
title: "sjain15_Assignment_3" author: "Sargam Jain" date: "10/16/2022" output:
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Objective Function Minimize TC = $622x_{11} + 614x_{12} + 630x_{13} + 641x_{21} + 645x_{22} + 649x_{23}$
These are subject to following constraints $x_{11} + x_{12} + x_{13} \geq 100$ $x_{21} + x_{22} + x_{23} \geq 120$
These are supply constraints

$x_{11} + x_{21} \geq 80$ $x_{12} + x_{22} \geq 60$ $x_{13} + x_{23} \geq 70$ These are demand constraints

These are all subject to non-negativity where $x_{ij} \geq 0$ where $i=1,2$ and $j=1,2,3$

```
library(Matrix)
library(tinytex)
library("lpSolve")

display <- matrix(c(22,14,30,600,100,
                    16,20,24,625,120,
                    80,60,70,"-", "210/220"),ncol=5,nrow=3,byrow=TRUE)
colnames(display) <- c("Warehouse1", "Warehouse2", "Warehouse3", "Production
Cost", "Production Capacity")
rownames(display) <- c("PlantA", "PlantB", "Monthly Demand")
display <- as.table(display)
display

##              Warehouse1 Warehouse2 Warehouse3 Production Cost
## PlantA              22          14          30          600
## PlantB              16          20          24          625
## Monthly Demand    80          60          70          -
##              Production Capacity
## PlantA              100
## PlantB              120
## Monthly Demand    210/220

display1 <- matrix(c(622,614,630,0,100,
                     641,645,649,0,120,
                     80,60,70,10,220),ncol=5,nrow=3,byrow=TRUE)
colnames(display1) <-
c("Warehouse1", "Warehouse2", "Warehouse3", "Dummy", "Production Capacity")
rownames(display1) <- c("PlantA", "PlantB", "Monthly Demand")
display1 <- as.table(display1)
display1

##              Warehouse1 Warehouse2 Warehouse3 Dummy Production Capacity
## PlantA              622          614          630          0          100
## PlantB              641          645          649          0          120
## Monthly Demand      80          60          70          10          220
```

#This table will satisfy the need for balanced problem. We have made total costs matrix below

```
totalcosts <- matrix(c(622,614,630,0,
                      641,645,649,0),nrow=2, byrow = TRUE)
```

#Identifying production capacity in the row of the matrix

```
row.rhs <- c(100,120)
row.signs <- rep("<=", 2)
```

#Identifying the monthly demand with double variable 10 at the end.

```
col.rhs <- c(80,60,70,10)
col.signs <- rep(">=", 4)
```

#Ready to run LP Transport command

```
lp.transport(totalcosts,"min",row.signs,row.rhs,col.signs,col.rhs)
```

Success: the objective function is 132790

#Here is the solution matrix

```
lp.transport(totalcosts, "min", row.signs, row.rhs, col.signs,
col.rhs)$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

This gives us the following that $Z = \$132790$. This gives us the following results for each of the variables: 60x12 which is the Warehouse 2 from plant A. 40x13 which is the Warehouse 3 from plant A. 80x21 which is the Warehouse 1 from plant B. 30x23 which is the Warehouse 3 from plant B. and because "10" shows up in the 4th Variable 10x24 is a "throw away variable". This would complete the answer for Question 1.

Question 2) We know that the number of variables in primal is equal to the number of constants in dual. The first question is the primal of the LP. Since we took the minimization in the primal we will maximize in the dual. let's use the variables "m" & "n" for the dual problem

```
display2 <- matrix(c(622,614,630,100,"m_1",
                    641,645,649,120,"m_2",
                    80,60,70,220,"-",
                    "n_1","n_2","n_3","-","-"),ncol=5,nrow=4,byrow=TRUE)
colnames(display2) <- c("W1","W2","W3","Prod Cap","Supply (Dual)")
rownames(display2) <- c("PlantA","PlantB","Monthly Demand","Demand (Dual)")
display2 <- as.table(display2)
display2
```

```
##           W1  W2  W3  Prod Cap  Supply (Dual)
## PlantA      622 614 630  100      m_1
## PlantB      641 645 649  120      m_2
```

```
## Monthly Demand 80 60 70 220 -
## Demand (Dual) n_1 n_2 n_3 - -
```

From here we are going to create our objective function based on the constraints from the primal. Then use the objective function from the primal to find the constants of the dual.

Maximize $Z = 100m_1 + 120m_2 + 80n_1 + 60n_2 + 70n_3$

This objective function is subject to following constraints:

$m_1 + n_1 \leq 622$ $m_1 + n_2 \leq 614$ $m_1 + n_3 \leq 630$ $m_2 + n_1 \leq 641$ $m_2 + n_2 \leq 645$ $m_2 + n_3 \leq 649$

These constants are taken from the transposed matrix of the primal of the linear programming function. An easy way to check is to transpose the f.con into the matrix and match to the constants above in the primal. These are unrestricted where m_k, n_l where $m=1,2$ & $n=1,2,3$

#Constants of the primal are now the objective function variables.

```
f.obj <- c(100,120,80,60,70)
#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                 1,0,0,1,0,
                 1,0,0,0,1,
                 0,1,1,0,0,
                 0,1,0,1,0,
                 0,1,0,0,1),nrow=6, byrow = TRUE)
#these change because we are MAX the dual not min
f.dir <- c("<=",
           "<=",
           "<=",
           "<=",
           "<=", "<=")
f.rhs <- c(622,614,630,641,645,649)
lp ("max", f.obj, f.con, f.dir, f.rhs)

## Success: the objective function is 139120

lp ("max", f.obj, f.con, f.dir, f.rhs)$solution

## [1] 614 633 8 0 16
```

So $Z=139,120$ dollars and variables are: $m_1 = 614$ which represents plant A $m_2 = 633$ which represents Plant B $n_1 = 8$ which represents Warehouse 1 $n_3 = 16$ which represents Warehouse 3

OBSERVATION

The minimal $Z=132790$ (Primal) and the maximum $Z=139120$ (Dual). What are we trying to max/min in this problem. We found that we should not be shipping from Plant(A/B) to all three Warehouses. We should be shipping from:

60x12 which is 60 Units from Plant A to Warehouse 2. 40x13 which is 40 Units from Plant A to Warehouse 3. 80x21 which is 60 Units from Plant B to Warehouse 1. 30x23 which is 60 Units from Plant B to Warehouse 3. Now we want to Max the profits from each distribution in respect to capacity.

Question 3)

$$m1 - n1 \leq 622$$

then we subtract $n1$ to the other side to get $m1 \leq 622 - n1$

To compute that value it would be $614 \leq (-8 + 622)$ which is true. We would continue to evaluate these equations:

$$\begin{aligned} m1 \leq 622 - n1 & \implies 614 \leq 622 - 8 = 614 = \text{TRUE} \\ m1 \leq 614 - n2 & \implies 614 \leq 614 - 0 = 614 = \text{TRUE} \\ m1 \leq 630 - n3 & \implies 614 \leq 630 - 16 = 614 = \text{TRUE} \\ m2 \leq 641 - n1 & \implies 633 \leq 641 - 8 = 633 = \text{TRUE} \\ m2 \leq 645 - n2 & \implies 633 \leq 645 - 0 = 645 = \text{NOT TRUE} \\ m2 \leq 649 - n3 & \implies 633 \leq 649 - 16 = 633 = \text{TRUE} \end{aligned}$$

#Also learning from the Duality-and-Sensitivity we can test for the shadow price by updating each of the column. We change the 100 to 101 and 120 to 121 in our LP Transport.

```
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=", 2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=", 4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=", 2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=", 4)
lp.transport(totalcosts,"min",row.signs,row.rhs,col.signs,col.rhs)

## Success: the objective function is 132790

lp.transport(totalcosts,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)

## Success: the objective function is 132771

lp.transport(totalcosts,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)

## Success: the objective function is 132790
```

Since we are taking the min of this specific function seeing the number go down by 19 means the shadow price is 19, that was found from the primal and adding 1 to each of the Plants. However with Plant B does not have a shadow price. We also found that the dual variable $n2$ where Marginal Revenue (MR) \leq Marginal Cost (MC). Recalling the equation which was $m2 \leq 645 - n2 \implies 633 \leq 645 - 0 = 645 = \text{NOT TRUE}$ which was found by using $m1 - n1 \leq 622$

```
lp ("max", f.obj, f.con, f.dir, f.rhs)$solution
```

```
## [1] 614 633    8    0   16
```

n_2 was = 0.

CONCLUSION: from the primal: 60x12 which is 60 Units from Plant A to Warehouse 2. 40x13 which is 40 Units from Plant A to Warehouse 3. 80x21 which is 60 Units from Plant B to Warehouse 1. 30x23 which is 60 Units from Plant B to Warehouse 3. from the dual We want the $MR=MC$. Five of the six $MR \leq MC$. The only equation that does not satisfy this requirement is Plant B to Warehouse 2. We can see that from the primal that we will not be shipping any AED device there.