MODULE I

LIMIT STATE METHOD OF DESIGN OF REINFORCED CONCRETE SECTION

- METHOD OF DESIGN

A reinforced concrete structure should be designed such that it full fills the intended purpose of the str during its life time with

1. Adequate safety, in terms of strength and ~~durabl~~ stability.
2. Adequate servicability in terms of stiffness and durability
3. Reasonable economy

The commonly used structures are designed by theoratical methods and the design methods used for a design of reinforced concrete structures are

1. Working stress method/ modular ratio method (WSM)
2. Ultimate load method/ Load factor method
3. Limit state method (LSM)

WORKING STRESS METHOD

It is the traditional method of design for reinforced concrete strs, structural steel and timber.

- This method is based on the linear elastic theory hooks law stress & strain
- ~~wit~~ The assumption of linear elastic behaviour is considered justifyable since the permissible stresses are kept below the ultimate strength of the material
- Reinforced concrete is a composite material. The WSM assumes strain compactibility ie, the strain in the reinforcing steel is assumed equal to that in adjoining concrete to which it is bonded. The stress in steel is linearly related to the stress in adjoining concrete by a constant factor called modular ratio.

Modular ratio is defined as the ratio of modulus of elasticity of steel to that of concrete. The WSM is therefore called as modular ratio method.

DEMERITS OF WSM

1. It doesn't show the true factor of safety (FOS)
2. It doesn't discriminate different types of load

that act simultaneously but act different uncertainities.

3. Becoz of creep (shrinkage) time dependent strain and non linear stress-strain relationship concrete does not have definite youngs modulus

4. The modular ratio design result in larger percentage of compression steel than that is given by the limit state design. Thus leading to uneconomical design.

MERITS

1. Simplicity both in concept as well as applicatⁿ
2. Result in relatively larger section of structural member compare to ULM
3. It is the only method to investigate RC sections for service stresses and for servicability state of deflection and cracking

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ULTIMATE LOAD METHOD

This method is based on ultimate strength of reinforced concrete structure at ultimate load. The ultimate load is obtained by ~~ch~~ enhancing the service load by some factor reffered to as load factor for giving a desired margin of safety. Therefore this method is also known as load factor method or ultimate strength method.

In this method stress condition at the state of collapse of the str is analysed thus considering non linear stress strain curves of concrete and steel.

The safety measure is ~~designed~~ obtained in ~~the~~ design using proper load factor. Therefore it is possible to use different load factors under combined loading conditions

- This method utilises a large strength in plastic region of ultimate strength of member thus resulting slender or thin elements. It is noted that the satisfactory strength performance at ultimate loads does not guarandee satisfactory servicablility performance at normal service loads

MERITS

1. The ULM uses fully the actual stress-strain curve.
2. The load factor gives exact margin of safety

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4. The fai ted usi with th results.

DEMERIT
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2. This m take the ct shrink
3. The u gth s result deflec width

LIMIT ST

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- In LSM ~~defined~~ safety checked lity at Thus re fit for use.
- LSM i ation the wa

against collapse.

3. The method allows to use different load factors for different types of loads & ~~or~~ their combination.
4. The failure load computed using ULM matches with the experimental results.

DEMERITS

1. The method does not take into consideration the servicability ~~to~~ criteria of deflection and cracking
2. This method doesnot take into consideratn the effects of creep & shrinkage
3. The use of high strength steel and concrete results in increase of deflection and crack width

LIMIT STATE METHOD

It is an ideal method which takes into account not only the ultimate strength of the structure but also the servicability & durability requirements

- In LSM, a structure is ~~defined~~ designed for safety against collapse & checked for its servicability at working loads. Thus rendering the str fit for its indended use.
- LSM includes consideration of str at both the working and the ultimate load levels with a view to satisfy the requirements of both safety and servicability.

The design value are derived from the characteristic value through the use of partial safety factor for loads and for material strengths.

TYPE OF LIMIT STATES

The acceptable limit for safety and servicability requirements before failure occurs in called a limit state. There are 2 catogory of limit state considered in the design.

(i) Limit states of collapse

This could be accessed from repture of one or more critical sections and from buckling due to elastic or plastic instability or over turning.

Different limit state of collapse are :-

(a) limit state of collapse in fluxture
(b) " in compression
(c) " in compression & uniaxial bending
(d) " in compression & biaxial bending
(e) " in shear
(f) " in bond
(g) " in torsion
(h) " in tension

ii **limit state of Servicability**

limit state of servicability relate to the performance & behaviour of the str at working loads. Durability is taken care by considering appropriate grade of concrete, nominal cover for various exposure condition, cement content, water content, types of aggregates use etc.

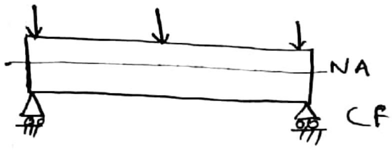
The various limit state of servicability are:-

(a) limit states of cracking
(b) " of deflection
(c) Other limit states such as fire resistance, vibration, durability etc

THEORY OF REINFORCED BEAMS & SLABS

Plain cement concrete has high compressive strength with its ~~so~~ tensile strength relatively low. Therefore the compressive strength of a beam which is made up of PCC is very low. Therefore it is reinforced by placing steel bars ~~with~~ in the tensile zone of concrete so that the compressive stress is carried by concrete & the tensile stress is carried by steel reinforcing bars only.

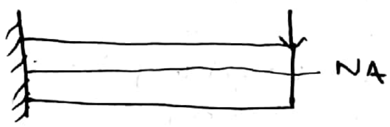
Figure(A) shows a simply supported RC beam subjected to transverse loads and bending downwards.

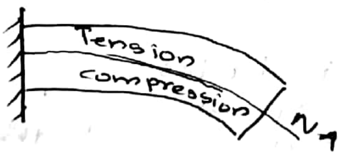


(fig. A)

In this the reinforcing steel barsis placed at a suitable depth below the neutral axis

fig(B) shows a cantilever beam bending downwards the tensile zone is above the neutral axis therefore the reinforcing steel bars are provided at a height above the neutral axis





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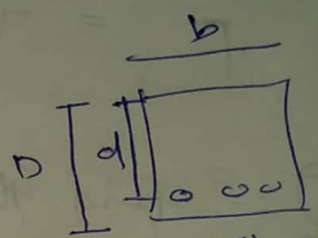
In both the cases & the reinforcement is provided in the tensile zone only such kind of section are reffered as ~~simply~~ singly reinforced section. In some cases reinforcing steel bars are provided in the compressive zone to carry the compressive loads also. Such kind of section are called as doubly reinforced section.

- The joint action of steel and concrete in a RC section depends on the following factors.

1. The bond b/w the concrete & steel bars
2. Absence of corrosion of steel bars embedded in the concrete.
3. Thermal expansion of both concrete & steel

A rectangular reinforced concrete beam has a width of 200mm & is reinforced width 2 bars of 20mm dia @ an effective depth of 400mm. If M-20 grade concrete and Fe-456 HYSD bars are used. Find the ultimate moment ~~for~~ of resistance of the section

Soln: Given.



$b = 200$ mm

$A_{st} = 2, 20mm\ \phi = 2 \times \frac{\pi}{4} D^2 = 2 \times \pi/4\ 20^2$
$= 628.3\ mm^2$

$d = 400m$

$M20 \rightarrow f_{ck} = 20 N/mm^2$

$Fe415 \rightarrow f_y = 415 N/mm^2$

From cl. G-1.1.a of IS 456:2000

$$\frac{x_u}{d} = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ b.d}$$

$$= \frac{0.87 \times 415 \times 628.318}{0.36 \times 20 \times 200 \times 400}$$

$$= 0.394$$

from note of cl. 38.1 of IS 456:2000

$$\frac{x_{umax}}{d} = 0.48$$

$\frac{x_u}{d} < \frac{x_{umax}}{d}$ ∴ It is an under reinforced section

From cl. G.1.1.b of IS 456 : 2000

$$M_u = 0.87\ f_y\ A_{st}\ d\left[1 - \frac{A_{st}\ f_y}{bd\ f_{ck}}\right]$$

$$M_u = 0.87 \times 415 \times 628.318 \times 400\left[1 - \frac{628.318 \times 41}{200 \times 400 \times 2}\right] \text{ Nmm}$$

$= 76.0415$ kNm

Q :- Find the moment of resistance of a singly reinforced concrete beam of 200 mm width & effective depth 400 mm reinforced with 3 bars of 16 mm dia of Fe-415 steel. Use M20 concrete redesign the beam if necessary.

$b = 200$ mm
$d = 400$ mm

$A_{st} = 3,\ 16\text{mm}\ \phi = 3 \times \frac{\pi}{4} \times 16^2 =$
$603.185\ mm^2$

$M_{20} \rightarrow f_{ck} = 20\ N/mm^2$
$Fe\ 415 \rightarrow f_y = 415\ N/mm^2$

From cl. G.1.1.a of IS 456 : 2000

$$\frac{x_u}{d} = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ b\ d} = \frac{0.87 \times 415 \times 603.185}{0.36 \times 20 \times 200 \times 400}$$

$= \cancel{151.23607}$ 0.378 no unit

From note of cl. 38.1 of IS code 456 : 2000

$\frac{x_{umax}}{d} = 0.48$

$\frac{x_u}{d} < \frac{x_{umax}}{d}$

$\therefore$ It is an underreinforced section.

From cl. G.1.1.b of IS code 456 : 2000

$$M_u = 0.87\ f_y\ A_{st}\ d\left[1 - \frac{A_{st}\ f_y}{bd\ f_{ck}}\right]$$

$$= 0.87 \times 415 \times 603.185 \times 400\left[1 - \frac{603.185 \times 415}{200 \times 400 \times 20}\right] \text{ Nmm}$$

$= 73483212.15$

$= 73.48$ kNm

Q :- 4 bars of 16 mm dia

$b = 200$ mm
$d = 400$ mm
$A_{st} = 4,\ 16\text{mm}\ \phi = 4 \times \frac{\pi}{4} \times 16^2 = 804.247\ mm^2$

$M_{20} \rightarrow f_{ck} = 20\ N/mm^2$
$Fe\ 415 \rightarrow f_y = 415\ N/mm^2$

From cl. G.1.1.a of IS 456 : 2000

$$\frac{x_u}{d} = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ b\ d} = \frac{0.87 \times 415 \times 804.247}{0.36 \times 20 \times 200 \times 400}$$

from cl. 38.1 of IS 456:2000

$$\frac{x_{umax}}{d} = 0.48$$

$$\frac{x_u}{d} > \frac{x_{umax}}{d}$$

It is an over reinforced section thus actual neutral axis depth is more than limiting value. Such a beam is over reinforced and is undesirable, since the failure is sudden without any warning. The code recomments that such a beam should be redesigned. the limitting value of moment of resistance is found from cl. G.1.1.c of IS 456:2000.

$$M_{u\,limit} = 0.36 \times \frac{x_{umax}}{d}\left[1 - 0.42\frac{x_{umax}}{d}\right] bd^2 f_{ck}$$

$$= 0.36 \times 0.48\left[1 - 0.42 \times 0.48\right] 200 \times 400 \times 20$$

$$M_{u\,limit} = 88296652.8 \text{ Nmm}$$

$$= 88.296 \text{ kNm}$$

From cl. G.1.1.b of IS 456:2000

$$M_u = 0.87 f_y A_{st}\, d\left[1 - \frac{A_{st} f_y}{bd f_{ck}}\right]$$

$$88.296 \times 10^6 = 0.87 \times 415 A_{st} \times 400\left[1 - \frac{A_{st} \times 415}{200 \times 400 \times 20}\right]$$

$ax^2 + bx + c = 0$

$$611.383 = A_{st} - \frac{A_{st}^2 \times 415}{200 \times 400 \times 20}$$

$$-A_{st}^2\, 2.593 \times 10^{-4} + A_{st} = 611.383$$

$$2.593 \times 10^{-4} A_{st}^2 - A_{st} + 611.383 = 0$$

~~Ast = 3094~~ $x_1 = 3094.629$

~~Ast = 4393.23~~ $x_2 = 761.907$

$A_{st} = 761.907 \text{ mm}^2$

DESIGN OF SINGLY REINFORCED BEAM SECTION

The design of singly reinforced section consist the determination of

1. Gross sectional dimensions b and d ~~and computation~~
2. Area of steel for developing ~~suitable~~ given moment of resistance.

Steps

1. Determine the cross sectional dimension breadth, depth, effective span
2. Determine loads & bending moment
3. Compute the depth of neutral axis & compare it with a limiting value.

4. Determine Ast if $\frac{x_u}{d}$ less than $\frac{x_{u,m}}{d}$.

from the eqn

$$M_u = 0.87\, f_y\, A_{st}\, d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

5. check the section for shear.
6. check the section for deflection
7. Compute the development length.
8. If $\frac{x_u}{d} > \frac{x_{u\,max}}{d}$, the ~~do~~ section is redesigned by changing the cross sectional dimensions & then follow the steps from 4 to 7

Q Design a beam using limit state method for the following data.

Given details

Clear span = 4m

width of the support = 300mm

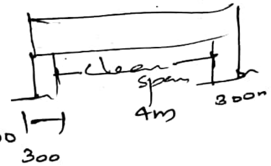
Service load = 5 kN/m

Materials used = Fe 415 HYSD bars & M-20 concrete

beam size 400x200

Step 1

1) computation of beam dimension



From cl. 23.2.1 a of IS 456:2000

the slenderness ratio

$l/d < 20$

Effective span

smaller { (i) c/c (ii) clear span + d.

Take $l/d = 12$ [thump rule]

$l/d = 12$

$\frac{4000}{d} = 12$

effective depth, $d = \frac{4000}{12} = 333.3$ mm

To find the overall depth

Assume ϕ of bars used as 20mm

From table 16 of IS 456:2000

for moderate climate

cover = 30mm

$D = d + c + \phi/2$

$= 333.33 + 30 + \frac{20}{2} = 373.33$ mm

$\approx \boxed{400 \text{ mm}}$

So revising effective depth.

$d = D - c - \phi/2$

$= 400 - 30 - 20/2 = \boxed{360 \text{ mm}}$.

width of beam $= [0.5 \text{ to } 0.67]\, D$

overall B $= 0.5\, D = 400 \times 0.5$

$= 200$ mm

To find effective span of a

cl. 22.2 a

(i) centre to centre distance.

$300/2 + 4000 + 300/2 = 4300$ mm

(ii) clear span + d

$4000 + 360 = 4360$ mm

$\therefore$ Effective span of beam is 4300 mm

assume 2 to 5 as live load.

KN/m ⇒ always UDL

STEP 2 : COMPUTATION OF LOADS & BM

Dead load, Live load

1). Dead load = [cross section Area] × unit weight of reinforced concrete. 25 kN

400 × 200

$= 0.2 \times 0.4 \times 25$

$= 2$ kN/m

2) Live load = 5 kN/m (Service load) Assumption.

$\therefore$ Total load $= 2+5 = 7$ kN/m

Bending moment $= \frac{wl^2}{8}$ (T.L, effective span)

$= \frac{7 \times 4.3^2}{8}$

$= 16.178$ kNm

Ultimate moment = BM × F.O.S (Factor of safety)

$= 16.178 \times 1.5$ (Table 11, 68)

$M_u = 24.267$ kNm

22.08.19 WEDNESDAY

~~STEP 3~~

~~COMPUTATION OF NEUTRAL AXIS~~

CHECK ~~WHEATHER~~ SINGLY | DOUBLY From Annex G cl. G.1.1.c of IS 456:2000

Singly → Down only tension

Doubly → Down & up → compression load.

Singly & Doubly and modulus that's we require

Singly: Under, Over, Balanced — Always sudden failure

$M_{u\,limit} = 0.36 \frac{x_u}{d}\left[d - 0.42\frac{x_u}{d}\right] bd^2 f_{ck}$

$= 0.36 \times 0.48 \left[1 - 0.42 \times 0.48\right] \times 200 \times 400^2 \times 20$

$= 88.296$ kNm

$x_u \rightarrow$ neutral axis depth

$M_u < M_{u\,limit}$

$\therefore$ It is singly reinforced section

STEP 3 : COMPUTATION OF NEUTRAL AXIS

From Annex G cl. G.1.1.c of IS 456:2000

to check $x_u < x_{u\,limit}$

there limit comes $x_{u\,max}$ region.

$M_u = 0.36 \frac{x_u}{d}\left[1 - 0.42\frac{x_u}{d}\right] bd^2 . f_{ck}$

24.267×10^6 Nmm $= 0.36 \frac{x_u}{d}\left[1 - 0.42\frac{x_u}{d}\right] 200 \times 360^2 \times 20$

$ax^2 + bx + c = 0$

$0.130032 = \frac{x_u}{d} - 0.42\left(\frac{x_u}{d}\right)^2$

$-0.42\left(\frac{x_u}{d}\right)^2 + \frac{x_u}{d} - 0.130012 = 0$

Smaller value $\frac{x_u}{d} = 0.138, 2.24$

T.L = 7 kN/m

M = 16.18 kNm

$M_u =$

From Cl. 38.1 note of IS 456:2000

$$\frac{x_{u\,max}}{d} = 0.48$$

A_{st} :- Area of steel in tension

$$\frac{x_u}{d} < \frac{x_{u\,max}}{d}$$

∴ The section is under reinforced.
Hence safe

STEP 4 : COMPUTATION OF AREA OF STEEL

from cl. G.1.1.b of IS 456:2000

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{bd f_{ck}}\right]$$

$$24.267 \times 10^6 = 0.87 \times 415 \times A_{st} \times 360 \left[1 - \frac{A_{st}\, 415}{20 \times 200 \times 360}\right]$$

$A_{st} = 197.99\ mm^2$

check for Ast → b/w $A_{st\,min}$ & $A_{st\,max}$

From cl. 26.5.1.1 (a) of IS 456:2000 pg: 46

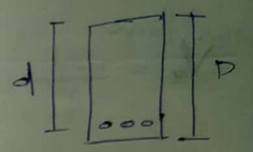
$$\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$$

$A_{st\,min} < A_{st}$
This will be given

$$A_{st\,min} = \frac{0.85\, bd}{f_y}$$

$$= \frac{0.85 \times 200 \times 360}{415}$$

$= 147.46\ mm$



$D = d +$ clear cover $+ \frac{1}{2}$ dia of bar

from cl. 26.5.1.1.(b) of IS 456:2000

$A_{st\,max} = 0.04\, bD$

$= 0.04 \times 200 \times 400$

$= 3200\ mm^2$

$A_{st\,min} < A_{st} < A_{st\,max}$

Refer step 3
↑
Assume ϕ 20 mm

~~STEP 5~~ : ~~CHECK FOR SHEAR~~

$$\text{No. of bars} = \frac{A_{st}}{\text{Area of 1 bar}} = \frac{197.99}{\frac{\pi}{4} \times 20^2}$$

→ Refer effective depth.

$= 0.63$

Assume 12 mm ϕ bars. $= \frac{197.99}{\frac{\pi}{4} \times 12^2}$

for 1000 range 99% area is 20mm dia

$= 1.75 = 2$ No.s
→ Always round forward

Spacing → next qn.

STEP 5 : STEP FOR SHEAR

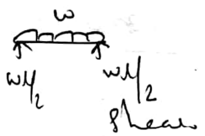
lateral dia (stirrups)
To prevent shear

From cl. 40.1. of IS 456:2000

Nominal shear stress

$$\tau_v = \frac{V_u}{b \times d}$$

Shear force $V_u = \frac{wl}{2}$



~~Total load~~ $= \frac{7 \times 4.3}{2} = 15.05$ kN

ultimate shear $V_u = V \times 1.5 = 15.05 \times 1.5$ → Factor of safety.

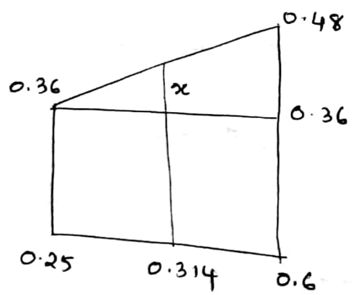
$= 22.575$ kN

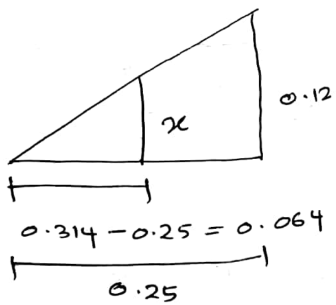
$\tau_v = \frac{V_u}{bd} = \frac{22.57 \times 10^3}{200 \times 360} = 0.313$ N/mm²

DESIGN SHEAR STRESS

From table 19 of IS 456 : 2000

$\tau_c \Rightarrow \frac{100 A_s}{bd} = \frac{100 \times 2 \times \pi/4 \times 12^2}{200 \times 360} = 0.314$





$\frac{0.12}{x} = \frac{0.25}{0.064}$

$x = 0.03072$

$\tau_c = 0.36 + 0.08 = 0.39072$

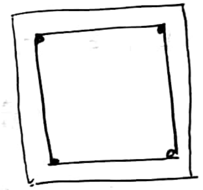
From table 20 of IS 456:2000

$\tau_{c\,max} = 28$ N/mm²

$\tau_v < \tau_c < \tau_{c\,max}$

If $\tau_v < \tau_y$, from cl. 40.3 of IS min. shear reinforcement is to be provided.

From cl. 26.5.16 of IS



$\frac{A_{sv}}{b s_v} = \frac{0.4}{0.87 f_y}$

Assume 2 legged stirrups of 8mm ϕ

$A_{sv} = 2 \times \pi/4 \times 8^2 =$

$= \frac{2 \times \pi/4 \times 8^2}{200 \times S_v} = \frac{0.4}{0.87 \times 415} = 453.7$ mm

From cl. 26.5.1.5

Max, spacing of shear reinforcement is 300 mm.

So provide 2 legged stirrups of 8 mm ϕ @ 300 mm c/c

STEP 7 : CHECK DEFLECTION

Slenderness ratio $= \frac{l}{d} = \frac{4300}{360}$

$= 11.94$

$\therefore$ safe.

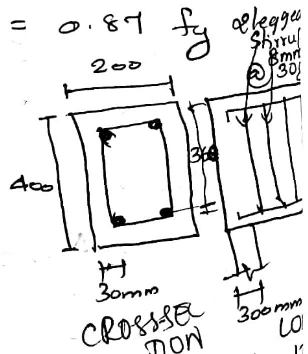
STEP 8 : CHECK FOR DEVELOPMENT LENGTH

From cl. 26.2.1 of IS 456:2000

$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$ $\quad \sigma_s = 0.87 f_y$

$= \frac{12 \times 0.87 f_y}{4 \times 12}$

$=$



26.08.19
Monday

Design a balanced singly reinforced beam for an applied moment of 60 kNm the width of the beam is limitted to 175mm. Use M20 concrete & Fe 415 steel

Soln: $b = 175mm$

$M = 60\ kNm \quad M_u = 60 \times 1.5\ kNm$

$f_y = 415\ N/mm^2$

$f_{ck} = 20\ N/mm^2$

Annex. cl. G.1.1.c of IS 456:2000 Balanced $\rightarrow x_u = x_{umax}$

Skipping first 4 steps.

$M_{u\,limit} = 0.36 \frac{x_{umax}}{d}\left[1 - 0.42 \frac{x_{umax}}{d}\right] b d^2 f_{ck}$

$60 \times 1.5 \times 10^6 = 0.36 \times 0.48 \left[1 - 0.42 \times 0.48\right] 175 \times d^2 \times 20$

$\boxed{d = \cancel{385.75}\ 431.83\ mm}$

Assume a clear cover of 30 mm & $\phi_b = 16mm$

$D = d + c + \phi/2$

$D = 431.83 + 30 + 16/2$

$= 469.83$

$D \approx 470mm$

$d = 470 - 30 - 8 = 432\ mm$

cl. G.1.1.b of IS 456:2000

$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$

$60 \times 1.5 \times 10^6 = 0.87 \times 415\ A_{st} \times 432 \left[1 - \frac{A_{st} \times 415}{175 \times 432 \times 20}\right]$

$577.02 = A_{st} - \frac{A_{st}^2\ 415}{175 \times 432 \times 20}$

$577.02 = A_{st} - 2.74 \times 10^{-4} A_{st}^2$

$2.74 \times 10^{-4} A_{st}^2 - A_{st} + 577.02 = 0$

$A_{st} = 2931.18, \cancel{718.451}\ 718.79\ mm^2$

pg 46 Cl. 26.5.1.1.(a)

$$\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$$

$$A_{st\,min} = \frac{0.85 \times 175 \times 432}{415}$$

$$A_{st\,min} = 154.843\ mm^2$$

Cl. 26.5.1.1.b

$$A_{st\,max} = 0.04\ bD$$

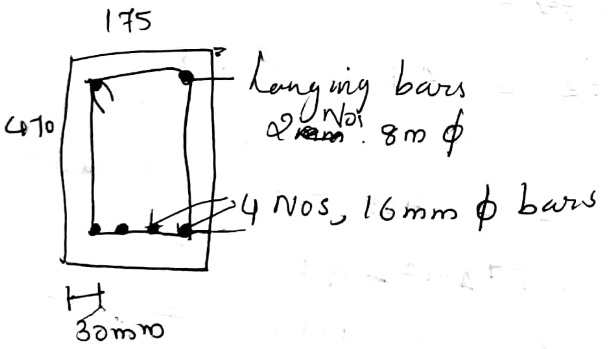
$$= 0.04 \times 175 \times 470$$

$$= 3290\ mm^2$$

$\therefore A_{st\,min} < A_{st} < A_{st\,max}$

$$\text{No. of Bar} = \frac{A_{st}}{\text{Area of 1 bar}} = \frac{718.479}{\frac{\pi}{4} \times 16^2}$$

$= 3.57 \approx 4$ No.s



8, 8, 12, 12, 20, 24, 25, 32

28.09.2019
WEDNESDAY

Q :- Find the reinforcement for the beam of effective depth 500mm, width 175mm applied moment 60kNm Use M20 concrete & Fe415 steel.

$d = 500mm$

$b = 175mm$

$M = 60kN/m$

$M_u = 60 \times 1.5\ kNm = 90\ kNm \Rightarrow$ ultimate moment (1.5 → FOS)

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

From cl. G.1.1.c of IS 456 : 2000

$$M_{u\,limit} = 0.36 \frac{x_{umax}}{d}\left[1 - 0.42 \frac{x_{umax}}{d}\right] bd^2 f_{ck}$$

$$= 0.36 \times 0.48 \left[1 - 0.42 \times 0.48\right] 175 \times 500^2 \times 20$$

$= 120.718080$ (6 decimal shift)

$M_u < M_{u\,limit}$

$\therefore$ It is a singly reinforced section.

(4) Check whether under reinforced $x_u < x_{umax}$

From cl. G.1.1.c of IS 456:2000

$$M_u = 0.36 \frac{x_u}{d}\left[1 - 0.42 \frac{x_u}{d}\right] bd^2 f_{ck}$$

$$90 \times 10^6\ Nmm = 0.36 \frac{x_u}{d}\left[1 - 0.42 \frac{x_u}{d}\right] 175 \times 500^2 \times 20 \ \cancel{bd^2 f_{ck}}$$

$$\frac{x_u}{d} - 0.4\left(\frac{x_u}{d}\right)^2 = 0.2857$$

Scanned by CamScanner

$$-0.4\left(\frac{x_u}{d}\right)^2 + \frac{x_u}{d} - 0.2857 = 0$$

$$x = 0.328,\ 2.171$$

$$\frac{x_u}{d} = 0.33$$

$$\frac{x_{umax}}{d} = 0.48 \qquad \frac{x_u}{d} < \frac{x_{umax}}{d}$$

It is under reinforced

The section is safe

* Reinforcement = ?

A_{st} = ?

From Cl. G.1.1.b

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$90\times10^6 = 0.87 \times 415 \times A_{st} \times 500 \left[1 - \frac{A_{st}\ 41}{175\times500\times2}\right]$$

$$498.546 = A_{st}\left[1 - \frac{A_{st}\ 415}{175\times500\times20}\right]$$

~~Ast – Ast² 87245~~

$$A_{st} - 2.371\times10^{-4} A_{st}^2 - 498.546 = 0$$

$$A_{st} = 577.66,\ 3639.96$$

$$A_{st} = 577.66\ mm^2$$

Cl. 26.5.1.1.a

$$\frac{A_{stmin}}{bd} = \frac{0.85}{f_y}$$

$$A_{stmin} = \frac{0.85\times175\times500}{415}$$

$$= 179.219\ mm^2$$

Cl. 26.5.1.1.b

$$A_{stmax} = 0.04\ bD$$

$$D = d + c + \frac{\phi}{2} \quad \text{Assumption}$$

$$= 500 + 30 + \frac{16}{2}$$

$$= 538$$

$$= 0.04\times538\times175$$

$$= 3766\ mm^2$$

$$A_{stmin} < A_{st} < A_{stmax}$$

$$179.219 \qquad 577$$

A_{st} safe.

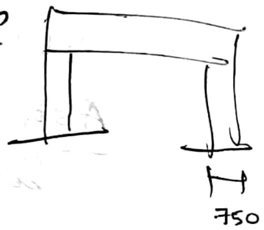
$$\text{No. of bars} = \frac{A_{st}}{\text{Area of 1 bar}} = \frac{577}{\frac{\pi}{4}\times16^2} = 2.86$$

$$= 3\ \text{No.s}$$

Q: A RC beam is supported on 2 walls 750mm thick spaced at a clear distance of 6m the beam carries a superimposed load of 9.8 kN/m Design the beam using M20 concrete & HYSD bars of Fe415 steel

$l = 6m = 6000mm$

width of support = 750 mm



live load = 9.8 kN/m

$f_y = 415 \; N/mm^2$

$f_{ck} = 20 \; N/mm^2$

Soln

1* Computation of beam dimension

$\frac{l}{d} = 12 \quad \frac{6000}{12} = d \qquad \frac{l}{d}$ max 20

(max 20) $\quad d = 500$ mm $\qquad$ Slew

Assume clear cover = 30 mm

$\phi \quad \phi_{bar} = 20mm$ (assumptn)

$D = d + c + \phi/2$

$= 500 + 30 + 10 = 540$ → Safe to round to 550 always round up.

$D = 540mm$

$b = 0.5 \times 540$

$= 270mm \qquad b = (0.5 \text{ to } 0.6)$

cl. 22.2 of IS 456 : 2000

$l_{effective}$ (i) c/c distance = 6000 + 750

$= 6750$ mm

(ii) $l_{clear\ span} + d = 6000 + 500$

$= 6500$ mm

$\therefore l_{eff} = 6500$ mm or 6.5 m

$d = 500$ mm $\quad \phi = 20$ mm $\quad D = 540$ mm

$l_{eff} = 6.5m \quad b = 270$ mm

Step 2 Load & B.M

Unit wgt of concrete $= 25 \; kN/m^3$

LL = 9.8 kN/m

D.L = b × D × 25

= 0.27 × 0.54 × 25

= 3.645 kN/m

load & BM kN/m

TL = LL + DL = 9.8 + 3.645

= 13.445 kN/m

$B.M = \frac{w l^2_{effective}}{8} = \frac{13.445 \times 6.5^2}{8}$

= 71.006 kNm

$M_u = 71.006 \times 1.5 = 106.509$ kNm

$= 106.509 \times 10^6$ Nmm

Step 3 check Singly / Doubly.

cl. G.1.1.c

$M_{u\,limit} = 0.36 \frac{x_{umax}}{d} \left[1 - 0.42 \frac{x_{umax}}{d}\right] bd^2 f_{ck}$

$= 0.36 \times 0.48 \left[1 - 0.42 \times 0.48\right] 270 \times 500^2 \times 20$

= 186250752

= 186.25

$M_u < M_{u\,limit}$ → Singly.

Step 4 : Under reinforcen $x_u < x_{umax}$

Cl. G.1.1. c

$$M_u = 0.36 \frac{x_u}{d} \left(1 - 0.42 \frac{x_u}{d}\right) bd^2 f_{ck}$$

$$106.509 \times 10^6 = 0.36 \frac{x_u}{d} \left[1 - 0.42 \frac{x_u}{d}\right] 270 \times 500^2 \times$$

$$-0.42 \frac{x_u}{d}^2 + \frac{x_u}{d} - 0.219 = 0$$

$$\frac{x_u}{d} = 0.244 , 2.13$$

$$\frac{x_u}{d} = 0.244 . \qquad \frac{x_{umax}}{d} = 0.48$$

$$\frac{x_u}{d} < \frac{x_{umax}}{d}$$

$\therefore$ Under reinforced

Safe section.

Step 5 Reinforcement = ?

A_{st} = ?

Cl. G.1.1.b

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{bd f_{ck}}\right]$$

$$106.509 \times 10^6 = 0.87 \times 415 \, A_{st} \, 500 \left[1 - \frac{A_{st} \, 415}{210 \times 500 \times 20}\right]$$

$$589.99 = A_{st} - A_{st}^2 \times 1.537 \times 10^{-4}$$

$$-1.537 \times 10^{-4} A_{st}^2 + A_{st} - 589.99 = 0$$

$$A_{st} = 656.166 , 5850.01$$

$$A_{st} = 656.166$$

cl. 26. 5.1.1.a

$$\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$$

$$A_{stmin} = \frac{0.85 \times 270 \times 500}{415}$$

$$= 276.506$$

cl. 26.5.1.1.b

$$A_{stmax} = 0.04 bD$$

$$= 0.04 \times 270 \times 540 = 5832$$

$A_{stmin} < A_{st} < A_{stmax}$

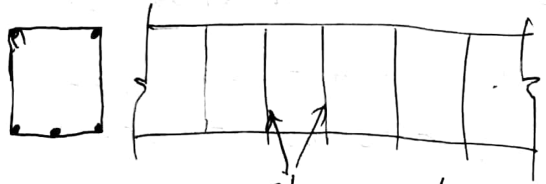
A_{st} safe

$$\text{No. of bars} = \frac{A_{st}}{\text{Area of 1 bar}} = \frac{656.166}{\pi/4 \times 20^2} = 2.08$$

3 No.s

Step 6 Check for Shear

pg: 72



Stirrups – shear reinforcement.

Cl. 40.3

$\tau_v < \tau_c$ 1) Minimum shear reinforcement

$\tau_v > \tau_c$ 2) Design shear – Inclined stirrups, vertical ", Bent up bars

$\tau_v \rightarrow$ Nominal shear $\rightarrow$ Loads

$\tau_c \rightarrow$ Design shear $\rightarrow$ Based on % of ste

$$\tau_v = \frac{V_u}{bd}$$

Shear force, $V_u = \frac{wl}{2} = \frac{13.445 \times 6.5}{2} \times 1.5$

$= 65.5\, kN$

$$\tau_v = \frac{V_u}{bd} = \frac{65.5 \times 10^3}{270 \times 500}$$

Table 19

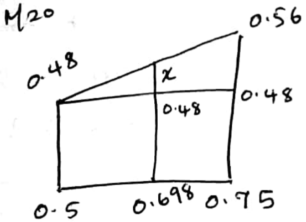
$\tau_v = 0.485\ N/mm^2$

$\tau_c \Rightarrow$

$$100\frac{A_s}{bd} = \frac{100 \times 3 \times \pi/4 \times 20^2}{270 \times 500} = 0.698$$

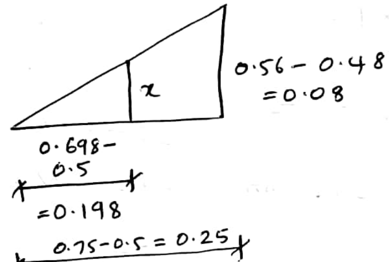
As → provide steel

M20



Similar Δles.

$\tau_c \Rightarrow 0.48 + x$



$$\frac{0.08}{x} = \frac{0.25}{0.198}$$

$$x = \frac{0.08 \times 0.198}{0.25}$$

$x = 0.063$

$\tau_v = 0.485\ N/mm^2$

$\tau_c = 0.48 + x$

$= 0.543\ N/mm^2$

$\tau_v < \tau_c$ (Cl. 40.3) $< \tau_{cmax}$

Table 20 → $\tau_{cmax} = 2.8\ N/mm^2$

∴ provide minimum shear reinforcement from cl. 26.5.1.6

Area of legs

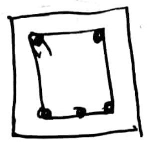
$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

$b \times s_v$

$s_v \rightarrow$ spacing of stirrups

$$\frac{A_{sv}}{b\, s_v} = \frac{0.4}{0.87 f_y}$$

unkno



Assume 2 legged stirrup of 8mm ϕ

A_{sv} = area of stirrups

$= 2 \times \pi/4 \times 8^2$

$$\frac{A_{sv}}{b \times s_v} = \frac{0.4}{0.87 f_y}$$

$$\frac{2 \times \pi/4 \times 8^2}{270 \times S_v} = \frac{0.4}{0.87 \times 415}$$

~~$S_v = 1344.322$~~

$S_v = 336.077$ mm

Cl. 26.5.1.5

Max spacing shall not exceed 300 mm

Check for Deflection

Slenderness ratio, $\frac{l}{d} = \frac{6500}{500}$

$= 13$

< 20

∴ section is safe

Step 8

Development length

Cl. 26.2.1

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$ ← cl. 26.2.1.1

$$= \frac{20 \times 0.87 \times 415}{4 \times 1.2} = 1504.375 \text{ mm}$$

$\sigma_s = 0.87 f_y$

overlapping or bending

Stress – N
Moment KN
KN
Force KN
Distance m
Dead load KN

W – KN
M – KNm

Equation
W – N
– mm

30.09.2019
MONDAY.

A beam simply supported over an eff. span of 7m carries a live load of 20 KN/m. Design the beam using M20 concrete & HYSD bars of grade Fe415. Keep the width of the beam = 1/2 the eff depth. Assume unit wt of concrete 25 KN/m³.

Soln:

L.L = 20 KN/m

f_{ck} = 20 N/mm²

f_y = 415 N/mm²

b = d/2

l_{eff} = 7m

Step 1
Computation of beam dimension

$\frac{l}{d} = 12$

$\frac{7000}{d} = 12$ $\quad \frac{7000}{12} = d$

d = 583.33 mm

Assume clear cover = 30 mm

ϕ_{bar} = 20 mm

$D = d + c + \phi/2$

= 583.3 + 30 + 10

= 623.3 mm

= 650 mm

d = 610

b = 610/2

= 305 mm

Step 2
2. Load & BM

L.L = 20 KN/m

D.L = b × D × 25

0.305 × 0.650 × 25

= 4.95625 KN

Total load = LL + DL

= 20 + 4.95

= 24.95625 KN

$BM = \frac{wl^2}{8}$

$= \frac{24.956 \times 7^2}{8}$

= 152.855 KNm

M_u = 152.855 × 1.5

= 229.283 KNm

M_u = 229.283 × 10⁶ Nmm

Step 3
Check singly / doubly

cl. G.1.1.c

$M_{u_{limit}} = 0.36 \frac{x_{umax}}{d}\left[1 - 0.42 \frac{x_{umax}}{d}\right] bd^2 f_{ck}$

$= 0.36 \times 0.48 \left[1 - 0.42 \times 0.48\right] 305 \times 610^2 \times 20$

= 313150977.3 Nmm

= 313.1509 KNm

$M_u < M_{u_{limit}}$

⇒ Singly.

Step 4:
Under reinforced

$x_u < x_{umax}$

$M_u = 0.36 \frac{x_u}{d}$ [1

229.283×10⁶ = 0.3

0.28 = $\frac{x_u}{d}$

$-0.42 \frac{x_u}{d}^2 +$

x = 0.324

$\frac{x_u}{d}$ = 0.3

$\frac{x_{umax}}{d}$ = 0.

$\frac{x_u}{d} < \frac{x_{u}}{}$

∴ Under
Safe

Step 5
Reinforcement

Ast = ?

cl. G.1.1.b

$M_u = 0.87 f_y$

229.283×10⁶ =

1041.057

x = 120

Ast =

$$M_u = 0.36 \frac{x_u}{d}\left[1 - 0.42\frac{x_u}{d}\right] bd^2 f_{ck}$$

$$229.283\times10^6 = 0.36\frac{x_u}{d}\left[1-0.42\frac{x_u}{d}\right]$$

$$= 305\times610^2\times20$$

~~0.40~~

$$0.28 = \frac{x_u}{d} - 0.42\frac{x_u}{d}^2$$

$$-0.42\frac{x_u}{d}^2 + \frac{x_u}{d} - 0.28 = 0$$

$$x = 0.324,\ 2.05$$

If redesign Make d unknown

$$\frac{x_u}{d} = 0.324$$

$$\frac{x_{umax}}{d} = 0.48$$

$$\frac{x_u}{d} < \frac{x_{umax}}{d}$$

∴ under reinforced Safe section.

Step 5

Reinforcement = ?

Ast = ?

Cl. G.1.1.b

$$M_u = 0.87 f_y A_{st} d\left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$229.283\times10^6 = 0.87\times415\times A_{st}\times610\left[1-\frac{A_{st}\times415}{305\times610\times20}\right]$$

$$1041.057 = A_{st} - 1.115\times10^{-4} A_{st}^2$$

$$x = 1202.2,\ 7766.4$$

$$A_{st} = 1202.2$$

cl. 26.5.1.1.a

$$\frac{A_{stmin}}{bd} = \frac{0.85}{f_y}$$

$$A_{stmin} = \frac{0.85\times305\times610}{415}$$

$$= 381.06$$

cl. 26.5.1.1.b

$$A_{stmax} = 0.04\ bD$$

$$= 0.04\times305\times650$$

$$= 7930$$

$$A_{stmin} < \underset{1202.2}{A_{st}} < A_{stmax}$$

Ast safe

$$\text{No. of bars} = \frac{A_{st}}{\text{Area of 1 bars}}$$ (with "Area" struck out before Ast)

$$= \frac{1202.2}{\pi/4\times20^2}$$

$$= 3.82$$

= 4 Nos

Step 6 : Check for shear

$$\tau_v = \frac{V_u}{bd}$$

shear force,

$$V_u = \frac{wl}{2}$$

$$= \frac{24.956\times7}{2}\times$$

$$= 131.019\ k$$

$\tau_v = \frac{V_u}{bd}$

$= \frac{131.019 \times 10^3}{305 \times 610}$

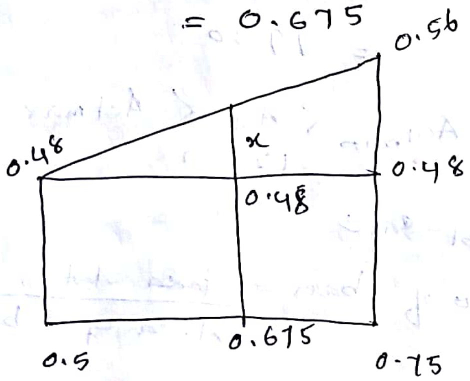
$\tau_v = 0.704$

Table 19 ⇒

τ_c ⇒

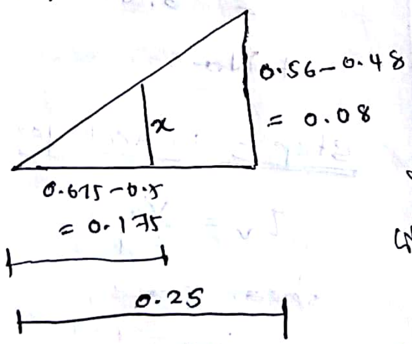
$100 \frac{A_s}{bd} = \frac{100 \times 4 \times \pi/4 \times 20^2}{305 \times 610}$

$= 0.675$



$\tau_c = 0.48 + x$

Similar Δles



$\frac{0.08}{x} = \frac{0.25}{0.175}$

$x = 0.056$

$\tau_c = 0.48 + x$
$= 0.536$

$\tau_v > \tau_c < \tau_{c\,max}$

$\tau_{c\,max} = 2.8$ (Table 20)

∴ Design shear reinforcement in the form vertical stirrups is to be provided.

From cl. 40.4. a

$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$?

Assume 2legged stirrups of 8mm ϕ.

$\left[V_u - \tau_c b d\right] = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 61}{S_v}$

$\left[131.109 \times 10^3 - 0.536 \times 305 \times 610\right] = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 6}{S_v}$

$S_v = 705.437$ mm

check for spaing [26.5.1.5]

smaller value.

i) 705.437 mm.

ii) 0.75d = 457.5

iii) 300 mm

∴ So provide 2 legged Stirrup of 8mm ϕ @ 300mm c/c

ep:7 Check for D

Slenderness ra

= 70 / 6

< 2

∴ sec

Step 8 : Deve

cl. 26.2.1

$L_d = \frac{\phi \sigma_s}{4 \tau_b}$

$= \frac{20 \times}{4 \times}$

= 150

1-10-2019
WEDNESDAY

DESIGN
REINFOR

Step 1 : Comput of beam – lons, b, overall effective

Step 2 : Comp and ben DL, LL,

Step 3 : che doubly If the a larger it is a Section

Step 4 : Com of nec $\frac{x_u}{d}$ &

step:1
Check for Deflection

Slendernous ratio , l/d

$= \frac{7000}{610} = 11.475$

< 20

$\therefore$ section is safe.

Step 8 : Development length

cl. 26.2.1

$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$ $\sigma_s = 0.87 f_y$.

$= \frac{20 \times 0.87 \times 415}{4 \times 1.2}$

$= 1504.375$ mm

9-10-2019
WEDNESDAY

DESIGN OF DOUBLY REINFORCED SECTION

Step 1: Computation of beam dimensions, b, effective depth, overall depth, width of beam, effective span.

strain corresponding
to design stress
fsc

Step 2: Computation of loads and bending moment
DL, LL, TL, ultimate moment

Step 3: check for singly / doubly reinfoced section
If the applied moment is larger than M_{ulimit} then it is a doubly reinforced Section.

Step 4: Computation of depth of neutral axis
x_u/d & x_{umax}/d

if $\frac{x_u}{d} < \frac{x_{umax}}{d}$ then section is underreinforced, & it is safe. If $\frac{x_u}{d} > \frac{x_{umax}}{d}$ the section is over reinforced & code recomments redesign

Step 5: compute the area of tensile reinforcement (A_{st1}) for the singly reinforced section for the moment M_{ulimit}

Step 6: Compute the value of compression reinforcement from the equation
$M_u - M_{ulimit} = f_{sc} A_{sc} (d - d')$

Step 7: compute the value of A_{st2} from the eqn

$A_{st2} = \frac{A_{sc} f_{sc}}{0.87 f_y}$

The total area of tensile reinforcement shall be obtained from the eqn

$A_{st} = A_{st1} + A_{st2}$

Step 8: check for Ast & Asc
[$A_{st\,min}$, $A_{st\,max}$, $A_{sc\,min}$, $A_{sc\,max}$]

compute the no. of bars in tension side and compression side.

Step 9: check for shear
[$\tau_v, \tau_c, \tau_{cmax}$]