1. Let
A = student knows the ans.
B = " guesses the ans.
C = " gets correct ans.

$\Rightarrow P(A) = 1/3$, $P(B) = 2/3$

$\Rightarrow P(C|B) = \frac{1}{5}$ (5 choices)

$\Rightarrow P(C|A) = 1$ (if student knows the ans, he will certainly get it correct)

TO FIND: prob. that he knows ans given that he gives the correct ans.

$P(A|C) = \frac{P(A) \cdot P(C|A)}{P(C)}$ (Bayes' Thm)

$\Rightarrow \frac{P(A) \cdot P(C|A)}{P(C|A) \cdot P(A) + P(C|B) \cdot P(B)}$ (using law of total prob.)

$\Rightarrow \frac{(1/3) \cdot (1)}{(1/3) + \left(\frac{1}{5}\right)\frac{2}{3}} = \frac{1/3}{\frac{5}{15} + \frac{2}{15}} = \frac{5}{7}$ Ans.

2. Let:
A: item comes from machine A
B: " " " " B
C: " " " " C
D: item is defective.

To find: probability that item comes from A given that it is defective, i.e. $P(A|D)$.

Bayes' Thm $\Rightarrow \left[P(A|D) = \frac{P(D|A) \times P(A)}{P(D)} \right]$

$P(D|A) = 0.02$ [$\because$ machine A produces 2% defective items]
$P(D|B) = 0.03$ (" " B " 3% " ")
$P(D|C) = 0.05$ (" " C " 5% " ")

• $P(D) = P(D|A) \cdot P(A) + P(D|B) \cdot P(B) + P(D|C) \cdot P(C)$ (Law of total prob.)

$\Rightarrow P(D) = (0.02) \times (0.25) + (0.03) \times (0.35) + (0.05) \cdot (0.4) = 0.035$.

$\therefore P(A|D) = (0.02 \times 0.25) / 0.035 = 0.005 / 0.035 = 0.143$.

3. ⓐ $P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} \Rightarrow \frac{P(A \cap (1-B))}{P(B^c)}$ [$\because B^c = 1 - B$]

$\Rightarrow \frac{P(A \cap (1-B))}{1 - P(B)} \Rightarrow \frac{P(A) - P(A \cap B)}{1 - P(B)}$.

ⓑ
$0 \leq P(A|B^c) \leq 1 \Rightarrow 0 \geq -P(A|B^c) \geq -1$

$\Rightarrow 0 \geq -P(A|B^c) \cdot (1 - P(B)) \geq -(1 - P(B))$ (Mult. throughout w/ $1 - P(B)$)

$\Rightarrow P(A) + 0 \geq P(A) - P(A|B^c) \cdot (1 - P(B)) \geq P(A) - (1 - P(B))$ (Adding $P(A)$ throughout

$\Rightarrow P(A \cap B) \geq P(A) + P(B) - 1$

$\Rightarrow P(A \cap B) \geq P(A) + P(B) - 1$

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4. (3B, 2W) (4B) (2B, 3W)

E_1 = draw a ~~black~~ white ball from urn 1 ~~~~

E_2 = draw is white ball " urn 2

E_3 = ~~get~~ " white ball " " urn 3

If all 3 draws r white from the urns.

(3B, 1W) (4B, 1W) (2B, 4W) — +1W, +1W

$P(E_1) = 2/5$, $P(E_2) = 1/5$

$P(E_3) = 2/3$

$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2) \cdot P(E_3)$

or $P(E_1 \cdot E_2 \cdot E_3)$ ($\because$ E_1, E_2 & E_3 r independent events.)

$\Rightarrow \left(\frac{2}{5}\right)\left(\frac{1}{5}\right)\left(\frac{2}{3}\right) = \frac{4}{75}$.

5. 1) $P(N_v = 2)$ prob. of exactly 2 voice calls $\Rightarrow P(\{vvd, vdv, dvv\}) = 0.3$.

2) $P(N_v \geq 1)$ prob of atleast one voice call $\Rightarrow P(\{vdd, dvd, ddv, vvd, vdv, dvv, vvv\})$

$= 6(0.1) + 0.2 = 0.8$.

3) $P(\{vvd\} \mid N_v = 2) \Rightarrow$ conditional prob. of 2 voice calls followed by data call given that there were exactly 2 voice calls

$\Rightarrow P(\{vvd\} \mid N_v = 2) = \dfrac{P(\{vvd\}, N_v = 2)}{P(N_v = 2)} = \dfrac{P(\{vvd\})}{P(N_v = 2)} = \dfrac{0.1}{0.3} = \dfrac{1}{3}$.

5) cond^n prob. of exactly 2 voice calls given atleast one voice call

$P(N_v = 2 \mid N_v \geq 1) = \dfrac{P(N_v = 2, N_v \geq 1)}{P(N_v \geq 1)} = \dfrac{P(N_v = 2)}{P(N_v \geq 1)} = \dfrac{0.3}{0.8} = \dfrac{3}{8}$.

6) cond^n prob of atleast one voice call given there were exactly 2 voice calls

$P(N_v \geq 1 \mid N_v = 2) = \dfrac{P(N_v \geq 1, N_v = 2)}{P(N_v = 2)} = \dfrac{\cancel{P(N_v = 2)}}{\cancel{P(N_v = 2)}} = 1$.

7) cond^n prob of 2 data calls foll. by voice call given there were 2 voice calls $P(\{ddv\} \mid N_v = 2) \Rightarrow \dfrac{P(\{ddv\}, N_v = 2)}{P(N_v = 2)} = 0$.

Given $P(L\cap F) = 0.5$, $P(B\cap W) = 0.2$, $P(B\cap F) = 0.2$

$S = \{BF, BW, LF, LW\}$

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a) $P(F) = P(LF) + P(BF) \Rightarrow P(F) = \underbrace{P(L\cap F)}_{0.5} + \underbrace{P(B\cap F)}_{0.2} \Rightarrow \underline{P(F) = 0.7}$

$\Rightarrow P(F) + P(W) = 1 \Rightarrow P(W) = 1 - P(F) \Rightarrow \underline{P(W) = 0.3}$

b) $P(B) = P(BF) + P(BW) = \underbrace{P(B\cap F)}_{0.2} + \underbrace{P(B\cap W)}_{0.2} \Rightarrow \underline{P(B) = 0.4}$

c) $P(W\cup B) = P(W) + P(B) - P(WB) = 0.3 + 0.4 - 0.2 = 0.3 + 0.2 = \underline{0.5}$

$P(H\cap F) = 0.2$, $P(M\cap W) = 0.1$, $P(F) = 0.5$,

a) $P(W) + P(F) = 1 \Rightarrow P(W) = 1 - P(F) = 1 - 0.5 \Rightarrow \underline{P(W) = 0.5}$

b) $P(MF) + P(HF) = P(F) \Rightarrow P(MF) = P(F) - P(HF) = 0.5 - 0.2 = \underline{0.3}$

c) $P(H) = P(HW) + \underbrace{P(HF)}_{0.2} = [1 - \underbrace{P(MW)}_{0.1} - \underbrace{P(HF)}_{0.2} - \underbrace{P(MF)}_{0.3}] + P(HF) = (1 - 0.6) + 0.2$
$= 0.4 + 0.2 = \underline{0.6}$

$[\because P(MW) + P(MF) + P(HF) + P(HW) = 1]$

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

$P(LH_0) = 0.1$, $P(BH_0) = 0.4$

$P(LH_1) = 0.1$, $P(BH_1) = 0.1$

$P(LH_2) = 0.2$, $P(BH_2) = 0.1$

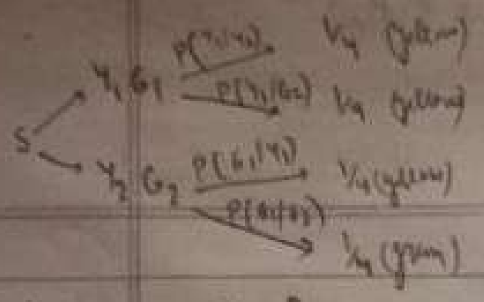
$P(H_0) = P(LH_0) + P(BH_0) = 0.1 + 0.4 = \underline{0.5}$

$P(B) = \cancel{P(} P(BH_0) + P(BH_1) + P(BH_2) = 0.4 + 0.1 + 0.1 = \underline{0.6}$

$P(L\cup H_2) = \underline{P(L)} + \underline{P(H_2)} - \underline{P(LH_2)} \Rightarrow (0.4) + (0.3) - 0.2 = \underline{0.5}$.

$[P(LH_0) + P(LH_1) + P(LH_2)] + [P(LH_2) + P(BH_2)] = 0.2$

9.



$P(\text{yellow peas}) = \frac{3}{4}$.

10. a) Acc. to the, $P(H_0/B)$ → prob of brief call which has no handoffs.

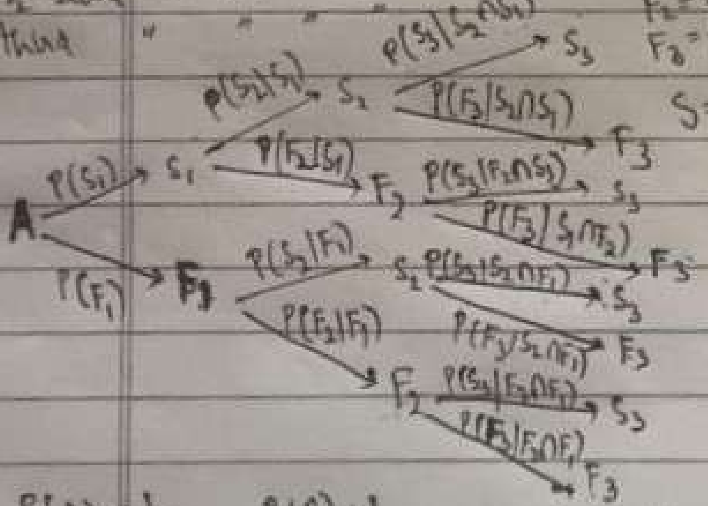
$\frac{P(H_0 B)}{P(B)} = \frac{0.4}{0.6} = \frac{2}{3}$.

b) $P(L/H_1) = \frac{1}{2}$

c) Prob. that a long call will have 1 or more handoffs $= P\frac{(0.1+0.2)}{(0.1+0.1+0.2)} = \frac{3}{4}$

11. S_1 = first paging attempt is successful.
S_2 = second " " " "
S_3 = third " " " "

F_1 = first paging attempt fails
F_2 = second " " "
F_3 = third " " "



$S = \{S_1S_2S_3, S_1S_2F_3, S_1F_2S_3, S_1F_2F_3, F_1S_2S_3, F_1S_2F_3, F_1F_2S_3, F_1F_2F_3\}$

E_1 = atleast one paging attempt is successful.

$P(E_1) = 1 - P(\overline{E_1}) = 1 - P(\{F_1F_2F_3\})$
$= 1 - (0.2)^3 = 1 - 0.008 = 0.992$

12. $P(A) = ½$, $P(B) = \frac{1}{2}$ [A: 1st coin selected by person is A; B: " " " " " " B]

H_A: Event that coin A comes up w/ head
H_B: " " " B " " " "

$P(H_{A_1}) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$; $P(H_{AB_2}) = \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$

$P(H_{B_1}) = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$; $P(H_{BA_2}) = \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$

$\because H_1$ & H_2 r independent , $P(H_1, H_2) = P(H_1) + P(H_2)$

$\Rightarrow P(H_1 H_2) = \frac{3}{32} + \frac{3}{32} = \frac{3}{16}$ Ans.

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