Bayan: A Branch-and-Cut Algorithm for Globally Maximizing Modularity

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Joint work with
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Mechanical & Industrial Engineering
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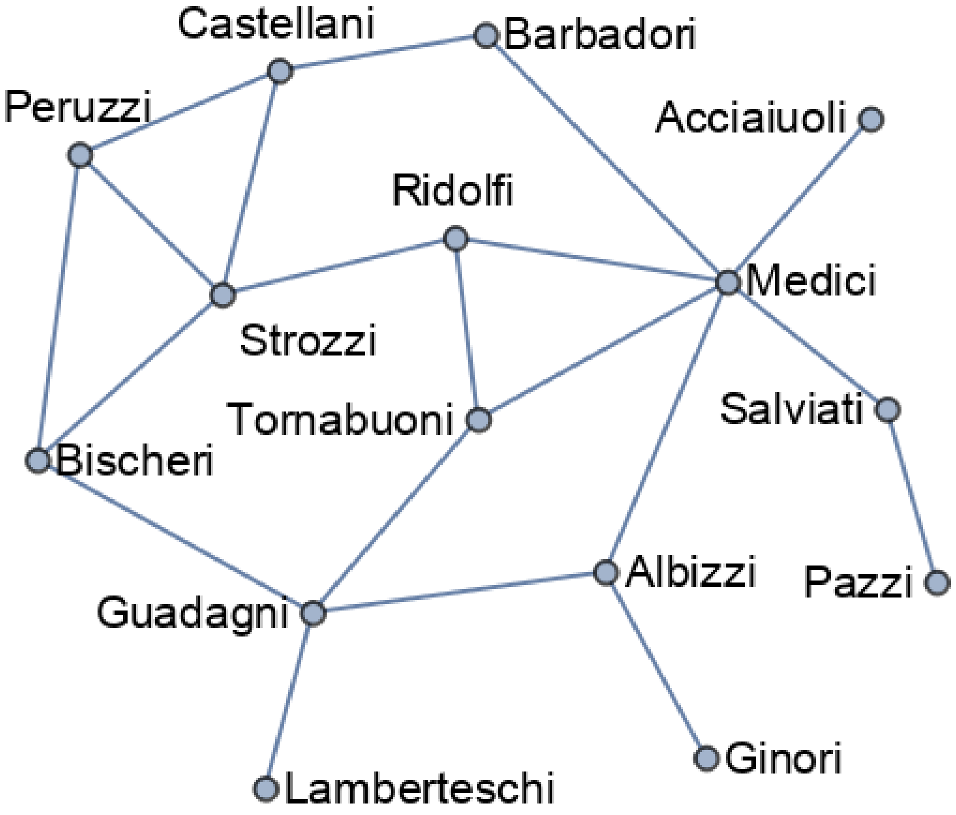
What is the maximum modularity of the Florentine families network? (γ=1)



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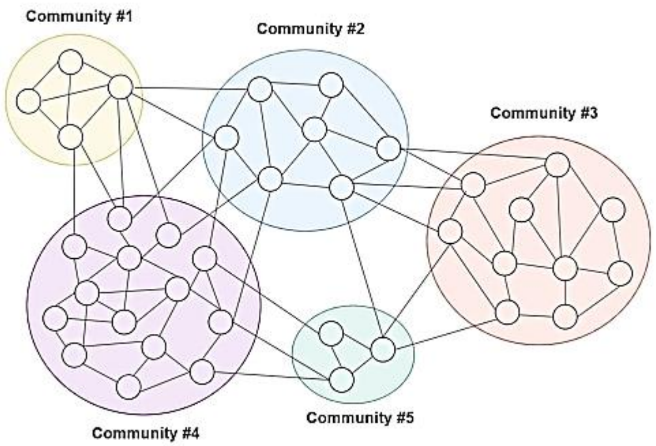
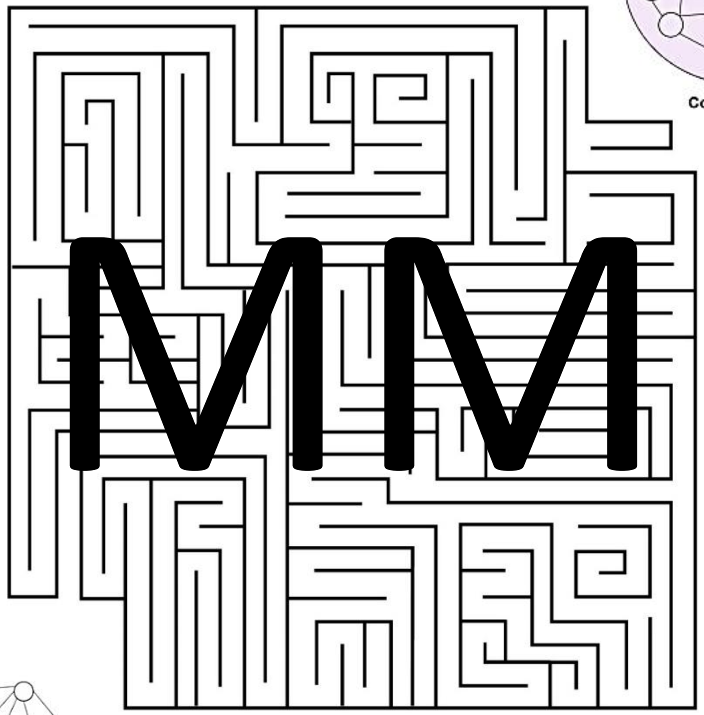
Output: communities

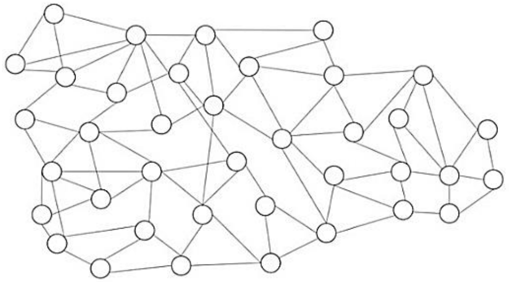


Detecting (assortative) communities via Modularity Maximization (MM)

Modularity b_{ij} : a function of degrees d_i, incidence a_{ij}, and the resolution parameter γ



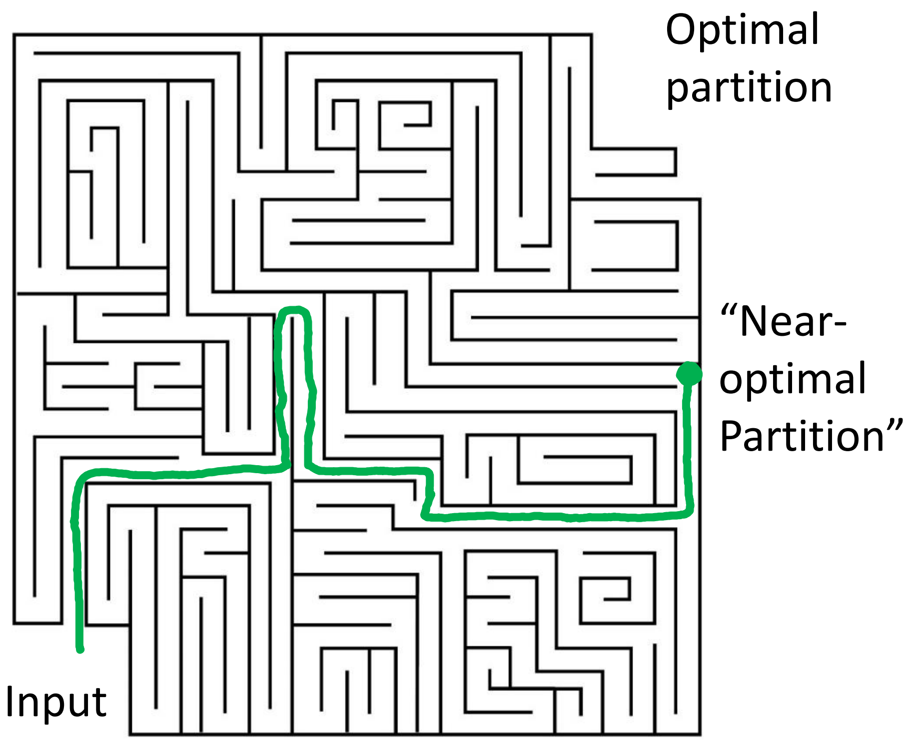
Input: a simple graph



Figures from timbr.ai

Approach 1: Modularity Maximization Heuristics

1. Edge Motif (EdMot) (Li et al. 2019)
2. Leiden (Traag et al. 2019)
3. Paris (Bonald et al. 2018)
4. Belief (Zhang & Moore 2014)
5. Combo (Sobolevsky et al. 2014)
6. Leicht-Newman (LN) (Leicht & Newman 2008)
7. Louvain (Blondel et al. 2008)
8. Greedy (CNM) (Clauset et al. 2004)



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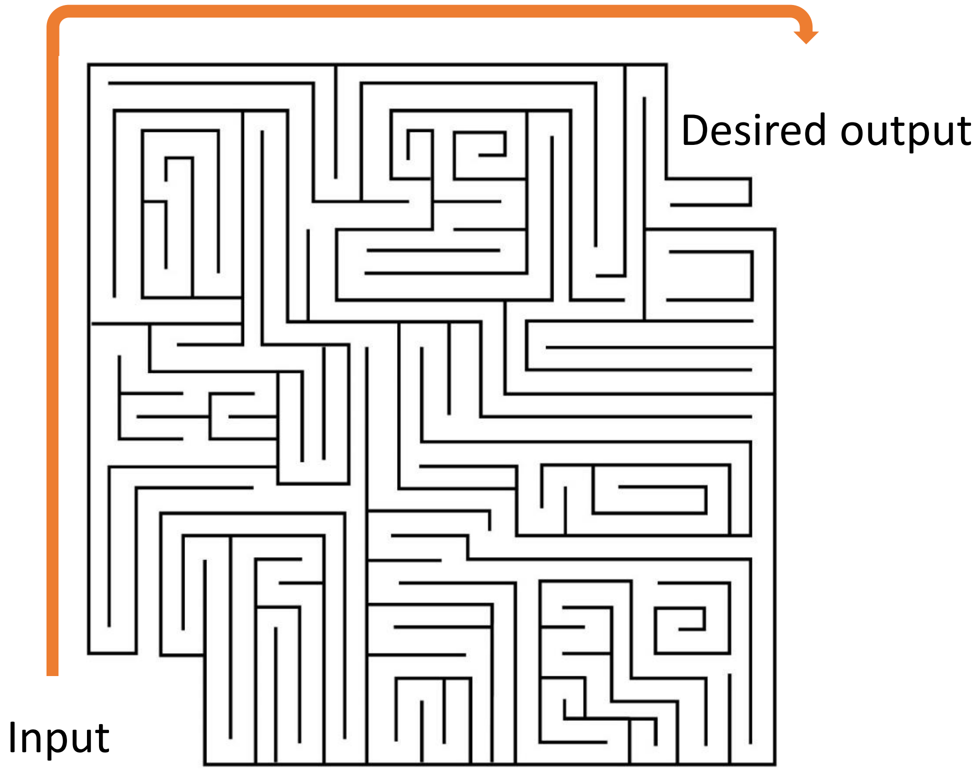
Approach 2: Not using modularity

1. Chinese whispers (Biemann et al. 2006)
2. RB (Reichardt & Bornholdt 2006)
3. Walktrap (Pons & Latapy 2006)
4. k-cut (Ruan & Zhang 2007)
5. Infomap (Rosvall & Bergstrom 2008)
6. Genetic Algorithm (Pizzuti 2008)
7. Label propagation (Cordasco & Gargano 2010)
8. CPM (Traag et al. 2011)
9. Significant scales (Traag et al. 2013)
10. Stochastic Block Model (SBM) (Peixoto 2014)
11. WCC (Prat-Pérez et al. 2014)
12. Surprise (Traag et al. 2015)
13. GemSec (Rozemberczki et al. 2019)



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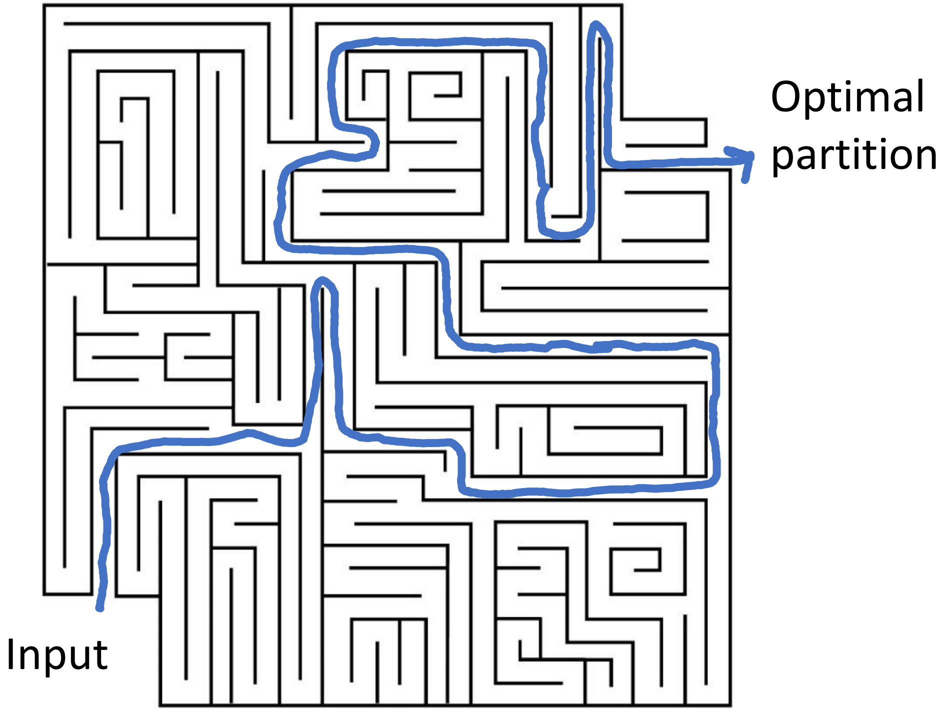
Approach 3: Exact/approximate modularity maximization

- Integer Programming - IP (Brandes et al. 2007)
- IP and LP rounding (Agarwal & Kempe 2008)
- Column generation (Aloise et al. 2010)
- Sparse IP and LP rounding (Dinh & Thai 2015)
- Approximation (Kawase et al. 2021)
- Our proposed method: Bayan (Aref et al. 2022)

$$\max_{x_{ij}} Q = \frac{1}{2m} \sum_{(i,j)\in V^2, i\leq j} b_{ij}(1 - x_{ij})$$

$$\text{s.t.} \quad x_{ik} + x_{jk} \geq x_{ij} \quad \forall k \in K(i,j) \subseteq V \setminus \{i,j\}$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in V^2, i \leq j$$



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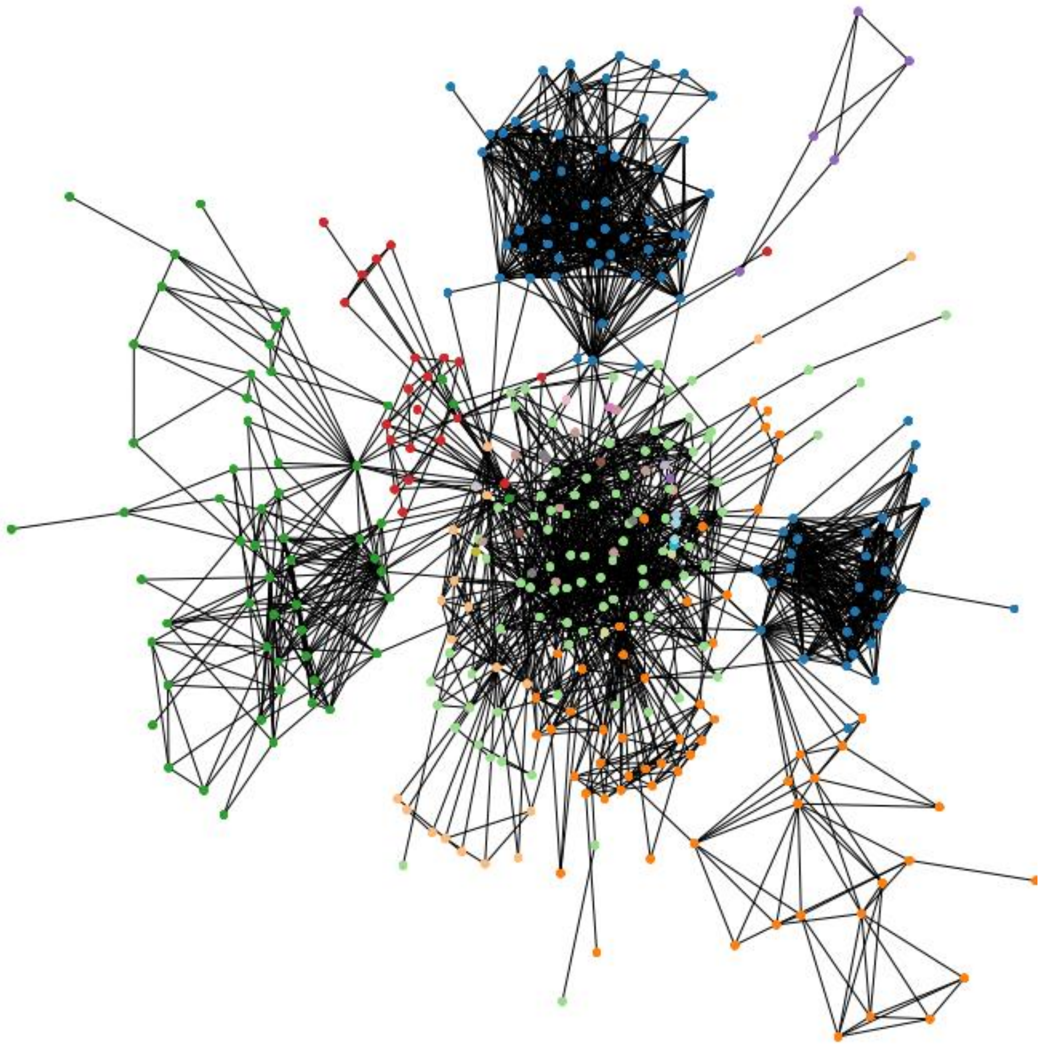
Tired of eye-balling the communities? Try Bayan.

Dataset:
facebook_friends*
m=1988

Q: modularity
k: number of communities
AMI: adjusted mutual information
(similarity to an optimal partition)



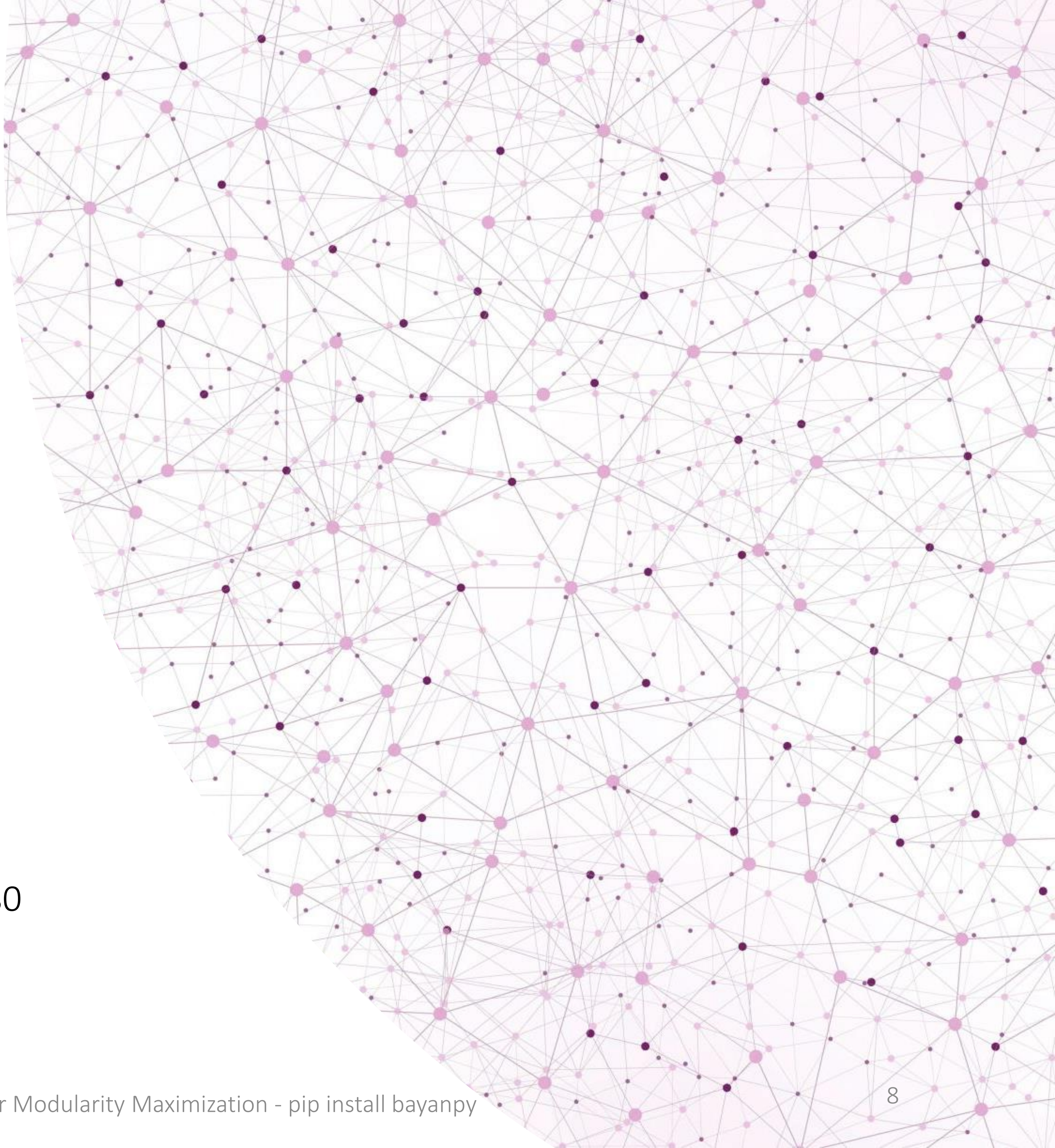
(a) Bayan, $Q^* = 0.7157714$, $k = 28$, AMI $= 1$

* Available in Netzschleuder

1

Heuristic modularity maximization algorithms rarely* maximize modularity.

*Only 16.9% of times according to our experiments on 80 networks

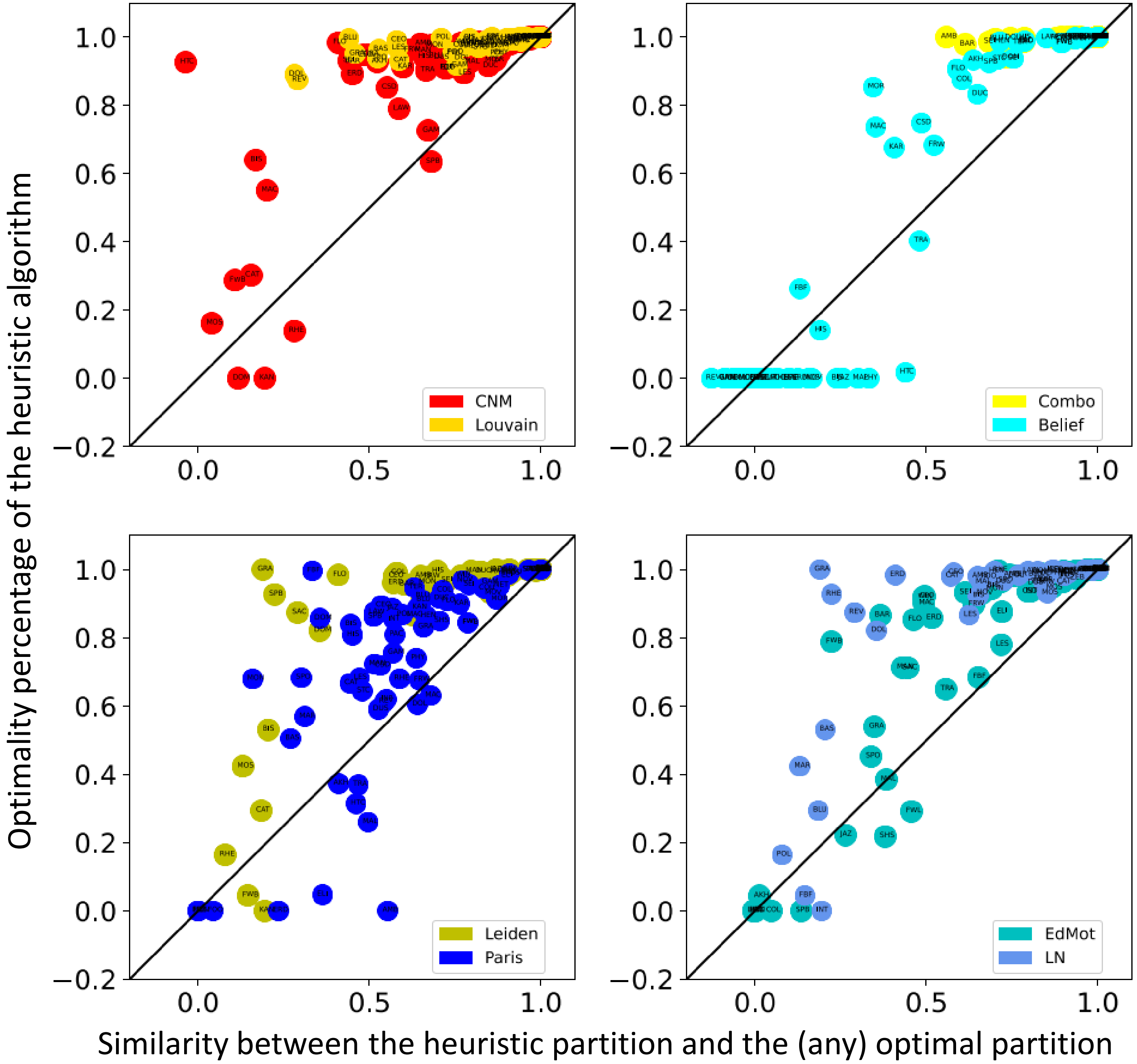


Test cases are 80 graphs with no more than 2812 edges:

- 60 real networks
- 10 Erdős–Rényi random graphs
- 10 Barabási–Albert random graphs

Each datapoint represents the performance of one algorithm on one test case.

1. Y-values: Many partitions are sub-optimal
2. X-values: Many partitions are dissimilar to any optimal partition
3. 45°-line: near-optimal partitions are not similar to any optimal partitions

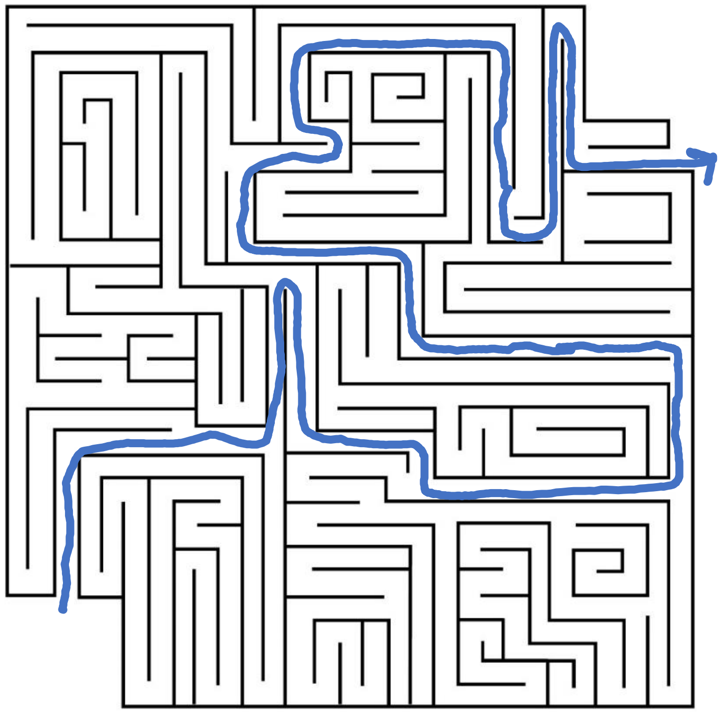
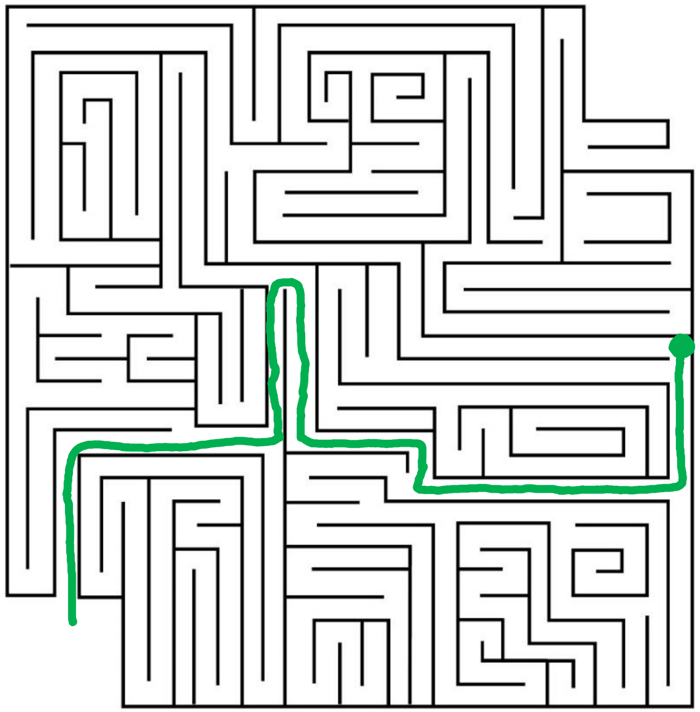
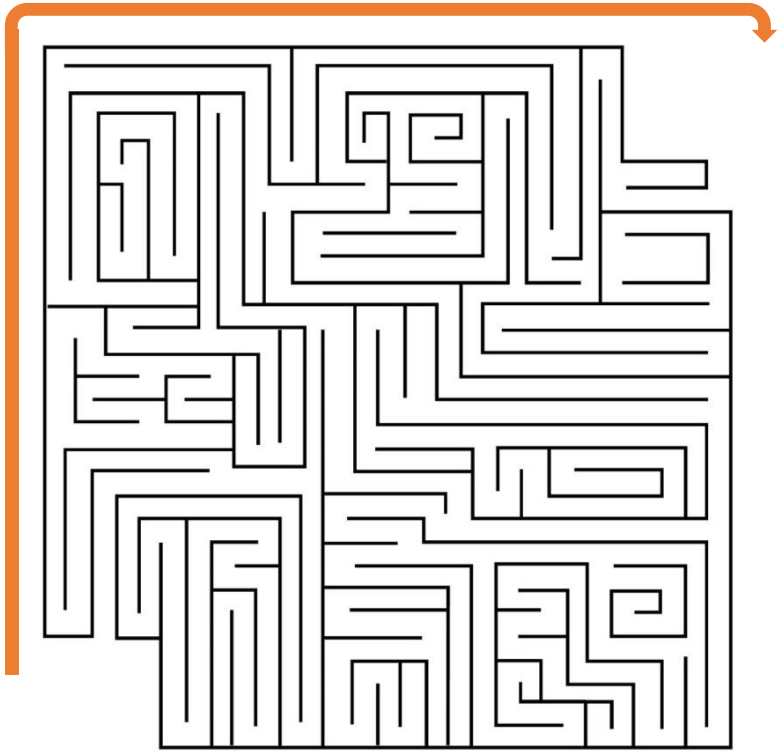


2

Suboptimal partitions of heuristic algorithms are disproportionately dissimilar to any optimal partition.

An x% suboptimality is often associated with a dissimilarity much larger than x% from any optimal partition.

Comparing Bayan with 21 other community detection algorithms

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Comparing Bayan with 21 other community detection algorithms (on random LFR graphs)

Test cases:

- 500 LFR random graphs (generated with planted communities) with up to 300 nodes
- Fraction of inter-community edges μ

$$\mu \in \{1\%, 10\%, 30\%, 50\%, 70\%\}$$

Performance measure:

- Similarity with the ground-truth communities (adjusted mutual information averaged over 100 graphs)

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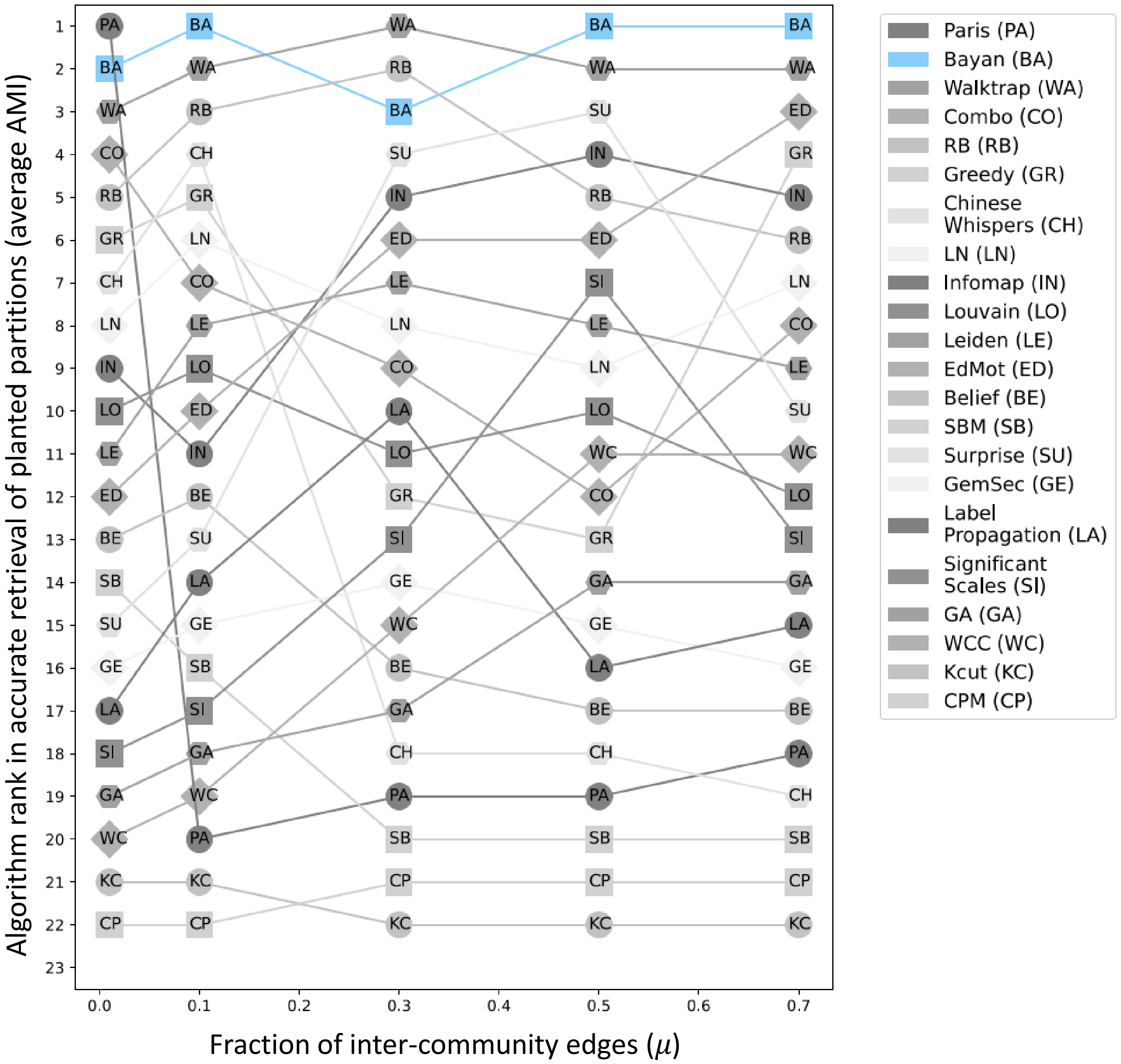

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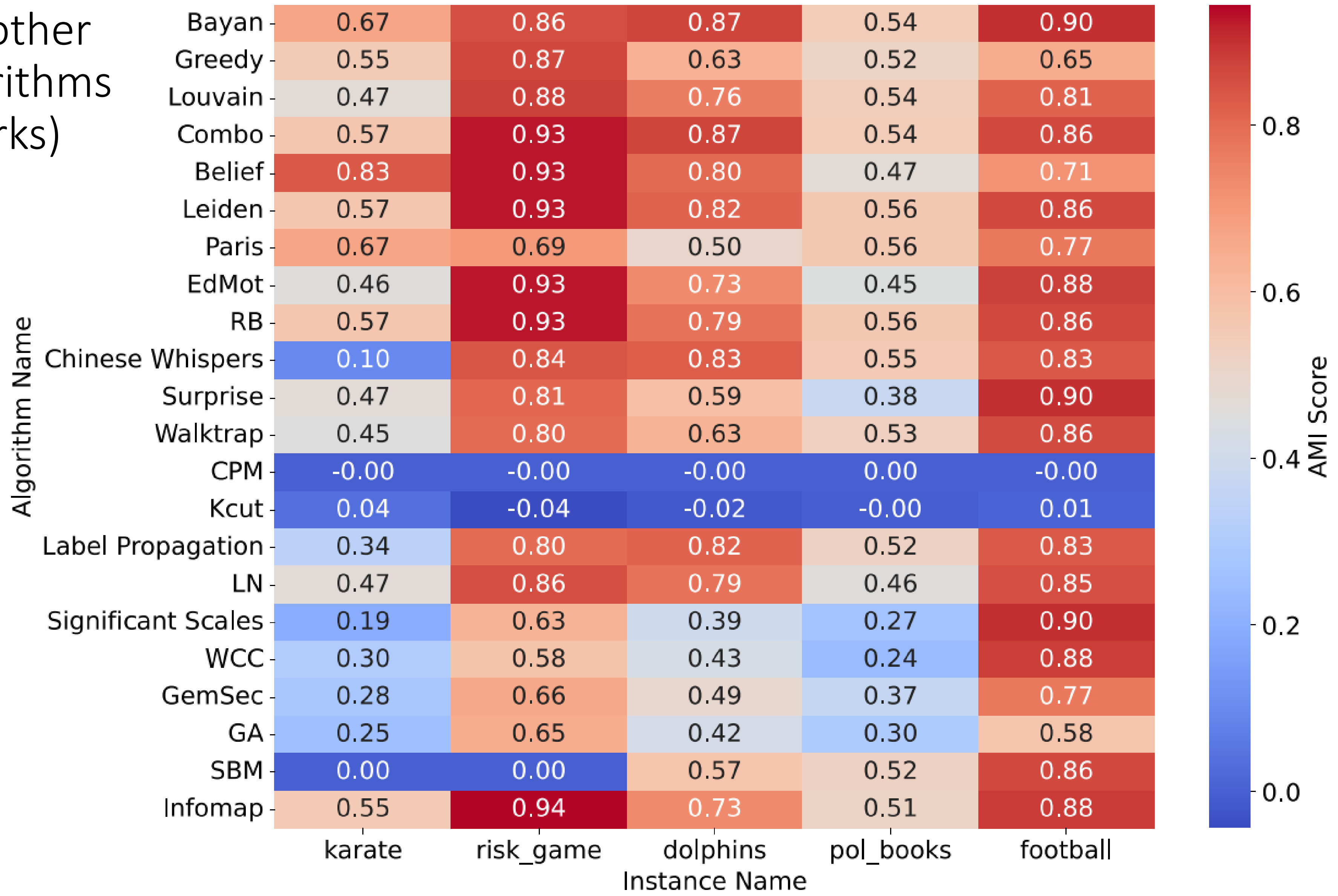
3

Bayan is more accurate than 21 other algorithms in retrieving planted communities of LFR graphs.

Bayan is several times faster than open-source and commercial solvers for maximizing modularity.

Comparing Bayan with 21 other community detection algorithms (on real network benchmarks)



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4

Bayan maximizes modularity in mid-sized* networks and approximates maximum modularity in larger networks on ordinary computers.

*up to 3000 edges in their largest connected component

You can start using Bayan in Python today!

```
%pip install bayanpy
```

```
import networkx as nx
import bayanpy
```

```
G = nx.barbell_graph(5,2)
```

```
bayanpy.bayan(G)
```

Paper 1: arxiv.org/abs/2302.14698 (ICCS'23)
Paper 2: arxiv.org/pdf/2209.04562
GitHub Repo: github.com/saref/bayan
Project website: bayanproject.github.io

Try Bayan on your smartphone ->

Google Colab examples: tinyurl.com/bayancolab

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Special thanks to my co-authors:

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Hriday Chheda
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4. Bayan maximizes modularity in mid-sized networks and approximates maximum modularity in larger networks on ordinary computers.

Try Bayan on your smartphone ->

Thank you!

Questions?

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