

Binary Trees-1

Lesson Plan



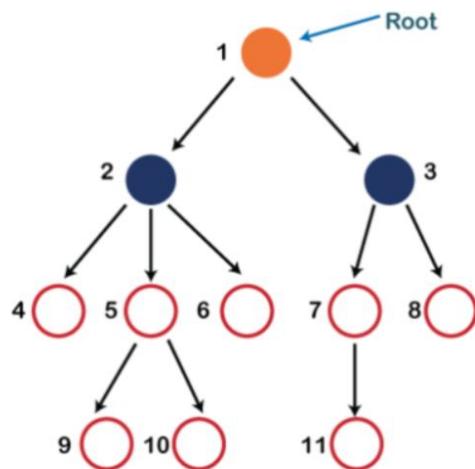
Today's Checklist

- What is a Tree data structure
- Representation
- Terminology
- Important properties of trees
- Types of Trees
- Applications of tree data structure
- What is a Binary Tree?
- Implementation
- Traversals
- Problems
- Types of Binary Trees

What is a Tree Data Structure?

- Tree is a data structure that is used to represent hierarchical data.
- A tree data structure is defined as a collection of objects or entities known as nodes that are linked together to represent the hierarchy.
- It is a hierarchical structure as elements in a Tree are arranged in multiple levels.
- Real life example: A family tree – Shows hierarchy in a family.

Representation of a Tree



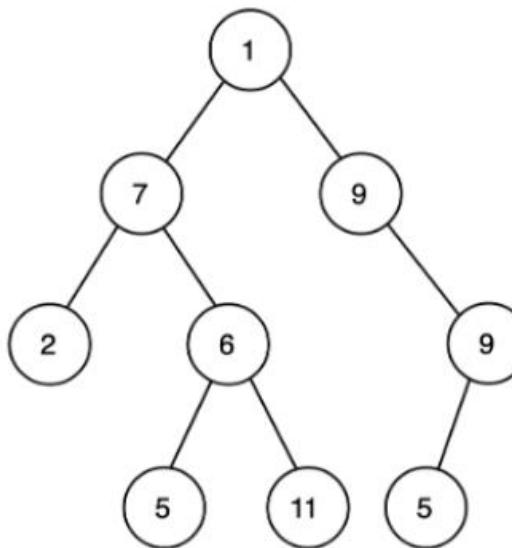
- The topmost node is known as a root node.
- Each node contains some data and the link or reference of other nodes that can be called children.
- Each node contains some data, and data can be of any type.
- The tree is represented by its root node, as all other nodes can be traversed if we know the root node.

Terms used in a Tree data structure

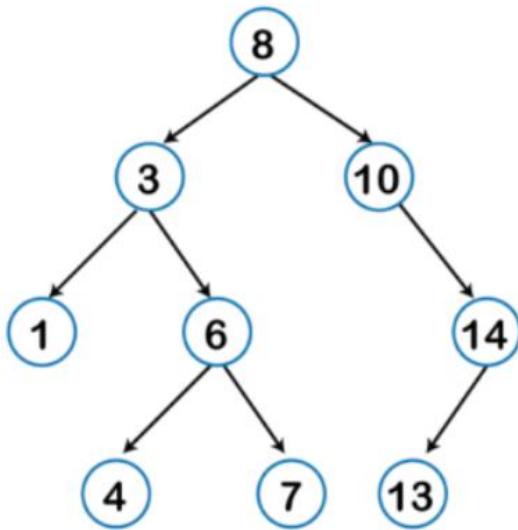
- **Root:** The root node is the topmost node in the tree hierarchy. In other words, the root node is the one that doesn't have any parent. In the above structure, node numbered 1 is the root node of the tree. If a node is directly linked to some other node, it would be called a parent-child relationship.
- **Child node:** If the node is a descendant of any node, then the node is known as a child node.
- **Parent:** If the node contains any sub-node, then that node is said to be the parent of that sub-node.
- **Sibling:** The nodes that have the same parent are known as siblings.
- **Leaf Node:** The node of the tree, which doesn't have any child node, is called a leaf node. A leaf node is the bottom-most node of the tree. There can be any number of leaf nodes present in a general tree. Leaf nodes can also be called external nodes.
- **Level** - Level of a node represents the generation of a node. If the root node is at level 0, then its next child node is at level 1, its grandchild is at level 2 and so on.
- **Number of edges**- If there are n nodes, then there would be n-1 edges. Each arrow in the structure represents the link or path. Each node, except the root node, will have at least one incoming link known as an edge. There would be one link for the parent-child relationship.
- **Height**- The height of a tree (also known as depth) is the maximum distance between the root node of the tree and the furthest leaf node.
- **Size**- Size of a tree is the total number of nodes in the tree.

Types of Trees:

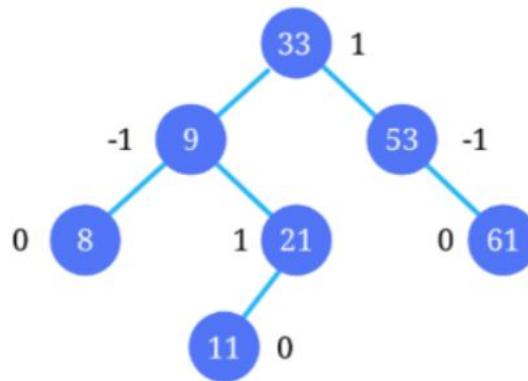
- **Binary tree:** Here, binary name itself suggests two numbers, i.e., 0 and 1. In a binary tree, each node in a tree can have utmost two child nodes. Here, utmost means whether the node has 0 nodes, 1 node or 2 nodes.



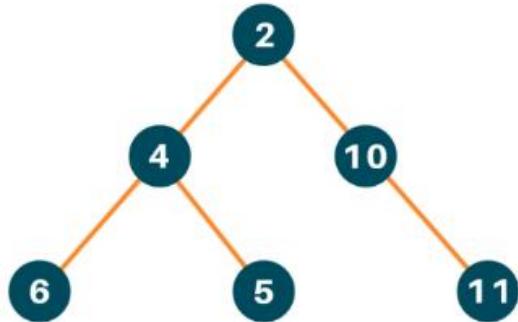
- **Binary search Tree:** Binary search tree is a non-linear data structure in which every node in the left subtree must contain a value less than the value of the root node, and the value of each node in the right subtree must be bigger than the value of the root node.



- **AVL Trees:** It is a self-balancing binary search tree. Here, self-balancing means balancing the heights of the left subtree and right subtree i.e.



1. For the tree given below, count the number of leaf nodes present in the tree

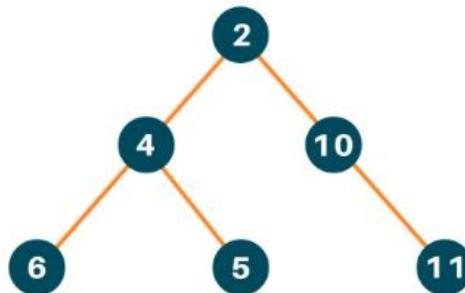


Output: 3

Explanation: The leaf nodes are 6,5,11

2. For the tree given below, find the height of the tree.

- **AVL Trees:** It is a self-balancing binary search tree. Here, self-balancing means balancing the heights of the left subtree and right subtree i.e.

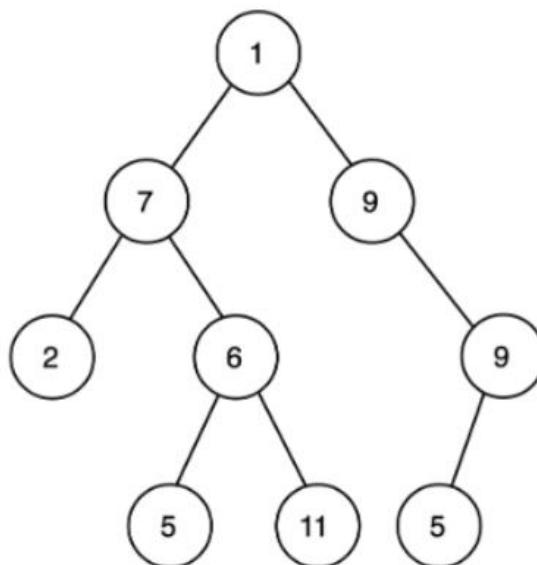


Output: 3

Explanation: The tree has 3 levels i.e. the height of the tree is 3.

What is a Binary Tree?

- A binary tree is a type of tree where each node can have at most 2 children.
- A node can either have 0(it is called leaf node), 1 or 2 child nodes.
- Every node in a binary tree has 2 pointers, i.e. left and right.
- These two pointers denote the Left and Right children of the node.
- In case, a node has no children, both the pointers point to null and in case a node has only 1 child, one of the pointers point to null respectively.



Implementation of Node Class

```

• Creating node class
class Node:
    def __init__(self, value):
        self.val = value
        self.left = None
        self.right = None
  
```

```
class BinaryTreeOperations:

    @staticmethod
    def display(root):
        if root is None:
            return
        print(root.val, end=' ')
        BinaryTreeOperations.display(root.left)
        BinaryTreeOperations.display(root.right)

    @staticmethod
    def sum_of_nodes(root):
        if root is None:
            return 0
        return root.val +
    BinaryTreeOperations.sum_of_nodes(root.left) +
    BinaryTreeOperations.sum_of_nodes(root.right)

    @staticmethod
    def find_max_node(root):
        if root is None or (root.left is None and
root.right is None):
            return root
        max_left =
    BinaryTreeOperations.find_max_node(root.left)
        max_right =
    BinaryTreeOperations.find_max_node(root.right)
        return max(max_left, max_right, root, key=lambda x:
x.val if x else float('-inf'))

    @staticmethod
    def height(root):
        if root is None:
            return -1
        return 1 +
max(BinaryTreeOperations.height(root.left),
BinaryTreeOperations.height(root.right))

    @staticmethod
    def size_of_tree(root):
        if root is None:
            return 0
        return 1 +
    BinaryTreeOperations.size_of_tree(root.left) +
    BinaryTreeOperations.size_of_tree(root.right)
```

```

# Example usage
root = Node(1)
root.left = Node(2)
root.right = Node(3)
root.left.left = Node(4)
root.left.right = Node(5)

print("Displaying the binary tree: ", end=' ')
BinaryTreeOperations.display(root)
print()

print("Sum of all nodes:",
BinaryTreeOperations.sum_of_nodes(root))

max_node = BinaryTreeOperations.find_max_node(root)
print("Node with maximum value:", max_node.val)

print("Height of the binary tree:",
BinaryTreeOperations.height(root))

print("Size of the binary tree:",
BinaryTreeOperations.size_of_tree(root))

```

Explanation:

- **Display:** Recursively traverses the tree in pre-order and prints the node values. Time complexity: $O(n)$, Space complexity: $O(n)$ due to the recursive stack.
- **Sum of Nodes:** Recursively computes the sum of all node values in the tree. Time complexity: $O(n)$, Space complexity: $O(n)$ due to the recursive stack.
- **Find Max Node:** Recursively finds the node with the maximum value in the tree. Time complexity: $O(n)$, Space complexity: $O(n)$ due to the recursive stack.
- **Height:** Calculates the height of the tree by recursively finding the maximum depth. Time complexity: $O(n)$, Space complexity: $O(n)$ due to the recursive stack.
- **Size of Tree:** Counts the number of nodes in the tree recursively. Time complexity: $O(n)$, Space complexity: $O(n)$ due to the recursive stack.

Ques: Diameter of Binary Tree [LeetCode 543]

Given the root of a binary tree, return the length of the diameter of the tree.

The diameter of a binary tree is the length of the longest path between any two nodes in a tree. This path may or may not pass through the root.

The length of a path between two nodes is represented by the number of edges between them.

Input: root = [1,2,3,4,5]
Output: 3

Explanation: 3 is the length of the path [4,2,1,3] or [5,2,1,3].

Example 2:

Input: root = [1,2]
Output: 1

class Solution:

```

def diameterOfBinaryTree(self, root):
    def depth(node):
        if not node:
            return 0
        left = depth(node.left)
        right = depth(node.right)
        self.diameter = max(self.diameter, left +
right)
        return max(left, right) + 1

    self.diameter = 0
    depth(root)
    return self.diameter

```

Explanation: The diameter of a tree is the maximum sum of the depths of left and right subtrees among its nodes. So we can compute the depths of the left and right subtrees of each node (recursively) and use these depths to compute the diameter at the current node.

Time: $O(n)$, dfs traverse each node, The function depth traverses each node exactly once.

Space: $O(n)$, It utilizes recursive calls that can go as deep as the height of the tree, leading to $O(n)$ space usage in the worst-case scenario

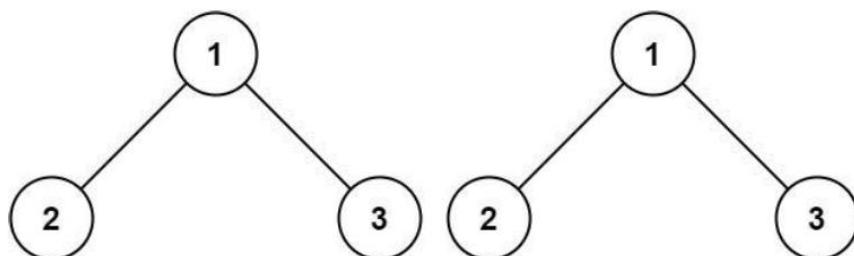
Ques: Same Tree [LeetCode 100]

Given the roots of two binary trees p and q, write a function to check if they are the same or not.

Two binary trees are considered the same if they are structurally identical, and the nodes have the same value.

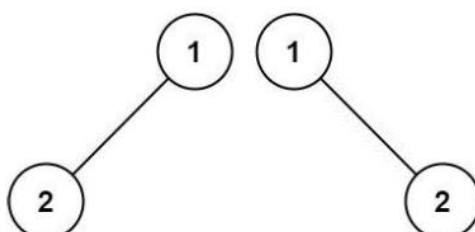
Input: p = [1,2,3], q = [1,2,3]

Output: true



Input: p = [1,2], q = [1,null,2]

Output: false



Code:

```

class Solution:
    def isSameTree(self, p, q):
        if not p and not q:
            return True
        if not p or not q:
            return False
        if p.val != q.val:
            return False
        return self.isSameTree(p.left, q.left) and
self.isSameTree(p.right, q.right)

```

Explanation: The intuition behind the solution is to recursively check if two binary trees are identical. If both trees are empty (null), they are considered identical. If only one tree is empty or the values of the current nodes are different, the trees are not identical. Otherwise, we recursively check if the left and right subtrees of both trees are identical.

Time: $O(n)$, It recursively traverses through the nodes, comparing values, until it either finds a mismatch or exhausts all nodes.

Space: $O(h)$, h is the height of smaller tree This is due to the recursive calls that occupy space on the call stack.

Ques: Invert Binary Tree [LeetCode 226]
Code:

```

class Solution:
    def invertTree(self, root):
        if root is None:
            return root
        root.left, root.right =
self.invertTree(root.right), self.invertTree(root.left)
        return root

```

Time Complexity: $O(n)$ – traverses every node once.

Space Complexity: $O(h)$ – where ' h ' is the height of the tree, due to recursive calls stored in the call stack.

Explanation: The function recursively inverts the left and right subtrees of each node in the tree. At each node, it swaps their left and right child pointers, effectively reversing the tree structure.

Ques: Binary Tree Paths [LeetCode 257]

Given the root of a binary tree, return all root-to-leaf paths in any order.

A leaf is a node with no children.

Input: root = [1,2,3,null,5]

Output: ["1->2->5","1->3"]

Code:

```

class Solution:
    def binaryTreePaths(self, root):
        def helper(node, path, paths):
            if not node:
                return
            path += str(node.val)
            if not node.left and not node.right:
                paths.append(path)
            else:
                path += '→'
                helper(node.left, path, paths)
                helper(node.right, path, paths)

        paths = []
        helper(root, '', paths)
        return paths
    
```

Example usage

```

root = Node(1)
root.left = Node(2)
root.right = Node(3)
root.left.right = Node(5)
    
```

```

solution = Solution()
print("Binary Tree Paths:", solution.binaryTreePaths(root))
    
```

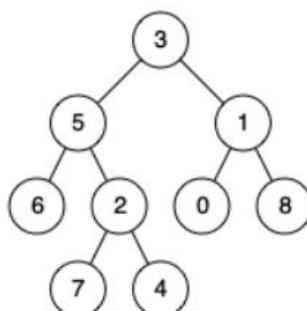
Explanation: The basic idea is to traverse the tree keeping the path in the string curr and when you find a leaf then substituting curr in the answer. We need vector answer to be same in all the calls in the call stack, therefore we use address operator while we need separate curr for each step, hence not using address operator.

- Time Complexity: $O(n)$ - Traverses every node once.
- Space Complexity: $O(h)$ - 'h' is the height of the tree, accounting for recursive calls stored in the call stack.

Ques: Lowest Common Ancestor of a Binary Tree [LeetCode 236]

Given a binary tree, find the lowest common ancestor (LCA) of two given nodes in the tree.

According to the definition of LCA on Wikipedia: "The lowest common ancestor is defined between two nodes p and q as the lowest node in T that has both p and q as descendants (where we allow a node to be a descendant of itself)."

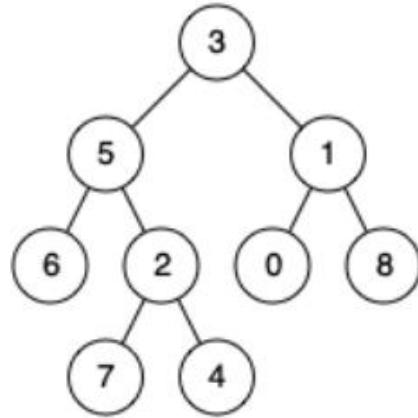
Example1:


Input: root = [3,5,1,6,2,0,8,null,null,7,4], p = 5, q = 1

Output: 3

Explanation: The LCA of nodes 5 and 1 is 3.

Example 2:



Input: root = [3,5,1,6,2,0,8,null,null,7,4], p = 5, q = 4

Output: 5

Explanation: The LCA of nodes 5 and 4 is 5, since a node can be a descendant of itself according to the LCA definition.

Code:

class Solution:

```

def lowestCommonAncestor(self, root, p, q):
    if not root or root == p or root == q:
        return root
    left = self.lowestCommonAncestor(root.left, p, q)
    right = self.lowestCommonAncestor(root.right, p, q)
    if left and right:
        return root
    return left if left else right
  
```

Explanation:

A node is the LCA if we find pq in left and right subtrees of a node. One has to be from the left and one has to be from the right

If we land on p or q before finding a node that fits #1 criteria above, then the node we landed on (p or q) is the LCA.

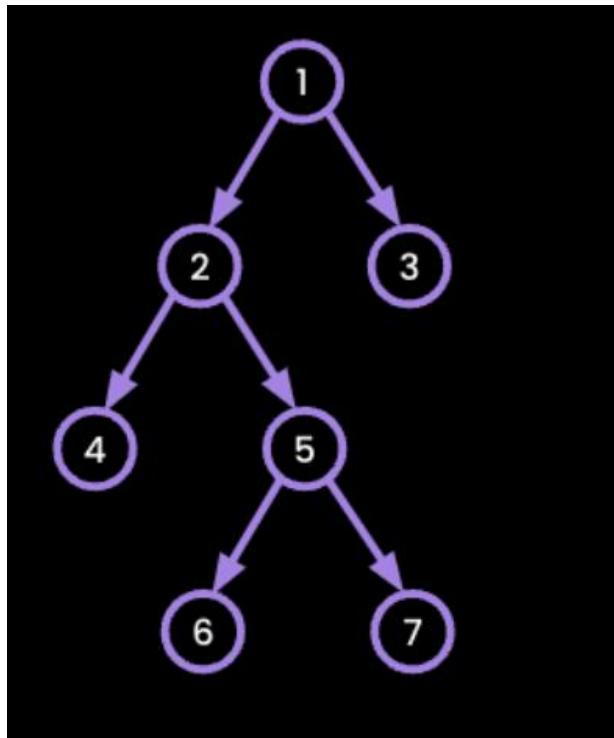
If we search the left and right subtrees of a node and don't find pq on a subtree (it returns NULL), that means we can ignore the subtree where we didn't find pq

- **Time Complexity:** $O(n)$ - Where 'n' is the number of nodes in the tree. This algorithm traverses each node once in the worst-case scenario.
- **Space Complexity:** $O(h)$ - 'h' represents the height of the tree. The space used by the recursive calls stored in the call stack can go up to the height of the tree, influencing the space complexity.

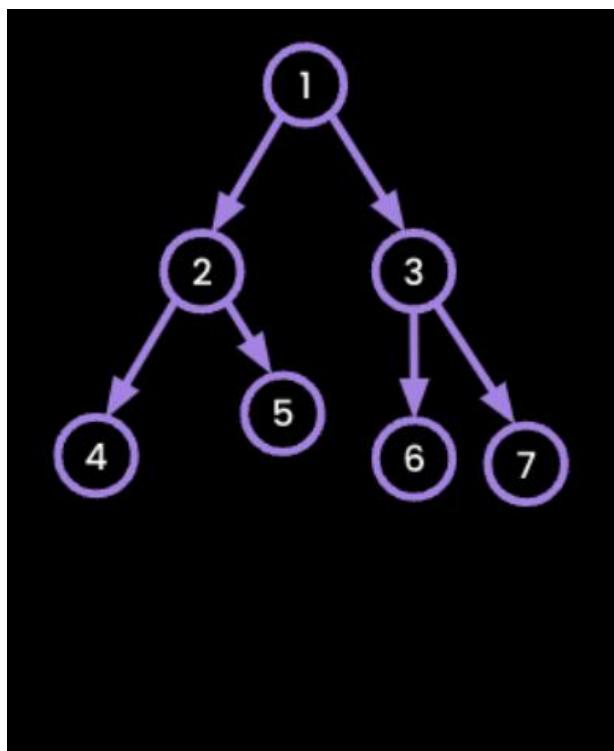
Types of Binary Trees

1. Full Binary Tree:

- Every node other than the leaf nodes has exactly two children. All nodes are either degree-0 (leaf) or degree-2.

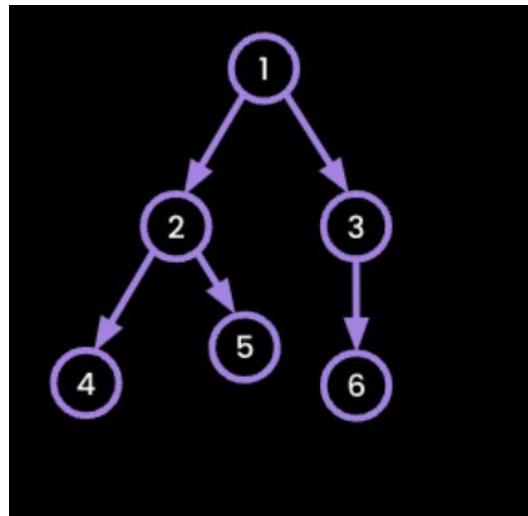


2. Perfect Binary Tree:



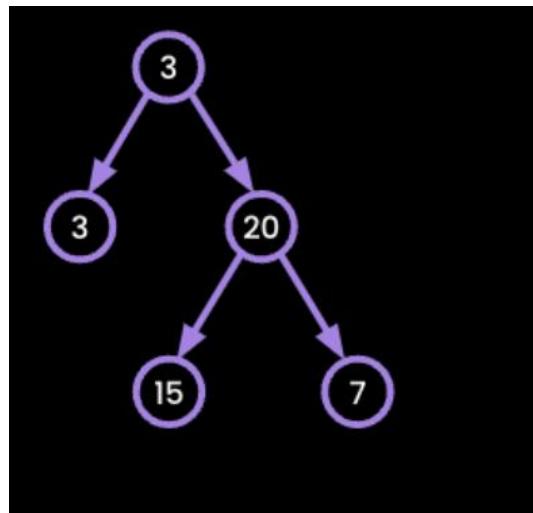
- A perfect binary tree is both full and complete.
- All interior nodes have two children.
- All leaves are at the same level.
- The number of nodes doubles at each level going down the tree.

3. Complete Binary Tree:



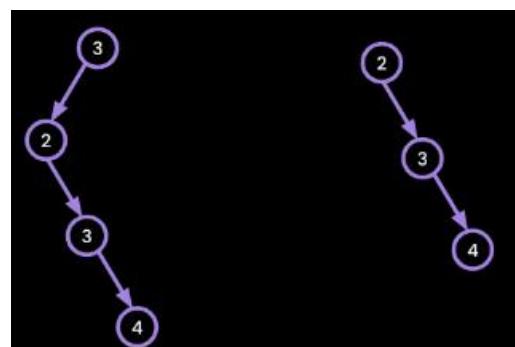
- All levels are completely filled except for the last level which is filled from left to right.
- Essentially, it's a tree that doesn't have any gaps in the sequence of nodes from the left.

4. Balanced Binary Tree:



- A balanced binary tree is a tree where the depth of the left and right subtrees of every node differ by no more than 1.
- This property helps maintain $O(\log n)$ time complexity for operations like search, insert, and delete.

5. Degenerate and Skewed Binary Tree:



- **Degenerate Tree:** Every parent node has only one associated child node. It essentially becomes a linked list.
- **Skewed Tree:** A skewed binary tree is a special case of a degenerate tree where all nodes have only one child, either left or right.