HW 3

3. a) $S = \{ddd, ddv, dvd, dvv, vdd, vdv, vvd, vvv\}$ $\boxed{R = X \cap Y}$

b) $S_X = \{0, 1, 2, 3\}$,

$S_Y = \{0, 1, 2, 3\}$, $S_R = \{0, 2\}$

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($X \to$ no. of voice calls
$Y \to$ no. of data calls)

b) $P_X(x) = P[X = x]$

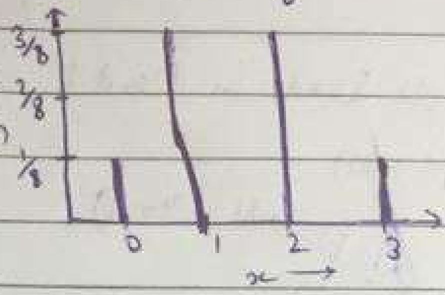
i) $P[X=0] = P(ddd)$ { when X (no of voice calls (v)) = 0) $\Rightarrow P[X=0] = 1/8$

ii) $P[X=1] = P(ddv) + P(dvd) + P(vdd) \Rightarrow P[X=1] = (1/8) \times 3 = 3/8$

iii) $P[X=2] = P(dvv) + P(vdv) + P(vvd) \Rightarrow P[X=2] = \frac{3}{8}$

iv) $P[X=3] = P(vvv) = 1/8$.

$$\text{PMF of } x \to P_X(x) = \begin{cases} 1/8 & \text{if } x = 0 \\ 3/8 & \text{if } x = 1 \\ 3/8 & \text{if } x = 2 \\ 1/8 & \text{if } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

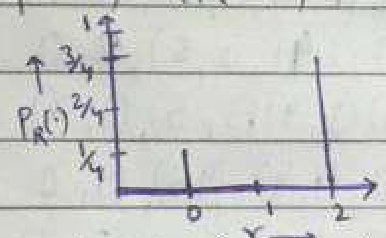


Now,

$P_R(r) = P[R = r]$

i) $P[R=0] = P(ddd) + P(vvv) = 1/4$ ii) $P(R=2) = 6\left(\frac{1}{8}\right) = 3/4$

$$\text{PMF of } r \to P_R(r) = \begin{cases} 1/4 & \text{if } r = 0 \\ 3/4 & \text{if } r = 2 \\ 0 & \text{otherwise} \end{cases}$$



c) i) $P[X=0] = P(ddd)$ (when no of voice calls (x) = 0) $= \frac{1}{8}$

ii) $P[X<3] = P(ddd) + P(ddv) + P(dvd) + P(dvv) + P(vdd) + P(vdv) + P(vvd) = \frac{7}{8}$.

iii) $P[R>1] \Rightarrow P[R=2] = 6 \times \left(\frac{1}{8}\right) = \frac{3}{4}$.

2) a) $\sum_{v=1}^{4} P_V(v) = 1 \Rightarrow P_V(1) + P_V(2) + P_V(3) + P_V(4) \Rightarrow c(1) + c(4) + c(9) + c(16) \Rightarrow 30c = 1$ $\boxed{c = 1/30}$

b) $U = \{u^2 \mid u = 1, 2, \dots\} \Rightarrow P[V \in U] = P_V(1) + P_V(4) = \frac{1}{30} + \frac{4^2}{30} = \frac{17}{30}$

c) $P[V \text{ is even}] = P_V(2) + P_V(4) = \frac{2^2}{30} + \frac{4^2}{30} = \frac{20}{30} = \frac{2}{3}$

d) $P[V>2] = P_V(3) + P_V(4) = \frac{3^2}{30} + \frac{4^2}{30} = \frac{25}{30} = \frac{5}{6}$

3. $P_X(x) = \begin{cases} c/x & , x = 2, 4, 8 \\ 0 & \text{otherwise} \end{cases}$

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a) $\sum_x P_X(x) = \frac{c}{2} + \frac{c}{4} + \frac{c}{8} = \frac{7c}{8} = 1 \Rightarrow \left(c = \frac{8}{7}\right)$

b) $P[X=4] = P_X(4) = \frac{c}{4} = \frac{8}{7 \cdot 4} = \frac{2}{7}$

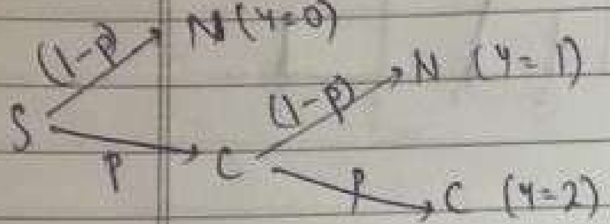
c) $P[X<4] = P_X(2) = \frac{8}{7 \cdot 2} = \frac{4}{7}$

d) $P(3 \leq X \leq 9) = P_X(4) + P_X(8) = \frac{8}{7}\left(\frac{1}{4} + \frac{1}{8}\right) = \frac{8}{7} \cdot \frac{3}{8} = \frac{3}{7}$

4. Y = no of pts scored in "1 and 1"

C : event that free throw is converted / shot goes

N: " " " " " not converted / shot doesn't go in



$\rightarrow P_Y(y) = \begin{cases} (1-p) & , y = 0 \\ p(1-p) & , y = 1 \\ p^2 & , y = 2 \\ 0 & , \text{otherwise} \end{cases}$

5. $S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

or $S = \{ (m,n) : m, n \in \mathbb{Z}$ and $1 \leq m, n \leq 6 \}$

i) $P[X=2] = P[(1,1)] = 1/36$

ii) $P[X=3] = P[(1,2), (2,1)] = 2/36$

iii) $P[X=4] = P[(1,3), (2,2), (3,1)] = 3/36$

iv) $P[X=5] = P[(1,4), (2,3), (3,2), (4,1)] = 4/36$

v) $P[X=6] = P[(1,5), (2,4), (3,3), (4,2), (5,1)] = 5/36$

vi) $P[X=7] = P[(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)] = 6/36$

vii) $P[X=8] = P[(2,6), (3,5), (4,4), (5,3), (6,2)] = 5/36$

viii) $P[X=9] = P[(3,6), (4,5), (5,4), (6,3)] = 4/36$

ix) $P[X=10] = P[(4,6), (5,5), (6,4)] = 3/36$

x) $P[X=11] = P[(5,6), (6,5)] = 2/36$

xi) $P[X=12] = P[(6,6)] = 1/36$

$$P_X(x) \text{ (PMF)} = \begin{cases} 1/36, & x=2 \\ 2/36, & x=3 \\ 3/36, & x=4 \\ 4/36, & x=5 \\ 5/36, & x=6 \\ 6/36, & x=7 \\ 5/36, & x=8 \\ 4/36, & x=9 \\ 3/36, & x=10 \\ 2/36, & x=11 \\ 1/36, & x=12 \\ 0, & \text{otherwise} \end{cases}$$

$S_X = \{2,3,4,5,6,7,9,10,11,12\}$ (range)

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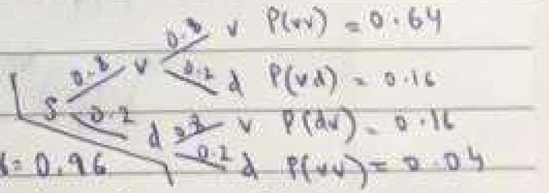
$$\Rightarrow P_X(x) = \begin{cases} 1/36 & x = 2, 12 \\ 2/36 & x = 3, 11 \\ 3/36 & x = 4, 10 \\ 4/36 & x = 5, 9 \\ 5/36 & x = 6, 8 \\ 6/36 & x = 7 \\ 0 & \text{otherwise} \end{cases}$$

6. $S_X = \{0,1,2,3,4,5,6,7,8,9,10\}$ (range)

$$P_X(x) \text{ (PMF)} = \begin{cases} 1/2^{10}, & x=0,10 \\ {}^{10}C_1/2^{10}, & x=1,9 \\ {}^{10}C_2/2^{10}, & x=2,8 \\ {}^{10}C_3/2^{10}, & x=3,7 \\ {}^{10}C_4/2^{10}, & x=4,6 \\ {}^{10}C_5/2^{10}, & x=5 \\ 0, & \text{otherwise} \end{cases}$$



7. a) $P[\{N_v = 2\}] \Rightarrow P[\,W\,] = 0.64$

& $P[\{N_v \geq 1\}] \Rightarrow P[\{vv, vd, dv\}] = 0.64 + 0.16 + 0.16 = 0.96$

$\Rightarrow P[(N_v = 2) \cap (N_v \geq 1)] = P[N_v = 2] = 0.64$

$\Rightarrow P[(N_v = 2)] \cdot P[N_v \geq 1] = (0.64)(0.96)$

$\{ \because P[(N_v = 2) \cdot (N_v \geq 1)] \neq P[(N_v = 2)] \cdot P[(N_v \geq 1)] \Rightarrow$ not independent $\}$

b) $P[(N_v \geq 1)] = 0.96$; $P[\{c_1 = v\}] \Rightarrow P[\{vv, vd\}] = 0.64 + 0.16 = 0.8$

$\Rightarrow P[\{N_v \geq 1\}] \cdot P[\{c_1 = v\}] = (0.96)(0.8)$

$\Rightarrow P[(N_v \geq 1) \cap (\{c_1 = v\})] \Rightarrow P[\{vv, vd\}] = 0.8$

$\left[\{N_v \geq 1\}\ \&\ \{c_1 = v\} \text{ are not independent.} \right]$

c) $P[\{c_2 = v\}] = P[\{vv, dv\}] = P(vv) + P(dv) = 0.64 + 0.16 = 0.8$

$P[\{c_1 = d\}] = P[\{dd, dv\}] = P(dd) + P(dv) = 0.04 + 0.16 = 0.2$

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$P[\{c_2 = v\} \cap \{c_1 = d\}] = P[\{vv, dv\} \cap \{dd, dv\}]$

$\Rightarrow P[\{dv\}] = 0.16 \Rightarrow \underline{P[\{c_2 = v\} \cap \{c_1 = d\}] = 0.16}$

& $\underline{P[\{c_2 = v\}] \cdot P[\{c_1 = d\}]} = (0.8)(0.2) = \underline{0.16}$

$[\because P(AB) = P(A) \cdot P(B)$
$\Rightarrow A$ & B r independent

$\because P[\{c_2 = v\} \cap \{c_1 = d\}] = P[\{c_2 = v\}] \cdot P[\{c_1 = d\}]$

$\Rightarrow \{c_2 = v\}$ & $\{c_1 = d\}$ r independent.

d) $P[\{c_2 = v\}] = 0.64 + 0.16 = 0.8$, $P[N_v \text{ is even}] = P(vv) + P(dd) = 0.64 + 0.04$
($N_v = 0$ or $N_v = 2$) $= \underline{0.68}$.

$\because P[\{dv, vv\}] = 0.8$, $P[\{vv, dd\}] = 0.68$

$P[\{dv, vv\}] \cdot P[\{vv, dd\}] \neq P[\{dv, vv\} \cap \{vv, dd\}]$

$\Rightarrow$ {Nv is even} and $\{c_2 = v\}$ r not independent.

8. C: event that chip works

p: event that transistor works.

$P(\text{chip work}) = [P(p)]^n$ ($\because$ all transistors shd work together)

$\Rightarrow P(M) = P[8 \text{ chips work}] + P[9 \text{ chips work}]$

$= {}^9C_1 [P(C)]^8 (1 - P(C)) + [P(C)]^9$

$= 9 (p)^{8n} (1 - p^n) + p^{9n}$

[illegible] lineup requires [1C, 2F, 2G] (F → Forward, G → Guard)

Swingman has 3 options - he plays either G or F or neither (on bench)

We have $3C_p$, $4F_p$, $4G_p$ [where p subscript = pure players]

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① If Swingman (S) plays Guard (G). Then we have to select 1 out of 3 C's, 2 out of 4 F's & only 1 out of 4 G's ∵ S is serving as one guard.

$\Rightarrow a = ({}^3C_1)({}^4C_2)({}^4C_1) = 3 \cdot 6 \cdot 4 = 72.$

② If S plays F. Then we have to choose 1 out of 3 c's, 2 out of 4 G's & only 1 out of 4 F's.

$\Rightarrow b = ({}^3C_1)({}^4C_1)({}^4C_2) = 3 \cdot 4 \cdot 6 = 72$

③ If S is on bench. We choose 1 out of 3 C's, 2 out of 4 G's & 2 out of 4 F's.

$c = ({}^3C_1)({}^4C_2)({}^4C_2) = 3 \cdot 6 \cdot 6 = 108.$

$\Rightarrow$ Total lineups $= a + b + c = 72 + 72 + 108 = \underline{252}$.

10 a) $P[G=2, Y=1, R=2] = \dfrac{5!}{(2!)(1!)(2!)} \left(\dfrac{7}{16}\right)^2 \cdot \left(\dfrac{1}{8}\right)\left(\dfrac{7}{16}\right)^2$ (multinomial)

b) $P[G=R] \Rightarrow$ prob. that no of green lights = no of red lights.

$\Rightarrow P[G=1, R=1, Y=3] + P[G=2, R=2, Y=1] + P[G=0, R=0, Y=5]$

$= \dfrac{5!}{(1!)(1!)(3!)} \left(\dfrac{7}{16}\right)\left(\dfrac{7}{16}\right)\left(\dfrac{1}{8}\right)^3 + \dfrac{5!}{(2!)(2!)(1!)} \left(\dfrac{7}{16}\right)^2\left(\dfrac{7}{16}\right)^2\left(\dfrac{1}{8}\right) + \dfrac{5!}{0!\,0!\,5!} \left(\dfrac{1}{8}\right)^5\left(\dfrac{7}{16}\right)^0\left(\dfrac{7}{16}\right)^0$

11 a) No. of grp 1 kickers = 3 $\quad P[G_1] = 1/3$

" " grp 2 " = 6 $\quad P[G_2] = 2/3$

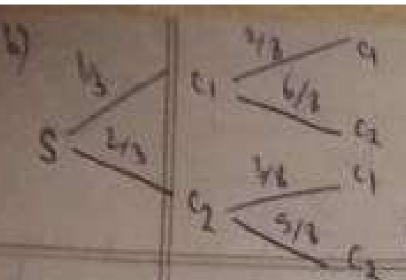
K: event that kicker kicks field goal.

$P[K \mid G_1]$ = kicker kicks field goal given that he is from grp 1 $\Rightarrow P[K \mid G_1] = 1/2$

$P[K \mid G_2]$ = " " " " " " " " grp 2 $\Rightarrow P[K \mid G_2] = 1/3$

$P[K] = P[K \mid G_1] \cdot P[G_1] + P[K \mid G_2] \cdot P[G_2] = \left(\dfrac{1}{2}\right)\left(\dfrac{1}{3}\right) + \left(\dfrac{1}{3}\right)\left(\dfrac{2}{3}\right) = \dfrac{7}{18}.$

b)



$P(c_1c_1) = 1/12$

$P(c_1c_2) = 1/4$

$P(c_2c_1) = 1/4$

$P(c_2c_2) = 5/12$

$P[K_1K_2 | C_{11}] = \left(\frac{1}{2}\right)^2$, $P[K_1K_2 | C_{12}] = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$

$P[K_1K_2 | C_{21}] = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$, $P[K_1K_2 | C_{22}] = \left(\frac{1}{3}\right)^2$

$P[K_1K_2] = P[K_1K_2|C_{11}]\,P[C_{11}] + P[K_1K_2|C_{12}]\,P[C_{12}] + P[K_1K_2|C_{21}]\,P[C_{21}] + P[K_1K_2|C_{22}]\,P[C_{22}] =$

$= \left(\frac{1}{4}\right)\left(\frac{1}{12}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{9}\right)\left(\frac{5}{12}\right) = \frac{15}{96}$.

Also, $P[K_2 | C_{11}] = \frac{1}{2}$, $P[K_2 | C_{12}] = \frac{1}{3}$, $P[K_2|C_{21}] = \frac{1}{2}$, $P[K_2|C_{22}] = \frac{1}{3}$

$P[K_2] = P[K_2|C_{11}]\,P[C_{11}] + P[K_2|C_{12}]\,P[C_{12}] + P[K_2|C_{21}]\,P[C_{21}] + P[K_2|C_{22}]\,P[C_{22}]$

$= \left(\frac{1}{2}\right)\left(\frac{1}{12}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{3}\right)\left(\frac{5}{12}\right) = \frac{7}{18}$ Ans.

$P[K_1] \cdot P[K_2] = \left(\frac{7}{18}\right) \cdot \left(\frac{7}{18}\right) = \left(\frac{7}{18}\right)^2$

&

$P[K_1K_2] = \frac{15}{96}$ $\Rightarrow P[K_1K_2] \neq P[K_1]P[K_2] \Rightarrow K_1$ & K_2 r not independent.

c) $P[M|G_1] = {}^{10}C_5 \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5$ prob of missing given kids from grp 1, $P(G_1) = \frac{1}{3}$

$P[M|G_2] = {}^{10}C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^5$ " " " " " " grp 2, $P(G_2) = \frac{2}{3}$

$P[M] = P[M|G_1] \cdot P[G_1] + P[M|G_2] \cdot P[G_2]$

$= {}^{10}C_5 \left[\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{10} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5 \right]$ Ans.

$F_X(-\infty) = 0.\ F_X(\infty) =$

12. P[winning 8 straight championships] = $(0.32)^8$

P[10 titles in 11 yrs] = ${}^{11}C_{10}\,(0.32)^{10}(0.68)$

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13. P[test +ve | not infected] = $\frac{1}{1000}$

P[test −ve | infected] = 0   To find: P[infected | +ve] = ?

$\frac{P(\text{infected} \cap +ve)}{P(+ve)} = P(\text{infected} \mid +ve)$

$\Rightarrow \frac{P(+ve \mid \text{infected})\, P(\text{infected})}{P(+ve)} \Rightarrow \frac{P(\text{infected})}{P(+ve)}$

$\Rightarrow \frac{P(\text{infected})}{P(\text{infected})P(+ve \mid \text{infected}) + P(+ve \mid \text{not infected})P(\text{not infected})}$

$= \frac{1/1000}{1/1000 + (1/1000)\times(999/1000)} = \frac{1000}{1999}$ Ans.

14. P[voice sample is scream] = p

P[app detected a scream | audio is not scream] = q

P[app detected a scream | audio is a scream] = d.

P[audio is a scream | app detected a scream] = r

P[audio is a scream] = $\frac{P[\text{audio scream} \cap \text{app detected scream}]}{P(\text{app detected scream})}$

$\Rightarrow \frac{P(\text{audio is a scream}) \times r}{P(\text{app detected a scream})} \Rightarrow \frac{p \times r}{P(\text{app detected scream} \mid \text{audio not scream}) + P(\text{detect scream} \mid \text{audio is scream})}$

$= \frac{p \times r}{r + q}$