sjain15_QMM assignment 4

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#Setting working directory

```
setwd("~/Desktop/QMM")
```

#Question 1 #The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labor (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third-party sources and the number of patient-days reimbursed privately.

1) Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH.
2) Determine the Peers and Lambdas under each of the above assumptions
3) Summarize your results in a tabular format
4) Compare and contrast the above results

#Using Bechmarking Libraries for DEA

#We will perform DEA analysis using benchmarking library. Install Benchmarking library.

```
library(Benchmarking)
```

```
## Loading required package: lpSolveAPI

## Loading required package: ucminf

## Loading required package: quadprog

##
## Loading Benchmarking version 0.30h, (Revision 244, 2022/05/05 16:31:31) ...

## Build 2022/05/05 16:31:40
```

```
library(readxl)
```

#Compute the Formulation #Here, we are going to create a matrix and values.

To create the vectors with our values

```
input <- matrix(c(150,400,320,520,350, 320, 200, 700, 1200, 2000, 1200, 700),ncol = 2)
output <- matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,
                   25000, 15000),ncol = 2)
```

Assign column names

```
colnames(output) <- c("staff_hours_daily","supplies_daily")
colnames(input) <- c("reimbursed_patient_daily", "privately_paid_patient-daily")
```

To see the values of Input

```
input
```

```
##      reimbursed_patient_daily privately_paid_patient-daily
## [1,]                      150                          200
## [2,]                      400                          700
## [3,]                      320                         1200
## [4,]                      520                         2000
## [5,]                      350                         1200
## [6,]                      320                          700
```

To see the values of Output

```
output
```

```
##      staff_hours_daily supplies_daily
## [1,]             14000           3500
## [2,]             14000          21000
## [3,]             42000          10500
## [4,]             28000          42000
## [5,]             19000          25000
## [6,]             14000          15000
```

As we can see, here we are getting the same values as the performance data table from the six nursing homes owned by Hope Valley Health Care Association.

In the following section, we will perform a Data Envelopment Analysis (DEA), which is an analytical tool that can help organizations to identify and allocate their resources to enhance their efficiency and have better practices.

#DEA Analysis using FDH

Now, we are going to formulate and compute the DEA analysis using FDH.

The Free disposability hull (FDH) is the assumption of dispose unwanted inputs and outputs. "Free dispos- ability means that we can always produce fewer outputs with more inputs." (DEA Slides)

Provide the input and output

```
analysis_fdh<- dea(input,output,RTS = "fdh")
```

Create a data frame with effciency values

```
eff_fdh <- as.data.frame(analysis_fdh$eff)
```

To assign an appropiate name

```
colnames(eff_fdh) <- c("efficiency_fdh")
```

Identify the peers

```
peer_fdh <- peers(analysis_fdh)
```

Identify the relative weights given to the peers using lambda function

```
lambda_fdh <- lambda(analysis_fdh)
```

To assign an appropiate column name for Lambda

```
colnames(lambda_fdh) <- c("L1_fdh", "L2_fdh", "L3_fdh", "L4_fdh", "L5_fdh", "L6_fdh")
```

Create a tabular data with peer, lambda, and efficiency

```
peer_lamb_eff_fdh <- cbind(peer_fdh, lambda_fdh, eff_fdh)
```

Show the summary chart

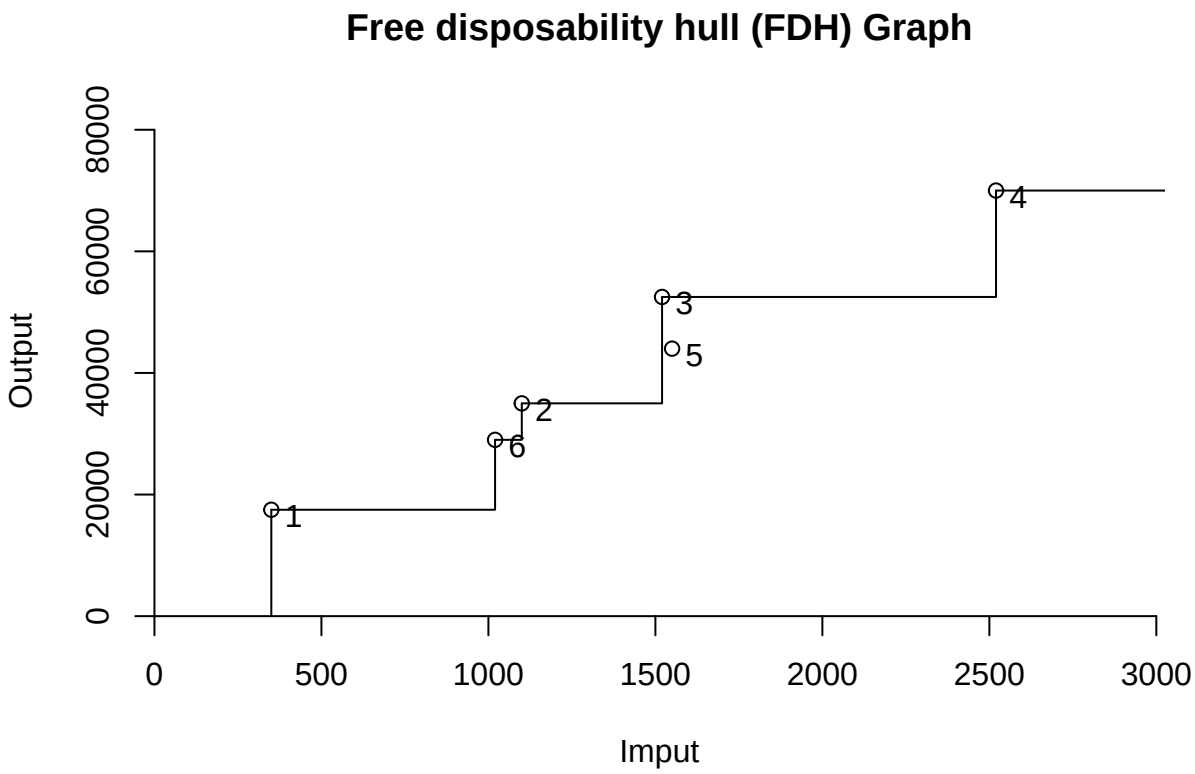
```
peer_lamb_eff_fdh
```

```
##   peer1 L1_fdh L2_fdh L3_fdh L4_fdh L5_fdh L6_fdh efficiency_fdh
## 1     1      1      0      0      0      0      0              1
## 2     2      0      1      0      0      0      0              1
## 3     3      0      0      1      0      0      0              1
## 4     4      0      0      0      1      0      0              1
## 5     5      0      0      0      0      1      0              1
## 6     6      0      0      0      0      0      1              1
```

As we learned during this module, peers are the way we could identify inefficient DMU or units, and Lambda values are the raw weights assigned from the peer units when solving the DEA model. The summary chart shown above, confirms that every DMU or facility is working using all its capacity and efficiency. Every peer was assigned one unit, for that reason, the Lambda values are 1, and efficiency are 1 as well. Now, let's see the graph.

Plot the results

```
dea.plot(input,output,RTS="fdh",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Free disposability hull (FDH) Graph")
```



#DEA Analysis using CRS

#Now, we are going to formulate and compute the DEA analysis using Constant Returns to Scale (CRS).

#The CRS is part of the scaling assumption, and it allows us to see if there is any possible combination to scale up or down.

Provide the input and output

```
analysis_crs <- dea(input,output,RTS = "crs")
```

To see the effciency values

```
eff_crs <- as.data.frame(analysis_crs$eff)
```

To assign an appropiate name

```
colnames(eff_crs) <- c("efficiency_crs")
```

Identify the peers

```
peer_crs <- peers(analysis_crs)
```

To assign an appropiate name

```
colnames(peer_crs) <- c("peer1_crs", "peer2_crs", "peer3_crs")
```

Identify the relative weights given to the peers using lambda function

```
lambda_crs <- lambda(analysis_crs)
```

To assign an appropiate column name for Lambda

```
colnames(lambda_crs) <- c("L1_crs", "L2_crs", "L3_crs", "L4_crs")
```

Create a tabular data with peer, lambda, and efficiency

```
peer_lamb_eff_crs <- cbind(peer_crs, lambda_crs, eff_crs)
```

Show the summary chart

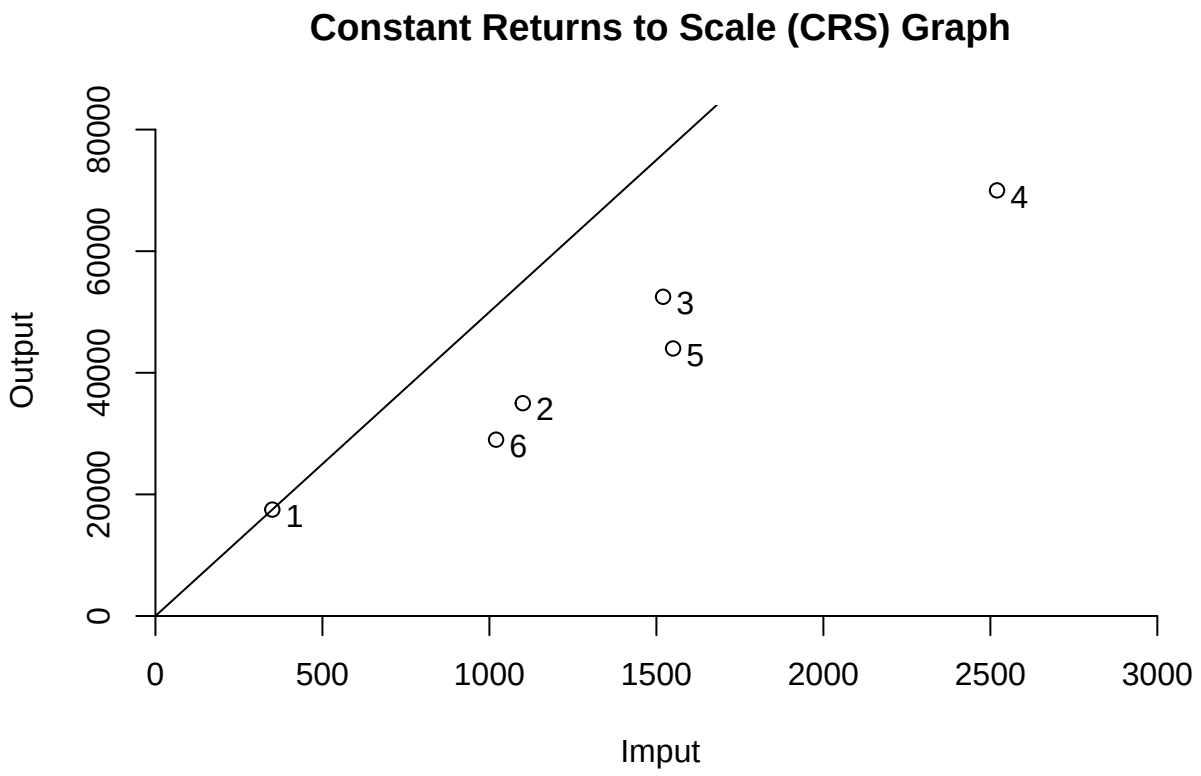
```
peer_lamb_eff_crs
```

```
##   peer1_crs peer2_crs peer3_crs    L1_crs     L2_crs L3_crs    L4_crs
## 1         1        NA        NA 1.0000000 0.00000000      0 0.0000000
## 2         2        NA        NA 0.0000000 1.00000000      0 0.0000000
## 3         3        NA        NA 0.0000000 0.00000000      1 0.0000000
## 4         4        NA        NA 0.0000000 0.00000000      0 1.0000000
## 5         1         2         4 0.2000000 0.08048142      0 0.5383307
## 6         1         2         4 0.3428571 0.39499264      0 0.1310751
##   efficiency_crs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      0.9774987
## 6      0.8674521
```

#Regarding Constant Returns to Scale (CRS), the facilities 1, 2, 3, and 4 are using all its efficiency as the lambdas and peers prove. Facility 5 and 6, on the other hand, need parts of 1, 2, and 4 as the peers and lambdas show above. It means these two facilities (5 and 6) have room to improve because they are getting an efficiency of 97.74% and 86.74% respectively.

Plot the results

```
dea.plot(input,output,RTS="crs",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Constant Returns to Scale (CRS) Graph")
```



##DEA Analysis using VRS

##Now, we are going to formulate and compute the DEA analysis using Variable Returns to Scale (VRS).

##VRS is also part of the scaling assumption, and it helps to estimate the efficiency of the variables whether an increase or decrease is not proportional.

Provide the input and output

```
analysis_vrs <- dea(input,output,RTS = "vrs")
```

To see the effciency values

```
eff_vrs <- as.data.frame(analysis_vrs$eff)
```

To assign an appropiate name

```
colnames(eff_vrs) <- c("efficiency_vrs")
```

Identify the peers

```
peer_vrs <- peers(analysis_vrs)
```

To assign an appropiate name

```
colnames(peer_vrs) <- c("peer1_vrs", "peer2_vrs", "peer3_vrs")
```

Identify the relative weights given to the peers using lambda function

```
lambda_vrs <- lambda(analysis_vrs)
```

To assign an appropiate column name for Lambda

```
colnames(lambda_vrs) <- c("L1_vrs", "L2_vrs", "L3_vrs", "L4_vrs", "L5_vrs")
```

Create a tabular data with peer, lambda, and efficiency

```
peer_lamb_eff_vrs <- cbind(peer_vrs, lambda_vrs, eff_vrs)
```

Show the summary chart

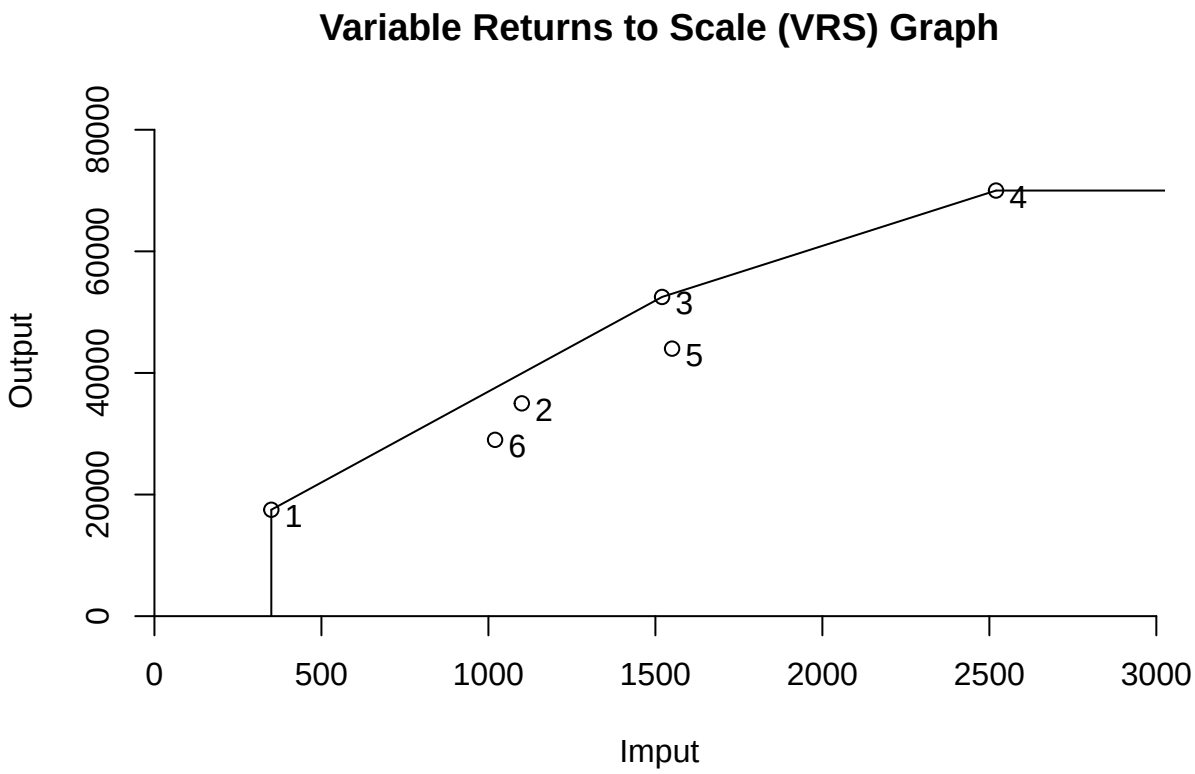
```
peer_lamb_eff_vrs
```

```
##   peer1_vrs peer2_vrs peer3_vrs    L1_vrs    L2_vrs L3_vrs L4_vrs    L5_vrs
## 1         1        NA        NA 1.0000000 0.0000000      0      0 0.0000000
## 2         2        NA        NA 0.0000000 1.0000000      0      0 0.0000000
## 3         3        NA        NA 0.0000000 0.0000000      1      0 0.0000000
## 4         4        NA        NA 0.0000000 0.0000000      0      1 0.0000000
## 5         5        NA        NA 0.0000000 0.0000000      0      0 1.0000000
## 6         1         2         5 0.4014399 0.3422606      0      0 0.2562995
##   efficiency_vrs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      1.0000000
## 6      0.8963283
```

#Now we run the Variable Returns to Scale (VRS), we can identify that facility 1, 2, 3, 4, and 5 are working in all its capacity or efficiency. However, that does not happen with facility 6, which has an efficiency of 89.63%. As peers and lambdas show, facility 6 needs part of facility 1, 2, and 5 to achieve better efficiency.

Plot the results

```
dea.plot(input,output,RTS="vrs",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Variable Returns to Scale (VRS) Graph")
```



#Now, we are going to formulate and compute the DEA analysis using Increasing Returns to Scale (IRS).
#IRS indicates if it is possible to increate the operation scale.

Provide the input and output

```
analysis_irs <- dea(input,output,RTS = "irs")
```

To see the effciency values

```
eff_irs <- as.data.frame(analysis_irs$eff)
```

To assign an appropiate name

```
colnames(eff_irs) <- c("efficiency_irs")
```

Identify the peers

```
peer_irs <- peers(analysis_irs)
```

To assign an appropiate name

```
colnames(peer_irs) <- c("peer1_irs", "peer2_irs", "peer3_irs")
```

Identify the relative weights given to the peers using lambda function

```
lambda_irs <- lambda(analysis_irs)
```

To assign an appropiate column name for Lambda

```
colnames(lambda_irs) <- c("L1_irs", "L2_irs", "L3_irs", "L4_irs", "L5_irs")
```

Create a tabular data with peer, lambda, and efficiency

```
peer_lamb_eff_irs <- cbind(peer_irs, lambda_irs, eff_irs)
```

Show the summary chart

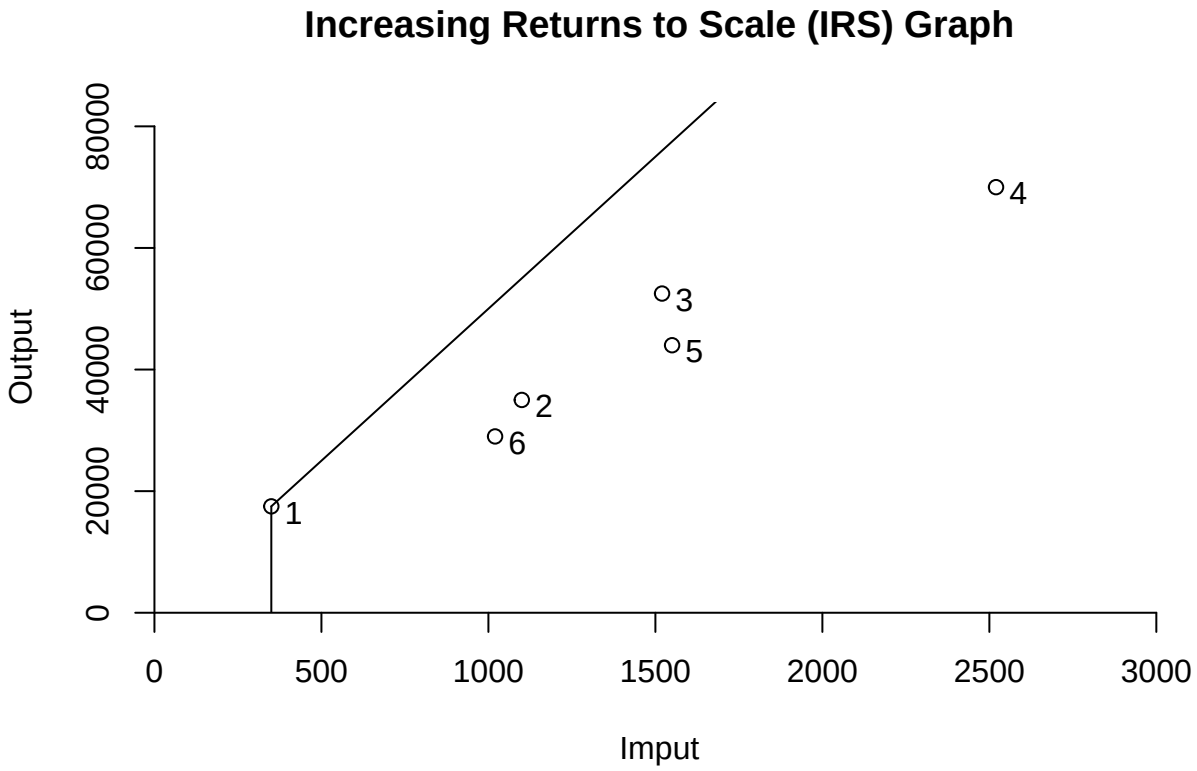
```
peer_lamb_eff_irs
```

```
##   peer1_irs peer2_irs peer3_irs    L1_irs    L2_irs L3_irs L4_irs    L5_irs
## 1         1        NA        NA 1.0000000 0.0000000      0      0 0.0000000
## 2         2        NA        NA 0.0000000 1.0000000      0      0 0.0000000
## 3         3        NA        NA 0.0000000 0.0000000      1      0 0.0000000
## 4         4        NA        NA 0.0000000 0.0000000      0      1 0.0000000
## 5         5        NA        NA 0.0000000 0.0000000      0      0 1.0000000
## 6         1         2         5 0.4014399 0.3422606      0      0 0.2562995
##   efficiency_irs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      1.0000000
## 6      0.8963283
```

#Increasing Returns to Scale (IRS) behives the same as Variable Returns to Scale (VRS) by getting facility 1, 2, 3, 4, and 5 are working all its efficiency, but facility 6 needs to improve needs from units 1, 2, and 5 to improve its efficiency which is 89.63%.

Plot the results

```
dea.plot(input,output,RTS="irs",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Increasing Returns to Scale (IRS) Graph")
```



DEA Analysis using DRS

#Now, we are going to formulate and compute the DEA analysis using Decreasing Returns to Scale (DRS).

#DRS is the opposite of IRS, which its goal is to decrease the operation scale on any possible production process.

Provide the input and output

```
analysis_drs <- dea(input,output,RTS = "drs")
```

To see the effciency values

```
eff_drs <- as.data.frame(analysis_drs$eff)
```

To assign an appropiate name

```
colnames(eff_drs) <- c("efficiency_drs")
```

Identify the peers

```
peer_drs <- peers(analysis_drs)
```

To assign an appropiate name

```
colnames(peer_drs) <- c("peer1_drs", "peer2_drs", "peer3_drs")
```

Identify the relative weights given to the peers using lambda function

```
lambda_drs <- lambda(analysis_drs)
```

To assign an appropiate column name for Lambda

```
colnames(lambda_drs) <- c("L1_drs", "L2_drs", "L3_drs", "L4_drs")
```

Create a tabular data with peer, lambda, and efficiency

```
peer_lamb_eff_drs <- cbind(peer_drs, lambda_drs, eff_drs)
```

Show the summary chart

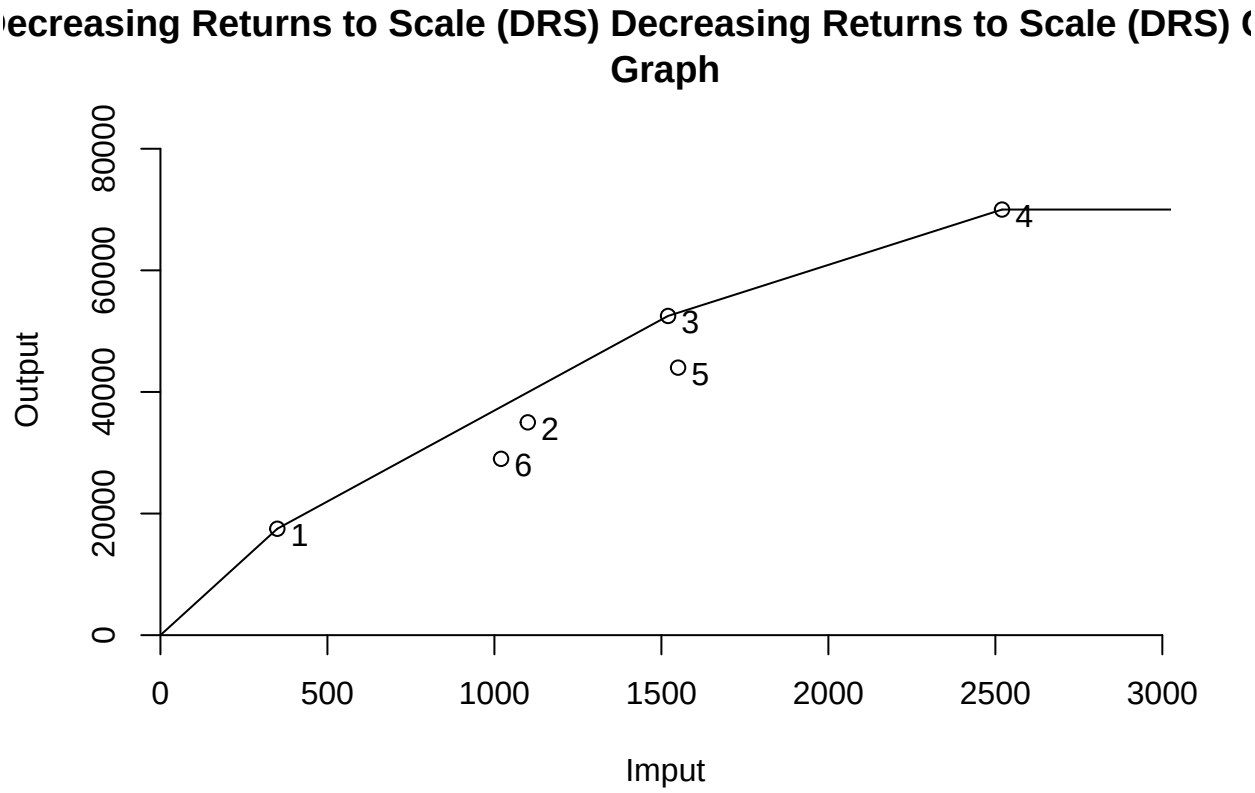
```
peer_lamb_eff_drs
```

```
##   peer1_drs peer2_drs peer3_drs    L1_drs     L2_drs L3_drs    L4_drs
## 1         1        NA        NA 1.0000000 0.00000000      0 0.0000000
## 2         2        NA        NA 0.0000000 1.00000000      0 0.0000000
## 3         3        NA        NA 0.0000000 0.00000000      1 0.0000000
## 4         4        NA        NA 0.0000000 0.00000000      0 1.0000000
## 5         1         2         4 0.2000000 0.08048142      0 0.5383307
## 6         1         2         4 0.3428571 0.39499264      0 0.1310751
##   efficiency_drs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      0.9774987
## 6      0.8674521
```

Decreasing Returns to Scale (DRS) has a good efficiency in facility 1, 2, 3, and 4. Regarding facility 5 and 6, there is room they can improve. Both of them need part of facilities 1, 2, and 4 to be able to achieve their highest efficiency of 1 as we can prove in the previous table.

Plot the results

```
dea.plot(input,output,RTS="drs",ORIENTATION="in-out", txt=TRUE, xlab = 'Imput', ylab= 'Output', main="D
Graph")
```



DEA Analysis using FRH

#Now, we are going to formulate and compute the DEA analysis using Free Replicability Hull (FRH).

#FRH as well as FDH use mixed integer programming, which refers that the variables must be integers to find the optimal solution. The goal of FRH is to replace deterministic data using random variables. (Majid, A. Reza, F., 2012, par. 2)

Provide the input and output

```
analysis_frh <- dea(input,output,RTS = "add")
```

To see the effciency values

```
eff_frh <- as.data.frame(analysis_frh$eff)
```

To assign an appropiate name

```
colnames(eff_frh) <- c("efficiency_frh")
```

Identify the peers

```
peer_frh <- peers(analysis_frh)
```

To assign an appropiate name

```
colnames(peer_frh) <- c("peer1_frh")
```

Identify the relative weights given to the peers using lambda function

```
lambda_frh <- lambda(analysis_frh)
```

To assign an appropiate column name for Lambda

```
colnames(lambda_frh) <- c("L1_frh", "L2_frh", "L3_frh", "L4_frh", "L5_frh", "L6_frh")
```

Create a tabular data with peer, lambda, and efficiency

```
peer_lamb_eff_frh <- cbind(peer_frh, lambda_frh, eff_frh)
```

Show the summary chart

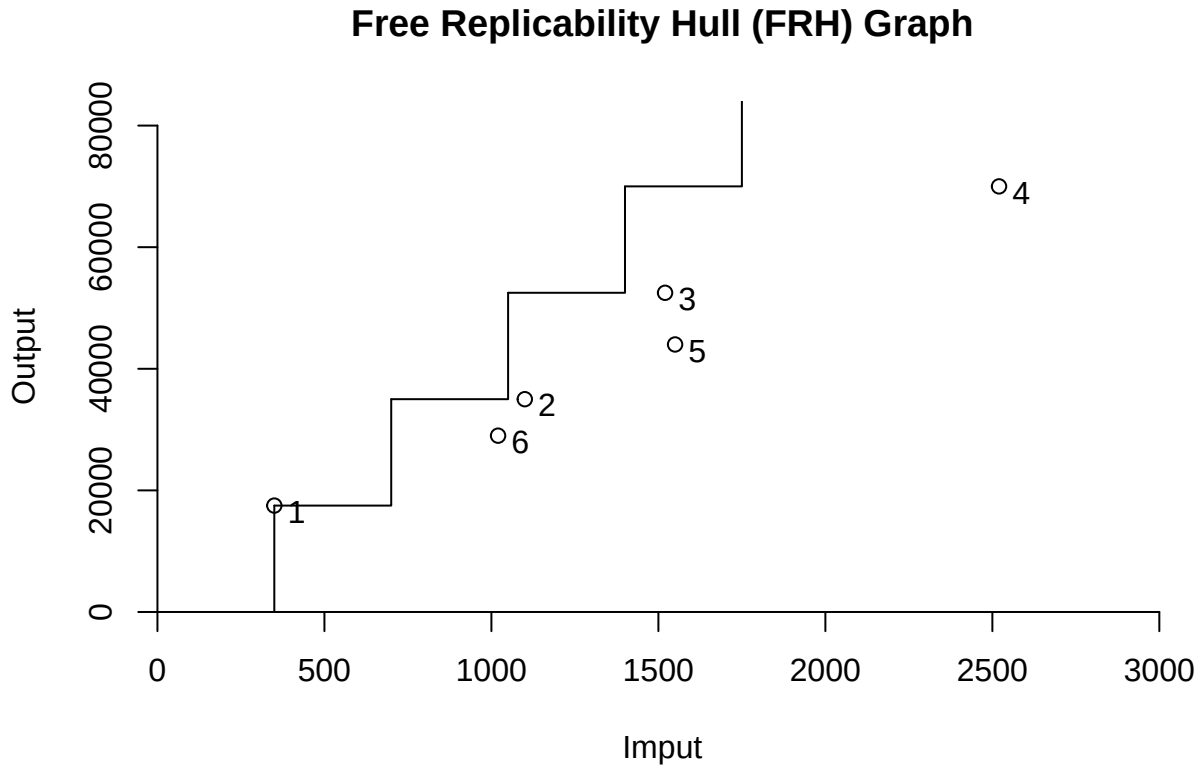
```
peer_lamb_eff_frh
```

```
##   peer1_frh L1_frh L2_frh L3_frh L4_frh L5_frh L6_frh efficiency_frh
## 1         1      1      0      0      0      0      0              1
## 2         2      0      1      0      0      0      0              1
## 3         3      0      0      1      0      0      0              1
## 4         4      0      0      0      1      0      0              1
## 5         5      0      0      0      0      1      0              1
## 6         6      0      0      0      0      0      1              1
```

#Free Replicability Hull (FRH) has a great efficiency in all its DMU. It behives the same as Free disposability hull (FDH), which all its values have their own peer, lambas and efficiency of 1.

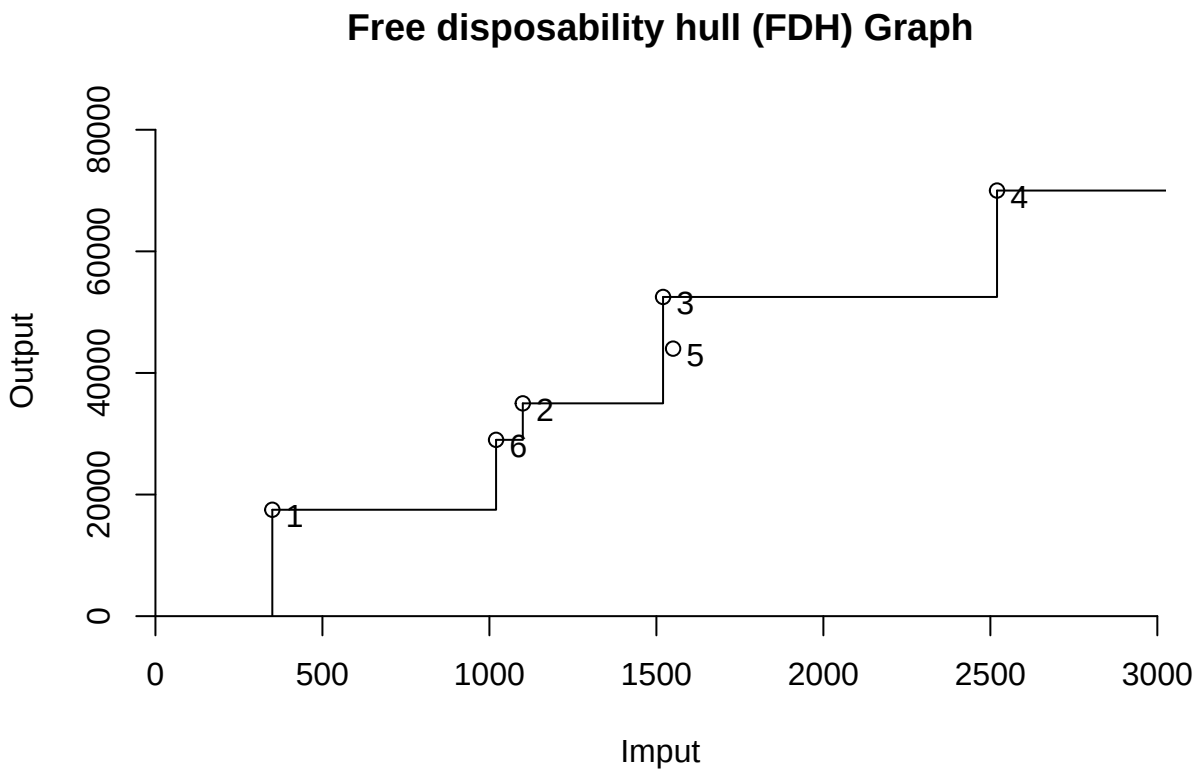
Plot the results

```
dea.plot(input,output,RTS="add",ORIENTATION="in-out", txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Fr
```

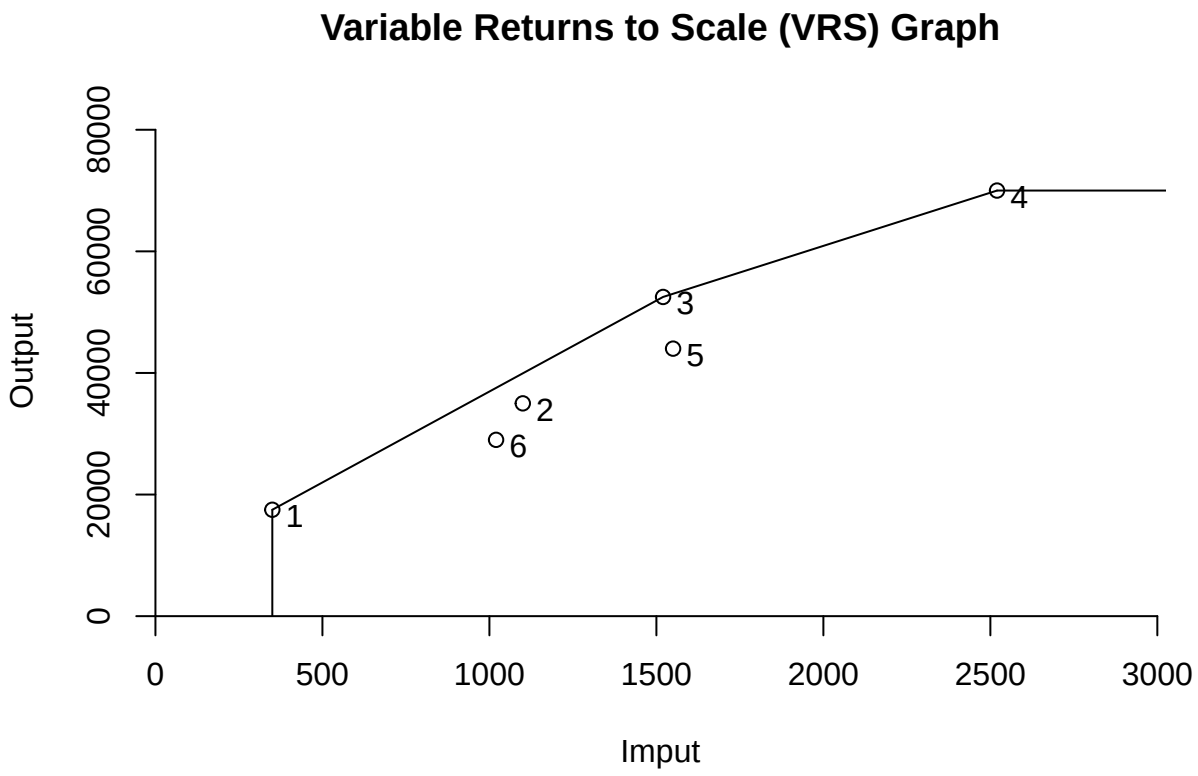


#Comparacion between different assumptions

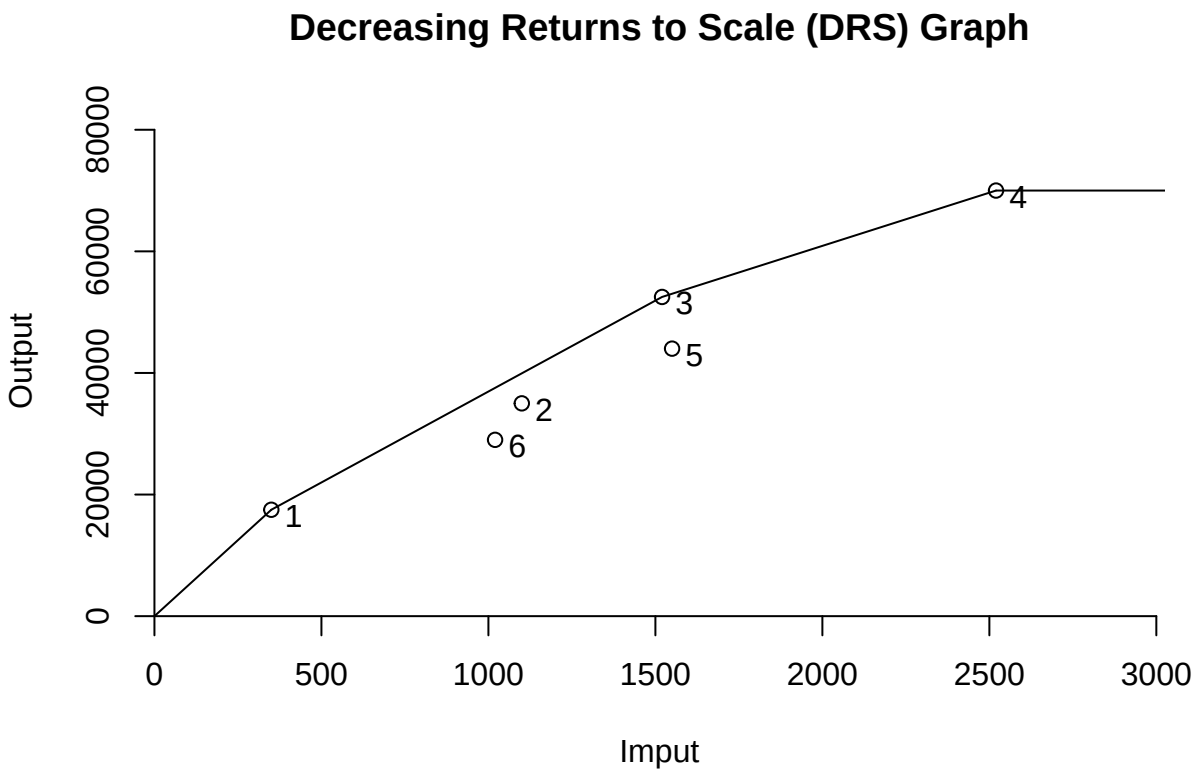
```
dea.plot(input,output,RTS="fdh",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Free disposability hull (FDH) Graph")
```



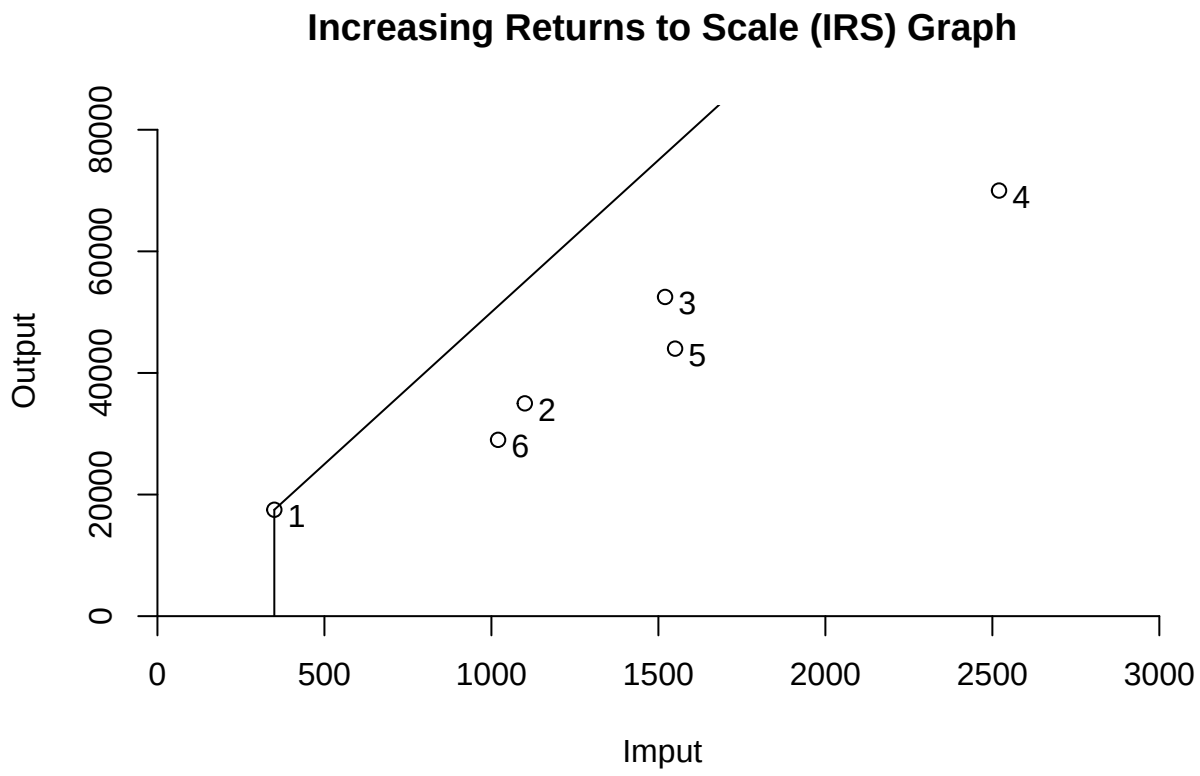

```
dea.plot(input,output,RTS="vrs",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Variable Returns to Scale (VRS) Graph")
```



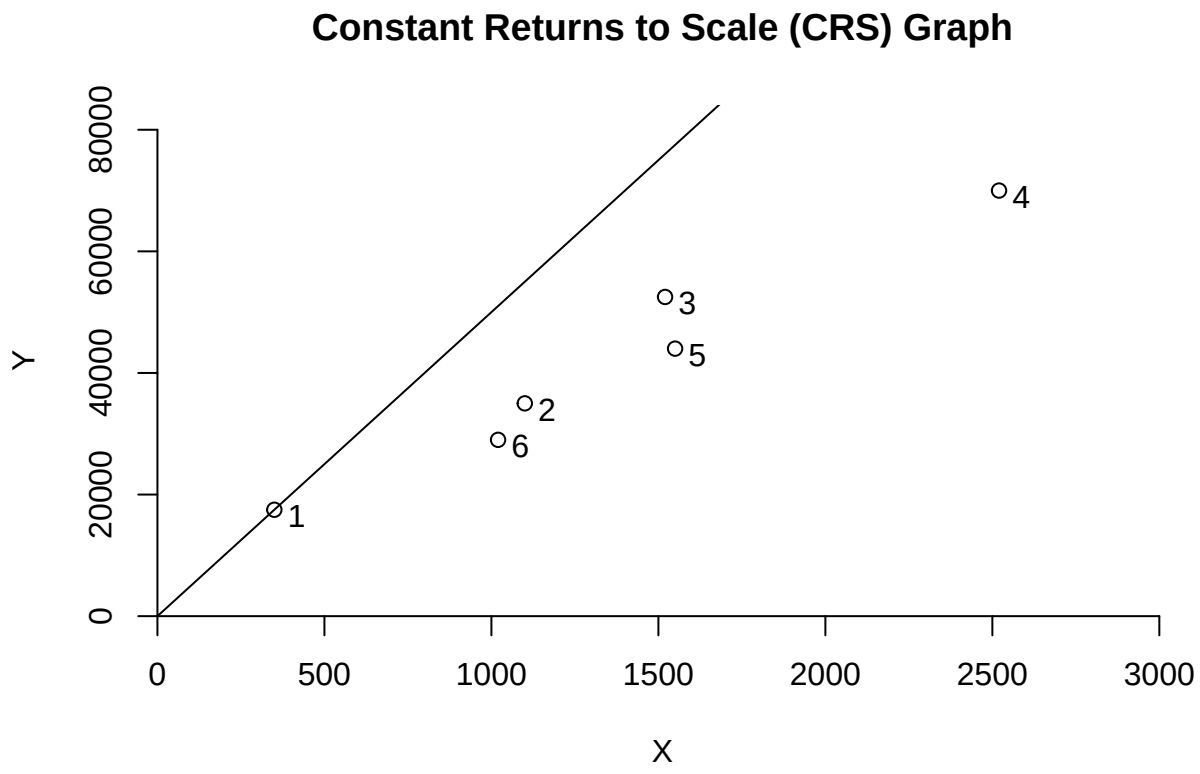

```
dea.plot(input,output,RTS="drs",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Decreasing Returns to Scale (DRS) Graph")
```



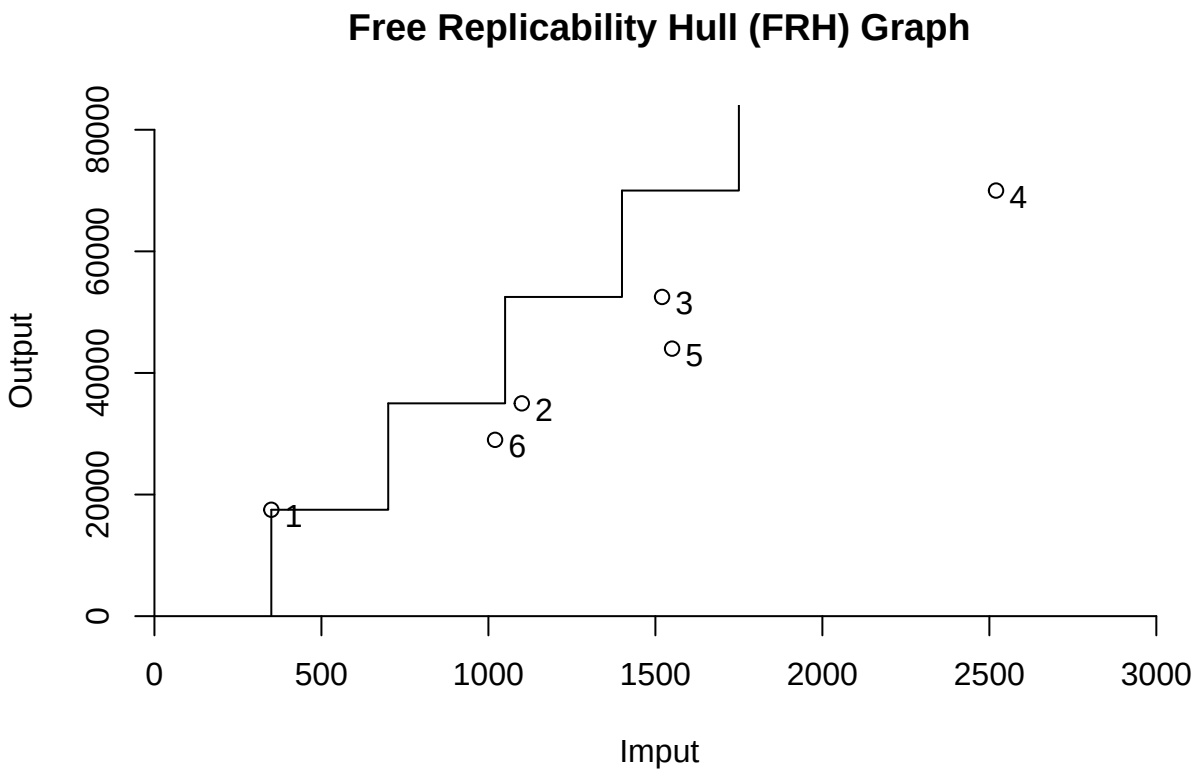

```
dea.plot(input,output,RTS="irs",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Increasing Returns to Scale (IRS) Graph")
```




```
dea.plot(input,output,RTS="crs",ORIENTATION="in-out",
txt=TRUE, main="Constant Returns to Scale (CRS) Graph")
```




```
dea.plot(input,output,RTS="add",ORIENTATION="in-out",
txt=TRUE, xlab = 'Imput', ylab= 'Output', main="Free Replicability Hull (FRH) Graph")
```



#These charts allow us to compare the results of each DEA model. #As we learned in this module, "all DEA models share the idea of estimating the technology using a minimal extrapolation approach" (DEA Slides). #As we can see FDH is the smallest technology set. It tries to produce fewer outputs (number of patient- days reimbursed by third-party sources and the number of patient-days reimbursed privately) with more inputs (staffing labor and the cost of supplies). FDH is usually the most wanted model by firms, however, it has some drawbacks due to its assumptions. As we can prove, all the efficiencies in this model are 1, but compared to other models it is not as efficient we think because we find areas/units to improve.

#VRS is larger than FDH because it "fills-out" the spaces that FDH reduced. Here we can see that unit 6 can improve its efficiency.

#DRS and IRS are larger than VRS as we can see in the charts. DRS tries to increase the set for less input value, while the IRS tries to increase the technology for large input values. DRS indicates that unit 5 and 6 could enhance their efficiency, and IRS shows that facility 6 may improve as well.

#CRS is the largest technology set, which allows us to see if there is any possible combination to scale up or down. Based on the efficiency values, units 5 and 6 need to improve.

#Regarding FRH, based on the arrow network discussed in class, it is larger than FDH but smaller than CRS, and its goal is to replace deterministic data using random variables.