TYPES OF PROBLEMS

→ Determination of moment of resistance of a given s/n.

→ Determination of actual stresses in the given s/n subjected to a given BM

→ Design of a given s/n to resist a given ultimate design moment.

PROBLEMS

1. A Rectangular reinforced concrete beam has a width of 200mm and is reinforced with 2 bars of 20mm ~~diagra~~ diameter @ an effective depth of 400mm. If M-20 grade concrete and Fe 415 HYSD bars are used, Find the ultimate moment of resistance of the section.

SOLUTION

Given :- $b = 200mm$

$d = 400mm$

$A_{st} = 2,\ 20mm\ \phi$

$= 2 \times \frac{\pi}{4} \times 20^2$

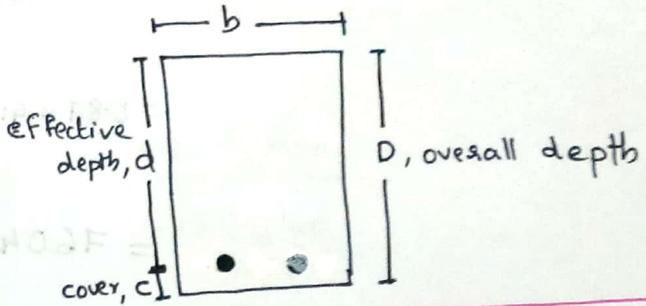
$= 628.3\ mm^2$

Grade of concrete = M_{20}

$\therefore f_{ck} = 20N/mm^2$

Grade of steel = Fe 415

$\therefore f_y = 415N/mm^2$



overall depth = effective depth + effective cover

$D = d + c$

[From cl. G.1.1.a of IS 456 : 2000]
Page = 96

Depth of N.A, $\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b \cdot d}$

$= \frac{0.87 \times 415 \times 628.3}{0.36 \times 20 \times 200 \times 400} = 0.394$

[From note below cl.38.1 of IS 456:2000
page.70]

limiting value of depth of N.A, for Fe 415 is

$$\frac{x_{u\,max}}{d} = 0.48$$

$$0.394 < 0.48$$

$$\frac{x_u}{d} < \frac{x_{u\,max}}{d}$$

∴ It is an under reinforced section.

[From cl.G.1.1.b of IS 456:2000, page 96]

Moment of Resistance,

$$M_u = 0.87 f_y A_{st}\, d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$= 0.87 \times 415 \times 628.318 \times 400 \left[1 - \frac{628.318 \times 415}{200 \times 400 \times 20}\right]$$

$$= 76041514.76\,Nmm$$

$$= 76.0415\,kNm$$

② Find the moment of resistance of a singly reinforced concrete beam of 200mm width and effective depth 400mm reinforced with 3 bars of 16mm diameter using Fe 415 steel. Use M_{20} Concrete. Redesign the beam if necessary.

ii) Find M_R of the beam if 4 bars of 16mm φ are used.

SOLUTION

GIVEN :- $b = 200mm$

$d = 400mm$

$A_{st} = 3, 16mm\ \phi$

$= 3 \times \frac{\pi}{4} \times 16^2$

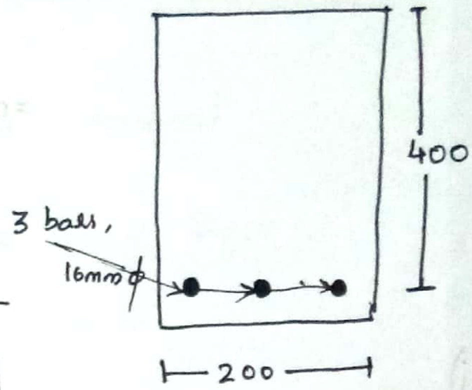
$= 603.185 mm^2$

Grade of concrete = M20

$\therefore f_{ck} = 20 N/mm^2$

Grade of steel = Fe 415

$\therefore f_y = 415 N/mm^2$



i) [From cl. G.1.1.a of IS 456:2000
page : 96]

Depth of N.A, $\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$

$= \frac{0.87 \times 415 \times 603.185}{0.36 \times 20 \times 200 \times 400}$

$= 0.378$

[From note below cl.38.1 of IS 456:2000
page : 70]

limiting value of depth of N.A for Fe 415 is

$\frac{x_{umax}}{d} = 0.48$

$0.378 < 0.48$

$\therefore \frac{x_u}{d} < \frac{x_{umax}}{d}$ $\therefore$ It is an under reinforced s/n.

[From cl. G.1.1.b of IS 456:2000, page 96]

Moment of Resistance,

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$= 0.87 \times 415 \times 603.185 \times 400 \left[1 - \frac{603.185 \times 415}{200 \times 400 \times 20}\right]$$

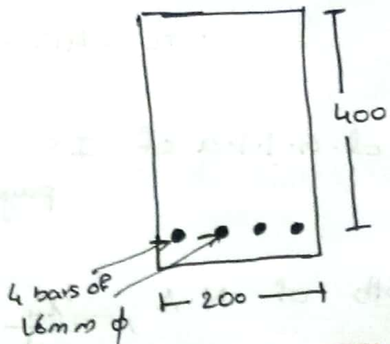
$$= 73483212.15 \text{ Nmm}$$

$$= \underline{\underline{73.48 \text{ kNm}}}$$

ii) A_{st} = 4 bars, 16mm ϕ

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2$$

$$= 804.247 \text{ mm}^2$$



[From cl. G.1.1.a of IS 456:2000, page 96]

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 \times f_{ck} b d}$$

$$= \frac{0.87 \times 415 \times 804.247}{0.36 \times 20 \times 200 \times 400}$$

$$= 0.504$$

[From Note below cl. 38.1 of IS 456:2000 page No: 70]

$$\frac{x_{u\,max}}{d} = 0.48$$

$$0.504 > 0.48 \quad \therefore$$

$\therefore \frac{x_u}{d} > \frac{x_{umax}}{d}$

$\therefore$ It is a over reinforced s/n.

[From cl. G.1.1.d of IS 456:2000, P.N-96]

The section should be redesigned

The limiting moment of resistance is found from

[cl. G.1.1.C of IS 456:2000, Page 96]

$$M_{ulim} = 0.36 \times \frac{x_{umax}}{d}\left[1 - 0.42\frac{x_{umax}}{d}\right] bd^2 f_{ck}$$

$$= 0.36 \times 0.48\left[1 - 0.42 \times 0.48\right] \times 200 \times 400^2 \times 20$$

$$= 88296652.8 \text{ Nmm}$$

$$\therefore M_{ulim} = 88.296 \text{ kNm}$$

From cl. G.1.1.b of IS 456:2000, page 96

$$M_u = 0.87 f_y A_{st} d\left[1 - \frac{A_{st} f_y}{bd f_{ck}}\right]$$

$$88.926 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400\left[1 - \frac{A_{st} \times 415}{200 \times 400 \times 20}\right]$$

$$611.383 = A_{st} - \frac{A_{st}^2 \times 415}{200 \times 400 \times 20}$$

$$2.593 \times 10^{-4} \times A_{st}^2 - A_{st} + 611.383 = 0$$

Solving, $A_{st} = 761.907 \text{mm}^2$

NOTE

$x_1 = 3094.629$
$x_2 = 761.907$
least value is considered among the

DESIGN OF SINGLY REINFORCED SECTION

Design of the singly reinforced section consist of the determination of

1) Cross-sectional dimensions b and d
2) Area of steel for developing given moment of resistance.

Steps

1. Determine the cross-sectional dimensions, breadth, depth, effective span etc.
2. Determine the loads and ~~B.M~~ bending moment.
3. Compute the depth of N.A & compare it with a limiting value.
4. Determine Ast if $\frac{x_u}{d} < \frac{x_{umax}}{d}$ from the equation,

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

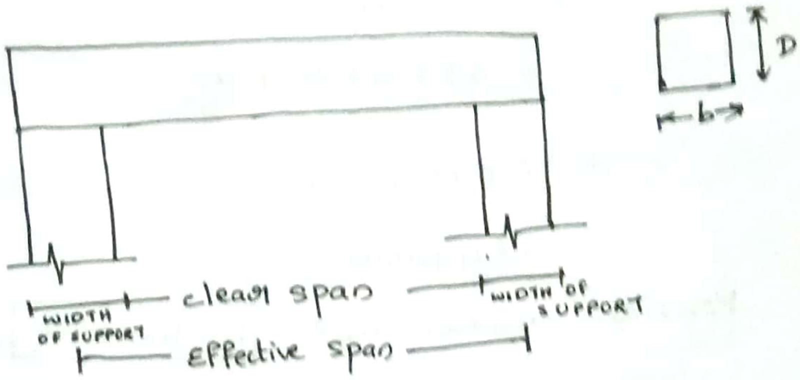
5. check the s/n for shear
6. check the s/n for deflection
7. Compute the development length.
8. If $\frac{x_u}{d} > \frac{x_{umax}}{d}$ the s/n should be redesigned by changing the c/s dimensions and then follow the steps from 4 to 7.

3. Design a beam using limit state method for the following data.

clear span = 4m, width of the support = 300mm

service load = 5kN/m, materials used = Fe 415 HYSD bars and M 20 concrete.

SOLUTION



given

$f_{ck} = 20 N/mm^2$ clear span, $l = 4m$

$= 4000 mm$

$f_y = 415 N/mm^2$

$L.L = 5kN/m$

STEP-1 : COMPUTATION OF BEAM DIMENSIONS [width, depth eff span]

From cl.23.2.1.a of IS 456:2000, Page 37

slenderness ratio for S.S beam should not be greater than 20

ie, $\frac{l}{d} < 20$

Take $\frac{l}{d} = 12$ [Thumb rule]

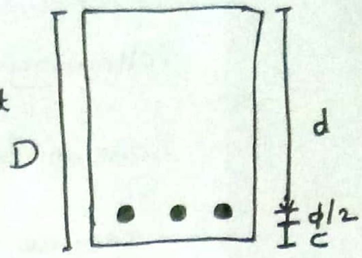
$\frac{4000}{d} = 12$

$\therefore d = \frac{4000}{12}$

$= 333.3 mm$

To find overall depth, $D = d + c + \phi/2$

Assume 20mm ϕ bars for reinforcement



[From Table 16 of IS 456:2000 page 47]

For moderate climate, nominal cover = 30mm

$$D = d + c + \phi/2$$

$$= 333.3 + 30 + \frac{20}{2}$$

$$= 373.33 \text{ mm}$$

$$\simeq 400\text{mm}$$

Provide overall depth, $\boxed{D = 400\text{mm}}$

$\therefore$ effective depth, $d = D - c - \phi/2$

$$= 400 - 30 - 10$$

$$\boxed{d = 360\text{mm}}$$

To find width of beam.

NOTE

width of beam = 0.5 to 0.67 D

Assume, $b = 0.5D$

$$\therefore b = 0.5 \times 400$$

$$\boxed{b = 200\text{mm}}$$

To find effective Span :-

[From cl. 22.2.a of IS 456:2000, page 34]

i) centre to centre distance b/w the supports

$$\frac{300}{2} + 4000 + \frac{300}{2} = 4300\text{mm}$$

ii) clear span + eff depth

$$4000 + 360 = 4360 mm$$

effective ~~depth~~ span is taken as the least of two cases.

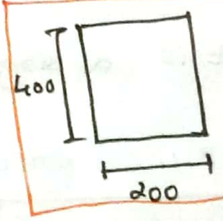
$\therefore$ effective span = 4300 mm

$$\boxed{l_{eff} = 4300 mm}$$

STEP: 2 : COMPUTATION OF LOADS & BM [D.L, L.L & B.M]

Dead load = [c/s Area] x unit weight of concrete.

NOTE



c/s Area = 0.2 x 0.4 = 0.08 m^2

UNIT WEIGHT OF CONCRETE = 25 KN/m^3

$$D.L = 0.2 \times 0.4 \times 25 = 2 KN/m$$

Live load, L.L = 5 KN/m [service load given]

$\therefore$ Total load = D.L + L.L = 2 + 5 = 7 KN/m

Bending moment $= \frac{wl^2}{8}$ | l :- effective span, w :- Total load

$$= \frac{7 \times 4.3^2}{8}$$

$$= 16.178 KN/m$$

ultimate moment = Bending moment x F.O.S

M_u = B.M x F.O.S

M_u = 16.178 x 1.5 [Table 18 of IS 456:2000 page 68]

$$\therefore M_u = 24.267 KNm$$

STEP:3 :- CHECK WHETHER DOUBLY / SINGLY

[From Annex.G., cl.G.1.1.c of IS 456:2000]

$$M_{u\,lim} = 0.36 \frac{x_{umax}}{d}\left[1 - 0.42\frac{x_{umax}}{d}\right] bd^2 f_{ck}$$

$$= 0.36 \times 0.48\left[1 - 0.42 \times 0.48\right] \times 200 \times 360^2 \times 200$$

$$= 88.296\,kNm$$

$\therefore$ $M_u < M_{ulim}$

It is a singly reinforced s/n.

STEP:4 :- CHECK COMPUTATION OF N.A

[From Annex.G. cl.G.1.1.c of IS 456:2000]

$$M_u = 0.36 \times \frac{x_u}{d}\left[1 - 0.42\frac{x_u}{d}\right] bd^2 f_{ck}$$

$$24.267 \times 10^6 = 0.36 \times \frac{x_u}{d}\left[1 - 0.42\frac{x_u}{d}\right] 200 \times 360^2 \times 20$$

$$0.130032 = \frac{x_u}{d}\left[1 - 0.42\frac{x_u}{d}\right]$$

$$0.130032 = \frac{x_u}{d} - 0.42\left(\frac{x_u}{d}\right)^2$$

$$0.42\left(\frac{x_u}{d}\right)^2 - \frac{x_u}{d} + 0.130012 = 0$$

solving,

$$\frac{x_u}{d} = 0.138 \text{ or } 2.24$$

smaller value is considered.

$$\therefore \frac{x_u}{d} = 0.138$$

[From Note below cl. 38.1 of IS 456:2000]
page 70

$$\frac{x_{u\,max}}{d} = 0.48$$

$0.138 < 0.48$

$$\therefore \frac{x_u}{d} < \frac{x_{u\,max}}{d}$$

The s/n is under reinforced. Hence safe.

STEP:5 : COMPUTATION OF AREA OF STEEL

[From cl. G.1.1.b of IS 456-2000, page 96]

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$22.267 \times 10^6 = 0.87 \times 415 \times A_{st} \times 340 \left[1 - \frac{A_{st} \times 415}{200 \times 340 \times 20}\right]$$

$$181.390 = A_{st}\left[1 - 3.05 \times 10^{-4} A_{st}\right]$$

$$181.390 = A_{st} - 3.05 \times 10^{-4} A_{st}^2$$

$$3.05 \times 10^{-4} - A_{st} + 181.390 = 0$$

Solving,

$$A_{st} = 197.99\,mm^2$$

Check for Ast

[From cl 26.5.1.1.a of IS 456:2000, page 46]

$$\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$$

$$A_{st\,min} = \frac{0.85\,bd}{f_y}$$

$$= \frac{0.85 \times 200 \times 360}{415}$$

$$= 147.46\ mm^2$$

[From cl. 26.5.1.1. b of IS 456:2000, page 47]

$$A_{st\,max} = 0.04\,bD$$

$$= 0.04 \times 200 \times 400$$

$$= 3200\,mm^2$$

$$147.46 < 197.99 < 3200$$

$$A_{st\,min} < A_{st} < A_{st\,max}$$

Hence safe.

$\therefore$ No:of tension bars, $n = \dfrac{A_{st}}{\text{Area of 1 bar}}$

$$= \frac{197.99}{\frac{\pi}{4} \times 20^2}$$

$$= 0.63$$

$\because$ the no:of bars are less

assume bars of 12 mm ϕ

$\therefore$ no:of bars, $n = \dfrac{197.99}{\frac{\pi}{4} \times 12^2}$

$$= 1.75$$

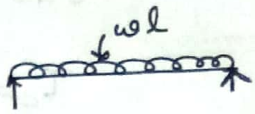
$$\approx 2 \text{ Nos}$$

$\therefore$ provide 2 bars of 12mm in the tensile zone

STEP: 6 - CHECK FOR SHEAR

[From cl. 40.1 of IS 456:2000, page 72]

→ Nominal shear stress, $\tau_v = \frac{V_u}{bd}$



Shear force, $S.F, V = \frac{wl}{2}$

$= \frac{7 \times 4.3}{2}$

$= 15.05\,kN$

ultimate shear force, $V_u = V \times F.O.S$

$= 15.05 \times 1.5$

$= 22.575\,kN$

$= 22.575 \times 10^3\,N$

$\therefore \tau_v = \frac{V_u}{bd}$

$= \frac{22.575 \times 10^3}{200 \times 360}$

$= 0.313\,N/mm^2$

→ Design shear stress

[From Table 19 of IS, 456:2000 page 73]

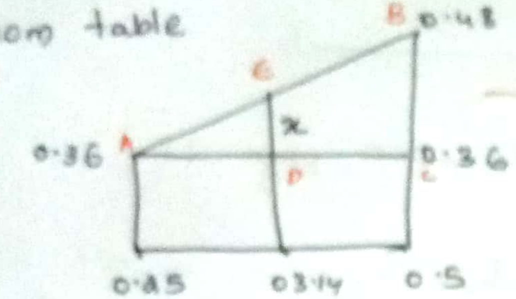
$\tau_c \Rightarrow 100 \frac{A_s}{bd}$

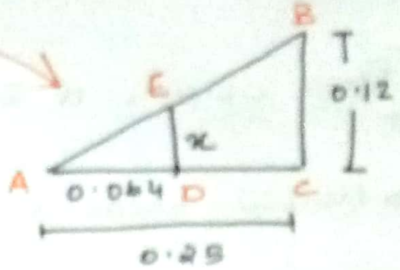
where A_s = Actual Area of steel provided

i.e, $A_s = 2 \times \frac{\pi}{4} \times 12^2$

$\frac{100 A_s}{bd} = \frac{100 \times 2 \times \frac{\pi}{4} \times 12^2}{200 \times 360} = 0.314$

From table





$\tau_c = 0.36 + x$

Considering similar Δ^{les},

$$\frac{0.25}{0.064} = \frac{0.12}{x}$$

$\therefore x = 0.03072$

$\therefore \tau_c = 0.36 + 0.03072$

$= 0.39072\ N/mm^2$

→ Max shear stress

[From Table 20 of IS 456:2000, page 73]

$\tau_{cmax} = 2.8 N/mm^2$

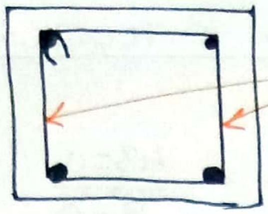
$\therefore$ We have,

$$\tau_v < \tau_c < \tau_{cmax}$$

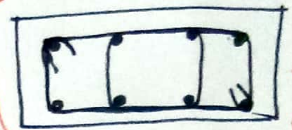
$\because \tau_v < \tau_c$ According to [cl.40.3 of IS-456:2000] minimum shear reinforcement is to be provided. according to cl.26.5.1.6

[From cl.26.5.1.6 of IS 456:2000, page 48]

$$\frac{A_{sv}}{b_{sv}} = \frac{0.4}{0.87 f_y}$$



FOUR LEGGED STIRRUPS



Here assume 2 legged stirrups of 8mm is provided

$$\therefore A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$\frac{A_{sv}}{bS_v} = \frac{0.4}{0.87 f_y}$$

[S_v is the stirrup spacing]

$$\therefore S_v = \frac{A_{sv} \times 0.87 f_y}{0.4 \times b}$$

$$= \frac{2 \times \frac{\pi}{4} \times 8^2 \times 0.87 \times 415}{0.4 \times 200}$$

$$\therefore S_v = 453.7mm$$

Check for spacing of stirrups,

[From cl. 26.5.1.5 of IS 456 : 2000, page 47]

the maximum spacing of shear reinforcement is 300mm.

$\therefore$ so provide 2 legged stirrups of 8mm ϕ @ 300mm c/c

STEP: 7 :- CHECK FOR DEFLECTION

slenderness ratio, $\frac{l}{d} = \frac{4300}{360}$

$= 11.94$

< 20 [cl. 23.2.1.a of IS 456:2000, page 37]

$\therefore$ the s/n is safe in deflection

STEP: 8 :- DEVELOPMENT LENGTH

[From cl.26.2.1 of IS 456:2000, page 42]

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

$\sigma_s = 0.87 f_y$

$= 0.87 \times 415$

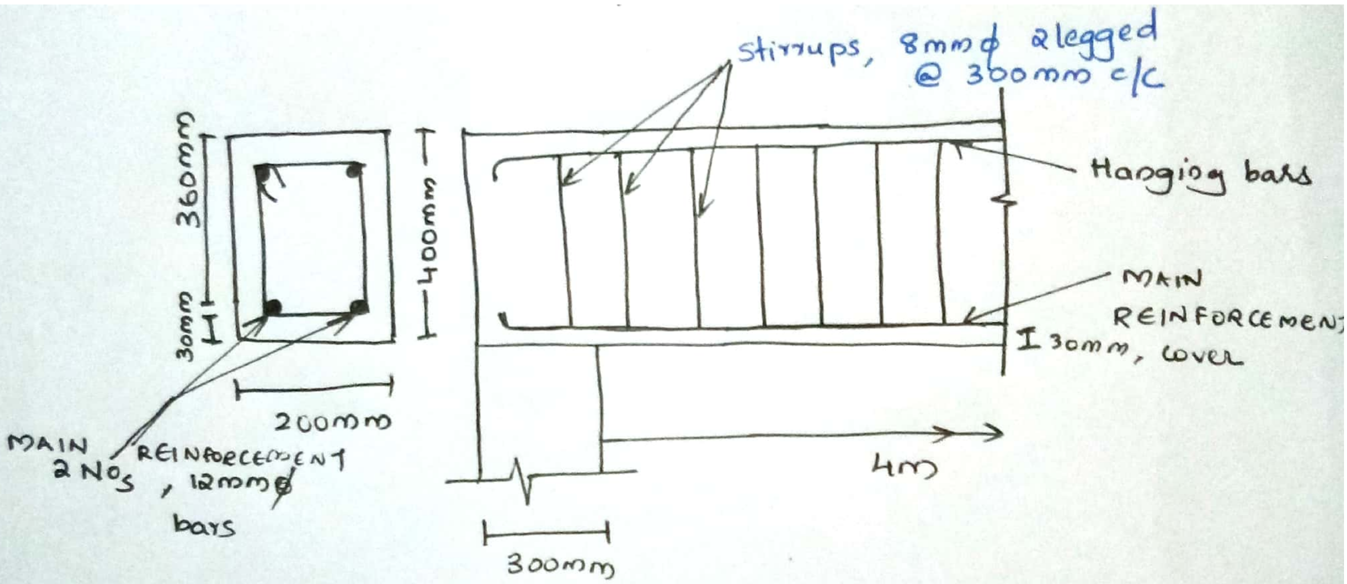
τ_{bd} [From Table of cl. 26.2.1.1 of IS 456:2000, page 43]

$\tau_{bd} = 1.2\ N/mm^2$

Substituting value of τ_{bd} & σ_s in L_d

we have $L_d = \frac{12 \times 0.87 \times 415}{4 \times 1.2}$

$L_d = 902.625 mm$


Stirrups, 8mm ϕ 2legged
@ 300mm c/c
Hanging bars
MAIN
REINFORCEMENT
30mm, cover
360mm
30mm
400mm
200mm
MAIN REINFORCEMENT
2 NOS , 12mm ϕ
bars
4m
300mm