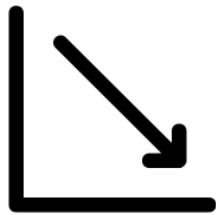
Schematically



Modifications



This modification leads to a saving of time and space concerning the calculations.



To calculate a single property, we go from **8 - 10 minutes** to **16 - 25 seconds**

Paper



The paper follows the following plan:

1. Introduction
2. Related Work
3. Model
4. Implementation
5. Evaluation
6. Conclusion



The first version of the paper is available on Github

Work incoming



Make some changes on the paper



See & evaluate other properties

References - Github

Source code: https://github.com/sardinhapatrack/PMC_PRISM

References

PRISM: <https://www.prismmodelchecker.org/>

Factorio wiki: https://wiki.factorio.com/Main_Page

Modélisation et simulation de flux de production:

Franck Fontanili, *Intégration d'outils de simulation et d'optimisation pour le pilotage d'une ligne d'assemblage multiproduit à transfert asynchrone*, Partie IV, page 87-133

References

FTA: https://en.wikipedia.org/wiki/Fault_tree_analysis

FTA via PMC: M. Ammar, G. B. Hamad, O. A. Mohamed and Y. Savaria, "*Efficient probabilistic fault tree analysis of safety critical systems via probabilistic model checking* "
<https://ieeexplore.ieee.org/abstract/document/7880373/metrics#metrics>

SML: https://fr.wikipedia.org/wiki/Systems_Modeling_Language

VP: <https://online.visual-paradigm.com/fr/diagrams/features/fault-tree-analysis-software/>

MDP: https://en.wikipedia.org/wiki/Markov_decision_process

References

FMTs:

M. Ammar, G. B. Hamad and O. Ait Mohamed, "Probabilistic High-Level Estimation of Vulnerability and Fault Mitigation of Critical Systems Using Fault-Mitigation Trees (FMTs)," *2019 IEEE Latin American Test Symposium (LATS)*, Santiago, Chile, 2019, pp. 1-6
<https://ieeexplore.ieee.org/document/8704589>

FTA via PMC
(case study):

M. Ammar, K. A. Hoque and O. A. Mohamed, "Formal analysis of fault tree using probabilistic model checking: A solar array case study," *2016 Annual IEEE Systems Conference (SysCon)*, Orlando, FL, USA, 2016, pp. 1-6
<https://ieeexplore.ieee.org/abstract/document/7490556>

Finite State
Machine Designer:

https://www.cs.unc.edu/~otternes/comp455/fsm_designer/

Probabilistic Model Checking with PRISM

Production Line

Patrick SARDINHA