Using integer programming models for the structural analysis of signed networks based on balance theory

Samin Aref (University of Toronto)

Joint work with Mark Wilson, Andrew Mason, Zachary Neal, Ly Dinh, Shadi Rezapour, and Jana Diesner

Centre interuniversitaire de recherche sur les reseaux d'entreprise, la logistique et le transport (Cirrelt), Université Laval, 2025-02-21

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Outline

Background (social networks)

1. Balance in signed networks
2. How can we measure balance

Methodology (optimization)

3. How can we measure balance efficiently

Empirical Research (political science)

4. Polarization in the US Congress
5. Hidden coalitions in the US Congress

Part 1

Background (social networks)

1. **Balance in signed networks**
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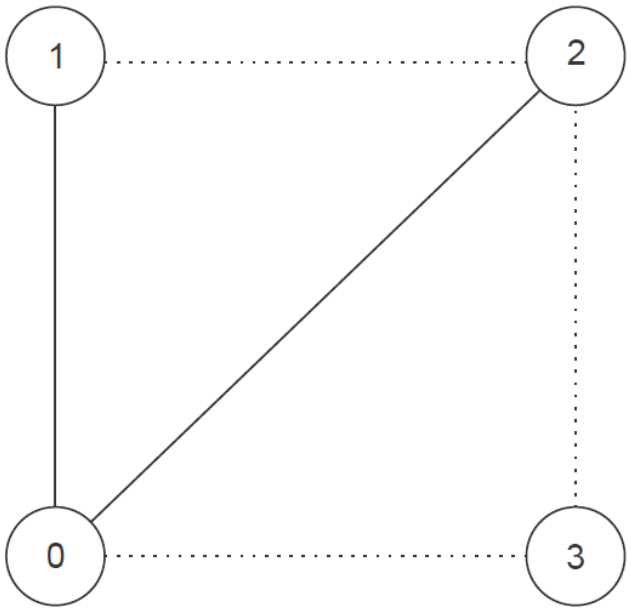
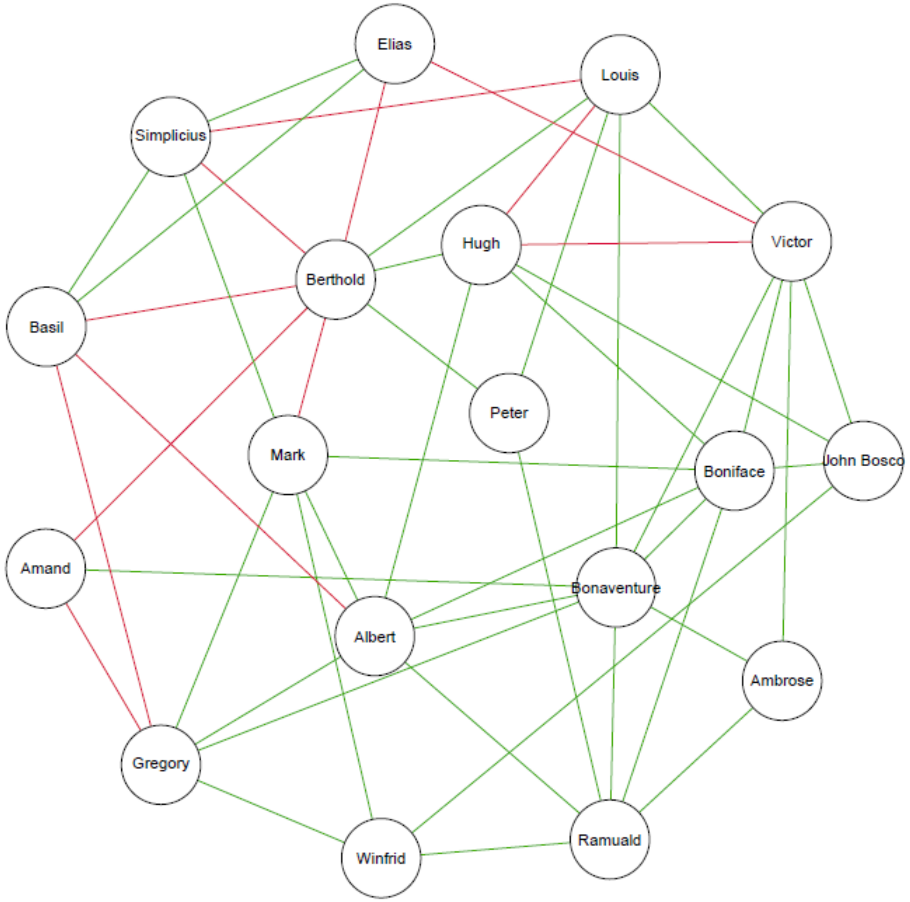
Signed network (signed graph)

Two types of edges:

-positive edges (solid line or green colour)

-negative edges (dotted line or red colour)





Theory of Structural Balance

(Heider 1944) (Cartwright and Harary 1956)

In *balanced* signed networks:

Enemy of an enemy = friend
Friend of a friend = friend
Enemy of a friend = enemy
Friend of an enemy = enemy

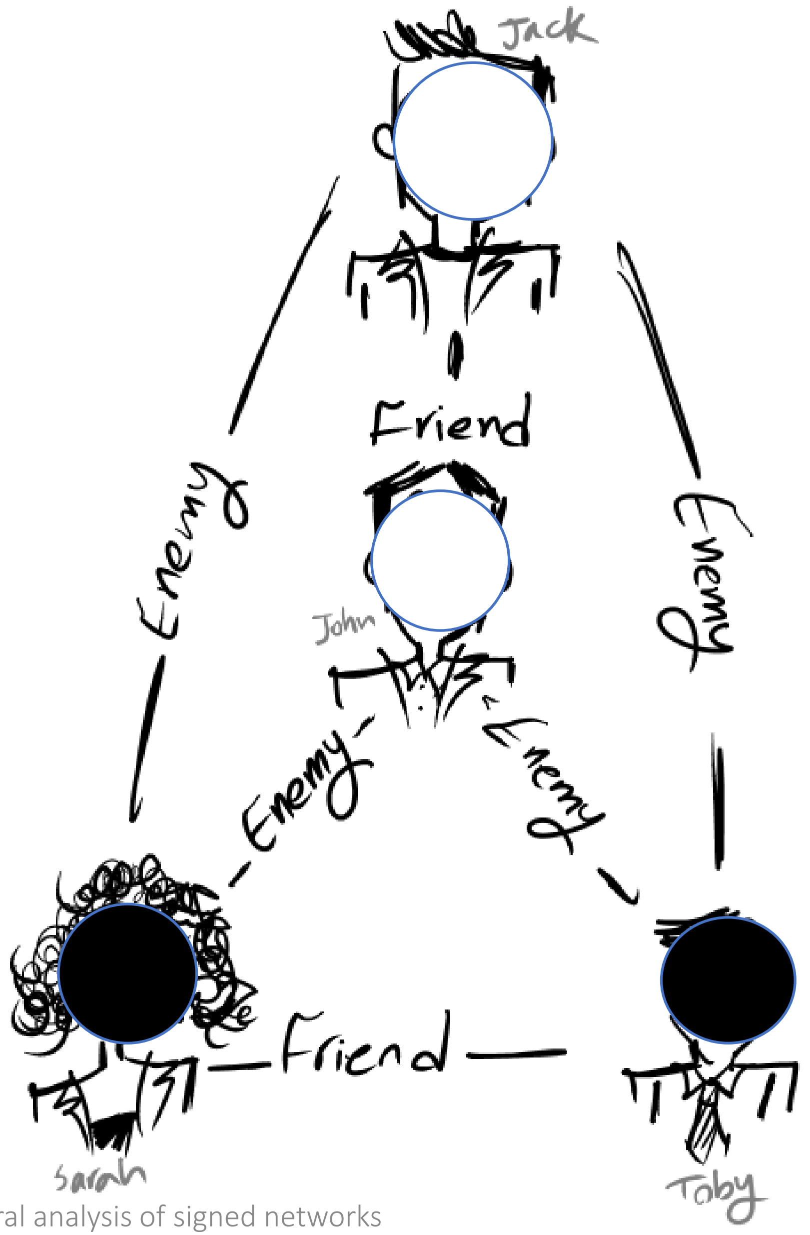
Balanced

Theory of Structural Balance

(Heider 1944) (Cartwright and Harary 1956)

In *balanced* signed networks:

Enemy of an enemy = friend
Friend of a friend = friend
Enemy of a friend = enemy
Friend of an enemy = enemy



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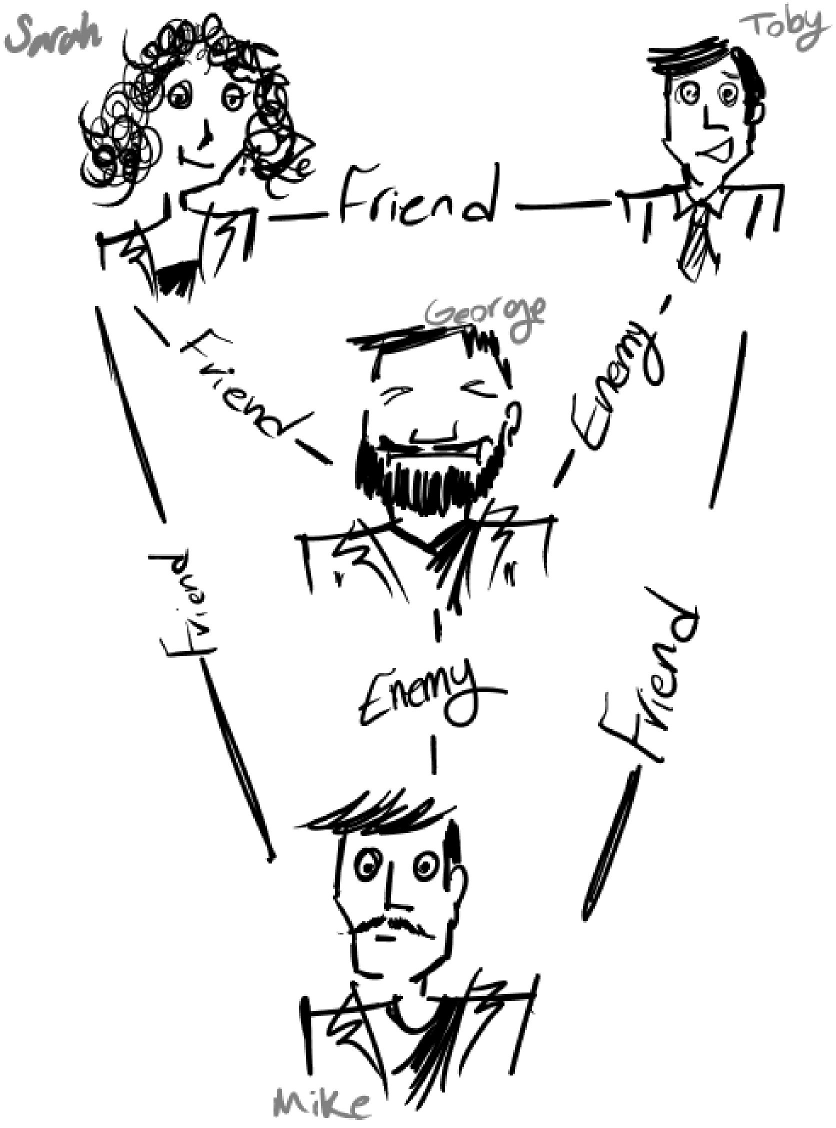
Unbalanced



Enemy of an enemy = friend ✓
Friend of a friend = friend ✗
Enemy of a friend = enemy ✗
Friend of an enemy = enemy ✗

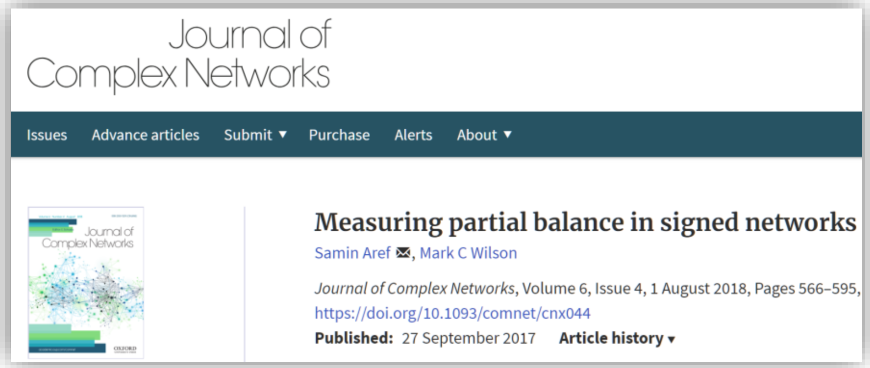
Balanced subgraph

It is 1 edge away from balance

Frustration index of a signed graph:

Minimum number of edges whose removal makes the resulting subgraph balanced.

- The *frustration index* is 1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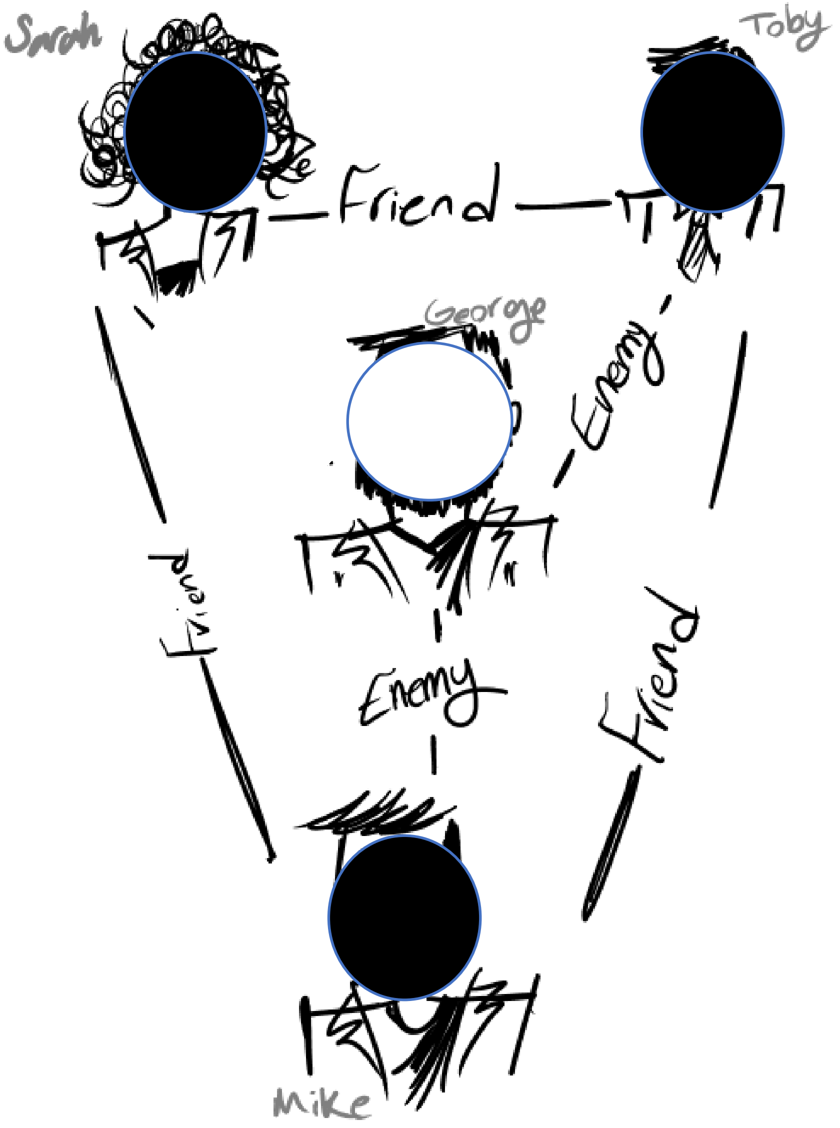
Measuring partial balance in signed networks

Samin Aref, Mark C Wilson

Journal of Complex Networks, Volume 6, Issue 4, 1 August 2018, Pages 566–595, https://doi.org/10.1093/comnet/cnx044

Published: 27 September 2017 **Article history** ▾



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Part 2

Background (social networks)

1. Balance in signed networks
2. **How can we measure balance**

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Undirected signed graph $G = (V, E, \sigma)$

$$\sigma : E \to \{-1, +1\}$$

$$|V| = n$$

$$|E| = m = m^+ + m^-$$

$$|E^+| = m^+$$

$$|E^-| = m^-$$

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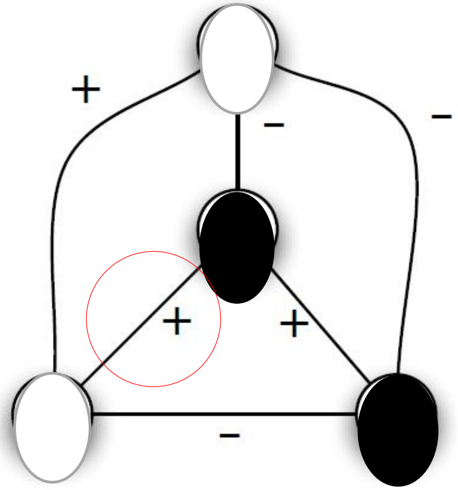
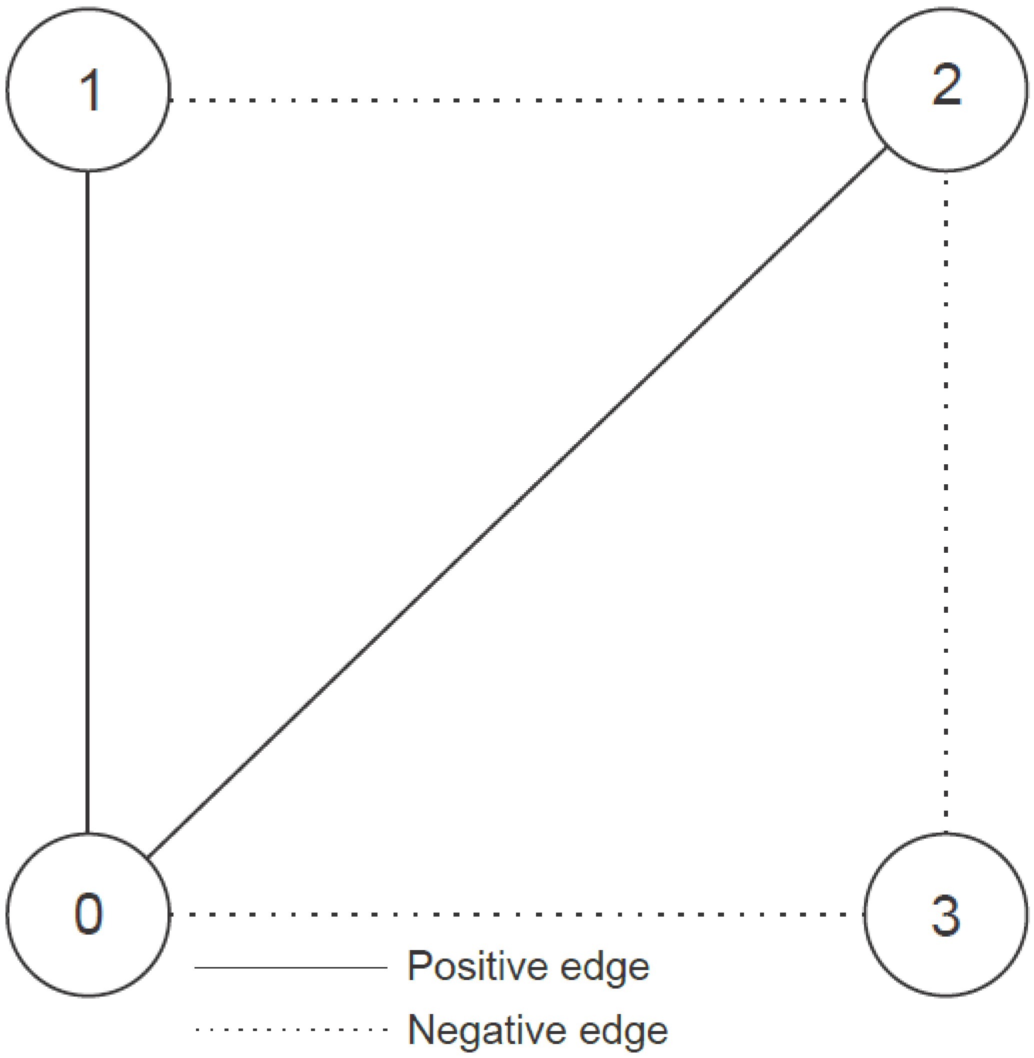
Defining frustration based on the clusters of nodes

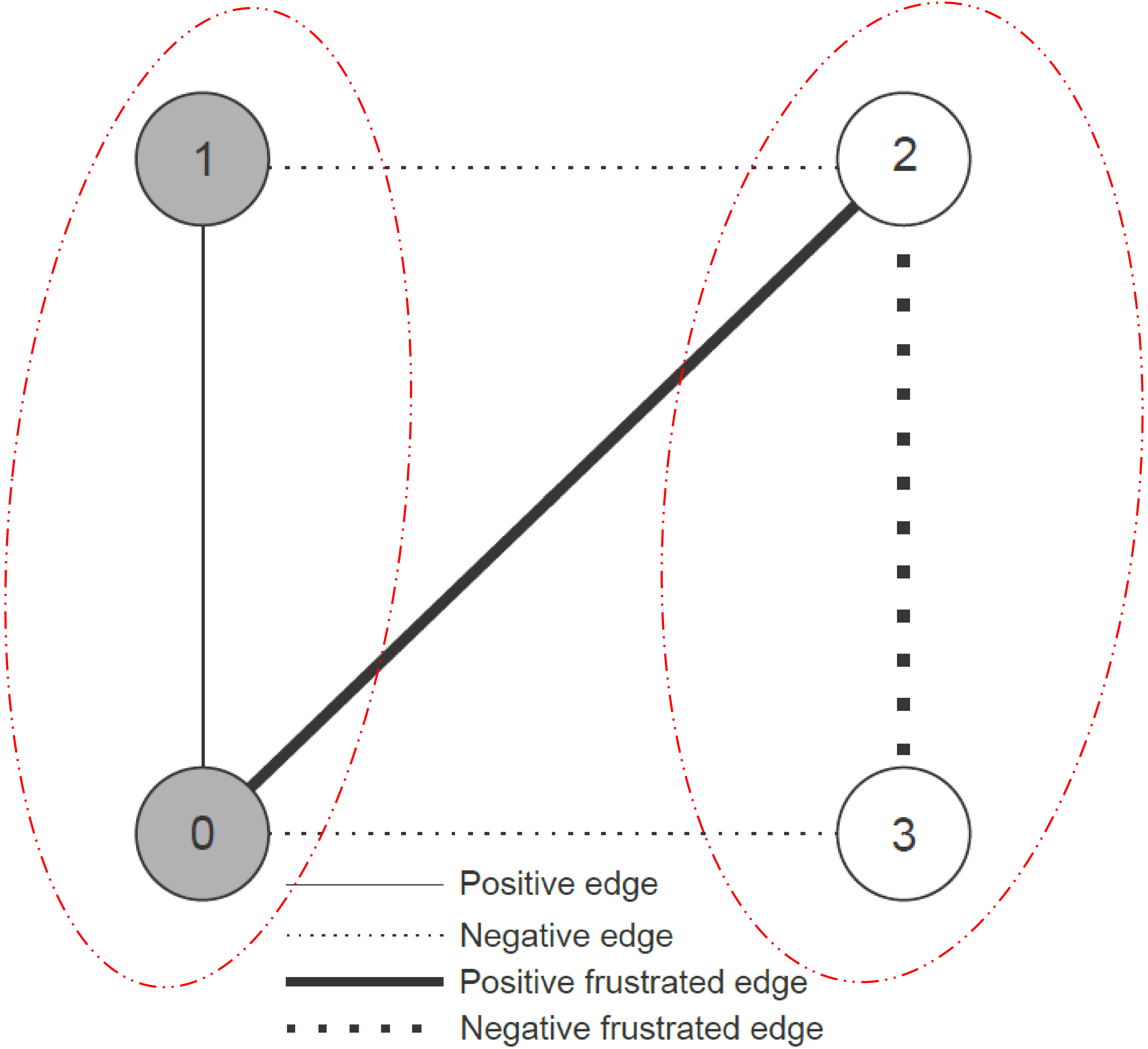
Frustrated edge:

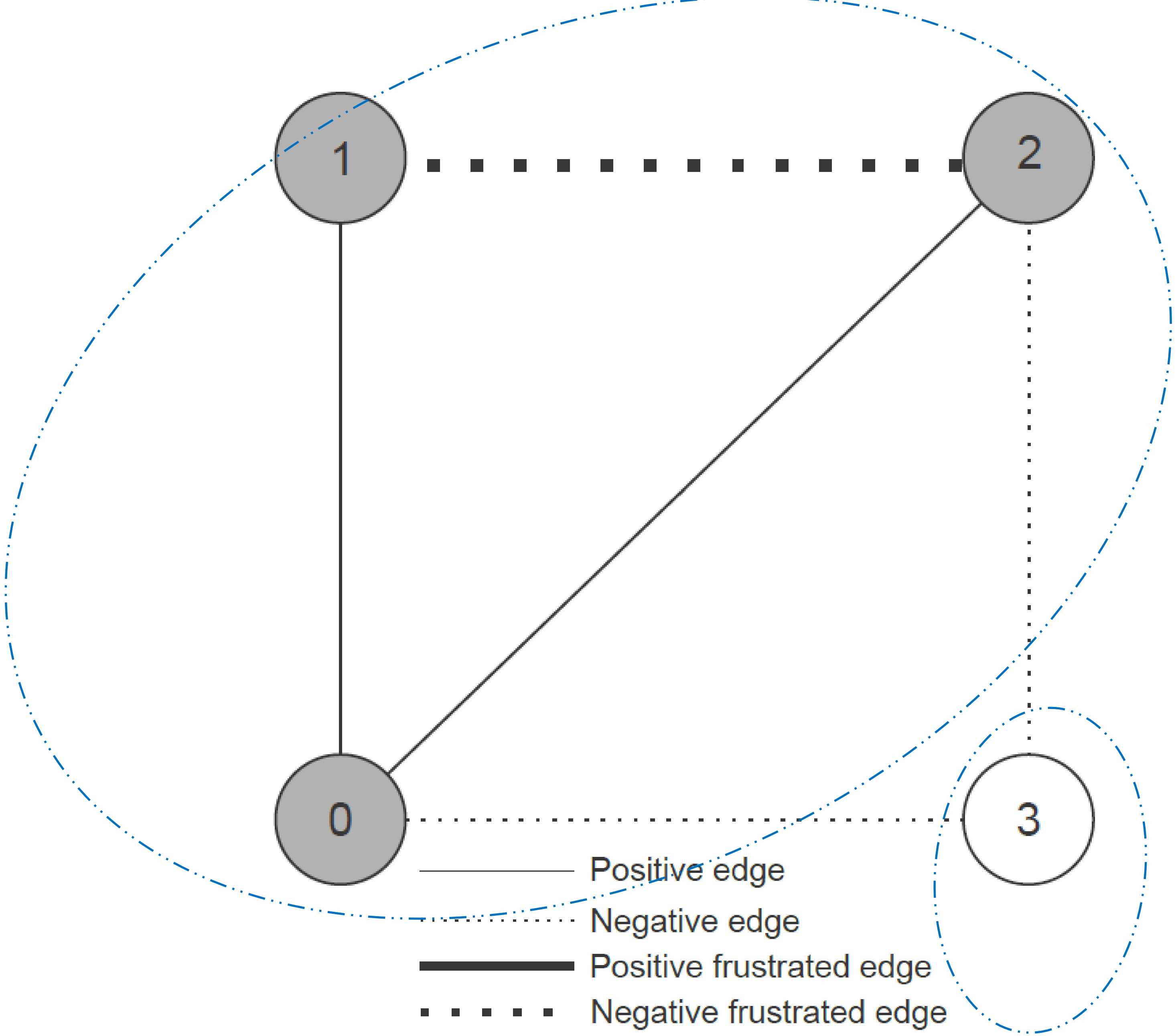
-positive edge with endpoints in different clusters

-negative edge with endpoints in the same cluster




1
2
0
3
Positive edge
Negative edge


1
2
0
3
Positive edge
Negative edge
Positive frustrated edge
Negative frustrated edge



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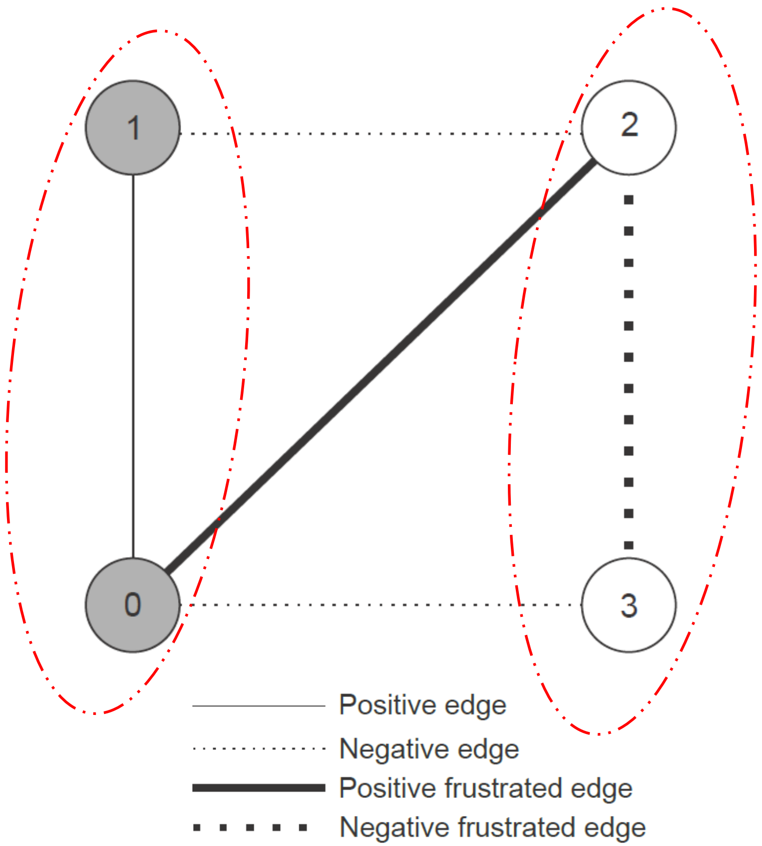
Frustration state of edge (i,j) under partition P=(X, V\X)

x_i : binary variable representing cluster of node i

$$x_i = 1 \text{ if } i \in X$$
$$x_i = 0 \text{ if } i \in V \setminus X$$

$$f_{ij}(X) = \begin{cases} 0, & \text{if } x_i = x_j \text{ and } (i,j) \in E^+ \\ 1, & \text{if } x_i = x_j \text{ and } (i,j) \in E^- \\ 0, & \text{if } x_i \neq x_j \text{ and } (i,j) \in E^- \\ 1, & \text{if } x_i \neq x_j \text{ and } (i,j) \in E^+ \end{cases}$$



Frustration count (of a graph G under partition P=(X, V\X))

$$f_G(X) := \sum_{(i,j)\in E} f_{ij}(X)$$

Frustration index (of graph G)

$$L(G) = \min_{X\subseteq V} f_G(X)$$

Minimizing the number of frustrated edges

x_i : binary variable representing cluster of node i

f_{ij} : binary variable representing the frustration of the signed edge (i,j) based on endpoint clusters

$$f_{ij} = x_i + x_j - 2x_i x_j \; \forall (i,j) \in E^+$$

$$f_{ij} = 1 - (x_i + x_j - 2x_i x_j) \; \forall (i,j) \in E^-$$

Model UBQP

$$\min_{x_i : i \in V} Z = \sum_{(i,j) \in E^+} x_i + x_j - 2x_i x_j + \sum_{(i,j) \in E^-} 1 - (x_i + x_j - 2x_i x_j)$$

$$\text{s.t.} \quad x_i \in \{0,1\} \quad \forall i \in V$$

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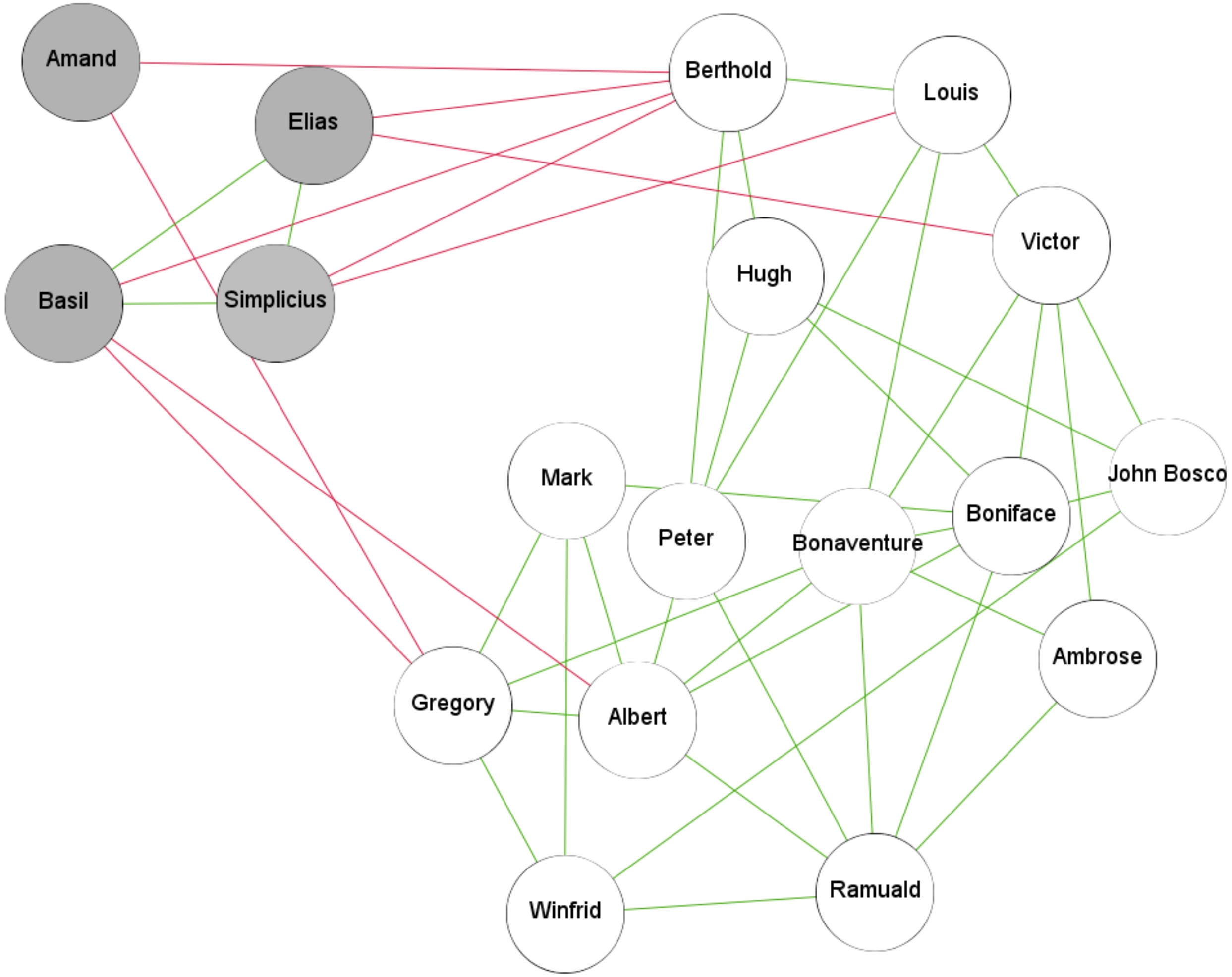
Sampson's monastery interactions

Social context

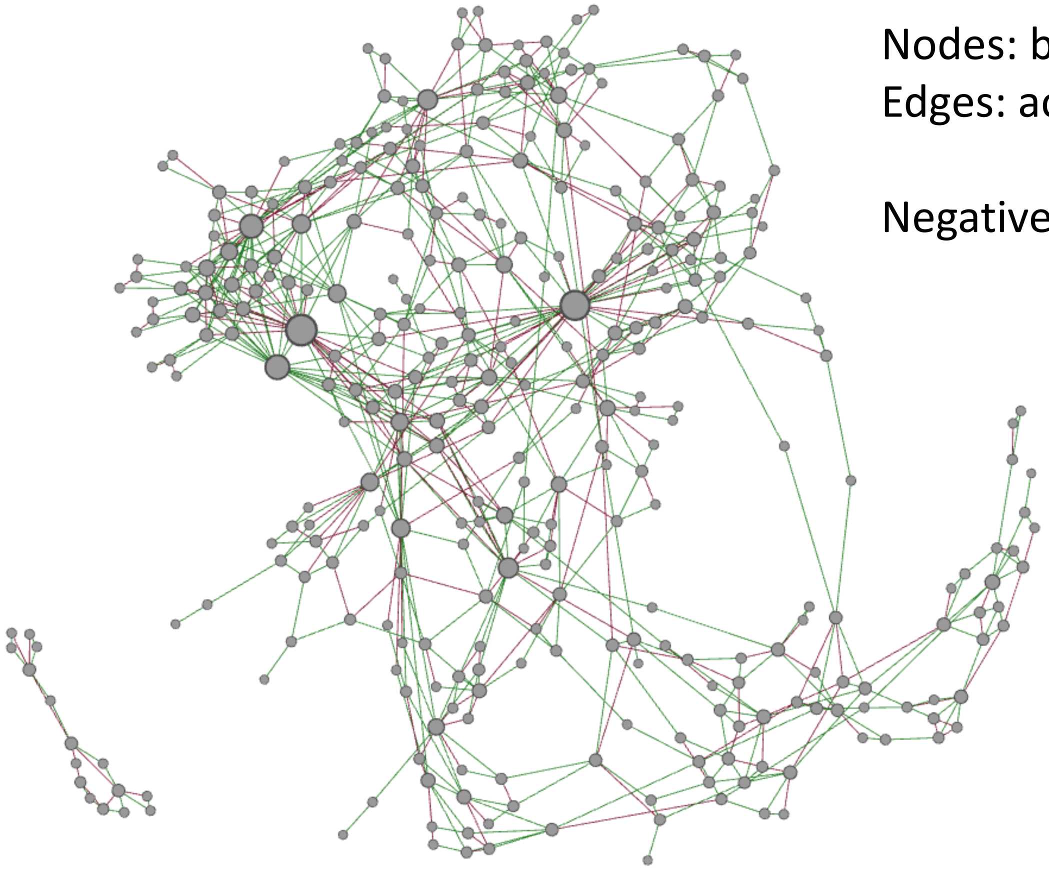
Nodes: people (monks)
Edges: top choices for like and dislike

Negative edges are shown in red



Signed network of a protein (EGFR)

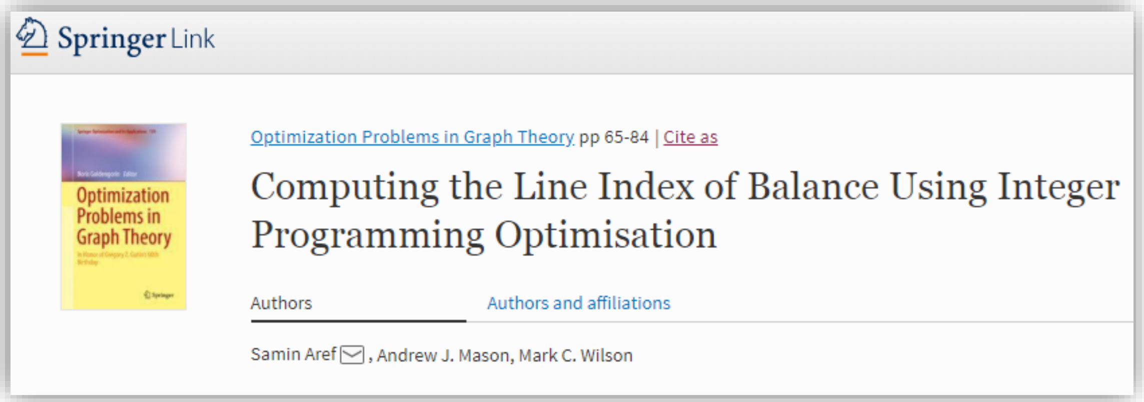
Biological context



Nodes: biological molecules
Edges: activation or inhibition relations

Negative edges are shown in red

n	m	m^-
329	779	264



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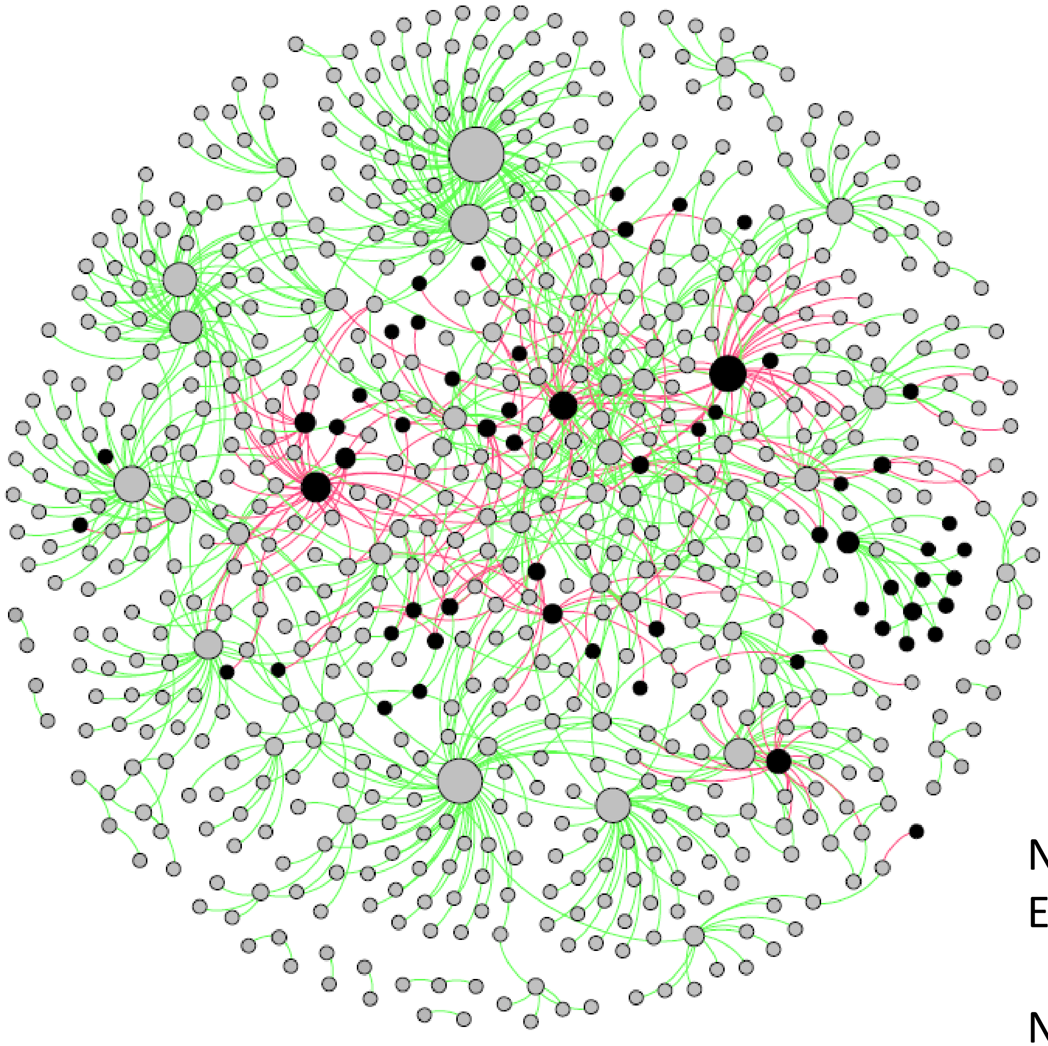
Baker's yeast

Biological context



n	m	m^-
690	1080	220

Nodes: biological molecules
Edges: activation or inhibition relations

Negative edges are shown in red

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Algorithm output for the biological networks

Author Reference	DasGupta et al. [13]	Hüffner et al. [34]	Iacono et al. [35]	Aref et al. [4]	AND	XOR	ABS
EGFR	[196, 219]*	210*	[186, 193]	193	193	193	193
Macro.	[218,383]	374*	[302, 332]	332	332	332	332
Yeast	[0, 43]	41	41	41	41	41	41
E.coli	[0, 385]	†	[365, 371]	371	371	371	371

* incorrect results

† the algorithm does not converge

(-1) x (-1) = (+1)

Transitivity of signed ties in networks

Balance is not restricted to social and biological networks though.

We observe it in certain financial networks, networks of international relations, some molecular networks, and in polarized political networks.

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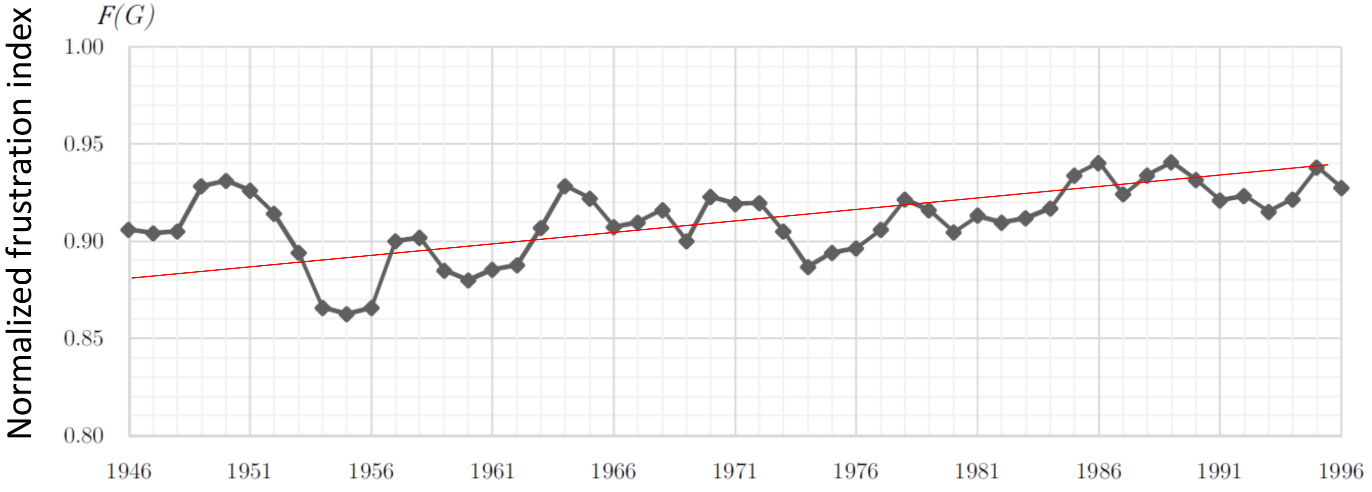
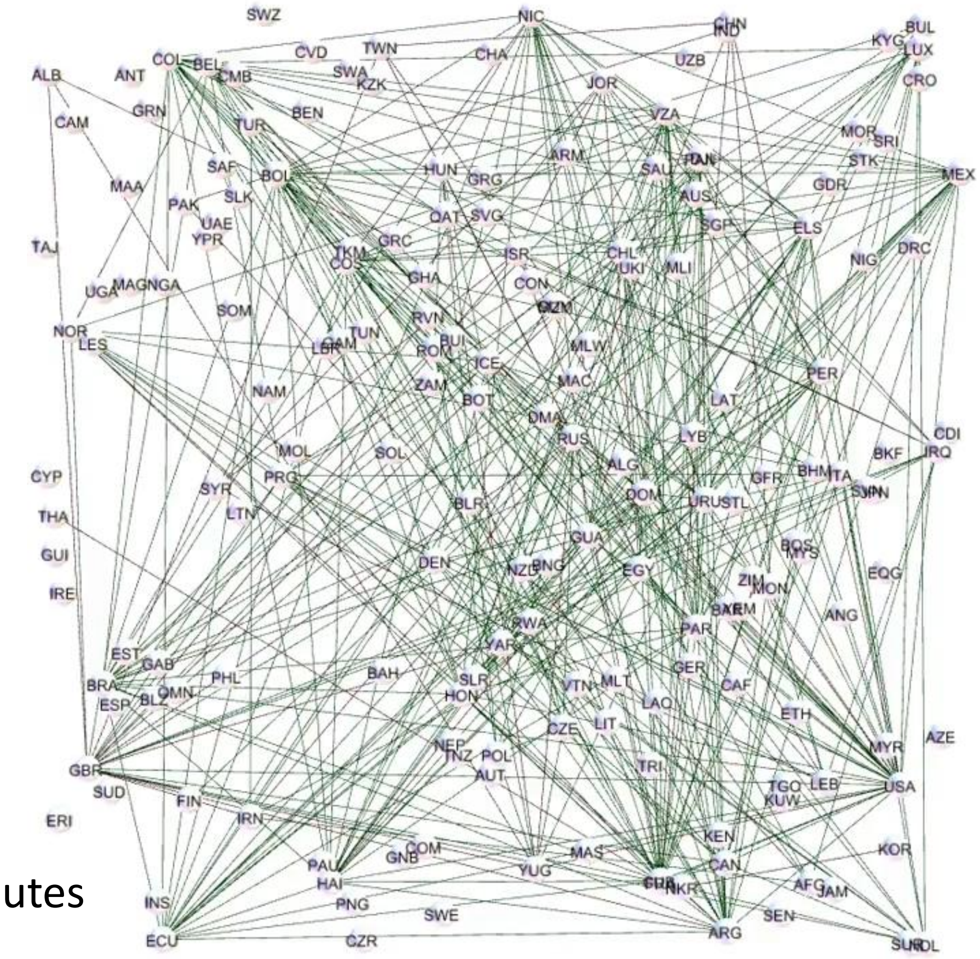
Signed international relations
Correlates of War dataset (1946-1996)

YouTube link: TinyURL.com/CoWDynamics





Nodes: Countries
Positive edges: Formal military alliances
Negative edges: Military conflicts, ideological conflicts, or border disputes

Part 3

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Linearizing the UBQP model (approach 1)

Model UBQP

$$\min_{x_i : i \in V} Z = \sum_{(i,j) \in E^+} x_i + x_j - 2x_ix_j + \sum_{(i,j) \in E^-} 1 - (x_i + x_j - 2x_ix_j)$$

$$\text{s.t.} \quad x_i \in \{0,1\} \quad \forall i \in V$$

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0,1 Integer Programming Model 1 (AND)

$$\min_{x_i: i \in V, x_{ij}:(i,j)\in E} Z = \sum_{(i,j)\in E^+} x_i + x_j - 2x_{ij} + \sum_{(i,j)\in E^-} 1 - (x_i + x_j - 2x_{ij})$$

$$\begin{aligned} \text{s.t.} \quad & x_{ij} \le x_i \quad \forall (i,j) \in E^+ \\ & x_{ij} \le x_j \quad \forall (i,j) \in E^+ \\ & x_{ij} \ge x_i + x_j - 1 \quad \forall (i,j) \in E^- \\ & x_i \in \{0,1\} \quad \forall i \in V \\ & x_{ij} \in \{0,1\} \quad \forall (i,j) \in E \end{aligned}$$

- n+m variables (binary)
  - Node variables represent node clusters
  - Edge variables represent AND of the node variables
- $2m^+ + m^-$ constraints (inequality)

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Linearizing the UBQP model (approach 2)

Model UBQP

$$\min_{x_i : i \in V} Z = \sum_{(i,j)\in E^+} x_i + x_j - 2x_i x_j + \sum_{(i,j)\in E^-} 1 - (x_i + x_j - 2x_i x_j)$$

$$\text{s.t.} \quad x_i \in \{0,1\} \quad \forall i \in V$$

Observe that $f_{ij} = \mathrm{XOR}_{(x_i, x_j)} \; \forall (i,j) \in E^+$

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0,1 Integer Programming Model 2 (XOR)

$$
\begin{aligned}
\min_{x_i: i \in V, f_{ij}:(i,j) \in E} \quad & Z = \sum_{(i,j) \in E} f_{ij} \\
\text{s.t.} \quad & f_{ij} \geq x_i - x_j \quad \forall (i,j) \in E^+ \\
& f_{ij} \geq x_j - x_i \quad \forall (i,j) \in E^+ \\
& f_{ij} \geq x_i + x_j - 1 \quad \forall (i,j) \in E^- \\
& f_{ij} \geq 1 - x_i - x_j \quad \forall (i,j) \in E^- \\
& x_i \in \{0,1\} \quad \forall i \in V \\
& f_{ij} \in \{0,1\} \quad \forall (i,j) \in E
\end{aligned}
$$

- n+m variables (binary)
  - Node variables represent node clusters
  - Edge variables represent the XOR of the node variables
- 2m constraints (inequality)

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Linearizing the model (approach 3)

$$\min_{x_i : i \in V} Z = \sum_{(i,j)\in E} f_{ij}(X)$$

$$\text{s.t.} \quad x_i \in \{0,1\} \quad \forall i \in V$$

$$f_{ij}(X) = \begin{cases} 0, & \text{if } x_i = x_j \text{ and } (i,j) \in E^+ \\ 1, & \text{if } x_i = x_j \text{ and } (i,j) \in E^- \\ 0, & \text{if } x_i \neq x_j \text{ and } (i,j) \in E^- \\ 1, & \text{if } x_i \neq x_j \text{ and } (i,j) \in E^+ \end{cases}$$

Observe that

$|x_i - x_j| = 1$ for a positive frustrated edge

$|x_i - x_j| = 0$ for a positive satisfied edge

$1 - |x_i - x_j| = |x_i + x_j - 1|$ gives the frustration state of a negative edge

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0,1 Integer Programming Model 3 (ABS)

$$\min_{x_i:i\in V, e_{ij}, h_{ij}:(i,j)\in E} Z = \sum_{(i,j)\in E} e_{ij} + h_{ij}$$

$$\begin{aligned}
\text{s.t.} \quad x_i - x_j &= e_{ij} - h_{ij} \quad \forall (i,j) \in E^+ \\
x_i + x_j - 1 &= e_{ij} - h_{ij} \quad \forall (i,j) \in E^- \\
x_i &\in \{0,1\} \quad \forall i \in V \\
e_{ij} &\in \{0,1\} \quad \forall (i,j) \in E \\
h_{ij} &\in \{0,1\} \quad \forall (i,j) \in E
\end{aligned}$$

- n+2m variables (binary)
- m constraints (equality)

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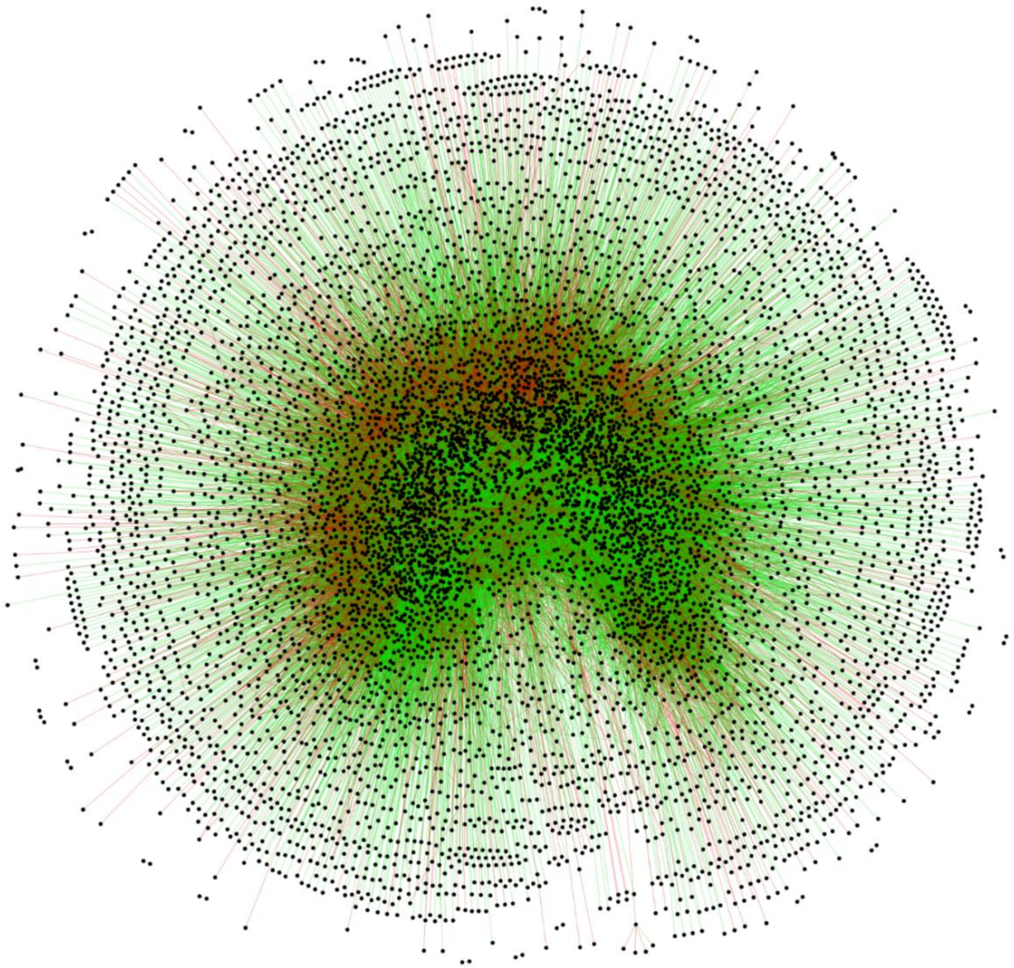
The four optimization models

	UBQP	AND	XOR	ABS
Variables	n	n+m	n+m	n+2m
Constraints	0	$2m^{+}+m^{-}$	2m	m
Variable type	binary	binary	binary	binary
Constraint type	-	linear inequality	linear inequality	linear equality
Objective function	quadratic	linear	linear	linear

Wikipedia Elections

Social context

n	m	m^{-}
7112	99917	21837



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Multilevel structural evaluation of signed directed social networks based on balance theory

Samin Aref, Ly Dinh, Rezvaneh Rezapour & Jana Diesner

Nodes: Wikipedia authors
Edges: approval or disapproval for promotion to administrator

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Simple speed-up ideas

1. Pre-processing (isolates, pendant nodes, articulation points)
2. **Branching priority and breaking symmetry**
3. **Valid inequalities (unbalanced triangles)**

2. Branching priority and breaking symmetry

- Breaking symmetry

$$x_k = 1 \quad k = \arg\max_{i \in V} d(i)$$

- Branching priority
  - Branching on node variables first, and in the decreasing order of node degrees

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3. Valid inequalities (unbalanced triangles)

$$f_{ij} + f_{ik} + f_{jk} \geq 1 \quad \forall (i, j, k) \in T^-$$

$$T^- = \{(i, j, k) \in V^3 \mid a_{ij} a_{ik} a_{jk} = -1\}$$

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Usefulness of the speed-up ideas

Table 2: Usefulness of the speed-up techniques based on 100 Erdős-Rényi graphs

	Average solve time (s)			Time improvement (%)		
	AND	XOR	ABS	AND	XOR	ABS
Without speed-up	14.80	41.60	19.71	-	-	-
With branching priority	5.90	4.91	5.50	60%	88%	72%
With triangle inequalities	9.21	31.72	17.26	38%	24%	12%
With both speed-up techniques	4.93	4.08	4.42	67%	90%	78%

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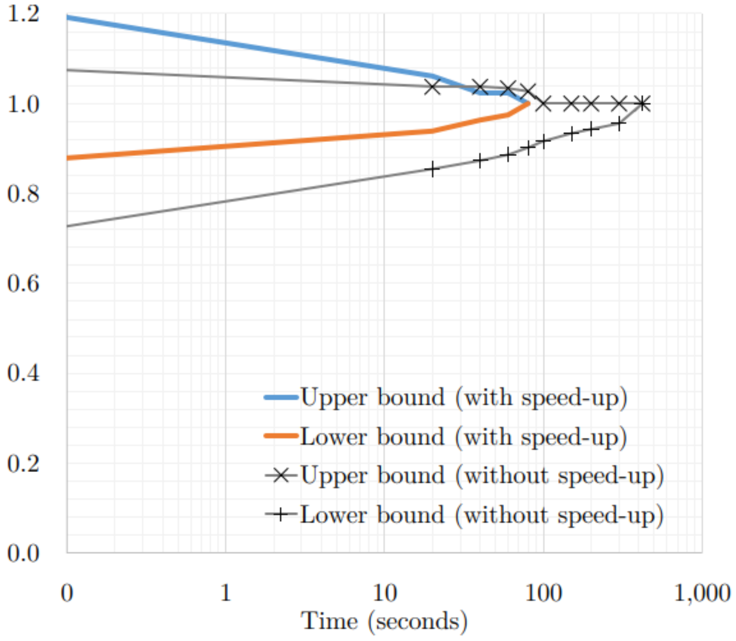
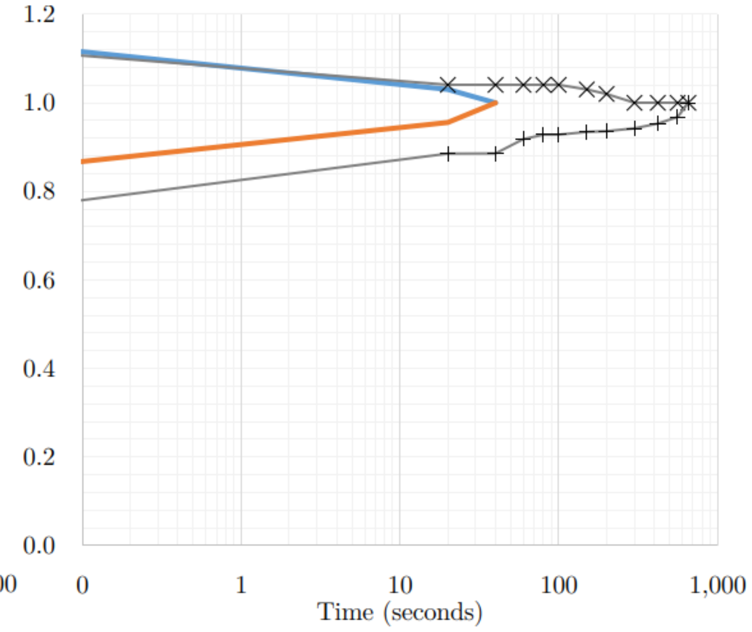
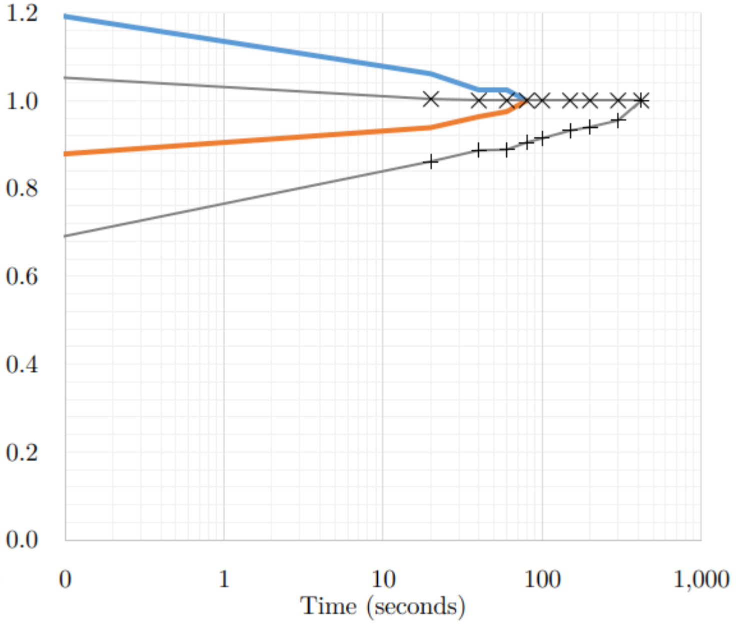
Convergence with and without speed-ups



(d) The AND model, BA graph



(e) The XOR model, BA graph



(f) The ABS model, BA graph

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Comparing the three models

Table 4: Performance measures for the three binary linear models with (and without) the speed-ups

	Graph	EGFR	Macro.	Yeast	E.coli
	n, m	329, 779	678, 1425	690, 1080	1461, 3215
Root node objective	AND	28.5	67	11.5	130.5
		(13)	(53)	(0)	(4)
	XOR	28.5	67	11.5	130.5
		(13)	(53)	(0)	(4)
	ABS	28.5	67	11.5	130.5
		(13)	(53)	(0)	(4)
Number of B&B nodes	AND	3	1	1	31
		(91)	(199)	(7)	(279)
	XOR	1	1	1	3
		(25)	(1)	(1)	(19)
	ABS	1	1	3	36
		(47)	(456)	(7)	(357)
Effective branching factor	AND	1.0010	1	1	1.0007
		(1.0041)	(1.0025)	(1.0011)	(1.0012)
	XOR	1	1	1	1.0002
		(1.0029)	(1)	(1)	(1.0006)
	ABS	1	1	1.0004	1.0006
		(1.0027)	(1.0022)	(1.0008)	(1.0010)

Algorithm output for the biological networks

Author Reference	DasGupta et al. [13]	Hüffner et al. [34]	Iacono et al. [35]	Aref et al. [4]	AND	XOR	ABS
EGFR	[196, 219]*	210*	[186, 193]	193	193	193	193
Macro.	[218,383]	374*	[302, 332]	332	332	332	332
Yeast	[0, 43]	41	41	41	41	41	41
E.coli	[0, 385]	†	[365, 371]	371	371	371	371

* incorrect results

† the algorithm does not converge

Solve time for the biological networks with and (without) the speed-up techniques

Author Reference	DasGupta et al. [13]	Hüffner et al. [34]	Iacono et al. [35]	Aref et al. [4]	AND	XOR	ABS
EGFR	420	6480	>60	0.68	0.27 (0.82)	0.21 (0.67)	0.23 (0.66)
Macro.	2640	60	>60	1.85	0.34 (1.24)	0.26 (1.37)	0.49 (1.30)
Yeast	4620	60	>60	0.33	0.18 (0.45)	0.11 (0.28)	0.15 (0.39)
E.coli	‡	†	>60	18.14	0.99 (1.91)	1.97 (4.73)	0.74 (1.86)

† the algorithm does not converge

‡ not reported

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A modeling and computational study of the frustration index in signed networks

Samin Aref, Andrew J. Mason, Mark C. Wilson

First published: 03 October 2019 | https://doi.org/10.1002/net.21907

Part 4

Background (social networks)

1. Balance in signed networks
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5. Hidden coalitions in the US Congress

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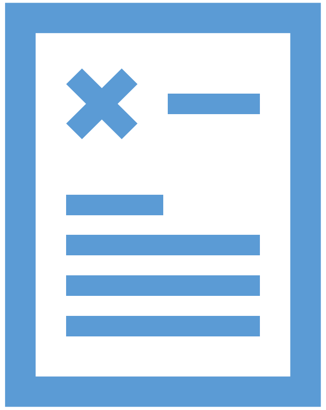
Networks of US Congress legislators
20 signed networks: one for each session (2-year period) of the Congress



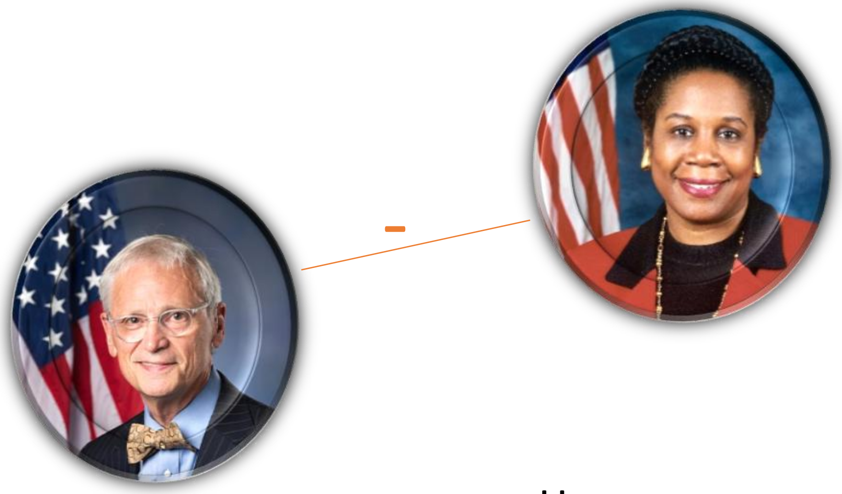
Bill co-sponsorship data

who has co-sponsored which bill

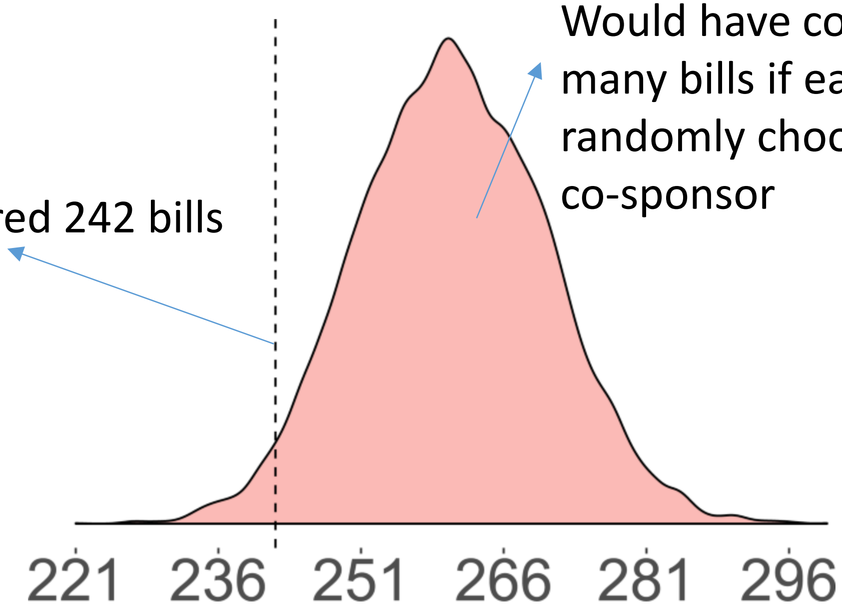
20 sessions: 1979-2017

~435 US Representatives × ~6000 bills

Inferring signed networks of US legislators



Have co-sponsored 242 bills



Would have co-sponsored these many bills if each legislator randomly chooses which bills to co-sponsor

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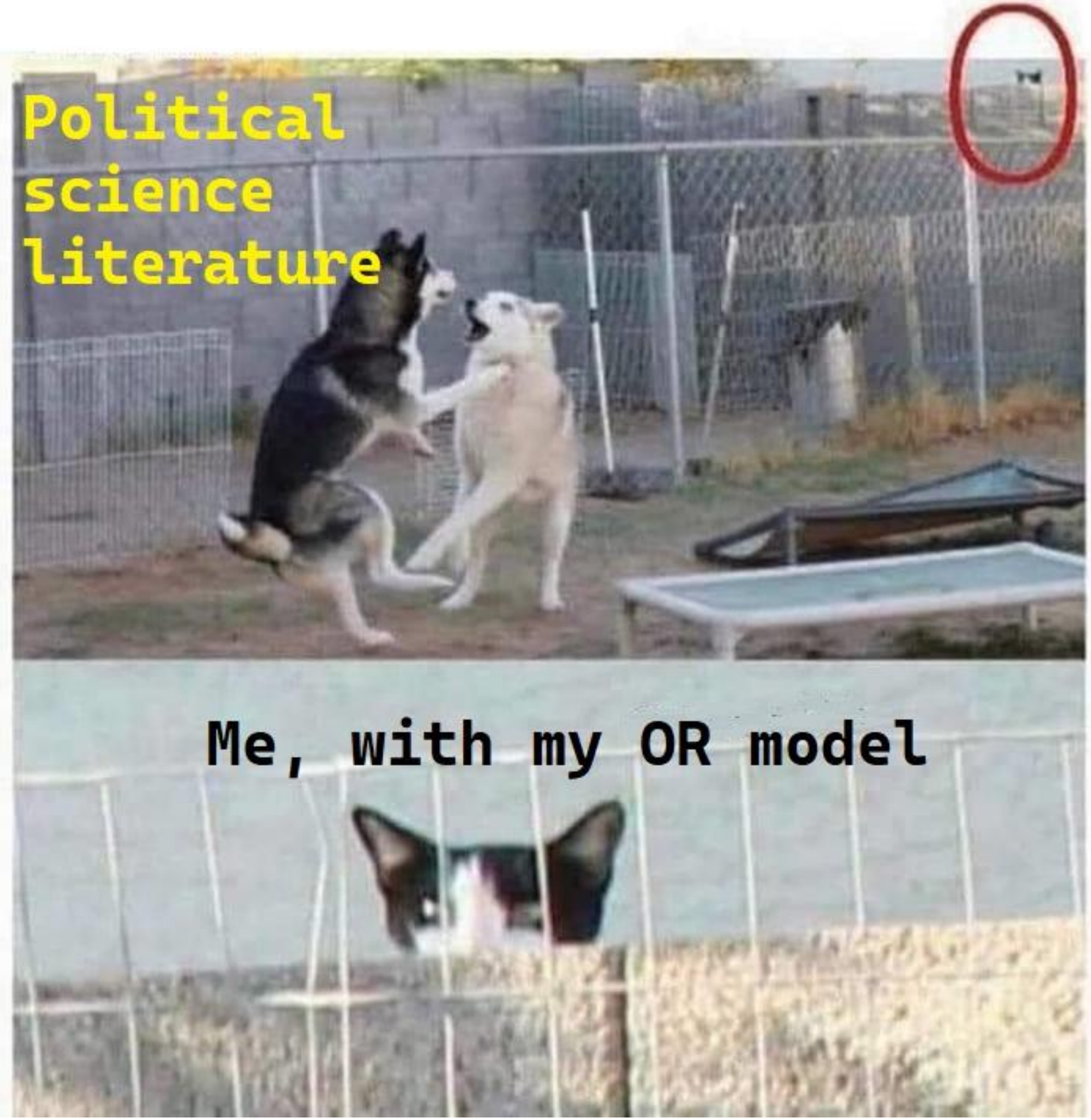

Political science literature
Me, with my OR model

US Senate network 1989-1991 (101^{st} session)

Node colors=party affiliation:

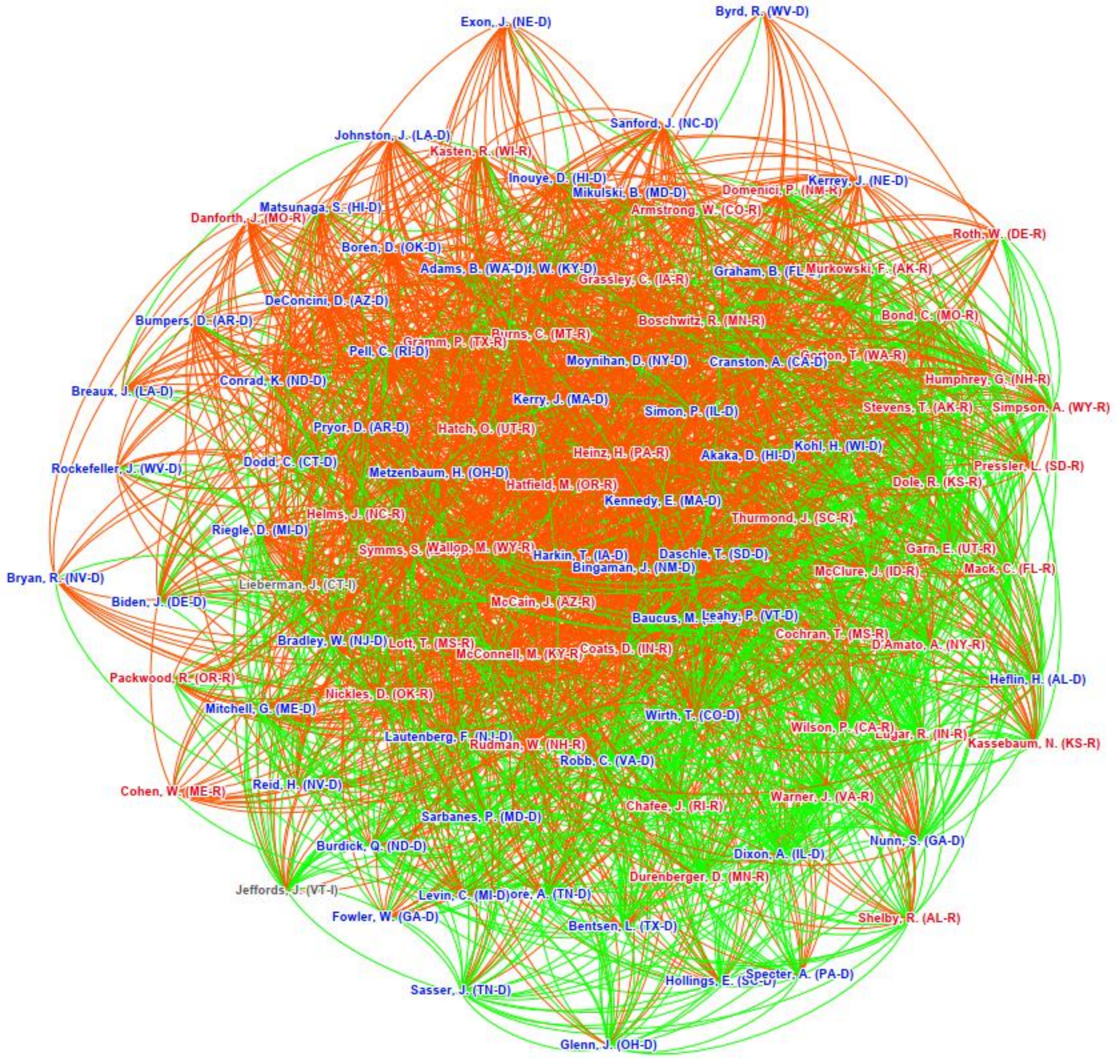
Republican

Democrat

Edge colors:

Significant collaboration

Significant lack of collaboration



1979

Node colors=party affiliation:

Republican

Democrat

Edge colors:

Significant collaboration

Significant lack of collaboration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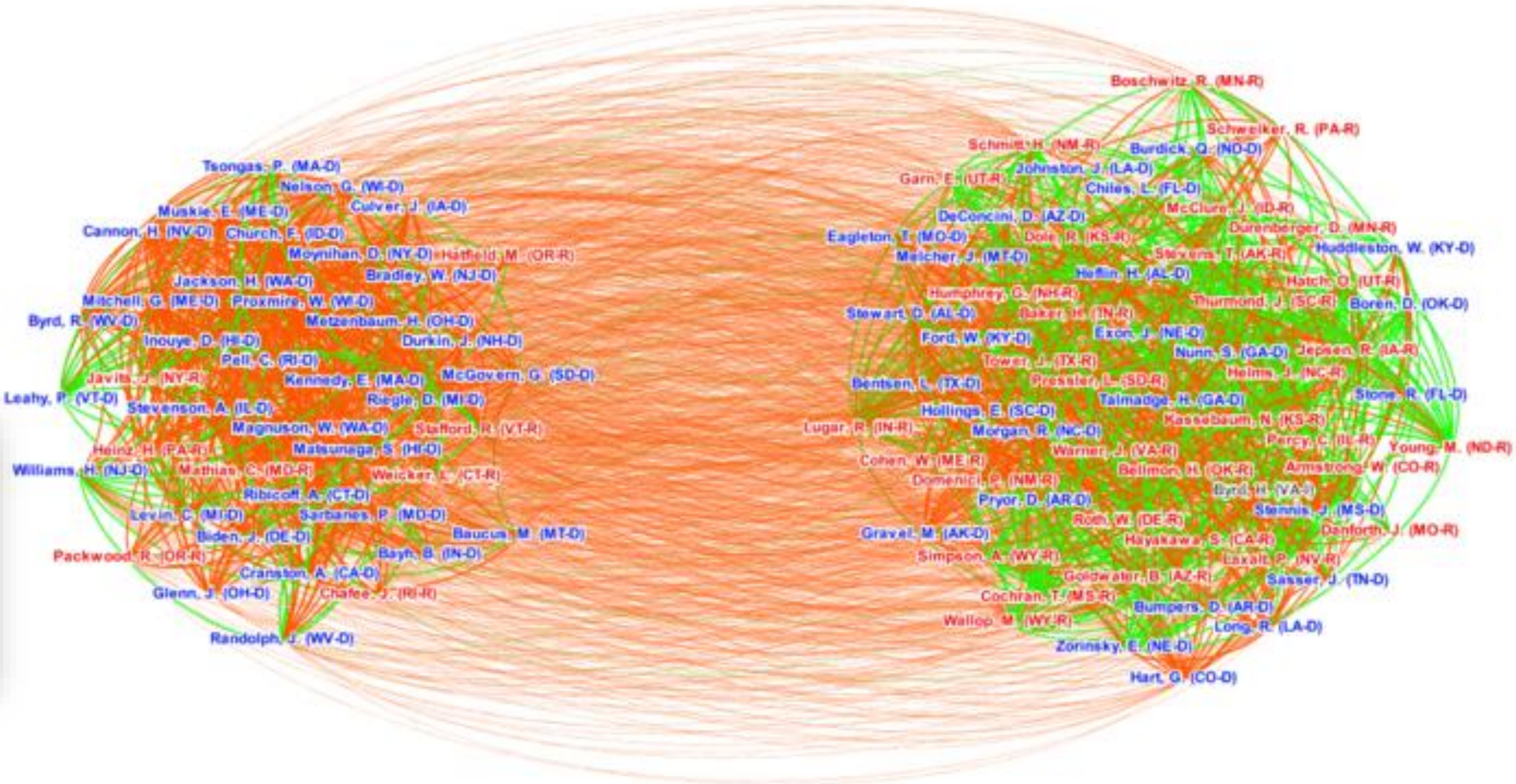
Detecting coalitions by optimally partitioning signed networks of political collaboration

Samin Aref & Zachary Neal



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Part 5

Background (social networks)

1. Balance in signed networks
2. How can we measure balance

Methodology (optimization)

3. How can we measure balance efficiently

Empirical Research (political science)

4. Polarization in the US Congress
5. Hidden coalitions in the US Congress

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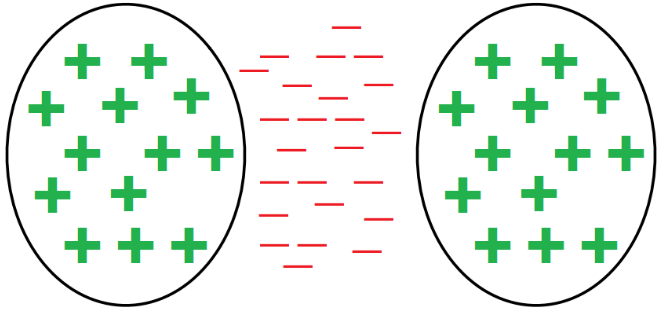
Classic balance vs. generalized balance

Structurally balanced if and only if:

The network can be partitioned into 2 clusters with positive edges being within clusters and negative edges being between clusters

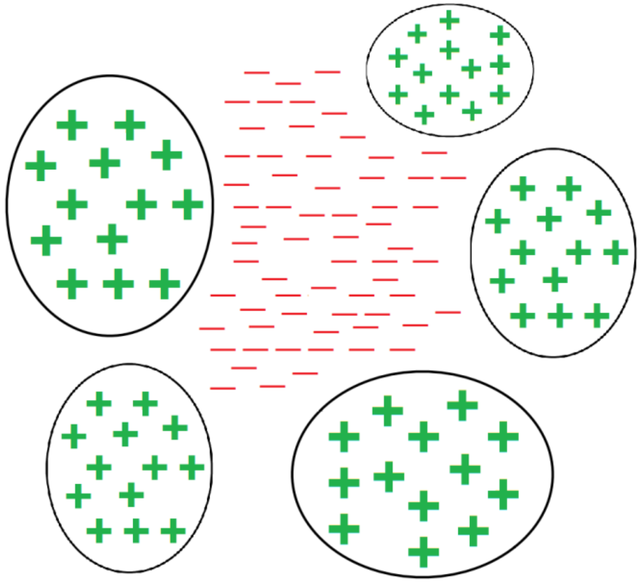
(Cartwright-Harary 1956)



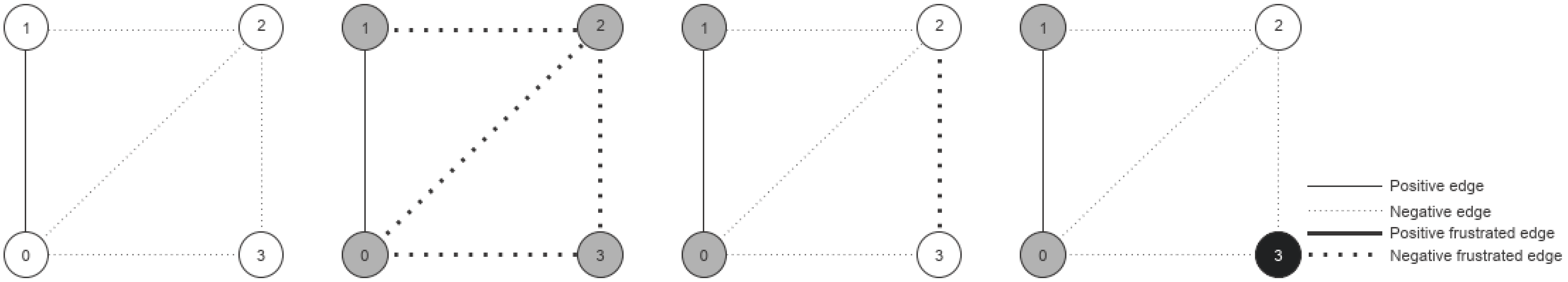
Weakly balanced if and only if:

The network can be partitioned into k clusters with positive edges being within clusters and negative edges being between clusters

(Davis 1967)



Generalized balance: more clusters are allowed



(a) An example graph with $n = 4$, $m^- = 4$, $m^+ = 1$

(b) One colour resulting in four frustrated edges

(c) Two colours resulting in one frustrated edge

(d) Three colours resulting in no frustrated edge

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Model for k-partitioning (requires k=|C|)

$$\min \sum_{(i,j)\in E} f_{ij}$$

$$\text{s.t.} \quad \sum_{c\in C} x_{ic} = 1 \quad \forall i \in V$$

$$f_{ij} \geq x_{ic} - x_{jc} \quad \forall (i,j) \in E^+, \forall c \in C$$

$$f_{ij} \geq x_{ic} + x_{jc} - 1 \quad \forall (i,j) \in E^-, \forall c \in C$$

$$x_{ic} \in \{0,1\} \quad \forall i \in V, \forall c \in C$$

$$f_{ij} \in \{0,1\} \quad \forall (i,j) \in E$$

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Model for partitioning (does not require *k*)

$$\min \sum_{(i,j)\in E} a_{ij}((a_{ij}+1)/2) - a_{ij}y_{ij}$$

$$\text{s.t.} \quad y_{ij} + y_{ik} \geq 1 + y_{jk} \quad \forall (i,j,k) \in T$$

$$y_{ij} + y_{jk} \geq 1 + y_{ik} \quad \forall (i,j,k) \in T$$

$$y_{ik} + y_{jk} \geq 1 + y_{ij} \quad \forall (i,j,k) \in T$$

$$y_{ij} \in \{0,1\} \quad \forall i \in V,\ j \in V,\ i < j$$

The ***correlation clustering*** model by Demaine et al. (2006) with minor modifications for non-complete graphs

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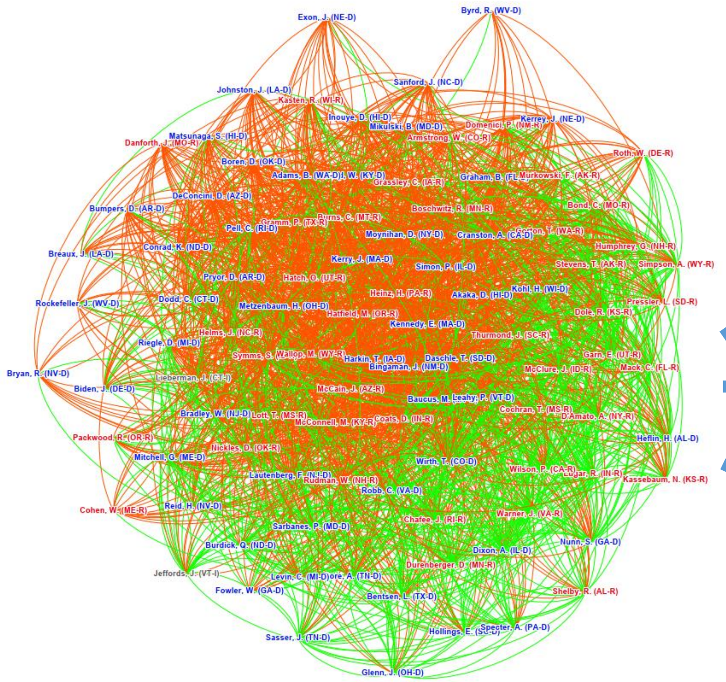
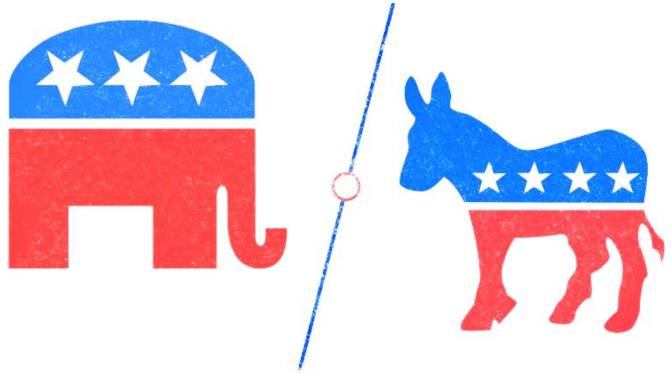
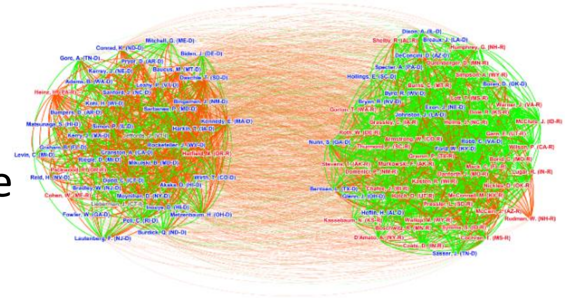


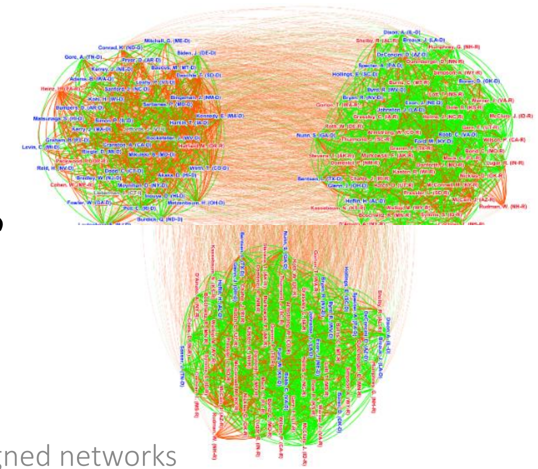
Partition legislators by party



Partition legislators by classic balance



Partition legislators by generalized balance



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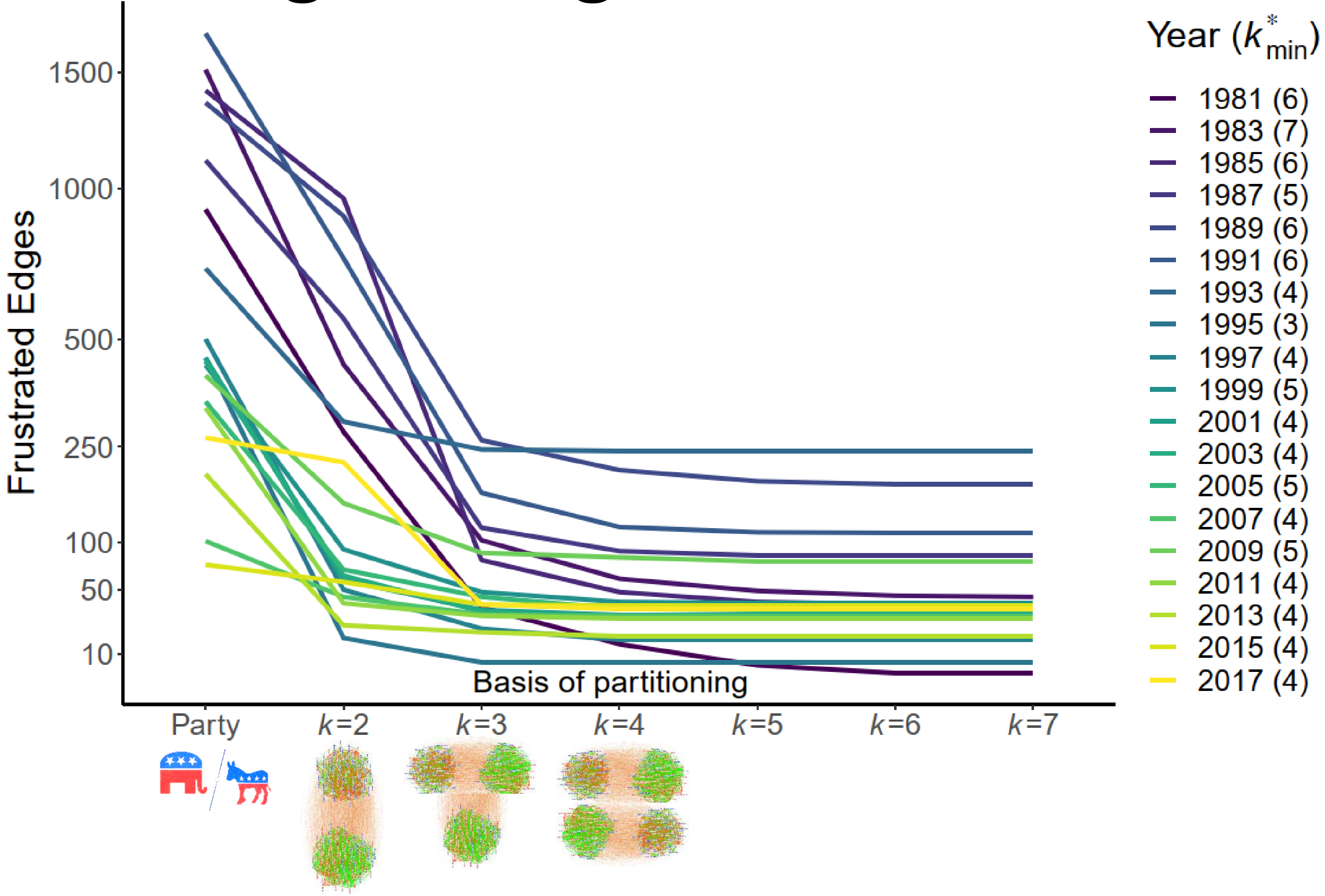
Partitioning the legislators into clusters



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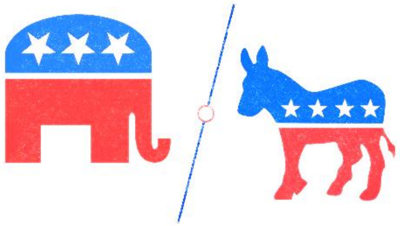
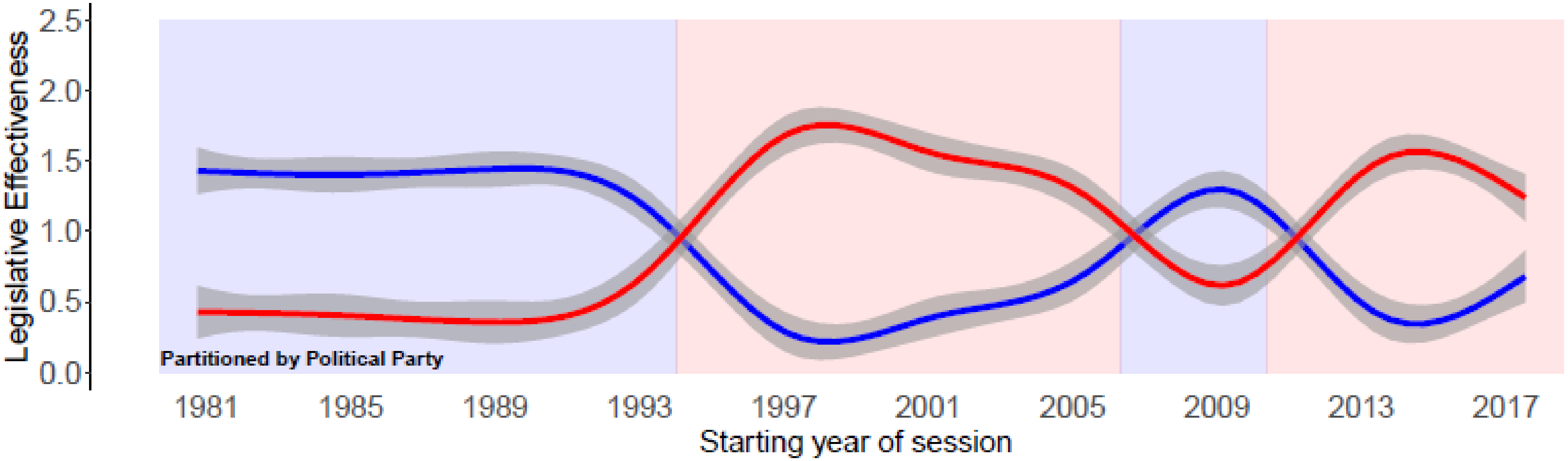
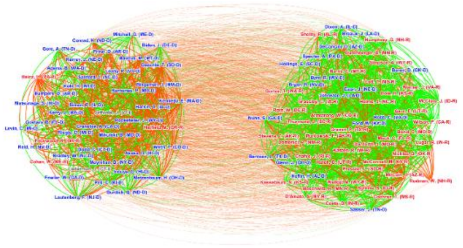
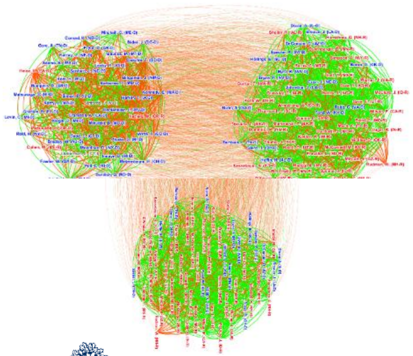
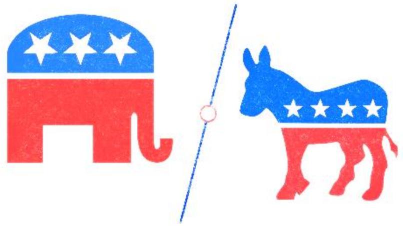


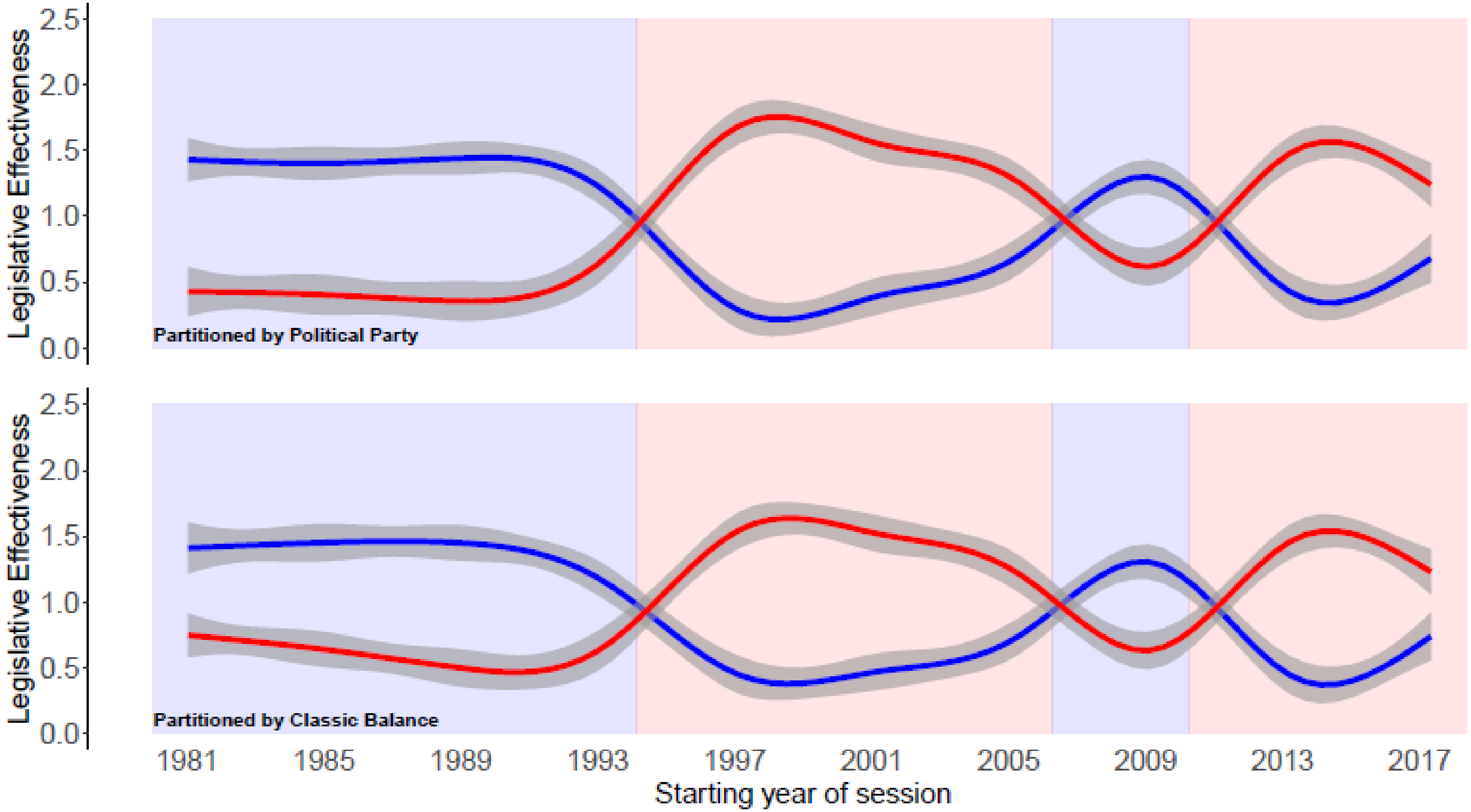


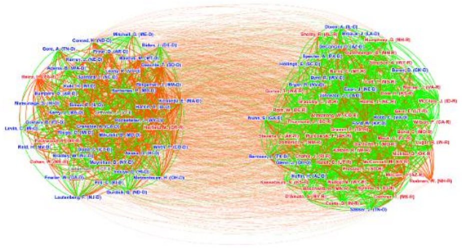
Legislative Effectiveness
2.5
2.0
1.5
1.0
0.5
0.0
Partitioned by Political Party
1981
1985
1989
1993
1997
2001
2005
2009
2013
2017
Starting year of session

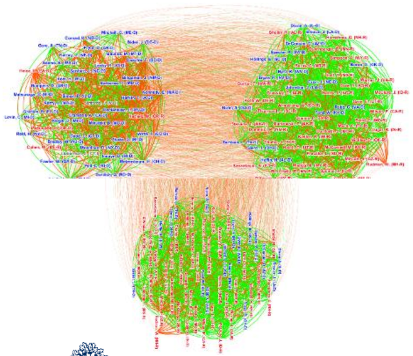












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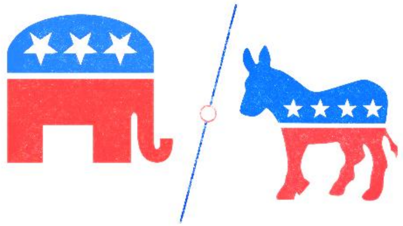

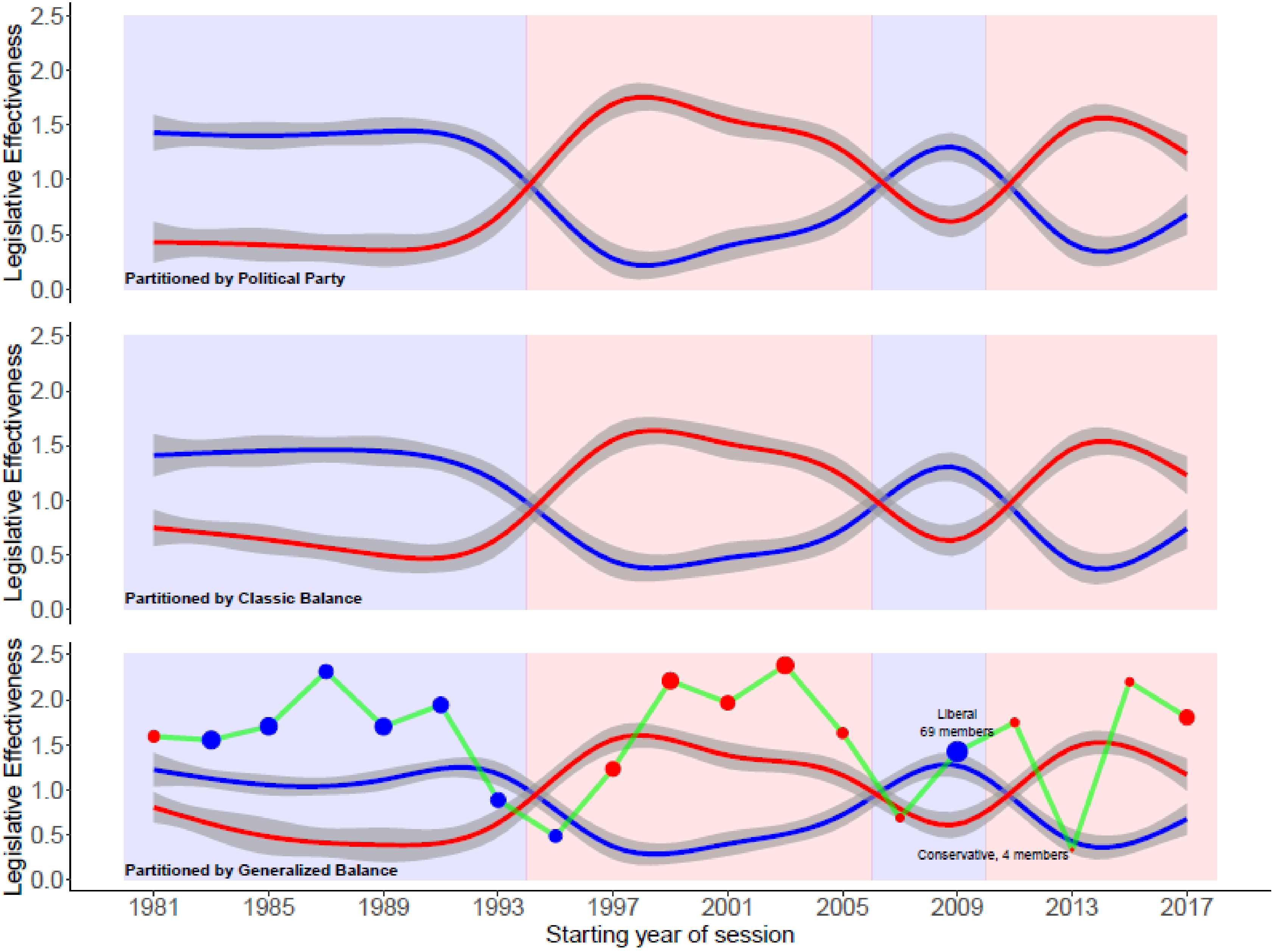

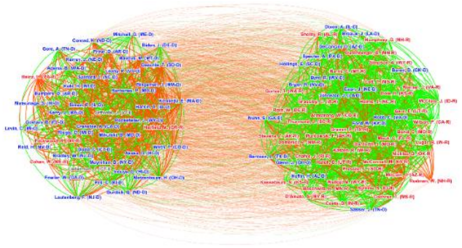

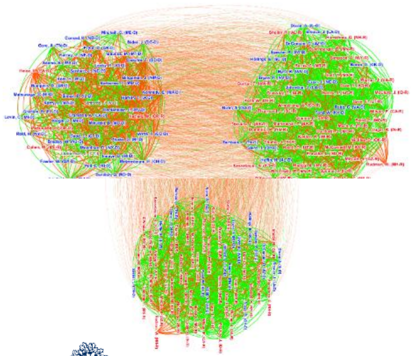

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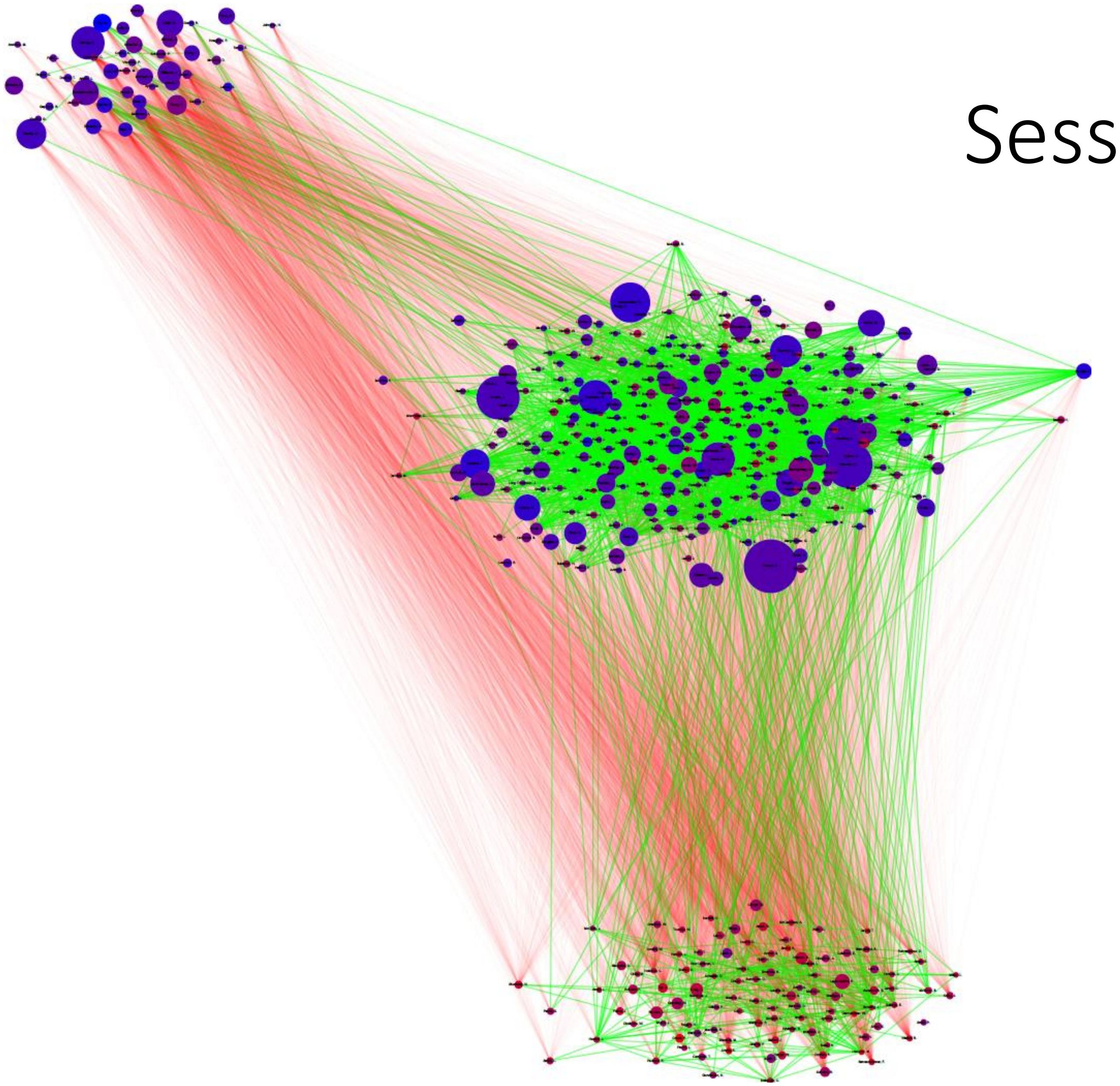
Session 101 (1989-1991)



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Identifying hidden coalitions in the US House of Representatives by optimally partitioning signed networks based on generalized balance

Samin Aref & Zachary P. Neal

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Peer-reviewed papers → ArXiv.org
Models and code → github.com/saref
Network data → FigShare and OSF saref.github.io

- Measuring balance arxiv.org/abs/1509.04037
- Computing frustration index arxiv.org/abs/1710.09876
- Frustration index of large networks arxiv.org/abs/1611.09030
- Applications of frustration and partitioning arxiv.org/abs/1712.04628
- Multilevel evaluation of balance arxiv.org/abs/2005.09925
- Partitioning dense signed networks of the US Congress arxiv.org/abs/1906.01696
- Partitioning signed networks based on generalized balance arxiv.org/abs/2105.01913

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