Supervised and unsupervised comparison of 30 community detection algorithms reveal that proximity to globally optimal partitions matters

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Mechanical & Industrial Engineering
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Clustering a network

Nodes: club members (n)

Edges: interactions outside club (m)

Communities: densely connected groups of nodes with sparser between-group connections

Community assignments are predicted by an optimization algorithm.

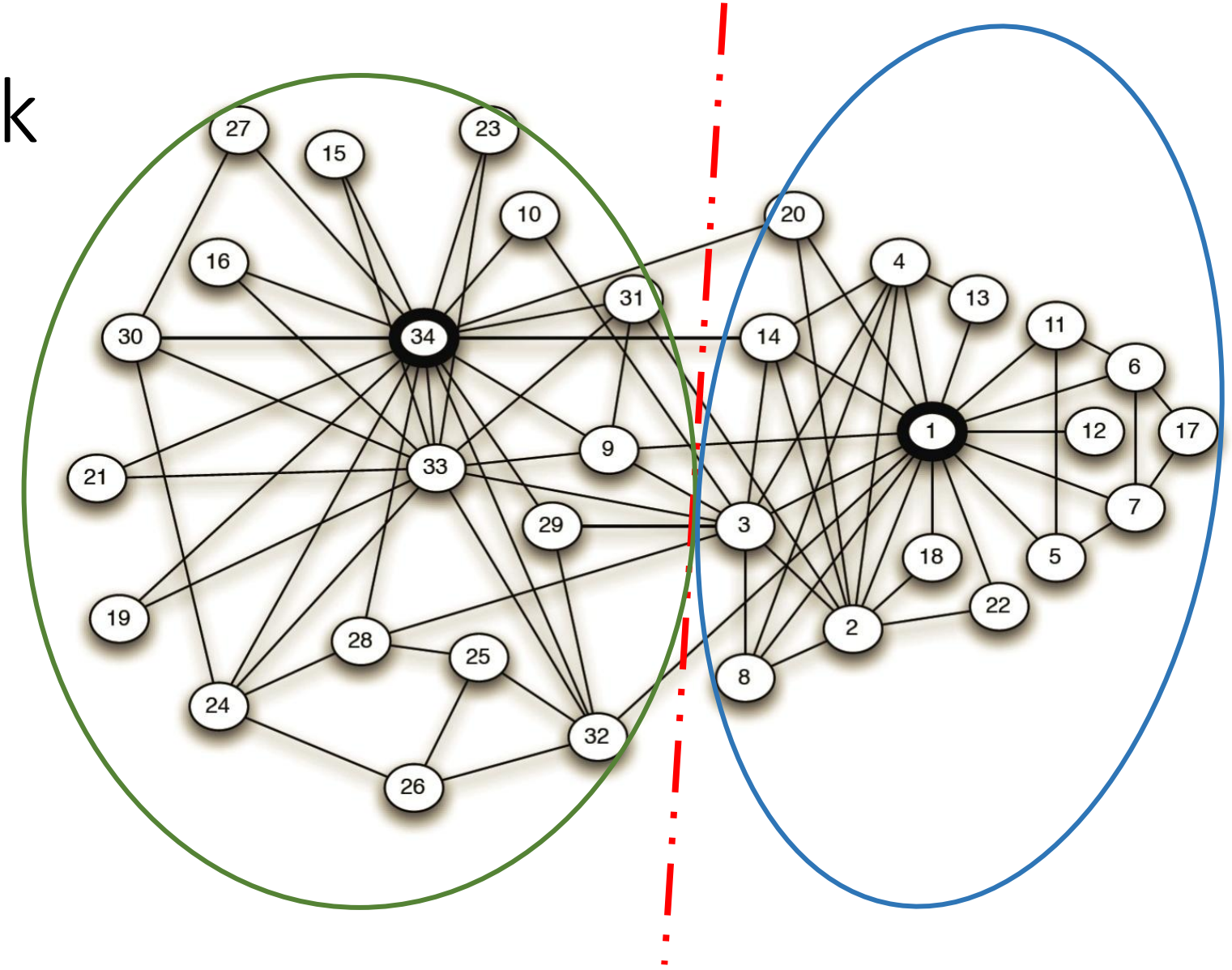

Figure from Leto Peel

Background

- There are varying degrees to the success of different algorithmic attempts at optimizing any function.

- Over the past 15-20 years, the term **modularity maximization (MM)** has been used for referring to **attempts at maximizing modularity** (whose success rates have remained unknown).

- If those MM attempts fail more than they succeed, we actually do not know much about maximum modularity partitions (for better or worse).

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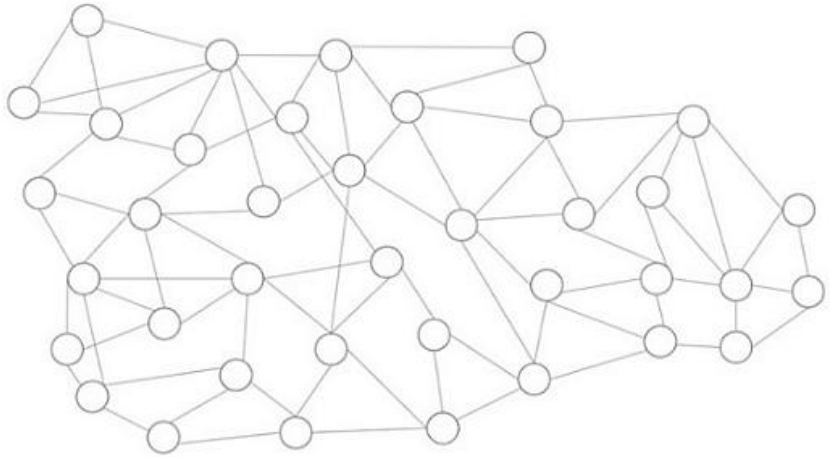
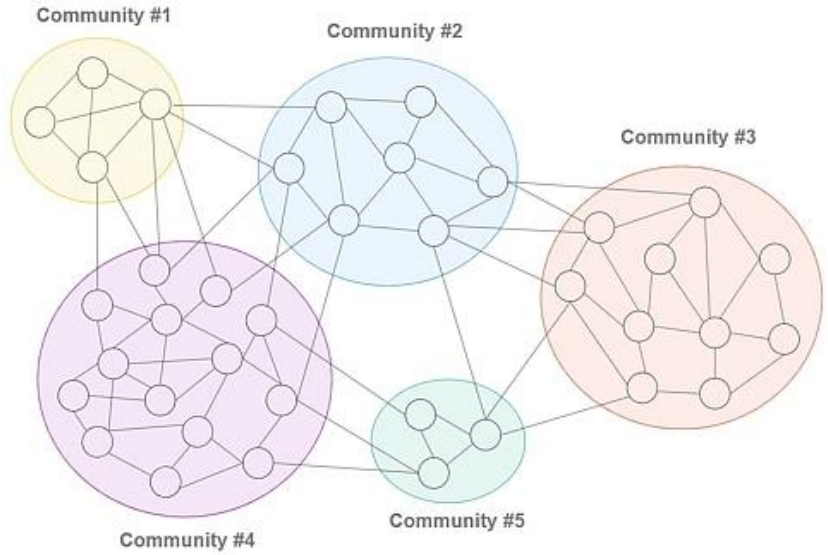
Problem definition

- Input: graph $G(V,E)$ with n nodes and m edges
- Method: a clustering (descriptive community detection) algorithm
- Output: a partition of the nodes into communities (node colours)
- Goal: grouping tightly connected nodes into communities (clusters) which can be gently connected to the rest of the network (Schaub et al. 2017).

Input



Output



Figures from timbr.ai

Schaub et al. (2017) The many facets of community detection in complex networks

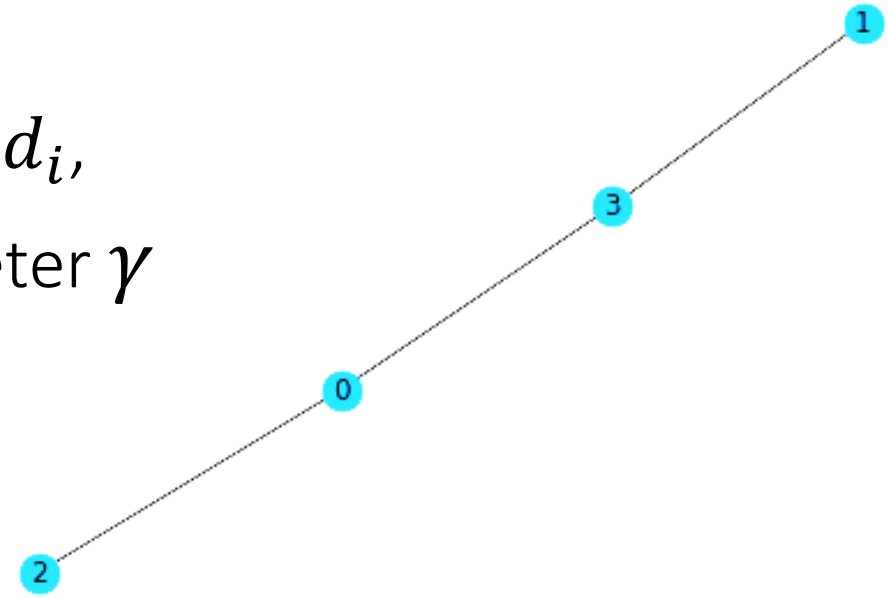
Modularity

Given a network (graph) *G(V,E),* find a partition of *V* into X={V_1, V_2, …, V_c} such that the modularity *Q(G,X)* is maximized.

Modularity entry b_{ij} : a function of degrees d_i,

connections a_{ij}, and the resolution parameter γ

$$b_{ij} = a_{ij} - \gamma \frac{d_i d_j}{2m}$$



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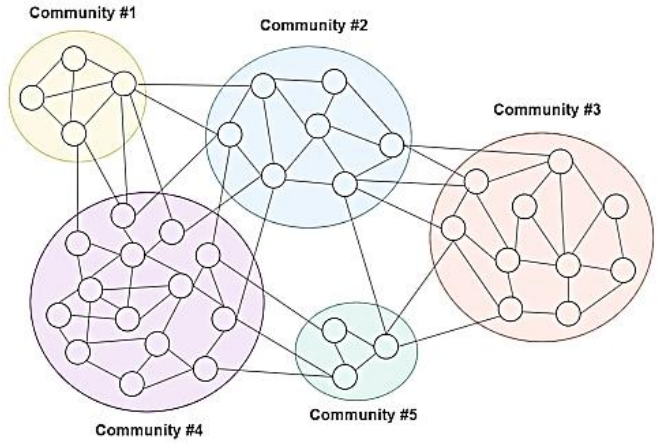
Detecting communities via Modularity Maximization (MM)

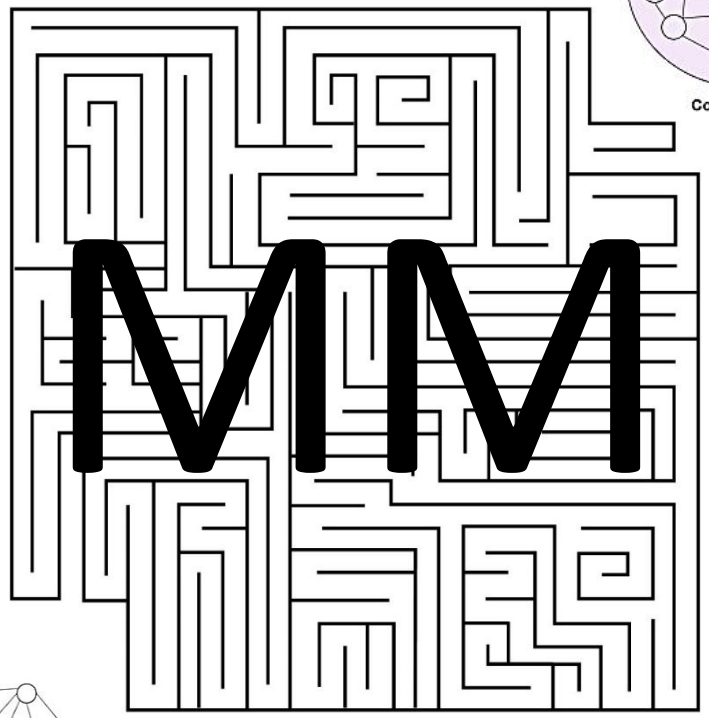
Given graph *G(V,E)* with modularity entries b_{ij}, find a partition of the node set *V* into $X=\{V_1, V_2, \ldots, V_k\}$ to maximizes the objective function *Q(G,X)*.

$$Q_{(G,X)} = \frac{1}{2m} \sum_{(i,j)\in V^2} b_{ij}$$

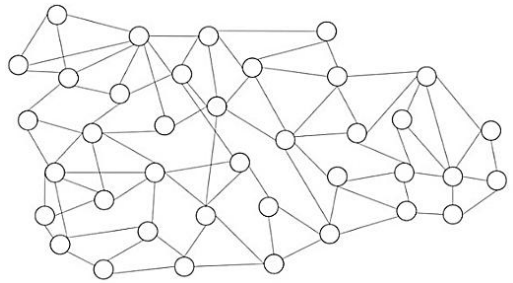
i and j are together in partition X

Output: communities





Input: a simple graph

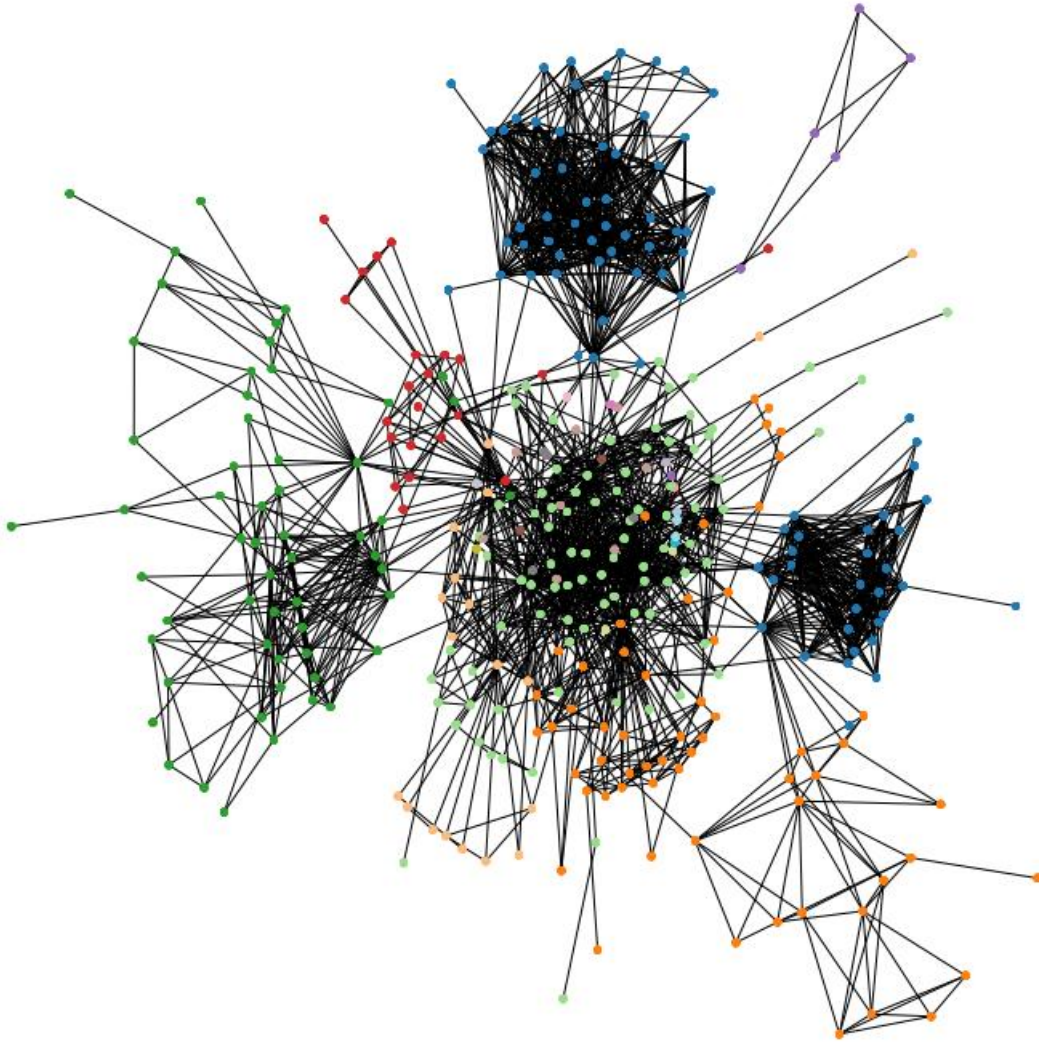


Figures from timbr.ai

Communities depend on the algorithm even if the objective function is the same.

Dataset:
facebook_friends
m=1988

Q: modularity for a partition
Q*: maximum modularity of the graph
k: number of communities
AMI: adjusted mutual information
(similarity to an optimal partition)



(a) Bayan, $Q^* = 0.7157714$, $k = 28$, AMI $= 1$

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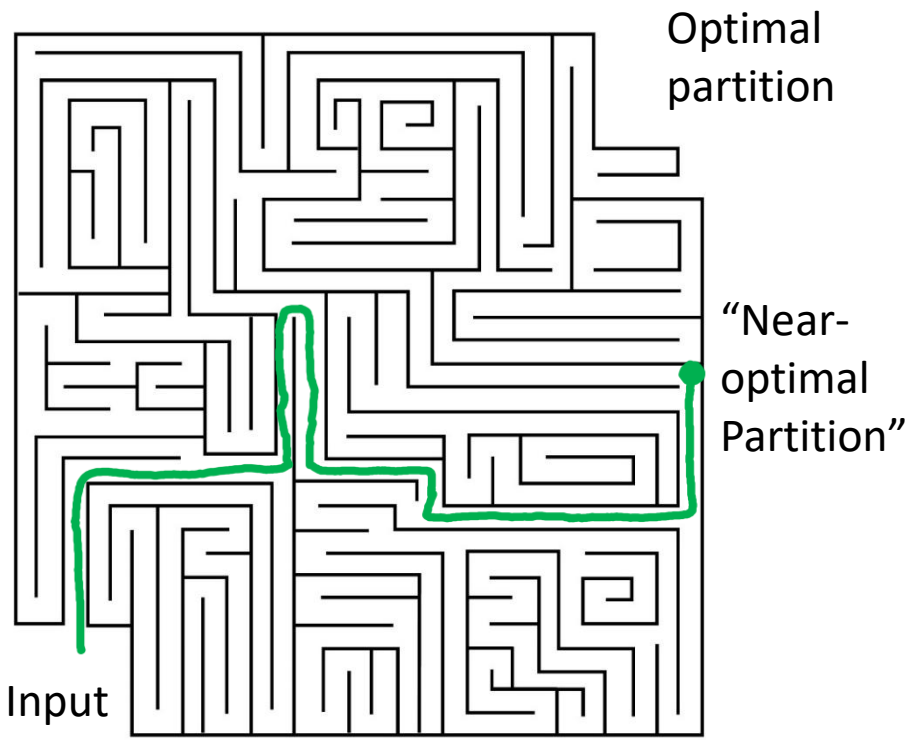
Approach 1: Modularity Maximization Heuristics

1. Edge Motif (EdMot) (Li et al. 2019)
2. Leiden (Traag et al. 2019)
3. Paris (Bonald et al. 2018)
4. Belief (Zhang & Moore 2014)
5. Combo (Sobolevsky et al. 2014)
6. Leicht-Newman (LN) (Leicht & Newman 2008)
7. Louvain (Blondel et al. 2008)
8. Greedy (CNM) (Clauset et al. 2004)
9. Graph Neural Network (GNN) (Sobolevsky et al. 2022)



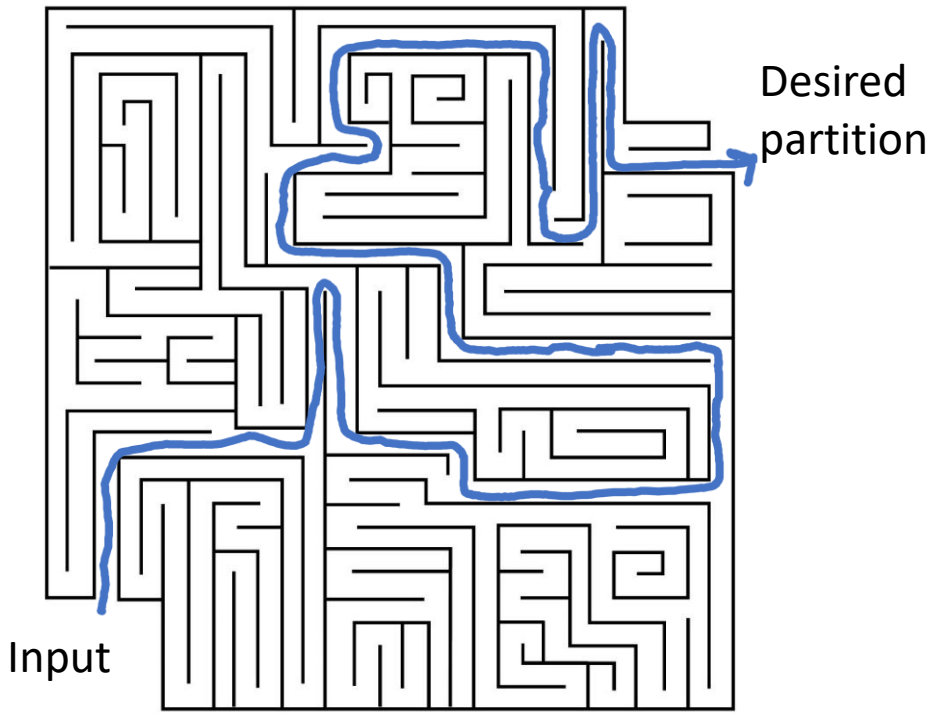
Approach 2: Exact/Approximate Modularity Maximization

- Integer Programming - IP (Brandes et al. 2007)
- IP and LP rounding (Agarwal & Kempe 2008)
- Column generation (Aloise et al. 2010)
- Sparse IP and LP rounding (Dinh & Thai 2015)
- Approximation (Kawase et al. 2021)
- Bayan (Aref et al. 2022) for graphs with m<3000

$$\max_{x_{ij}} Q = \frac{1}{2m}\left(\sum_{(i,j)\in V^2, i<j} 2b_{ij}(1-x_{ij}) + \sum_{i\in V} b_{ii}\right)$$

$$\text{s.t.} \quad x_{ik} + x_{jk} \geq x_{ij} \quad \forall (i,j) \in V^2, i<j, k \in K(i,j)$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in V^2, i<j$$



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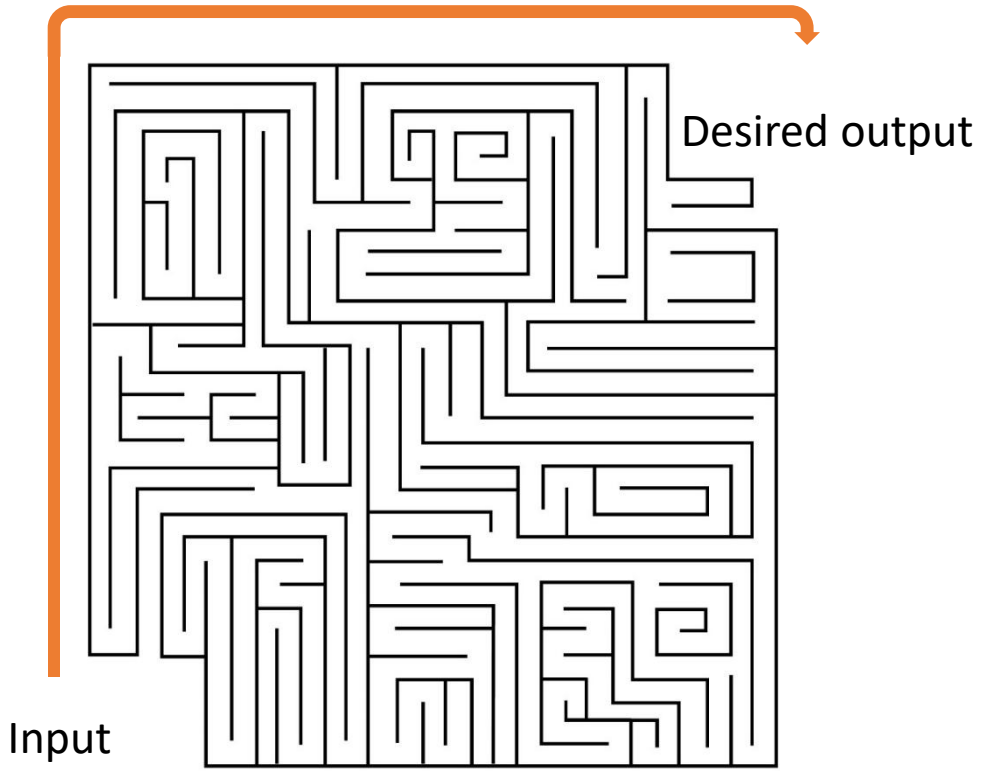
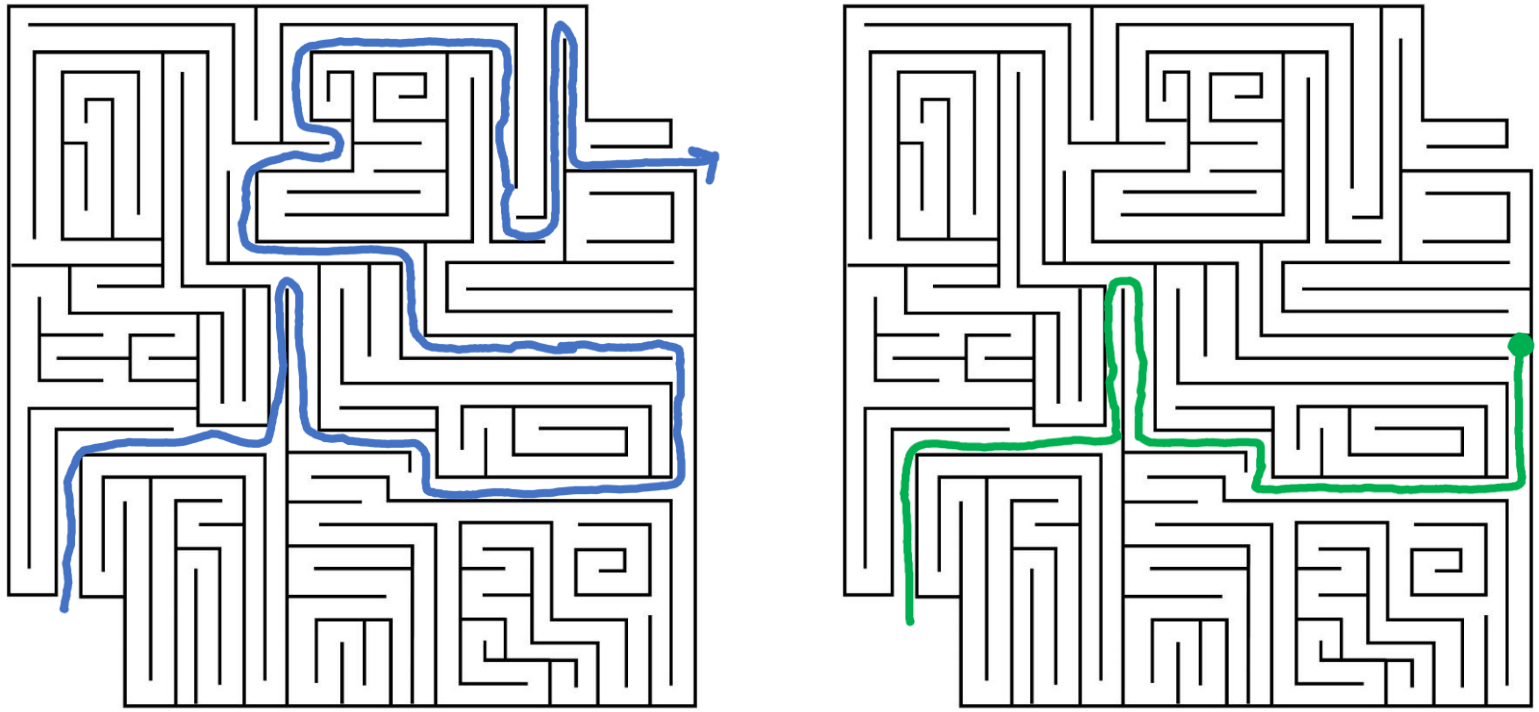
Approach 3: Other Methods

1. Kernighan-Lin bisection (Kernighan and Lin 1970)
2. RB Potts model with Erdős–Rényi as null (Reichardt & Bornholdt 2006)
3. Chinese whispers (Biemann et al. 2006)
4. Walktrap (Pons & Latapy 2006)
5. k-cut (Ruan & Zhang 2007)
6. Asynchronous label propagation (Raghavan et al. 2007)
7. Infomap (Rosvall & Bergstrom 2008)
8. Genetic Algorithm (Pizzuti 2008)
9. Semi-synchronous Label propagation (Cordasco & Gargano 2010)
10. Constant Potts Model (CPM) (Traag et al. 2011)
11. Significant scales (Traag et al. 2013)
12. Stochastic Block Model (SBM) (Peixoto 2014a)
13. SBM with Monte Carlo Markov Chain (MCMC) (Peixoto 2014b)
14. WCC (Prat-Pérez et al. 2014)
15. Surprise (Traag et al. 2015)
16. Diffusion Entropy Reducer (DER) (Kozdoba and Mannor 2015)
17. GemSec (Rozemberczki et al. 2019)
18. Bayesian Planted Partition (BPP) (Zhang and Peixoto, 2020)
19. Markov Stability (PyGenStability) (Arnaudon et al., 2023)



Assessing the optimality of partition among ten modularity-based algorithms



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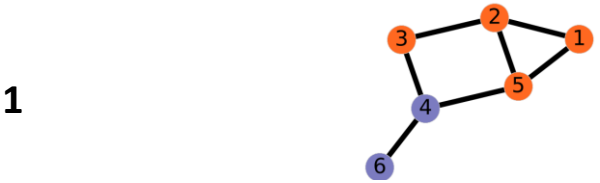
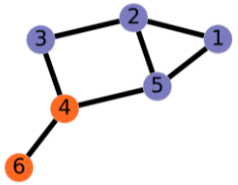
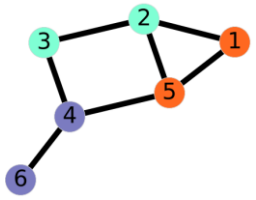
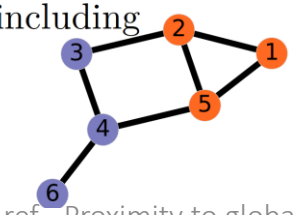
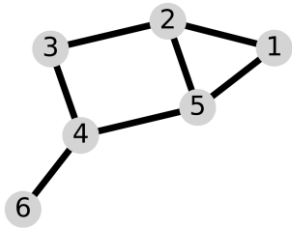
Evaluating ten modularity-based algorithms

Ten algorithms (and their variations)	Partition (obtained by an MM algorithm)	Modularity	Optimality percentage (GOP)	Maximum similarity to any optimal partition - AMI (or RMI or ECS)
Method 1		Q_1=0.122	Q_1/Q^*=60%	36%
Method 2		Q_2=0.122	Q_2/Q^*=60%	36%
...	...	...	...	...
Method 19		Q_{19}=0.092	Q_{19}/Q^*=45%	30%
Exact method (Integer Programming)	All optimal partitions including 	Q*=0.204	100%	100%
			Y-axis	X-axis

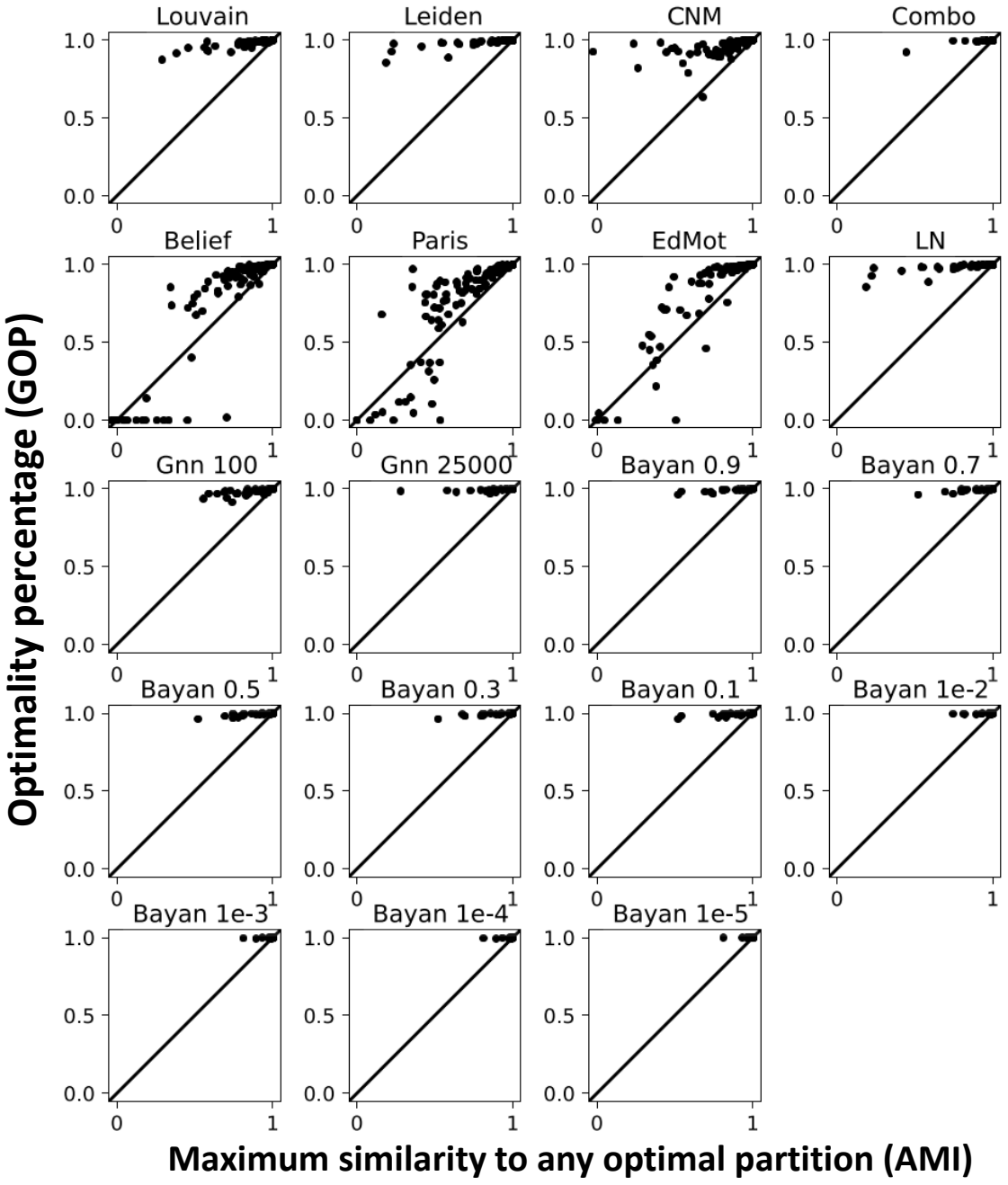


Test cases are 104 graphs with modular structure:

- 54 real networks
- 20 LFR random graphs
- 30 ABCD random graphs

Each datapoint represents the performance of one algorithm on one test case.

1. Y-values: Most partitions obtained from heuristics are sub-optimal
2. X-values: Many partitions are dissimilar to any optimal partition
3. 45°-line: near-optimal partitions are not similar to any optimal partitions



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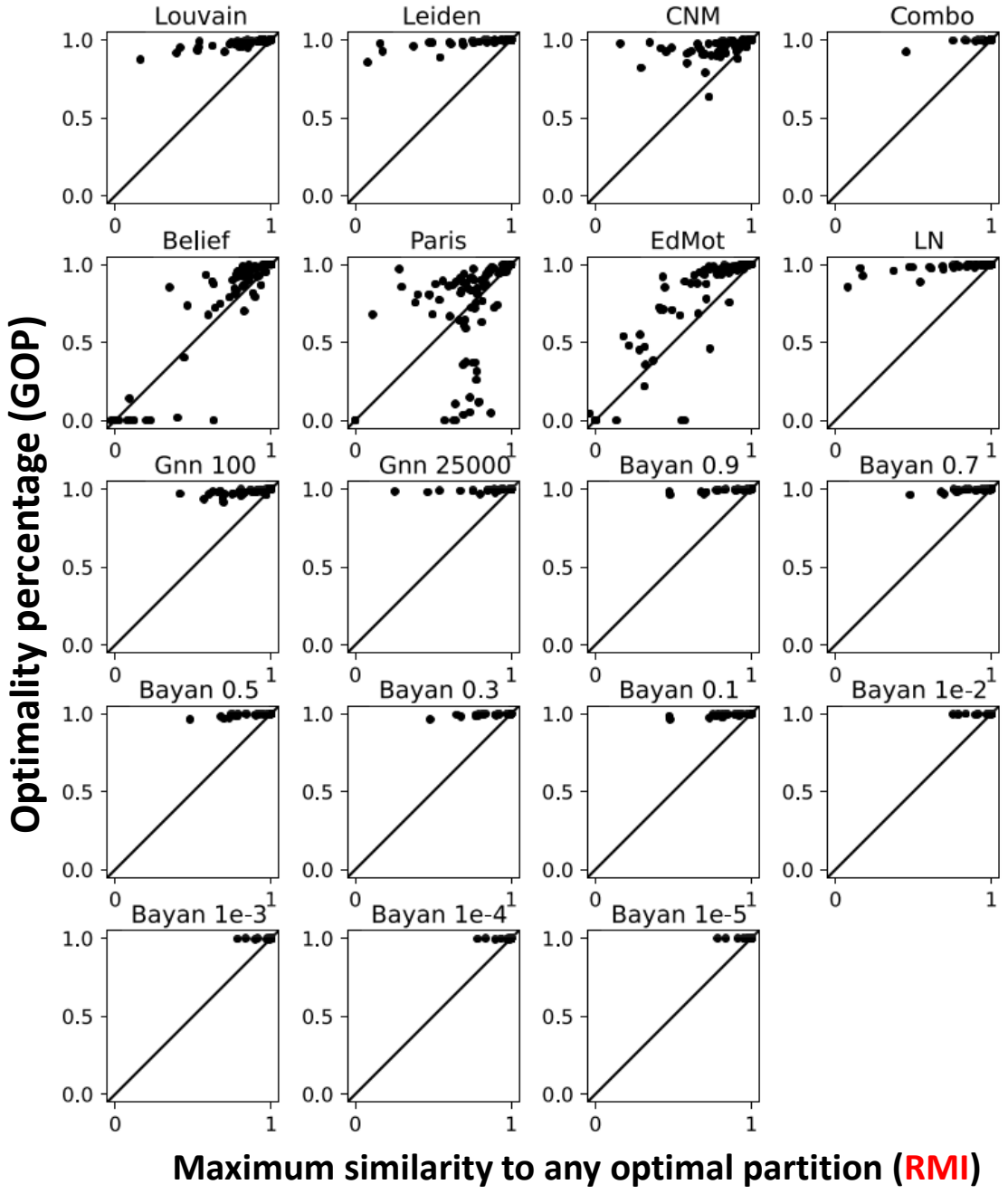

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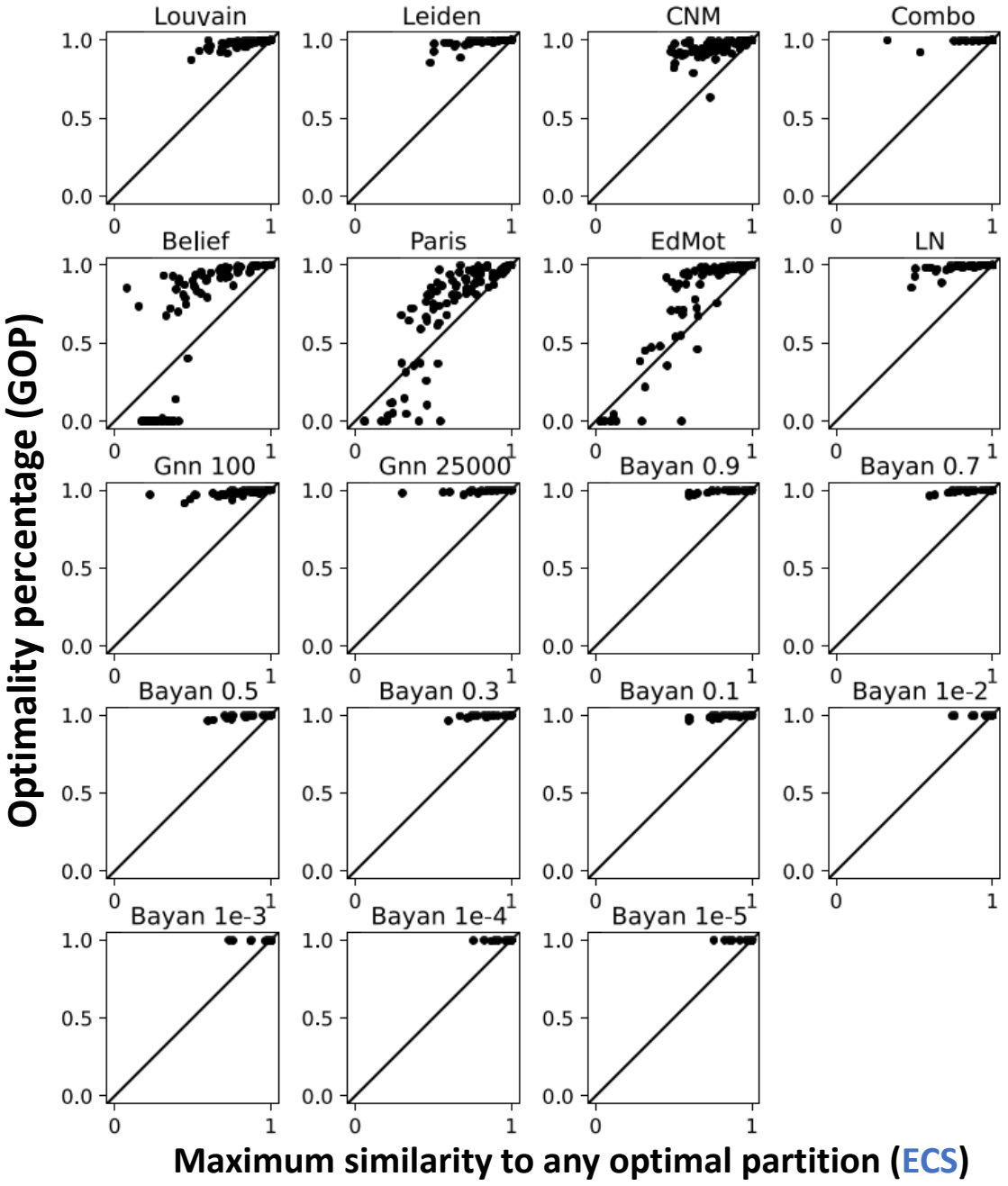
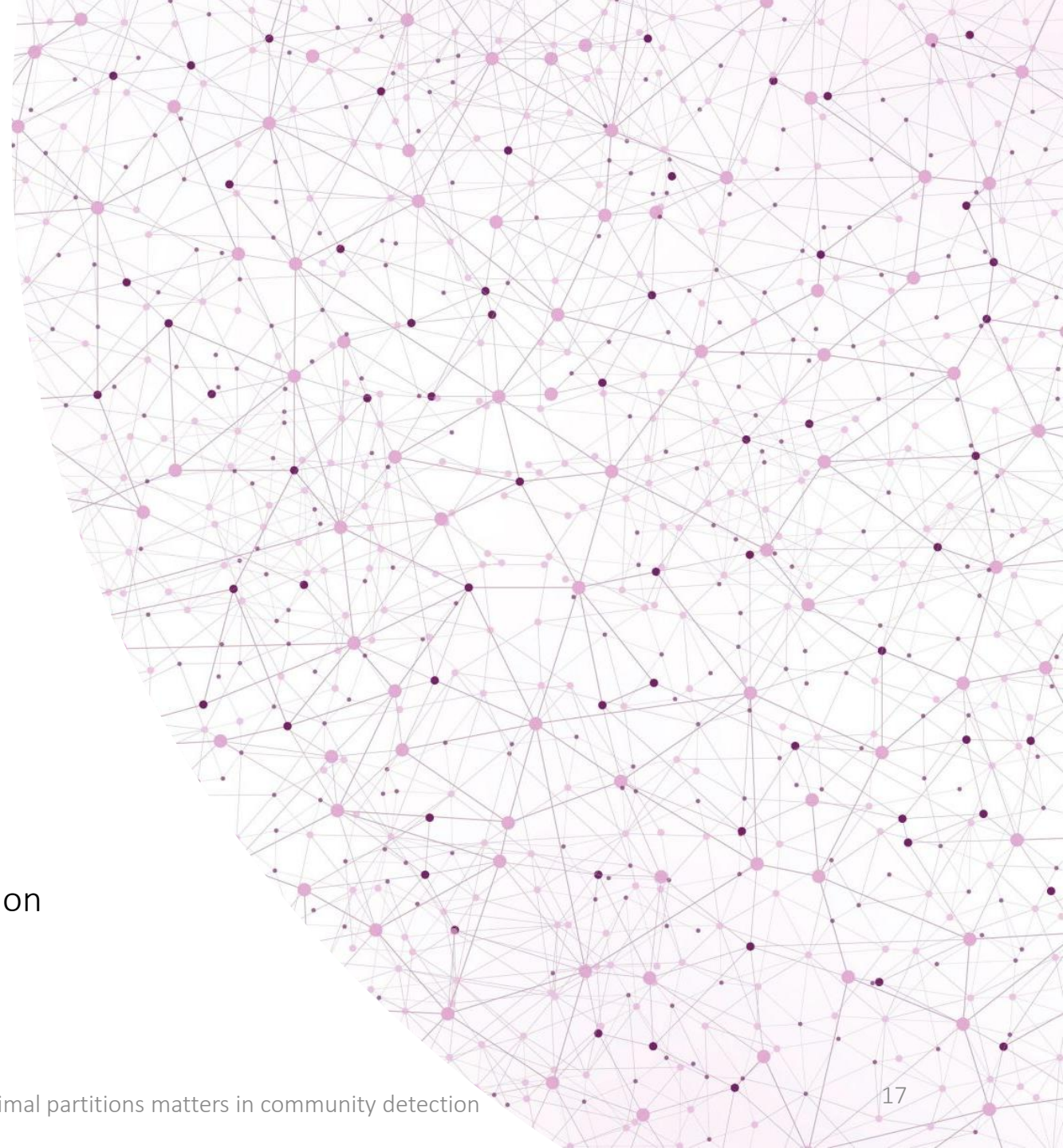
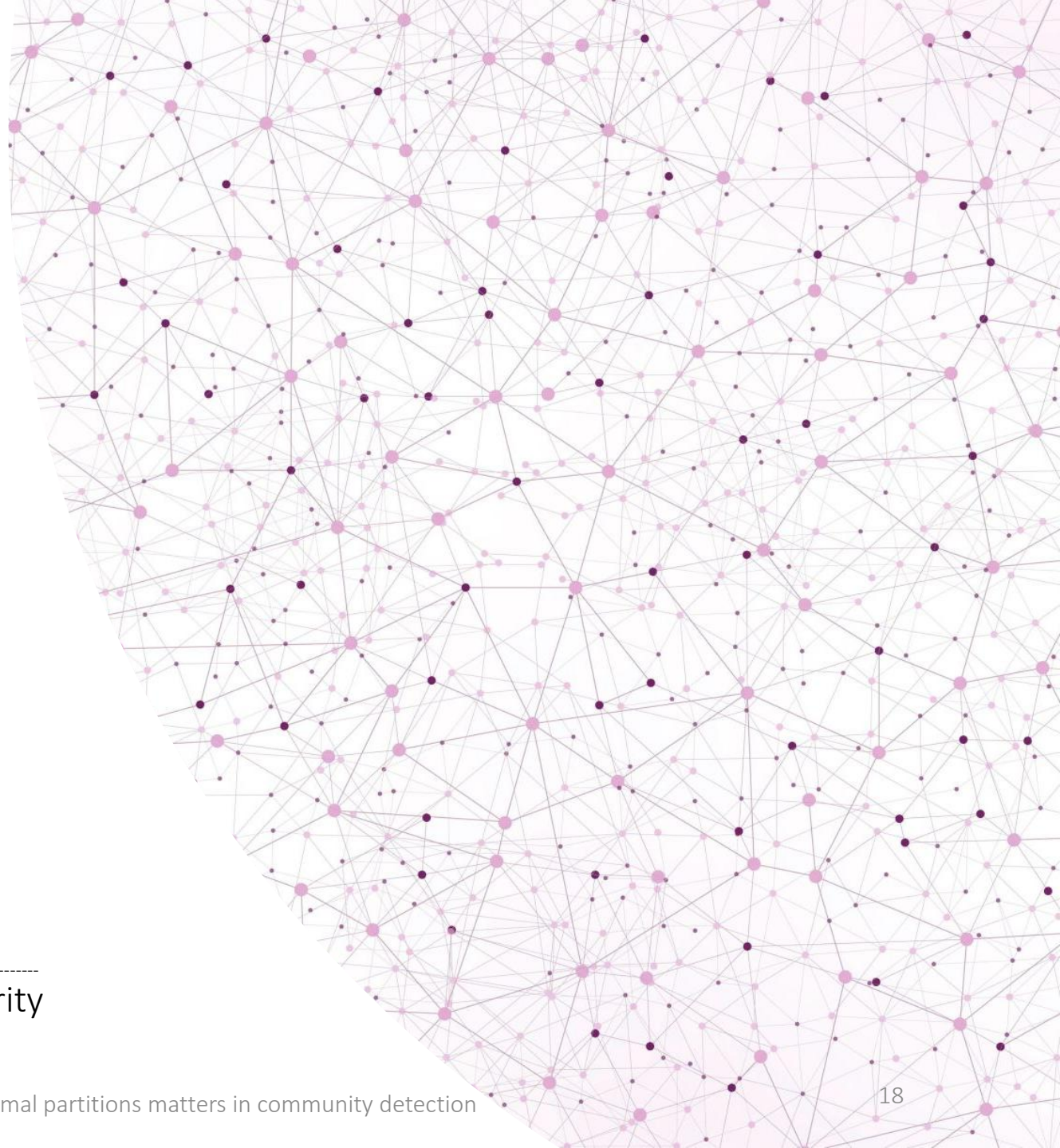




1

Heuristic modularity maximization algorithms rarely* maximize modularity.

*Only 43.9% of the times according to our experiments on 104 synthetic and real networks.



2

Suboptimal partitions of heuristic algorithms are disproportionately dissimilar to any optimal partition.

An x% suboptimality is often associated with a dissimilarity much larger than x% from any optimal partition.

Can we globally maximize modularity?

- YES, for some networks we can!
- Bayan maximizes modularity in networks with fewer than 3000 edges and approximates maximum modularity in slightly larger networks on ordinary computers.

```
%pip install bayanpy
```

```
import networkx as nx
import bayanpy
```

```
G = nx.barbell_graph(5,2)
```

```
bayanpy.bayan(G)
```

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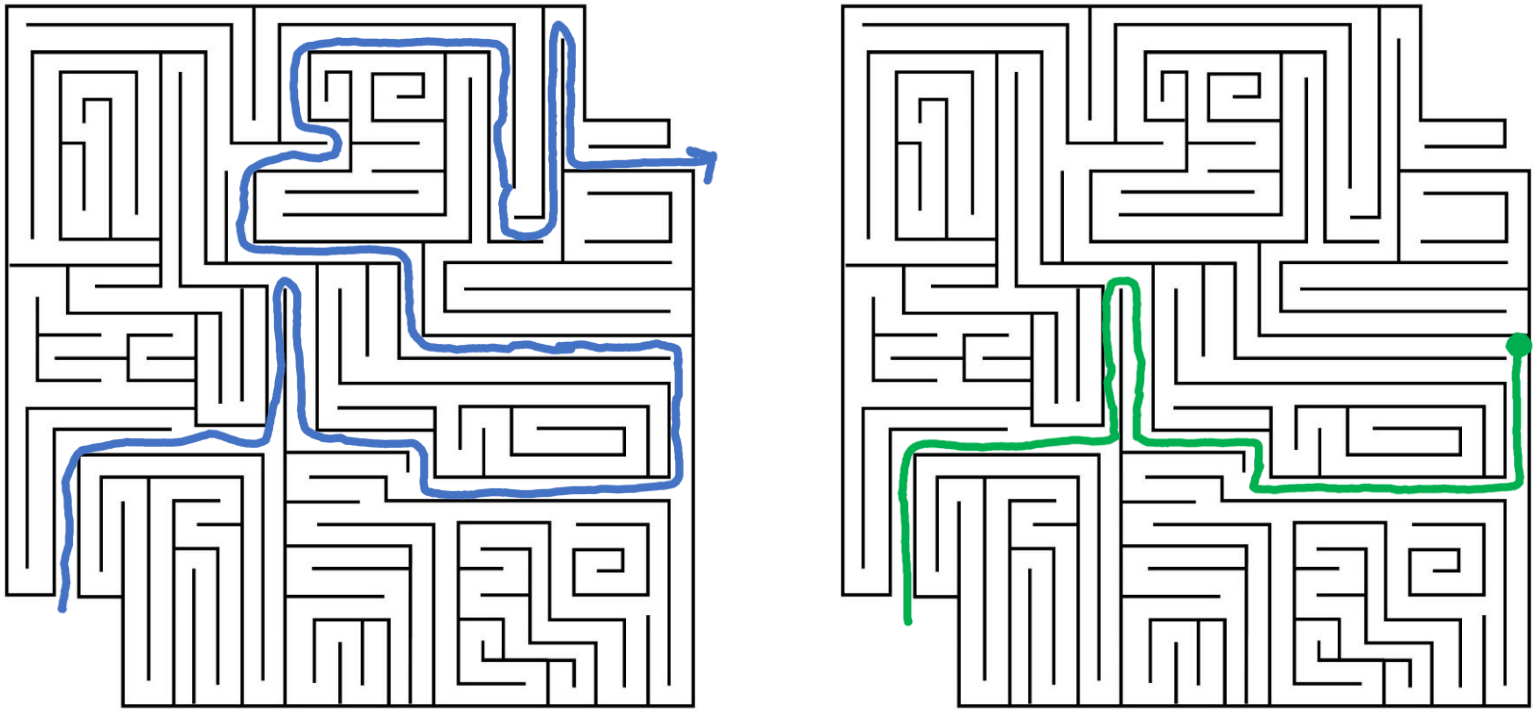
Assessing the run time of ten modularity-based methods (and their variations)



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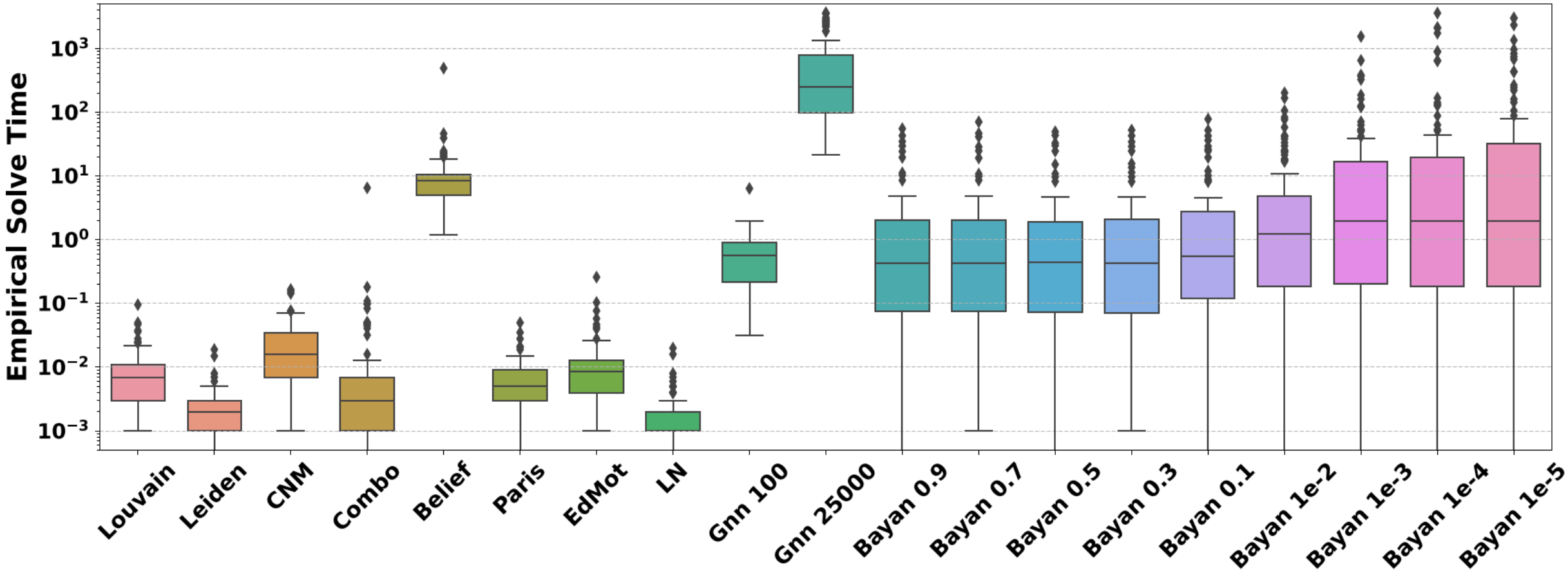
Solve time of ten modularity-based methods



How does Bayan work? Solving integer programming by branching on triples

There are 3 constraints for every triple $(i, j, k) \in S \subset V^3$.

All decision variables x_{ij} are binary.

$$x_{ij} + x_{jk} \geq x_{ik}$$
$$x_{ij} + x_{ik} \geq x_{jk}$$
$$x_{ik} + x_{jk} \geq x_{ij}$$

$$x_{ij} + x_{ik} + x_{jk} = 0$$

or

$$x_{ij} + x_{ik} + x_{jk} \geq 2$$

1. Solve the linear programming relaxation
2. Branch on a triple
3. Add a cut
4. Go to 1

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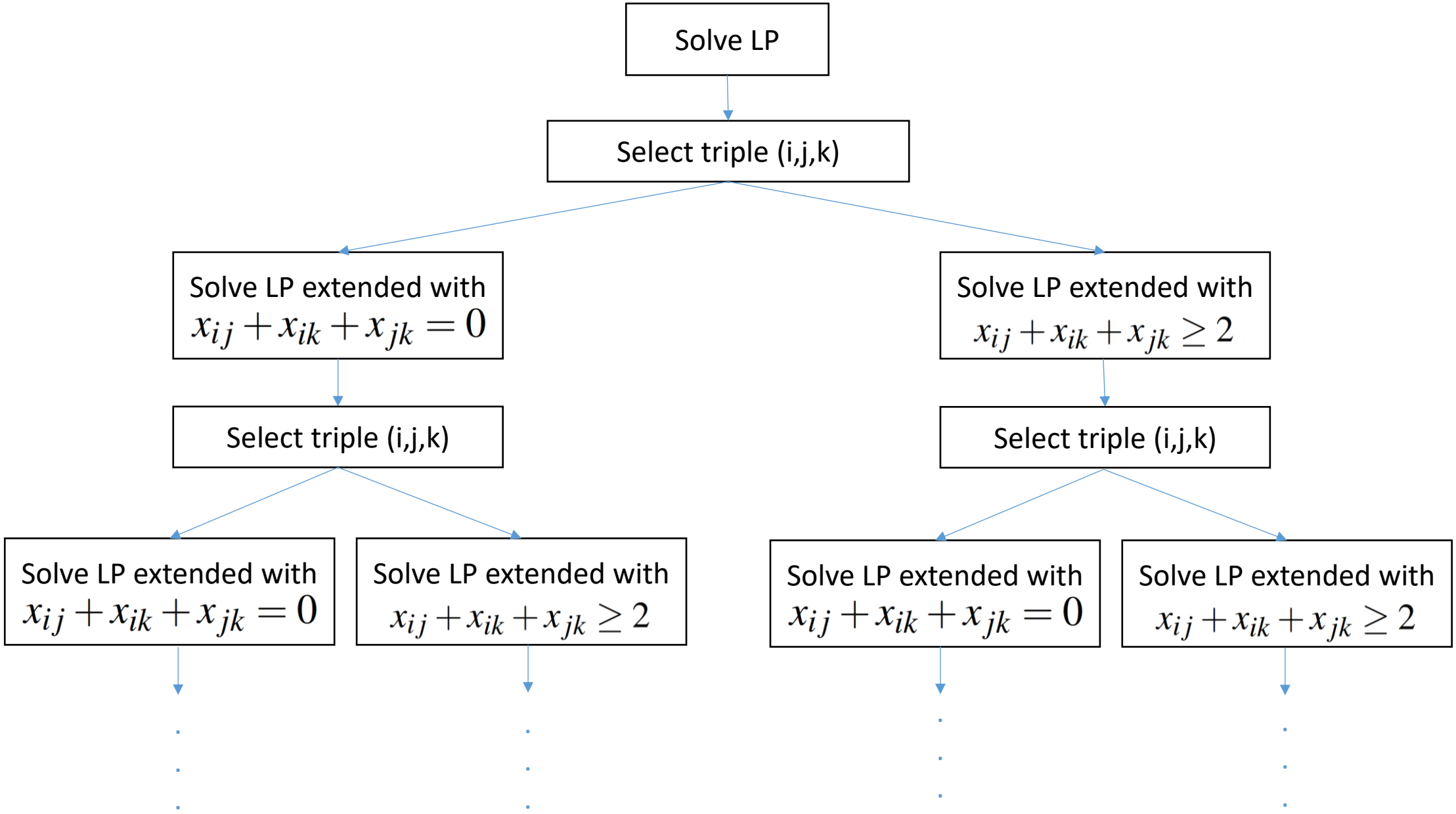
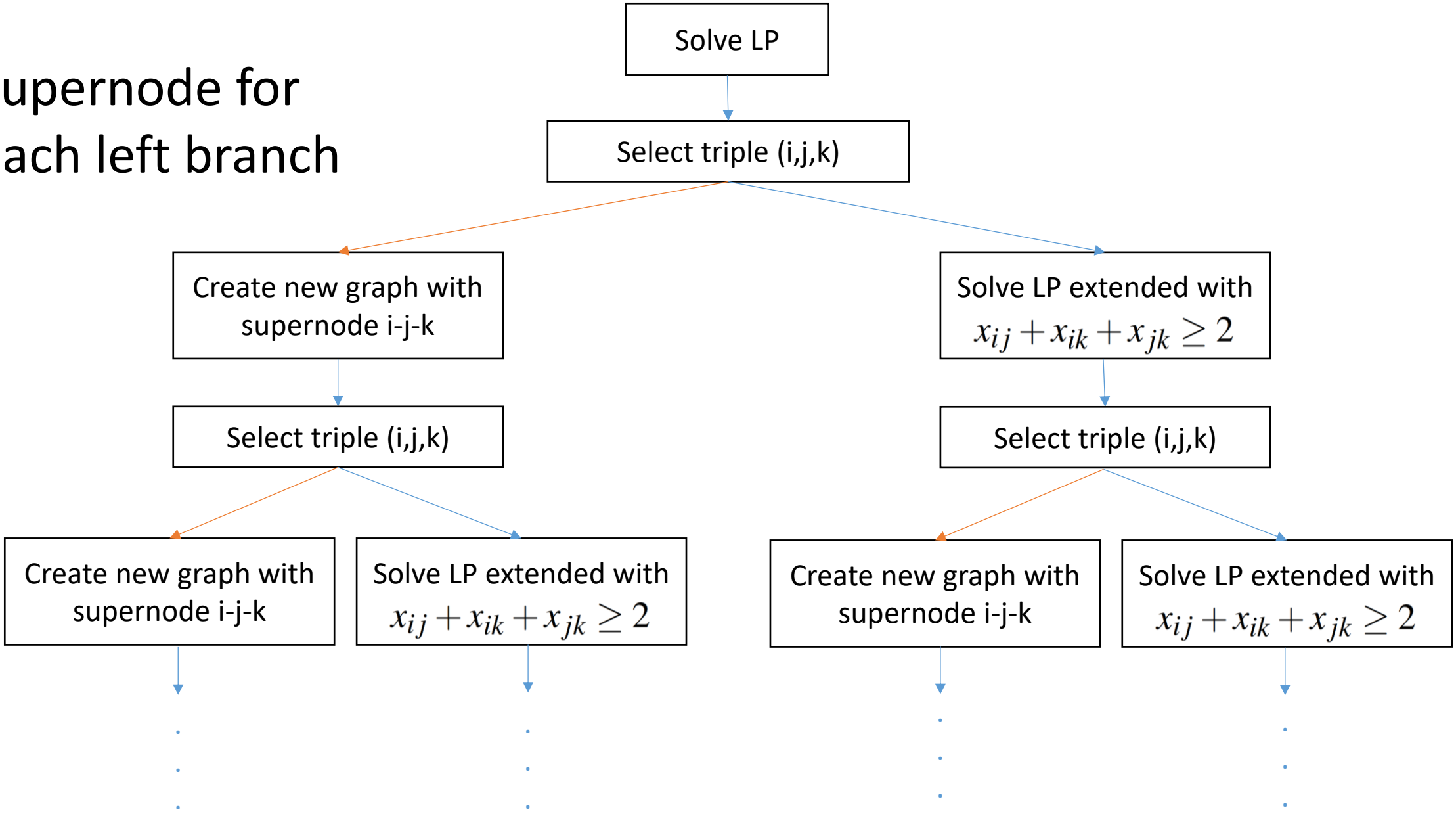

Solve LP
Select triple (i,j,k)
Solve LP extended with $x_{ij} + x_{ik} + x_{jk} = 0$
Solve LP extended with $x_{ij} + x_{ik} + x_{jk} \geq 2$
Select triple (i,j,k)
Select triple (i,j,k)
Solve LP extended with $x_{ij} + x_{ik} + x_{jk} = 0$
Solve LP extended with $x_{ij} + x_{ik} + x_{jk} \geq 2$
Solve LP extended with $x_{ij} + x_{ik} + x_{jk} = 0$
Solve LP extended with $x_{ij} + x_{ik} + x_{jk} \geq 2$

Supernode for each left branch



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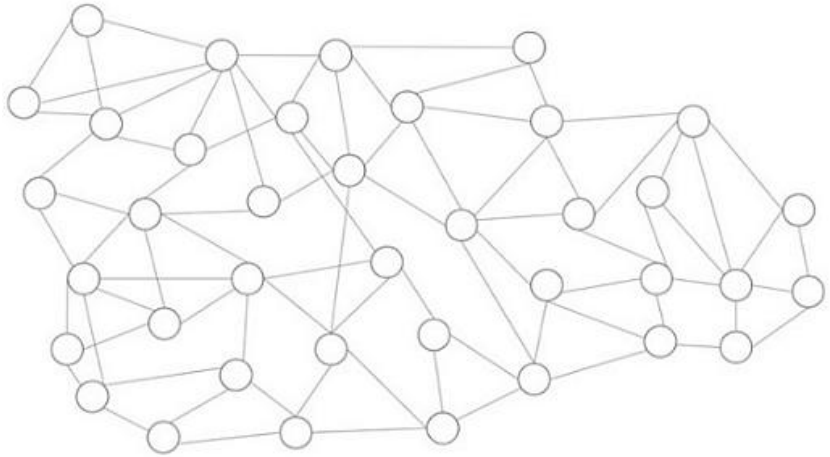
Wait! But clustering is not about modularity maximization!

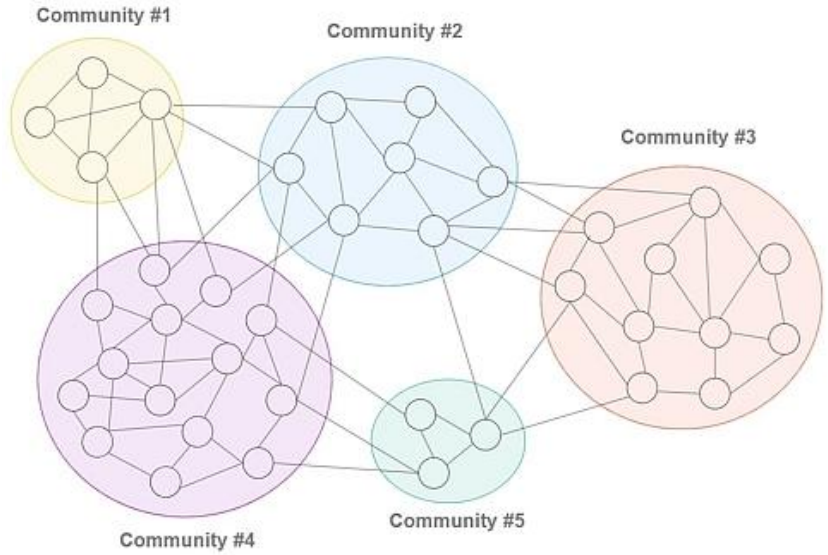
A supervised comparison of 30 CD algorithms

- Input: graph
- Method: a network clustering algorithm
- Output: a partition
- Test: **retrieval of planted partitions** in synthetic benchmark networks (e.g. LFR)

Input

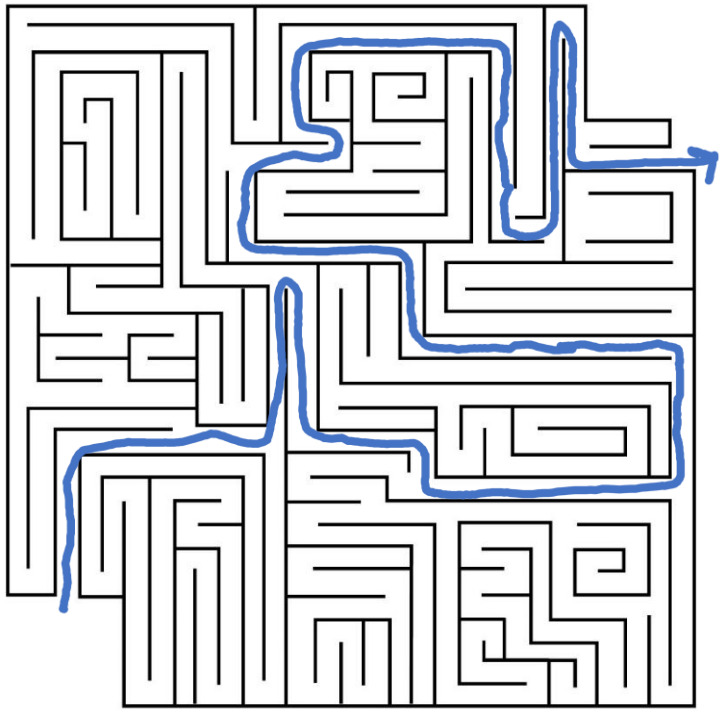


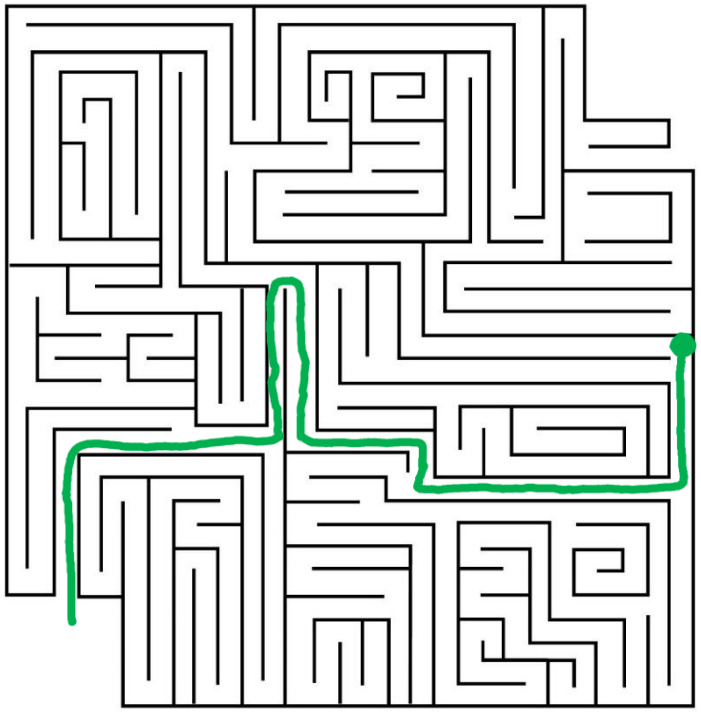
Output

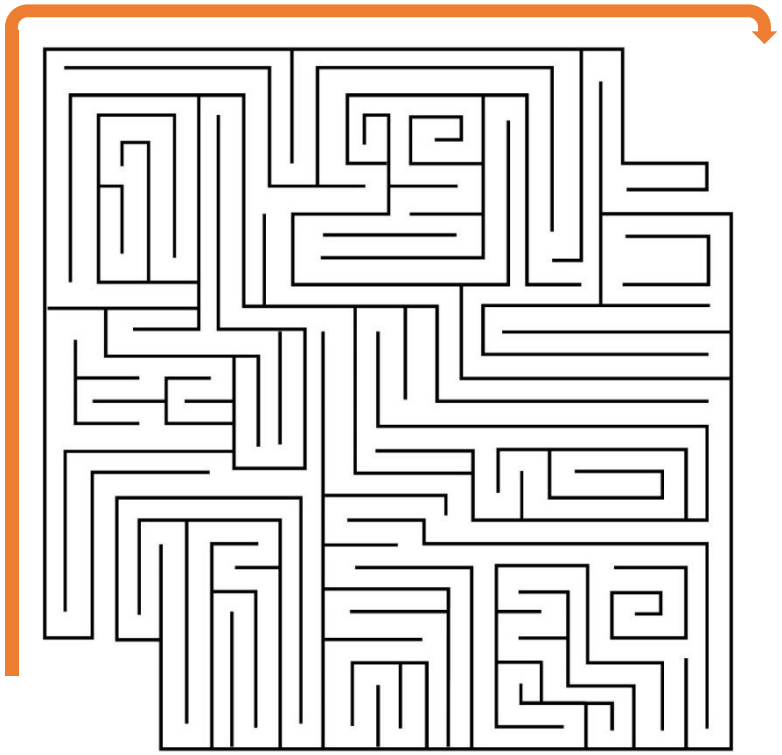


Figures from timbr.ai

Comparing 30 algorithms based on retrieving planted partitions







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Planted partition retrieval assessment

Standard benchmark graphs:

- LFR benchmarks (**L**ancichinetti-**F**ortunato-**R**adicchi benchmarks)
- ABCD benchmarks (**A**rtificial **B**enchmark for **C**ommunity **D**etection)

Retrieval performance indicator:

- Similarity with the planted partitions

adjusted mutual information (or RMI[1] or ECS[2]) averaged over 100 graphs

1. RMI: Reduced Mutual Information (Newman et al. 2020, Jerdee and Newman 2023)
2. ECS: Element-Centric Similarity (Gates et al. 2019)

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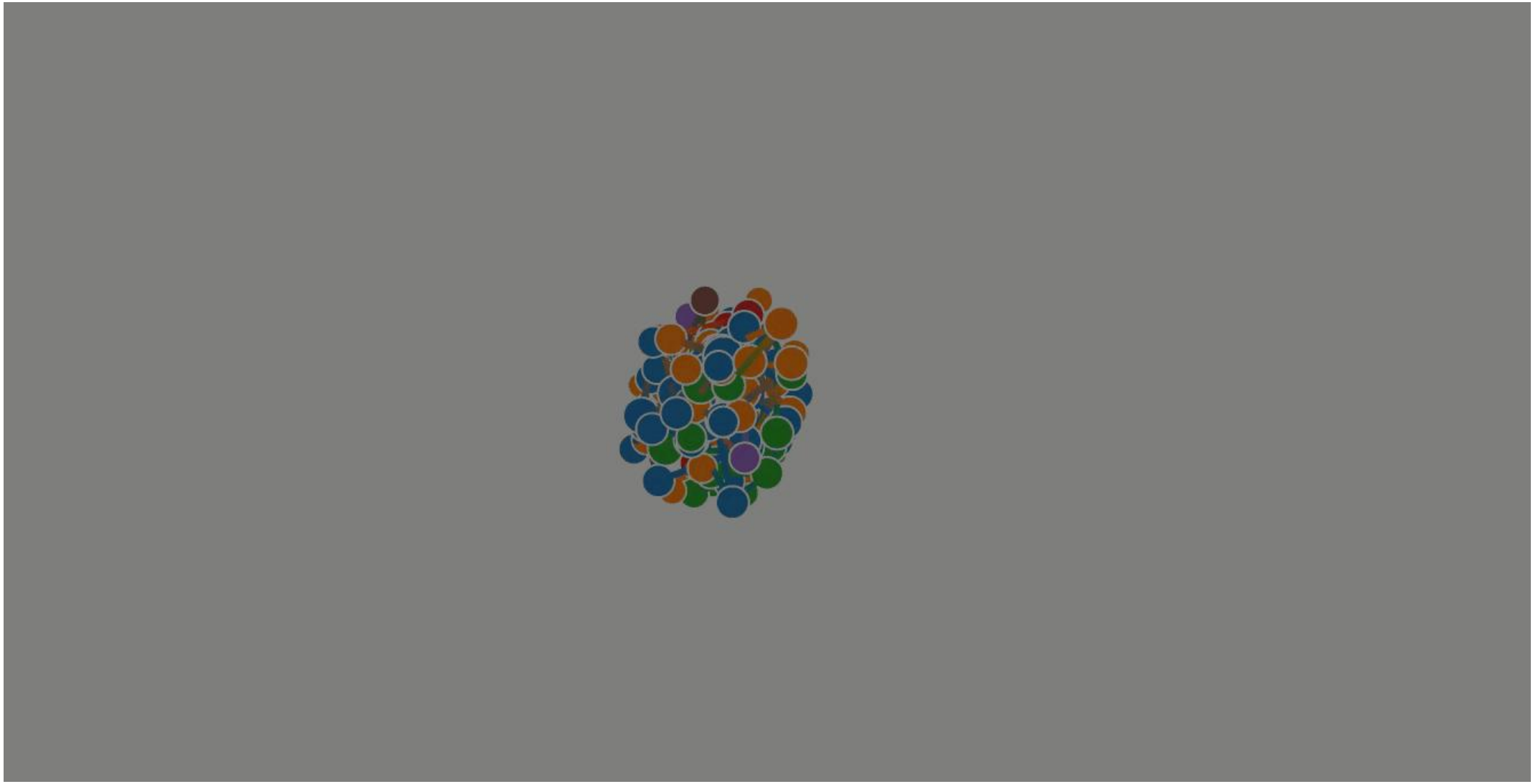
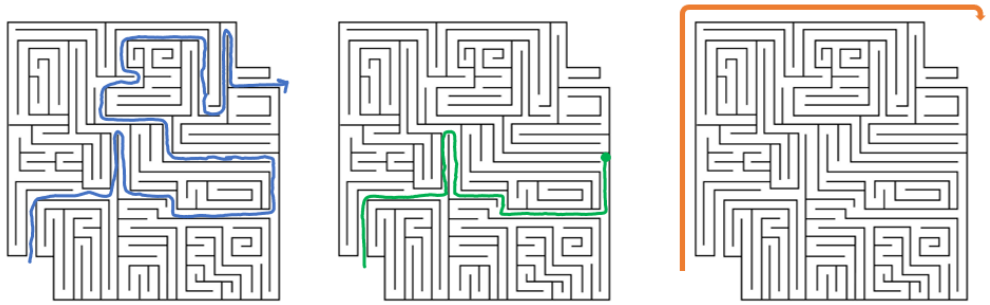
LFR benchmark with **low** noise

LFR benchmark with **high** noise



Comparing 30 algorithms



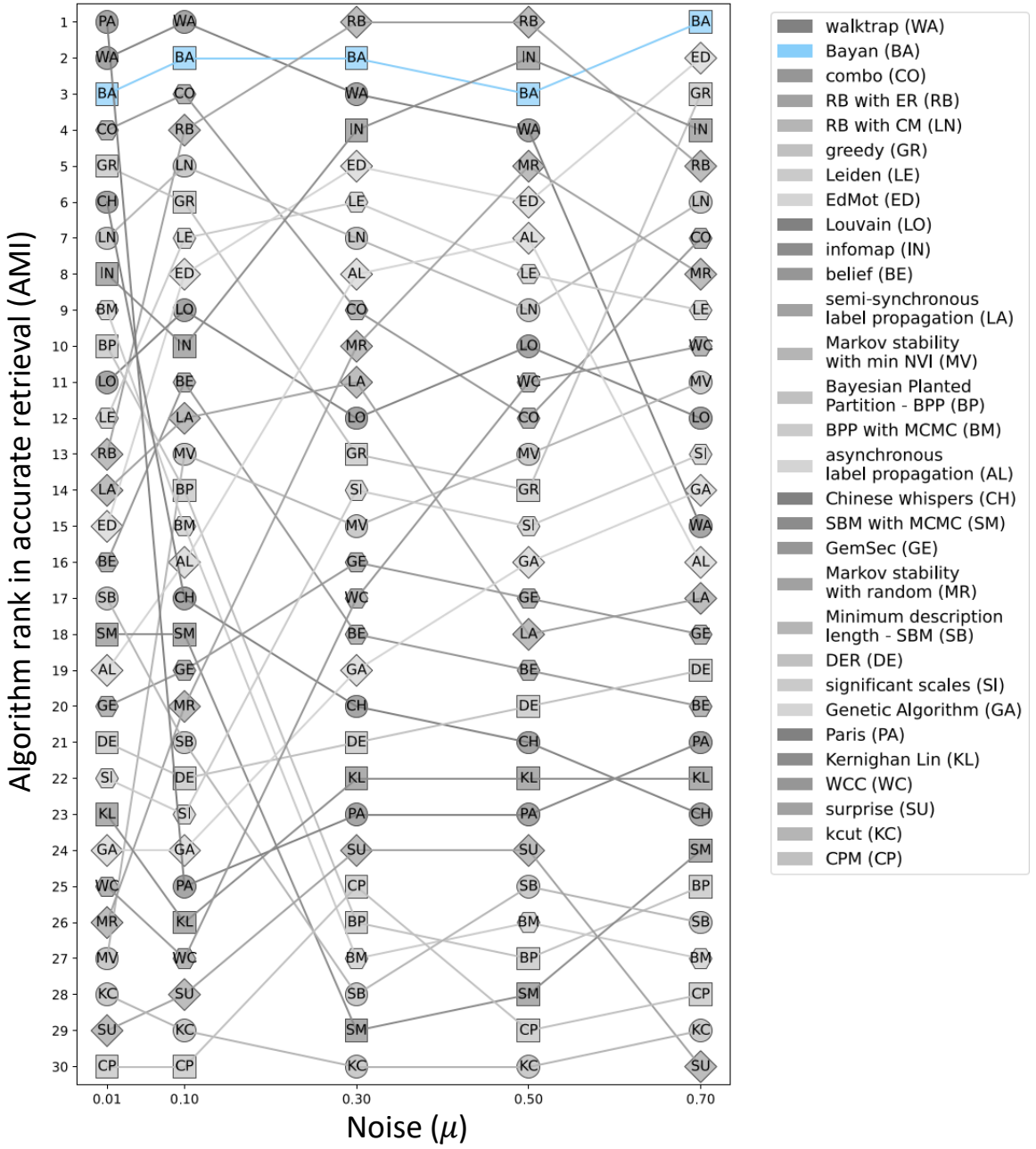
Test cases:

- 500 LFR benchmark graphs with up to 1000 edges (generated with planted partitions)
- Noise μ (fraction of inter-community edges)

$$\mu \in \{1\%, 10\%, 30\%, 50\%, 70\%\}$$

Comparing 30 algorithms

Based on retrieving the planted partitions of LFR benchmarks



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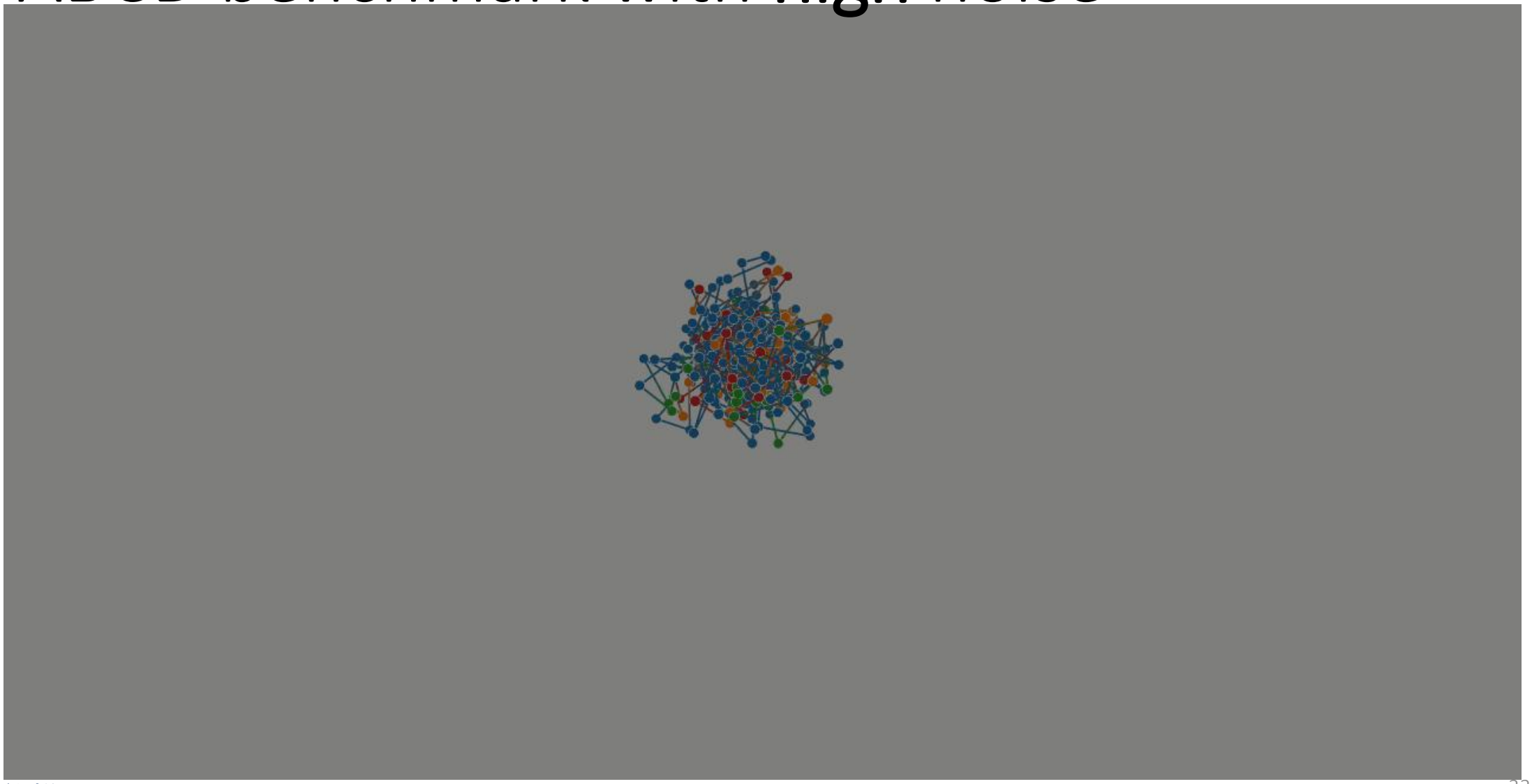
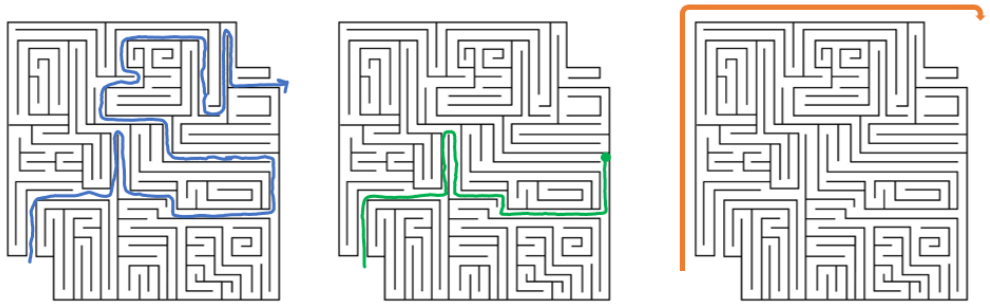
ABCD benchmark with **low** noise

ABCD benchmark with **high** noise



Comparing 30 algorithms



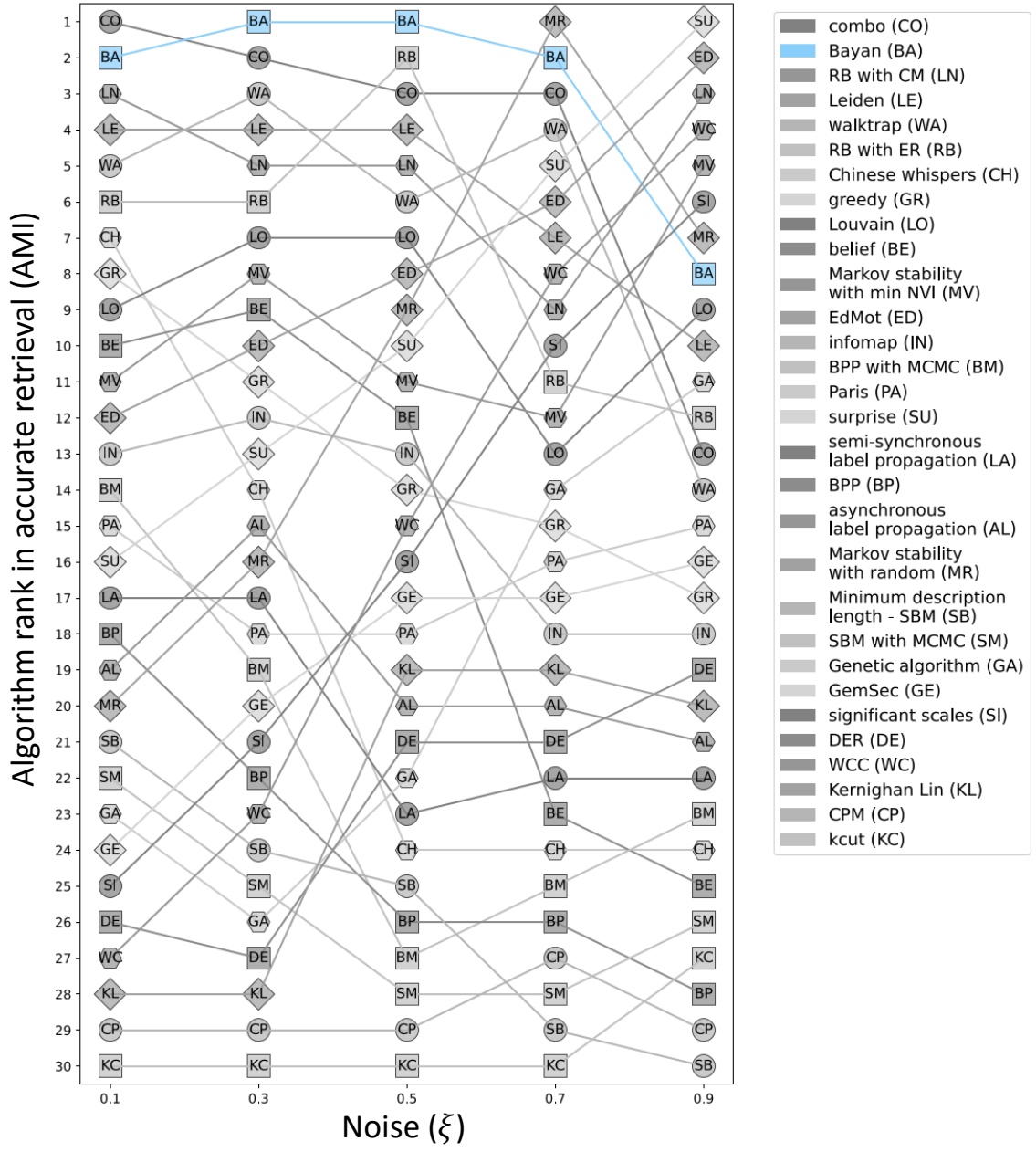
Test cases:

- 500 ABCD benchmark graphs with up to 1000 edges (generated with planted partitions)
- Noise ξ (mixing parameter)

$$\xi \in \{10\%, 30\%, 50\%, 70\%, 90\%\}$$

Comparing 30 algorithms

Based on retrieving the planted partition of ABCD benchmark



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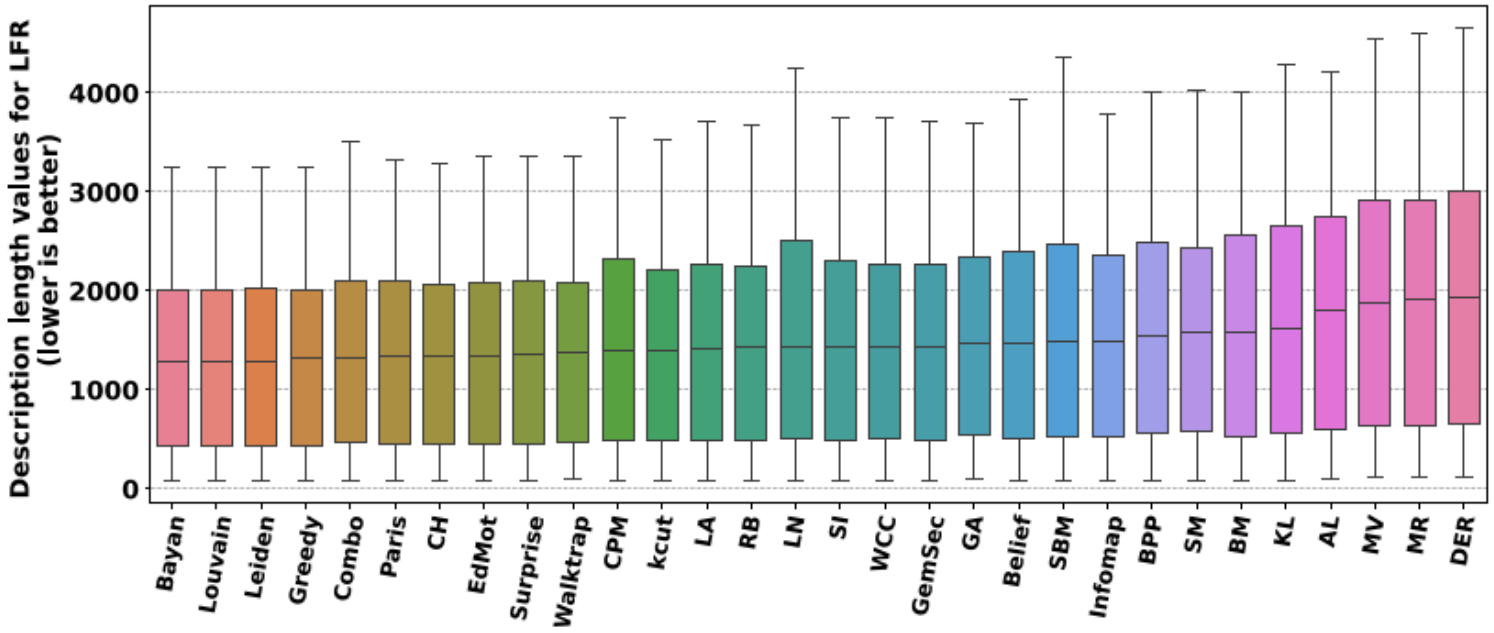
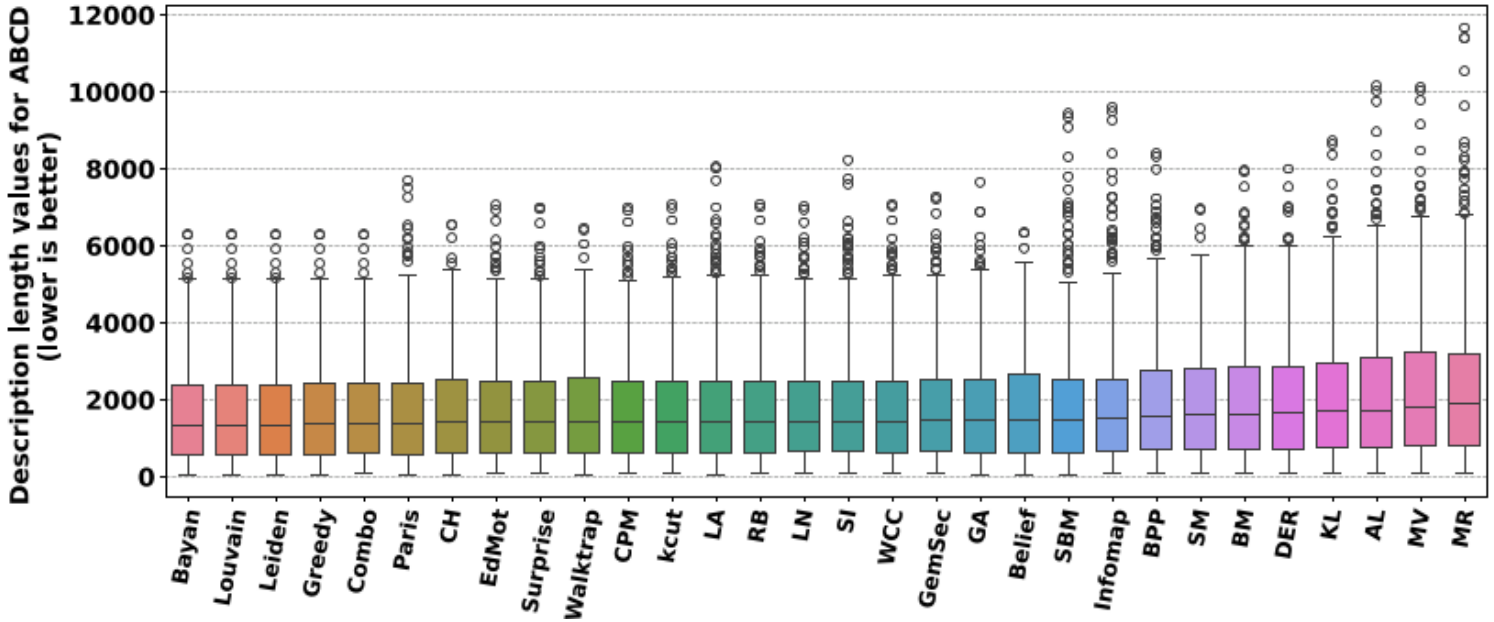
Unsupervised assessment: description length





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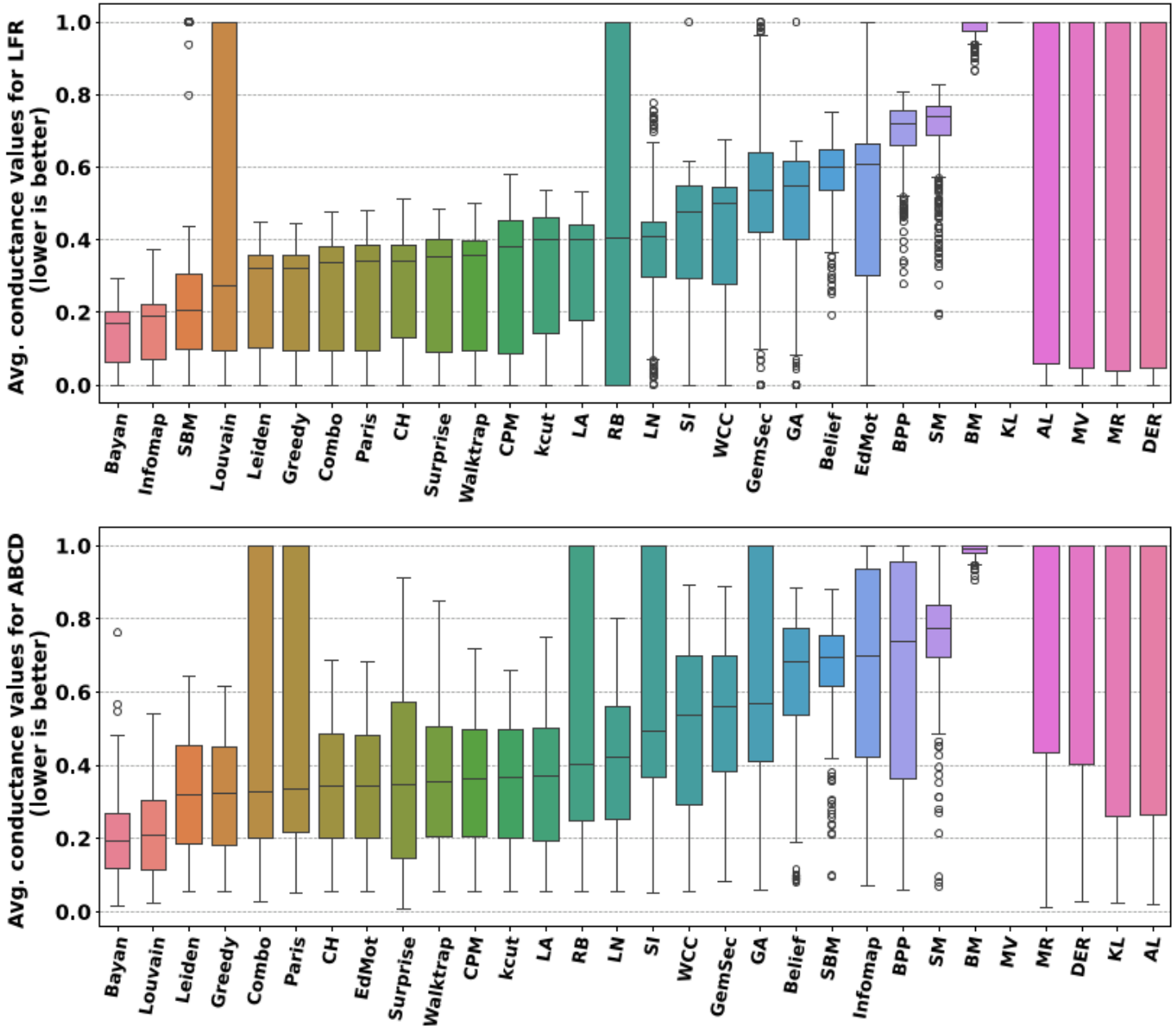
Unsupervised assessment: average conductance



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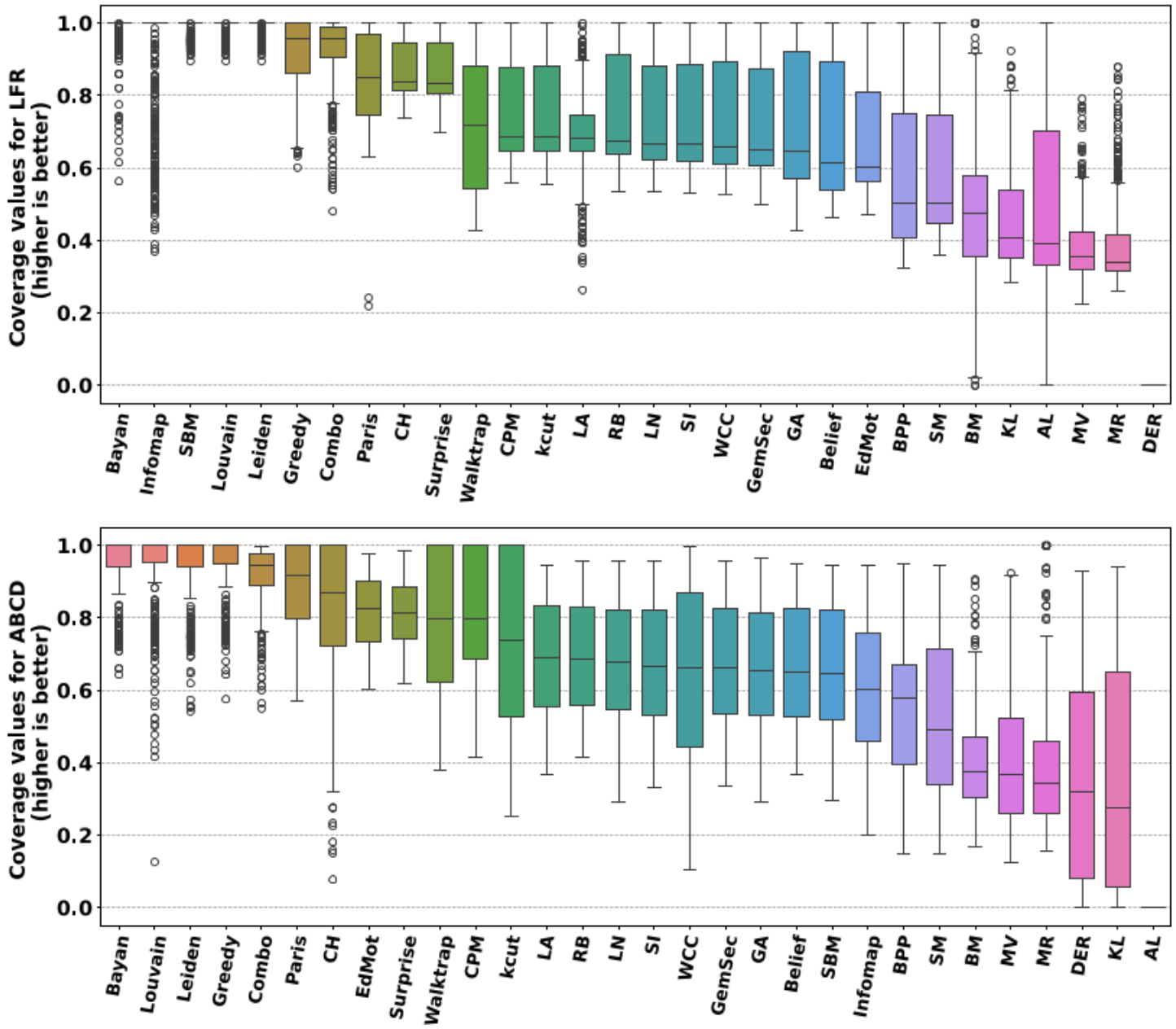
Unsupervised assessment: partition coverage



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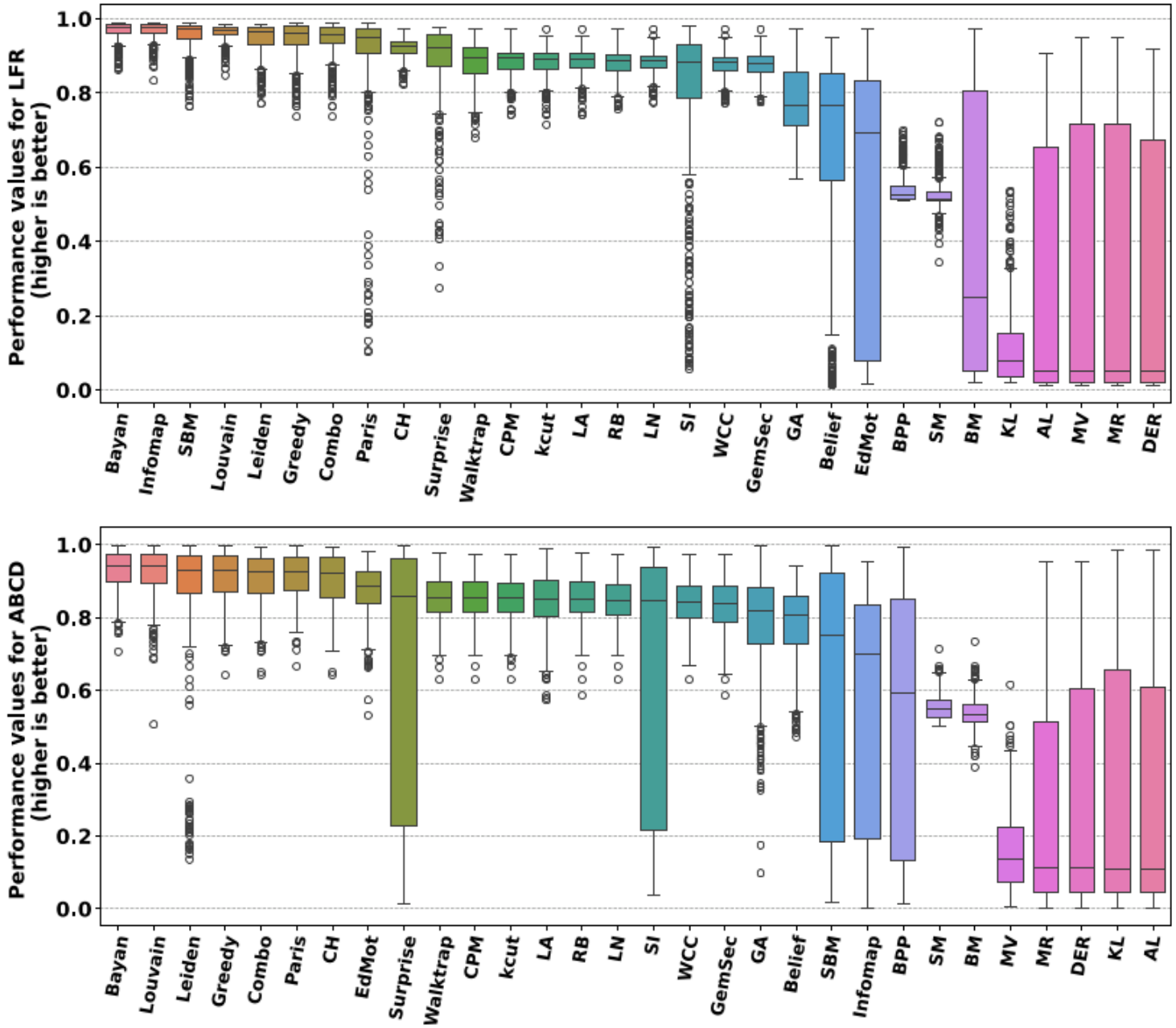
Unsupervised assessment: partition performance



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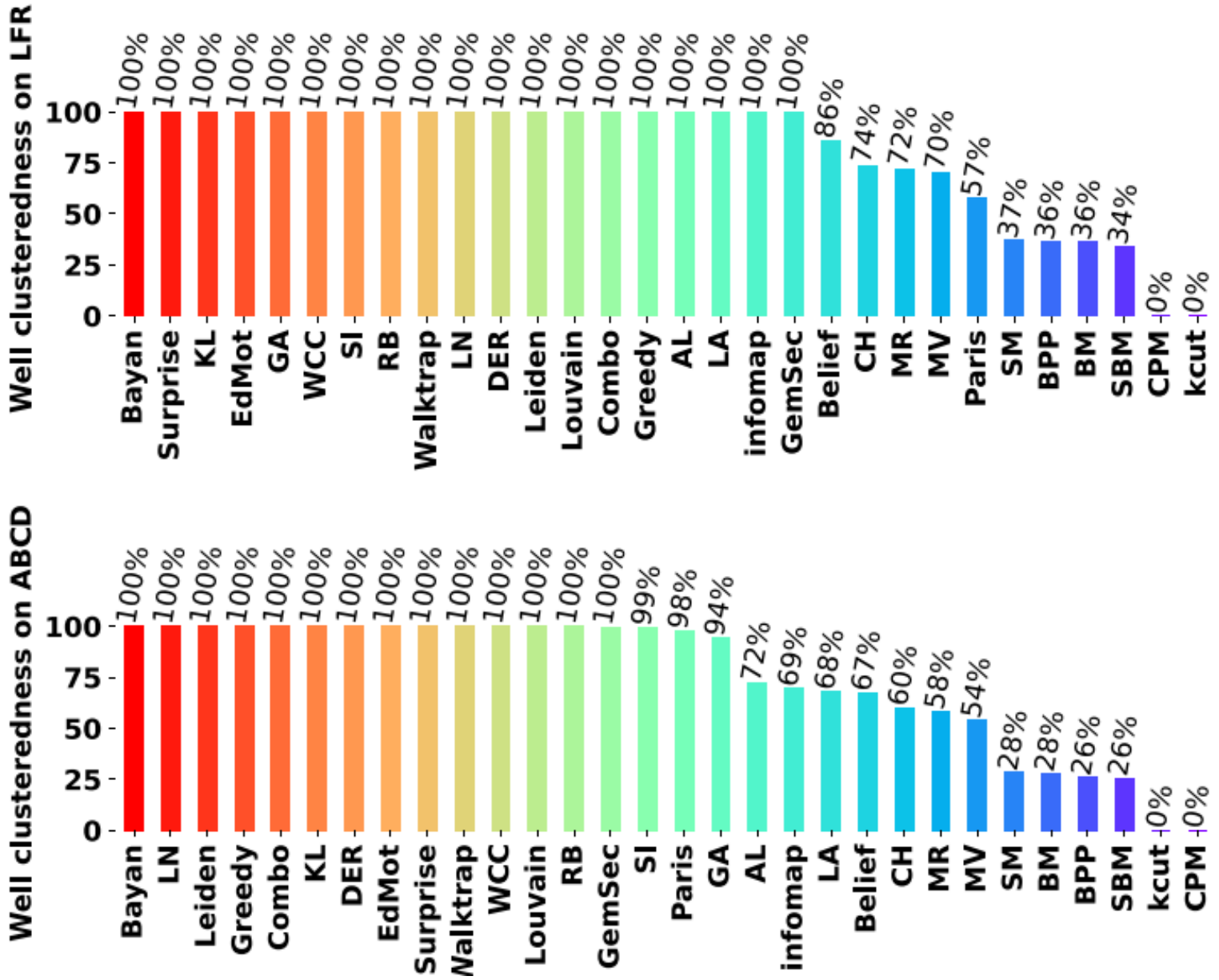
Unsupervised assessment: well-clusteredness



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Resources on this project

```
%pip install bayanpy
```

```
import networkx as nx
import bayanpy
```

```
G = nx.barbell_graph(5,2)
```

```
bayanpy.bayan(G)
```

Paper 1: doi.org/10.1007/978-3-031-36027-5_48
Paper 2: doi.org/10.1016/j.jocs.2024.102283
Paper 3: doi.org/10.1103/PhysRevE.110.044315
GitHub Repo: github.com/saref/bayan
Project website: bayanproject.github.io

Google Colab examples:
tinyurl.com/bayancolab

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1. The average modularity-based heuristic yields optimal partitions for only 43.9% of the 104 networks analyzed.

2. Suboptimal partitions of heuristic algorithms are disproportionately dissimilar to any optimal partition.

3. Bayan maximizes modularity in mid-sized networks and approximates maximum modularity in slightly larger networks on ordinary computers.

4. Modularity-maximum partitions retrieve planted partitions of LFR and ABCD graphs at rates higher than most other algorithms.

See Bayan on your smartphone ->

Thank you!

Questions?

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