Principles of Voltage Source Converter Control Using Clarke and Park Transformations

Explore dynamic real time simulations of Clarke and Park transformations, complete with harmonic analysis and advanced signal behaviors, all available on my website. Take your understanding beyond static models with interactive tools designed for engineers, students, researchers and anyone eager to appreciate the mathematical elegance behind these transformations.

Visit: https://saricmilos.com/

Author: Miloš Sarić **Advisor: Eduardo Prieto Araujo**

Contents

Problem 1: MATLAB script which visualizes three-phase electrical signals with harmonics

Problem 1
Open the file 'Script_Clarke_3D_Base.m' and proceed with the following tasks:

- Understand the link between abc-$\alpha\beta$ in time domain
- Understand the abc-$\alpha\beta$ in the 3D cubic representation
- Add harmonics and observe the impact both in time domain and in the 3D view

Space Phasors and Two-Dimensional Frames

Question 1: Understand the link between $abc - \alpha\beta$ – frames in time domain

Answer 1:

A sinusoidal voltage or current with a constant frequency is defined by two key parameters: its peak value and its phase angle. A voltage

$$v(t) = V_{max} \cos(\omega t + \delta)$$

has a maximum value V_{max} and a phase angle δ when referenced to $cos(\omega t)$. The root-mean- square (RMS) value, also called effective value, of the sinusoidal voltage is

$$V_{eff} = \frac{V_{max}}{\sqrt{2}}$$

Euler's identity

$$e^{j\theta} = \cos\delta + j\sin\delta$$

Is a fundamental equation in complex analysis that allows us to represent sinusoids using phasors. For the above voltage,

$$v(t) = Re\left[V_{max} e^{j(\omega t+\delta)}\right] = Re[\sqrt{2}\left(V_{eff} e^{j\delta}\right) e^{j\omega t}]$$

Where $j = \sqrt{-1}$ and Re denotes real part.

The RMS phasor representation of the voltage is given in three forms:

1. Exponential:

$$V = V_{rms} e^{j\delta}$$

2. Polar:

$$V = V_{rms} \angle\delta$$

3. Rectangular:

$$V = V_{rms} \cos\delta + jV_{rms} \sin\delta$$

We can easily convert a phasor between different forms. The phasor diagram in Figure 2.1 illustrates the process of changing from polar to rectangular form. Euler's identity also helps in transforming an exponential expression into its rectangular equivalent.

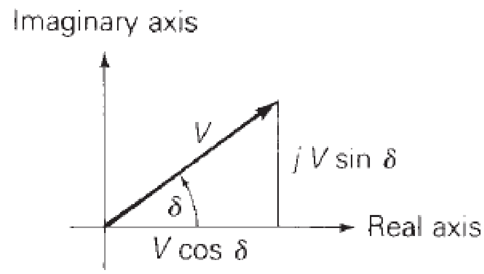


Figure 1. Phasor Diagram

Space-Phasor Representation of a Balanced Three-Phase Function

Consider the following balanced, three-phase, sinusoidal function:

$$f_a(t) = f\cos(wt + \theta_0)$$

$$f_b(t) = f\cos\left(wt + \theta_0 - \frac{2\pi}{3}\right)$$

$$f_c(t) = f\cos\left(wt + \theta_0 - \frac{4\pi}{3}\right)$$

Where: f - Amplitude, θ_0- Initial phase angle, w - angular frequency.

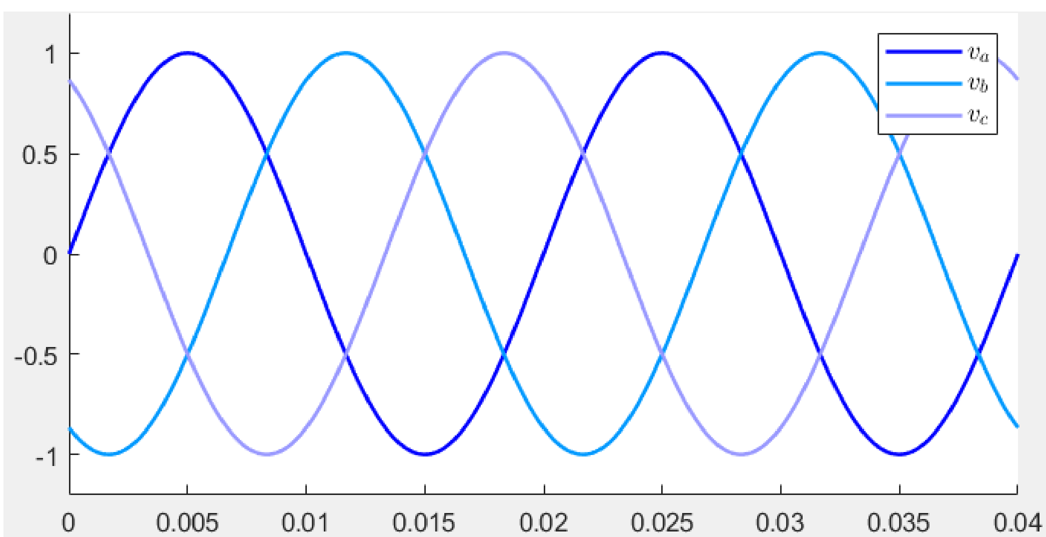


Figure 2. Three-Phased balanced system in abc time domain

Question 2: Understand the link between $abc - \alpha\beta$ – frames in 3D cubic representation

Answer 2 and continue of Answer 1:

On our 3D representation we can notice cartesian coordinate system (In Black) with three axis a, b and c with 90 degrees between them:

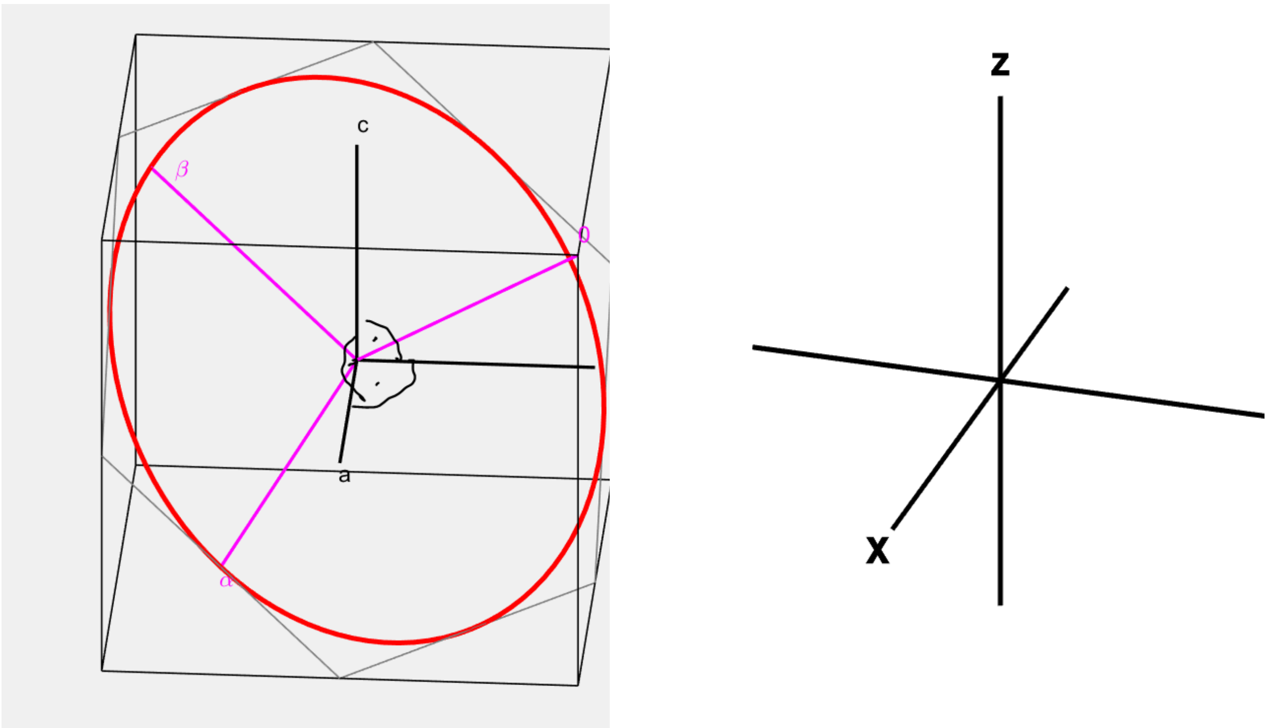


Figure 3. Cartesian Coordinate System

If we add time values in our sinusoidal function and plot results for 3 functions on their own axis (Values of fa plotted on a-axis, values of fb plotted on b-axis and values we get from fc on c axis) we would get this red circle:

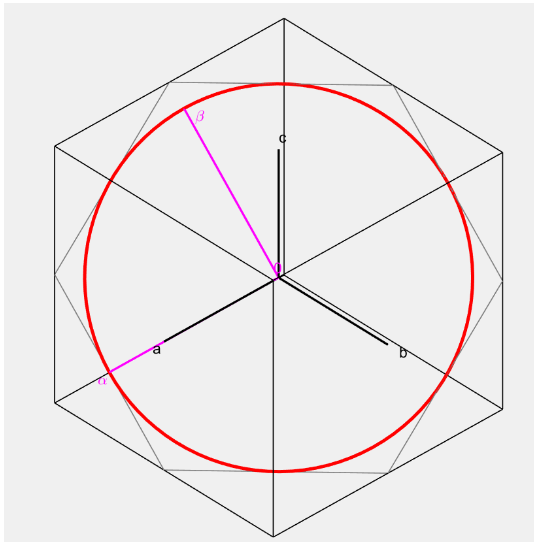


Figure 4. Plotting Results for each time value

Space phasor is defined as:

$$\vec{f}(t) = \frac{2}{3}[e^{j0}f_a(t) + e^{j\frac{2\pi}{3}}f_b(t) + e^{j\frac{4\pi}{3}}f_b(t)]$$

Using identities:

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

$$e^{j0} + e^{j\frac{2\pi}{3}} + e^{j\frac{4\pi}{3}} = 0$$

We obtain:

$$\vec{f} = (fe^{j\theta_0})e^{j\omega t} = \underline{f}e^{j\omega t}$$

Where:

$\underline{f} = fe^{j\theta_0}$ – Complex quantity which can be represented as a vector in the complex plane.

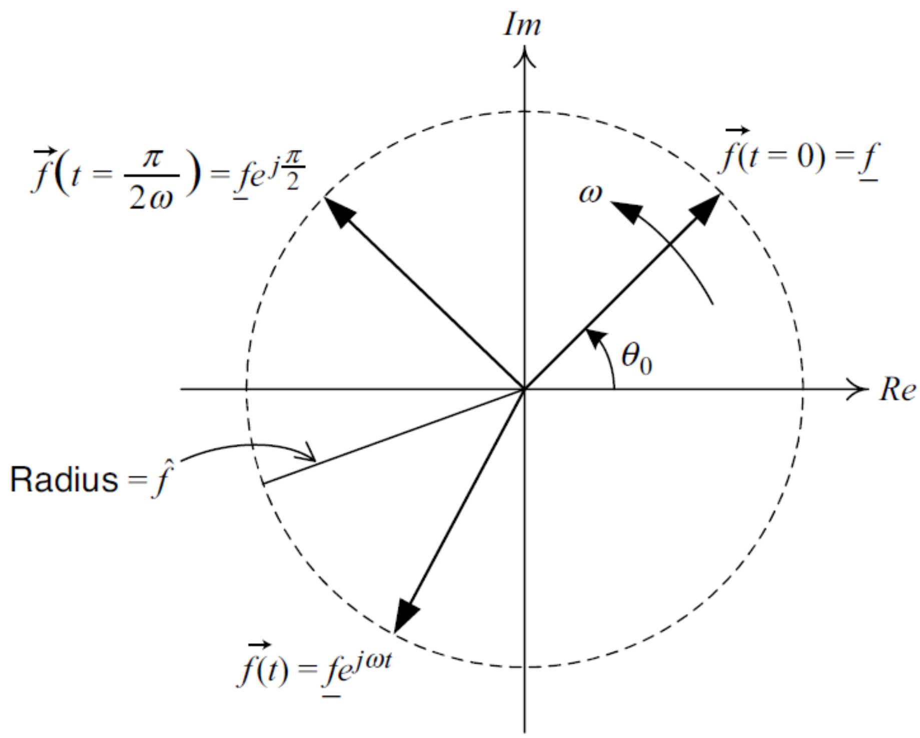


Figure 5. Space-phasor representation in the complex plane

Space phasor $\vec{f}(t)$ is the same as phasor $\underline{f}$ that rotates counter clock wise with the angular speed ω. A space phasor, in its most general form, contains all the information about the amplitude, phase angle, and frequency of the related three-phase signal.

The real-valued components $f_a(t), f_b(t), f_c(t)$ can be retrieved from the corresponding space phasor, based on following equations:

$$f_a(t) = Re\,\{\vec{f}(t)e^{-j0}\}$$

$$f_b(t) = Re\,\{\vec{f}(t)e^{-j\frac{2\pi}{3}}\}$$

$$f_b(t) = Re\,\{\vec{f}(t)e^{-j\frac{4\pi}{3}}\}$$

Which mean $f_a(t), f_b(t), f_c(t)$ are projections on real axis of the complex plane.

$\alpha\beta$ Representation of a Space Phasor

Considering the space phasor:

$$\vec{f}(t) = \frac{2}{3}[e^{j0}f_a(t) + e^{j\frac{2\pi}{3}}f_b(t) + e^{j\frac{4\pi}{3}}f_b(t)]$$

Where $f_a(t) + f_b(t) + f_c(t) = 0$.

$\vec{f}(t)$ can be decomposed into it's real and imaginary components as:

$$\vec{f}(t) = f_\alpha(t) + jf_\beta(t)$$

Substituting these two equations:

$$f_\alpha(t) + jf_\beta(t) = \frac{2}{3}\left[e^{j0}f_a(t) + e^{j\frac{2\pi}{3}}f_b(t) + e^{j\frac{4\pi}{3}}f_b(t)\right]$$

And equating the corresponding real and imaginary parts of both sides of the resultant we deduce:

$$\begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \end{bmatrix} = \frac{2}{3}\mathbf{C}\begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix}$$

Where:

$$\mathbf{C} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

We transform our balanced three-phased system into $\alpha\beta$domain.

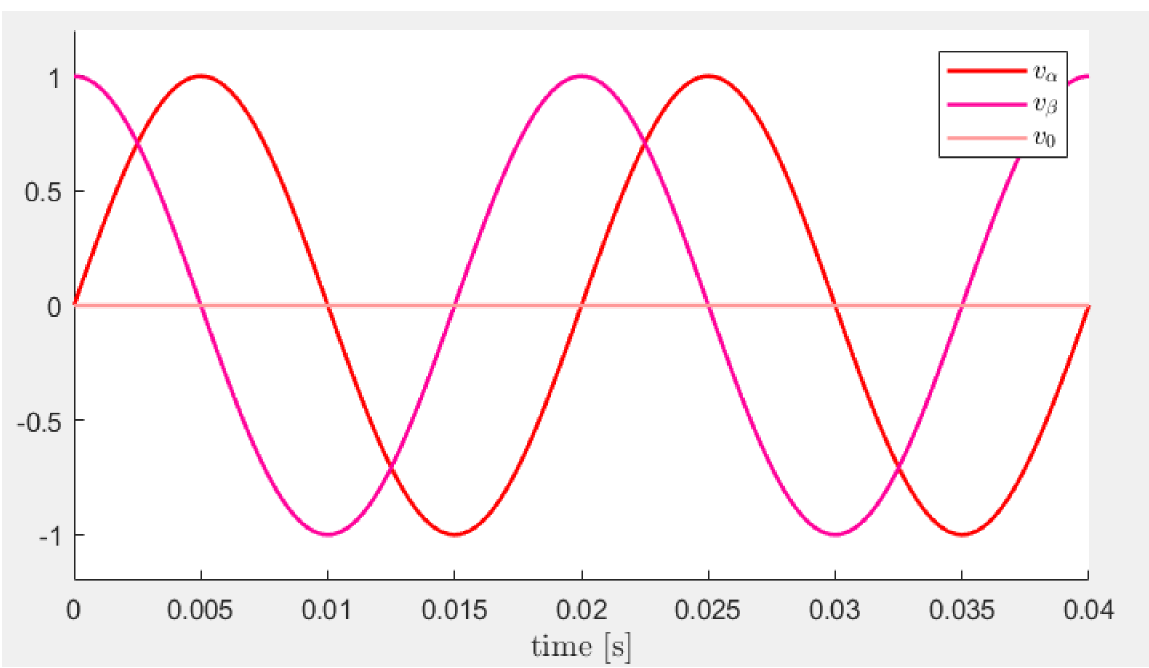


Figure 6. Three-Phased signal transformed into alpha-beta domain

Question 3: Add harmonics and observe the impact both in time domain and in the 3D view

Answer 3:

So far, we had a clean three-phased system with fundamental frequency. To add third harmonics, we'll need to add sine terms with 3 times the fundamental frequency. Third harmonics are in phase with each other (this is characteristic in three-phase systems). We'll now see the zero-sequence component (v_0) in the Clarke transformation is non-zero due to the third harmonics.

ADDED THIRD HARMONIC:

```
% Adding harmonics, etc.
 va = 1 * sin(w*t) + 0.5 * sin(3*(w*t))% + rand * 0.25 * sin(5*(w*t+360));
 vb = 1 * sin(w*t-2*pi/3) +  0.5 * sin(3*(w*t)) %+ rand * 0.25 * sin(5*(w*t+360));
 vc = 1 * sin(w*t+2*pi/3) +  0.5 * sin(3*(w*t)) %+ rand * 0.25 * sin(5*(w*t+360));
```

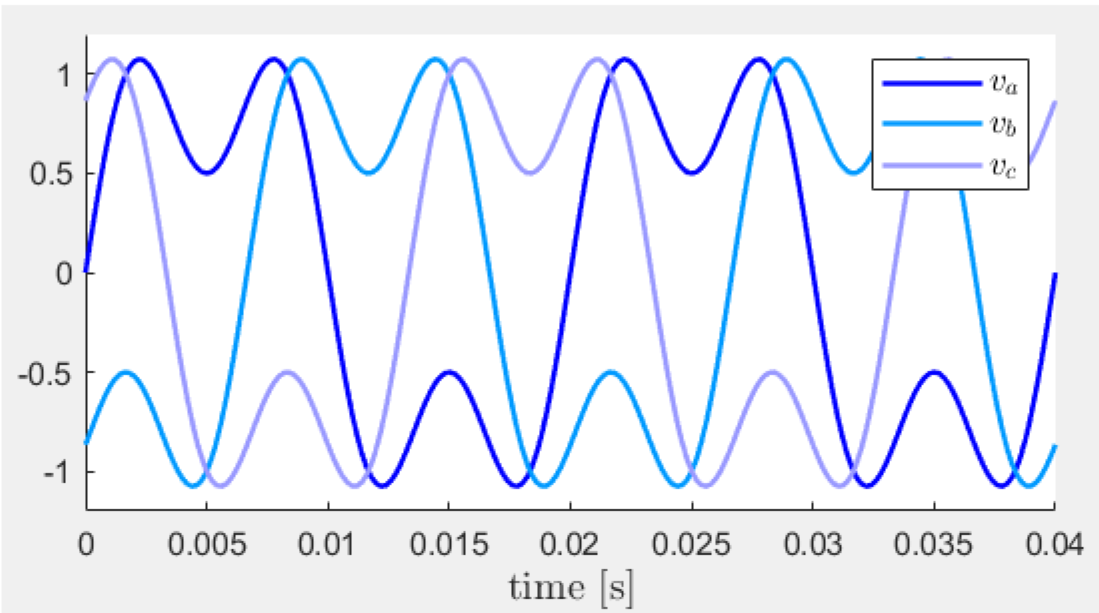


Figure 7. Grid Voltages

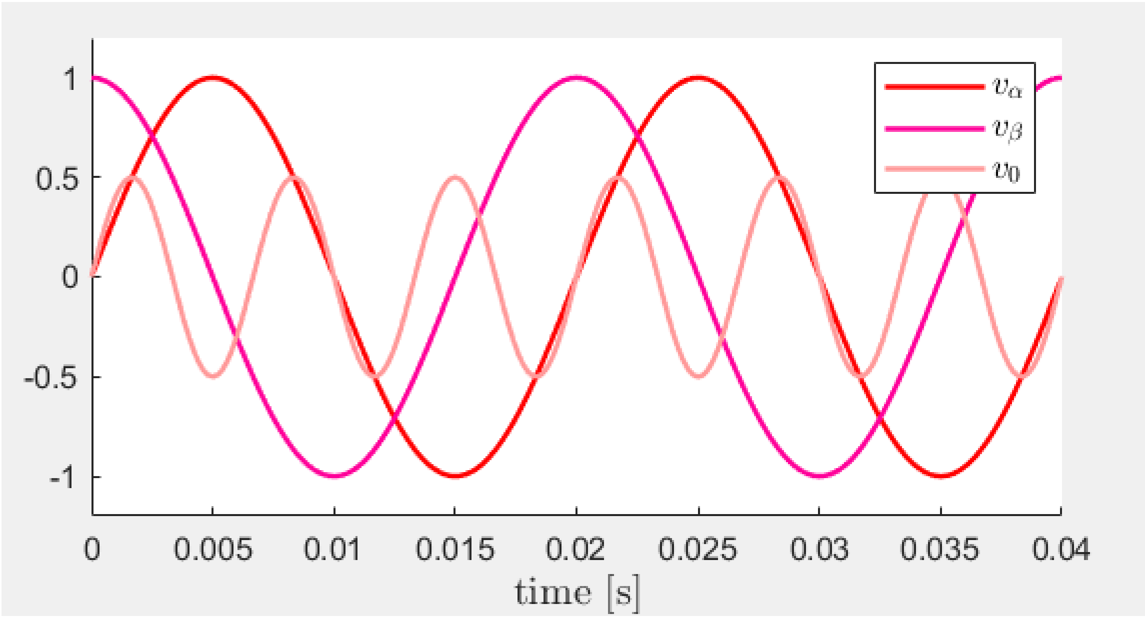


Figure 8. Alpha Beta frame

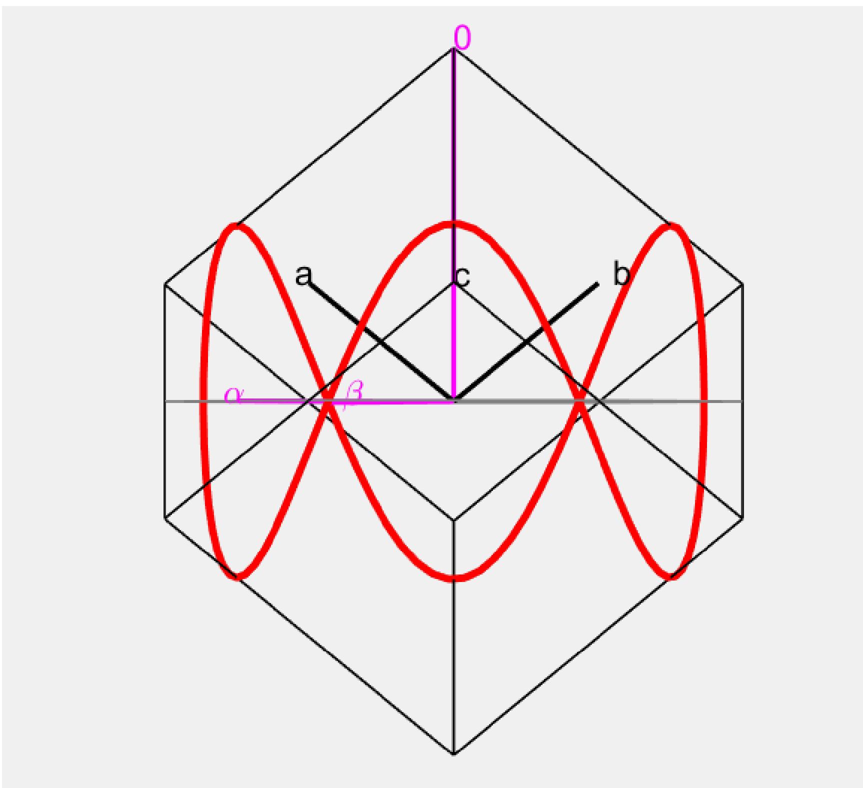


Figure 9. Clarke 3D frame

Added fifth harmonic:

```
% Adding harmonics, etc.
 va = 1 * sin(w*t) +  0.25 * sin(5*(w*t));
 vb = 1 * sin(w*t-2*pi/3) +  0.25 * sin(5*(w*t));
 vc = 1 * sin(w*t+2*pi/3)  +   0.25 * sin(5*(w*t));
```

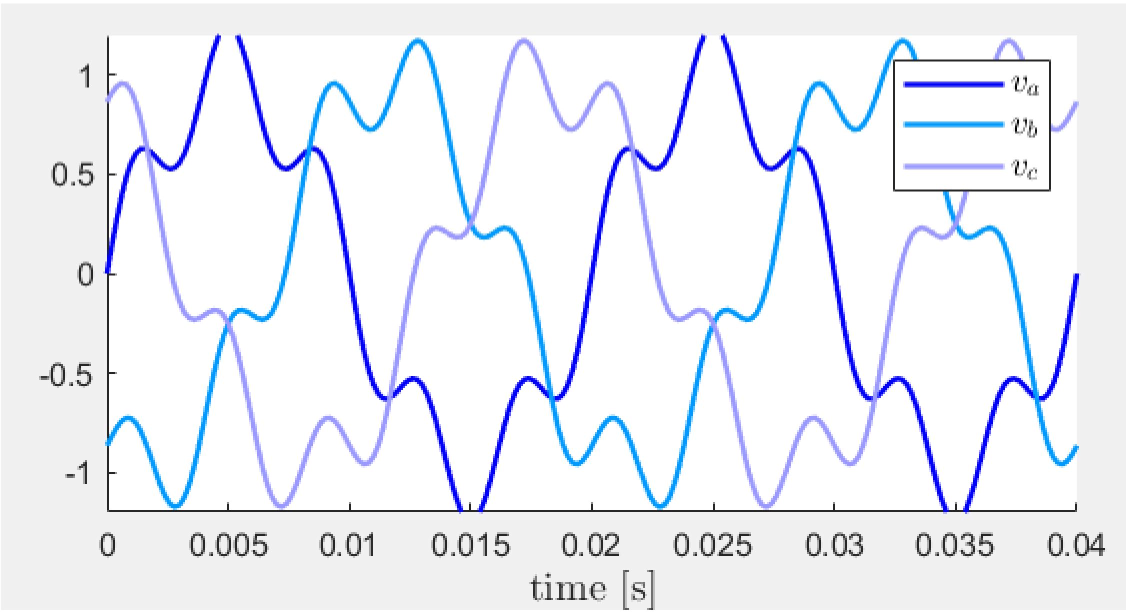


Figure 10. ABC

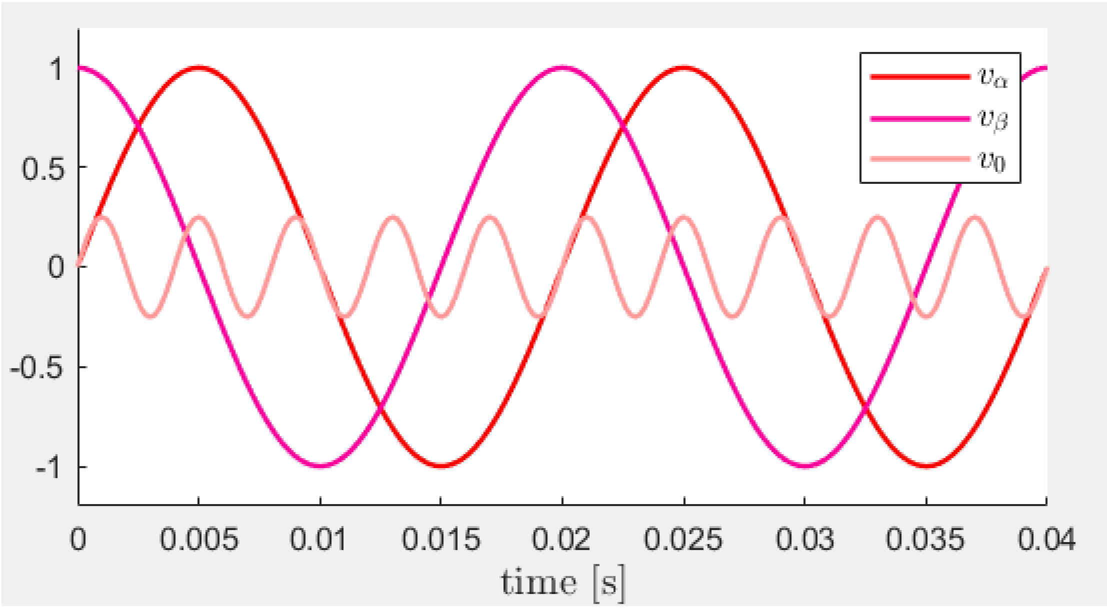


Figure 11. Apha Beta

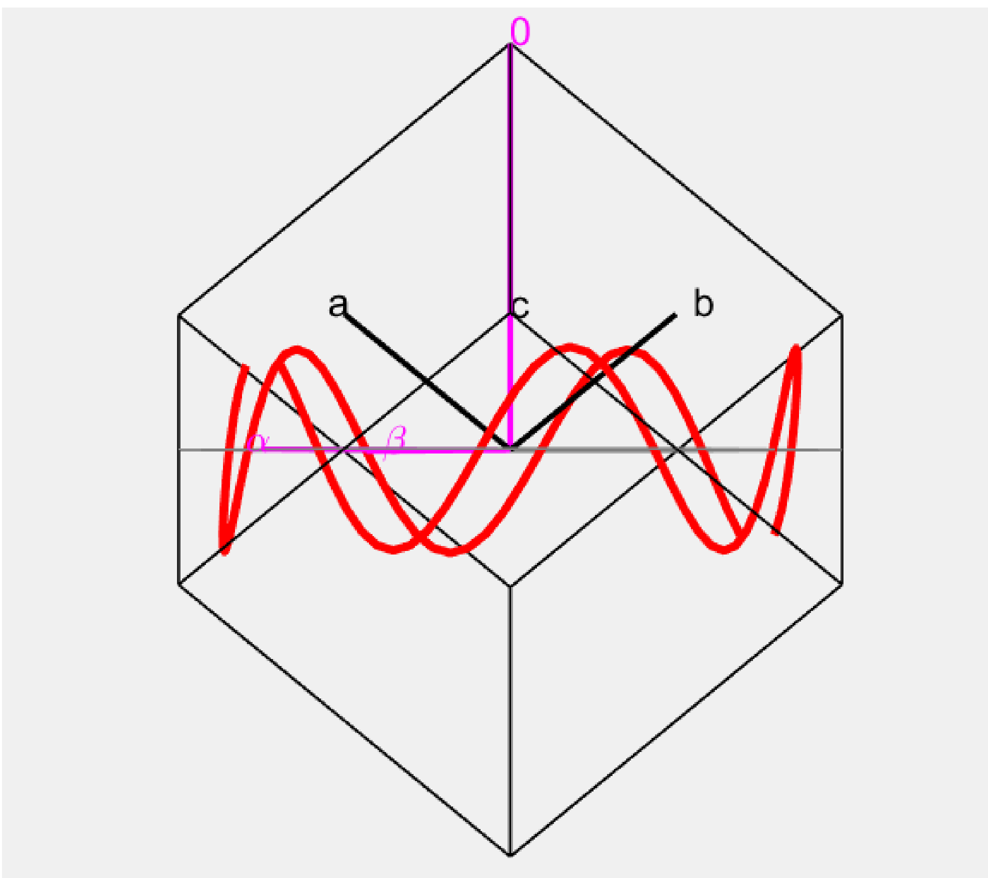


Figure 12. Alpha Beta 3D

Problem 2: Model0_PWM_Base

Problem 2
Open the file 'Model0_PWM_Base.slx' and proceed with the following tasks:

- Study how the modulation is implemented
- Observe the voltage applied by the converter (without filtering it)
- Observe the voltage applied by the converter (filtering it)
- Apply the phase different and observe the impact on the: grid voltage, converter voltage and network current

Question 1: Study how the modulation is implemented

Answer 1:

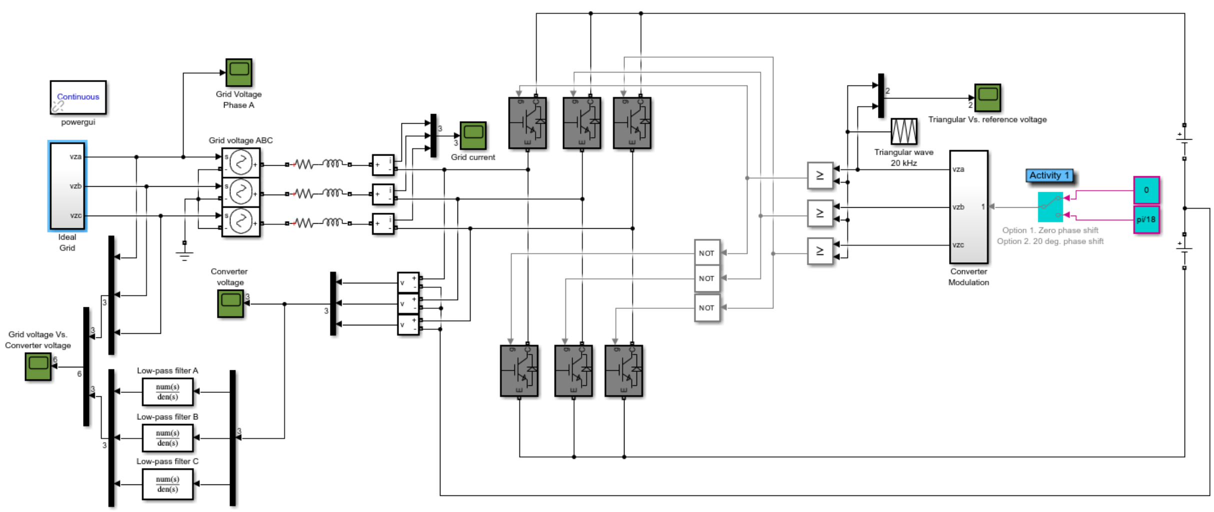


Figure 2.13. Simplified power circuit diagram of full-bridge AC/DC Converter

Figure 1. presents a schematic of an AC/DC full-bridge converter, which consists of six switch cells. Each switch cell is made up of a fully controllable unidirectional switch connected in antiparallel with a diode. This configuration, commonly referred to as a *reverse-conducting switch*, is commercially available as an IGBT (*Insulated-Gate Bipolar Transistor*) or IGCT (*Integrated Gate Commutated Thyristor*). We can refer to the fully controllable switch as the *transistor*. Thus, the upper switch cell consists of the transistor S1 and the diode D1. Similarly, the lower switch cell is composed of the transistor S4 and the diode D4.

Figure 1. shows a three-phase AC ideal grid (voltage source) feeding a balanced DC source (battery). Figure 1. is connected to two identical DC voltage sources, each with a voltage of VDC/2. The common point of the voltage sources is labeled as node 0. We refer to this node as the DC-side midpoint and choose it as the voltage reference node. On the AC side, the full-bridge converter is connected to the voltage source modeled in Figure 2.2. The negative terminal of this voltage source is linked to the DC-side midpoint. The connection between the AC-side terminal and the AC-side voltage source is facilitated

by an interface reactor, represented as a series RL branch. The inductance L and internal resistance R of the interface reactor serve to filter the AC-side current, reducing ripple and ensuring smoother operation. The three-phase source is NOT assumed to be ideal and source impedances are NOT neglected. The three-phase source is not ideal because it may introduce a potential short circuit between the AC and DC sides. This consideration is necessary for accurately modeling real-world conditions, where source impedances and non-idealities impact system behavior.

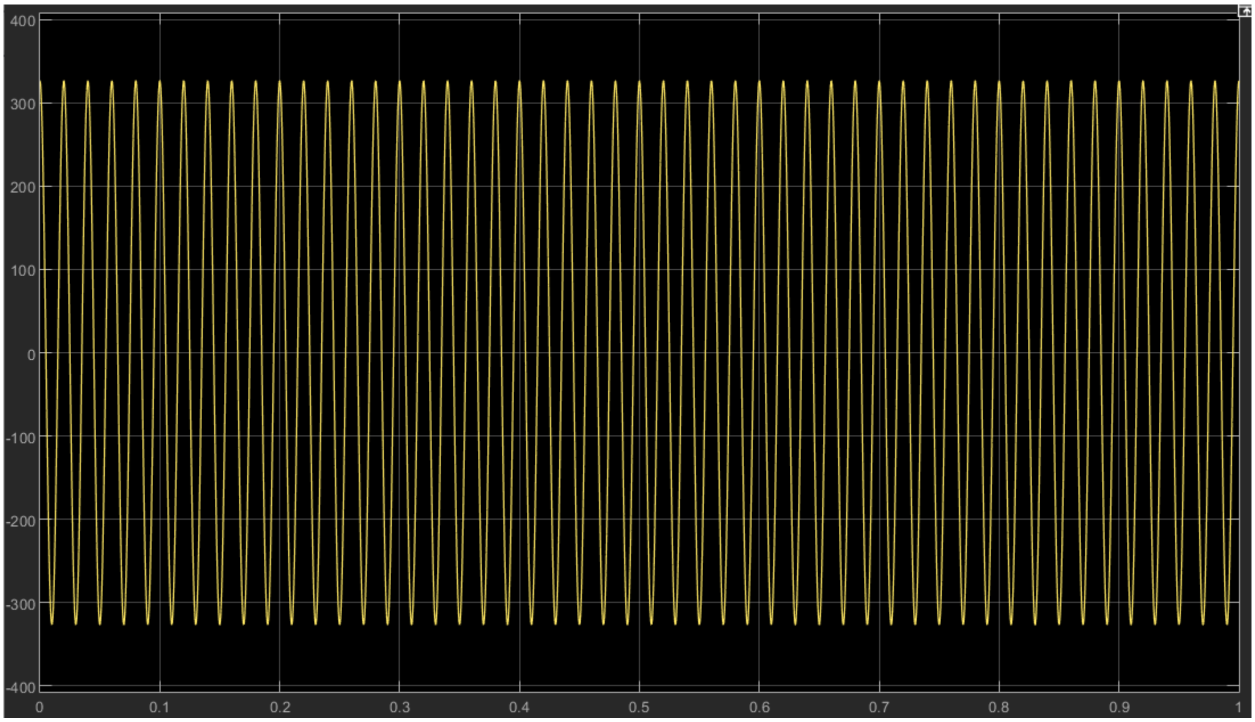


Figure 14. Grid Voltage Phase A

Phase shift of AC side is 0.

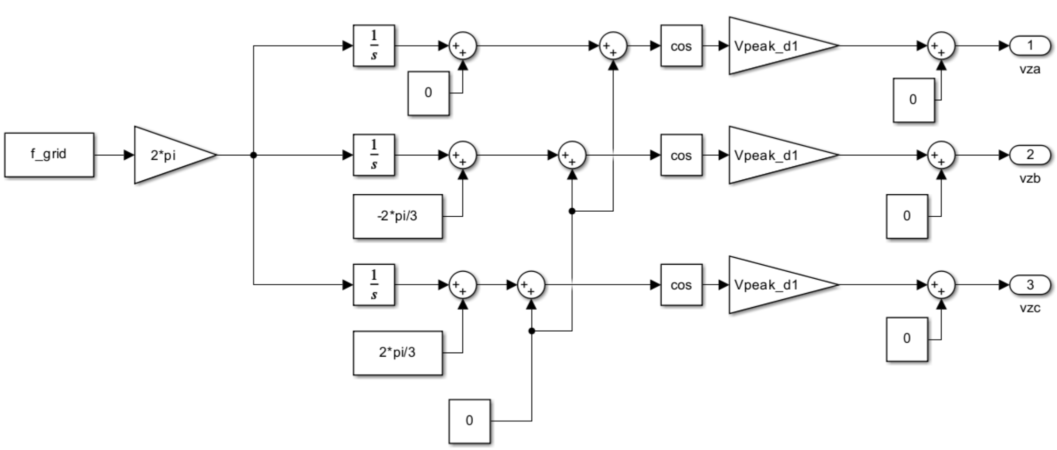


Figure 15.2. Ideal Grid Subsystem

To derive the formula for input AC voltage $v(t)$ in MATLAB SIMULINK, we start by multiplying the grid frequency f_{grid} by 2π, which applies to all three phases. Integrating this expression gives $\int f_{grid} \cdot 2\pi \, dt = wt$ Next, we introduce the phase shifts for each phase, since we have balanced Three-Phase System. The three-phase voltages (Va, Vb, Vc) have equal magnitudes and are phase-shifted by **120°**.

Phase A: 0 shift

Phase B: 120 shift (2*pi/3)

Phase C: 240 (-120) shift (+- 2*pi/3)

Additionally, we account for the phase shift of the input AC voltage, which is set to zero. Taking the cosine of the resulting expression and multiplying it by the voltage amplitude gives the final waveform.

Question 1: Study how the Pulse Width Modulation is implemented

Answer 1:

The full-bridge converter operates by alternately switching S1 and S4 (similarly, S2 and S5, as well as S3 and S6). The switching commands for S1 and S4 are generated using a pulse-width modulation (*PWM*) strategy.

Various *PWM* techniques exist, but the most commonly used approach involves comparing a high-frequency periodic triangular waveform, known as the carrier signal, with a slow-varying waveform called the modulating signal. The carrier signal oscillates between -1 and 1 with a period Ts, and its intersections with the modulating signal determine the switching moments for S1 and S4. This *PWM* process is illustrated in Figure 2.3.

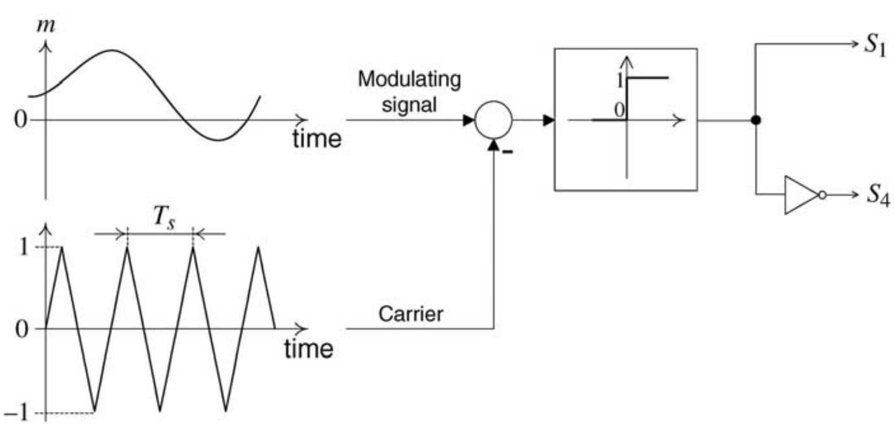


Figure 2.3. Schematic diagram of the mechanism to generate PWM gating pulses for S1 and S4

As shown in Figure 2.2, when the modulating signal exceeds the carrier signal, a turn-on command is sent to S1, while the turn-on command for S4 is canceled.

When the modulating signal falls below the carrier signal, the turn-on command for S1 is blocked, and a turn-on command is issued for S4. However, it is important to note that a switch does not necessarily conduct just because it receives a turn-on command. Conduction occurs only if both the turn-on command is active and the current direction aligns with the switch's characteristics. For instance, when an IGBT receives a turn-on command, it will conduct only if the current flows from the collector to the emitter. Figures 2.3(b) and 2.3(c) illustrate the switching waveforms of S1 and S4 according to the *PWM* strategy shown in Figure 2.2. As depicted in Figure 2.3, the switching functions satisfy the condition s1(t) + s4(t) = 1.

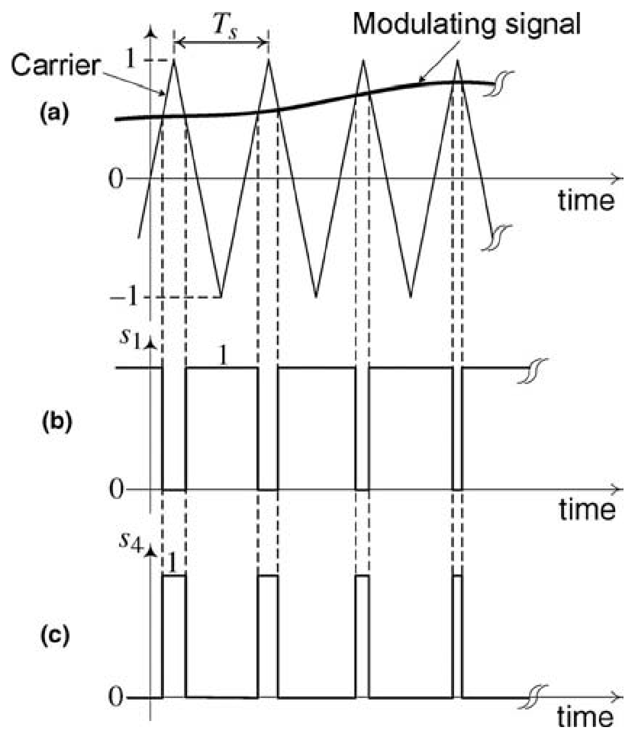


Figure 16. Signals based on PWM switching strategy

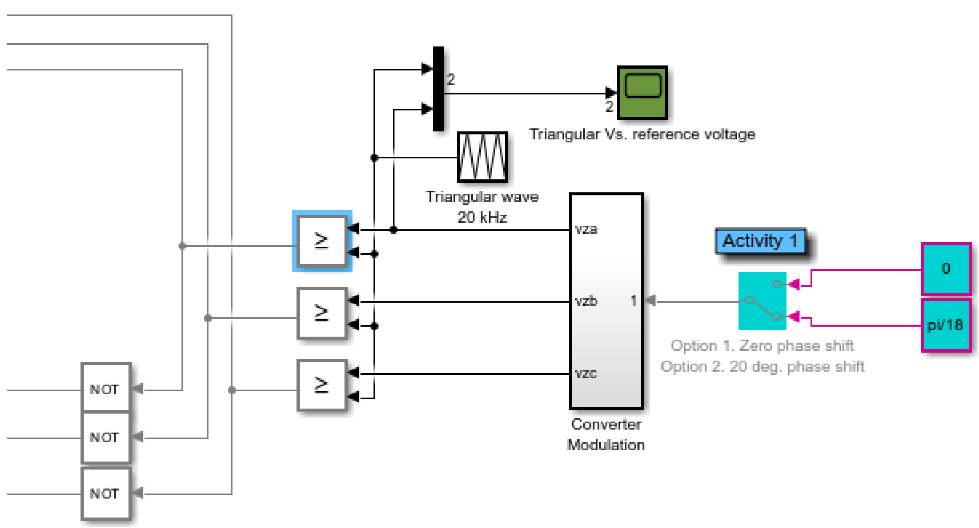


Figure 17. Implementation of PWM

In Simulink, the modulation signal is implemented similarly to the input AC voltage, with the key difference being the addition of a phase shift. While the input AC voltage had its phase shift set to 0, we now have two options for the modulation signal:

1. Keeping the phase shift at 0, the same as the input grid voltage.
2. Introducing a 20-degree phase shift and observing its effects.

By simulating both cases, we can analyze how the phase shift influences system behavior.

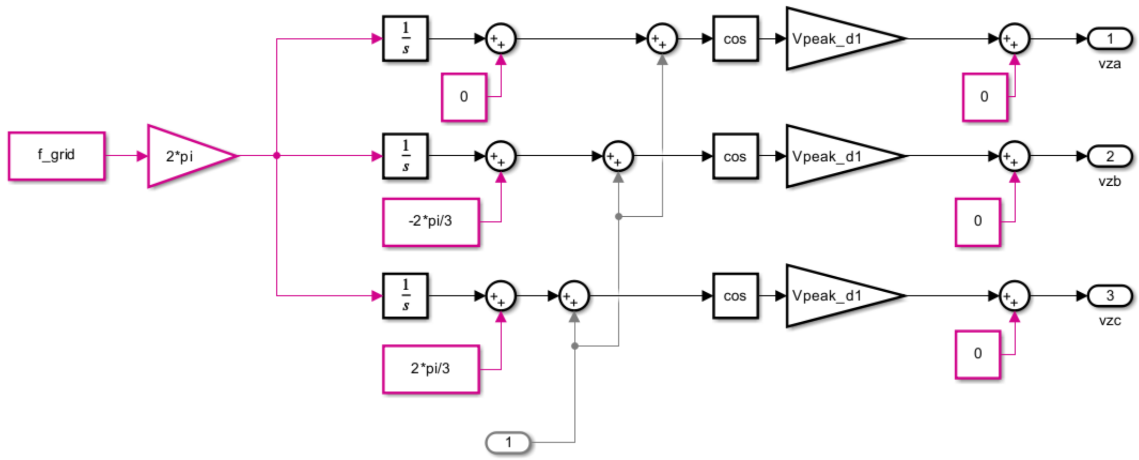
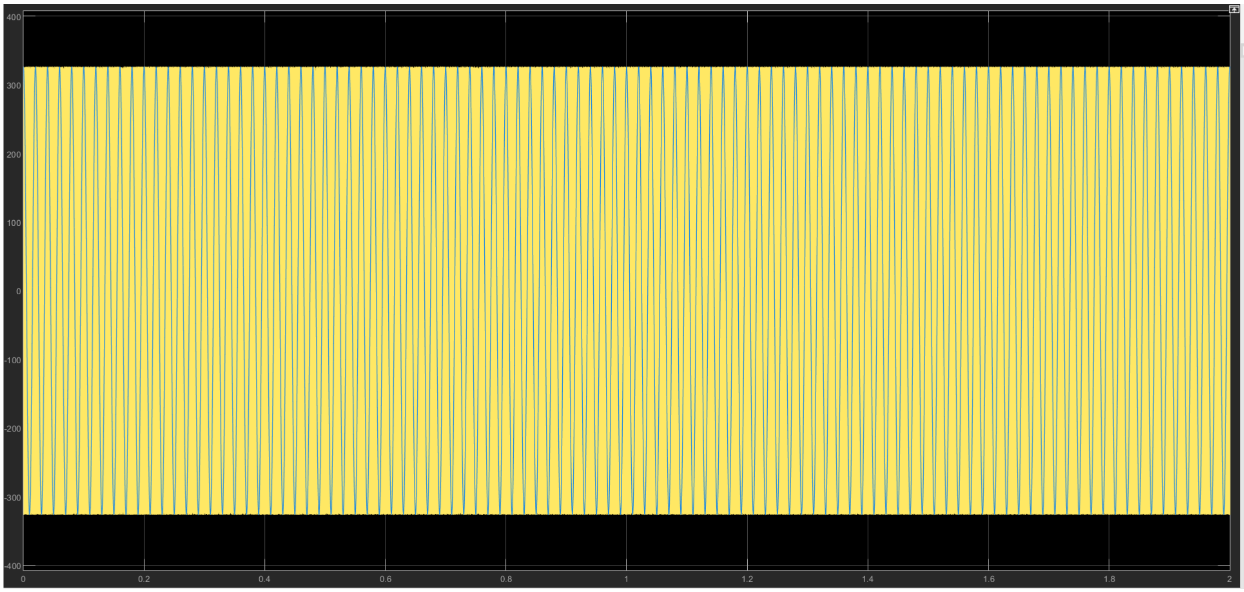
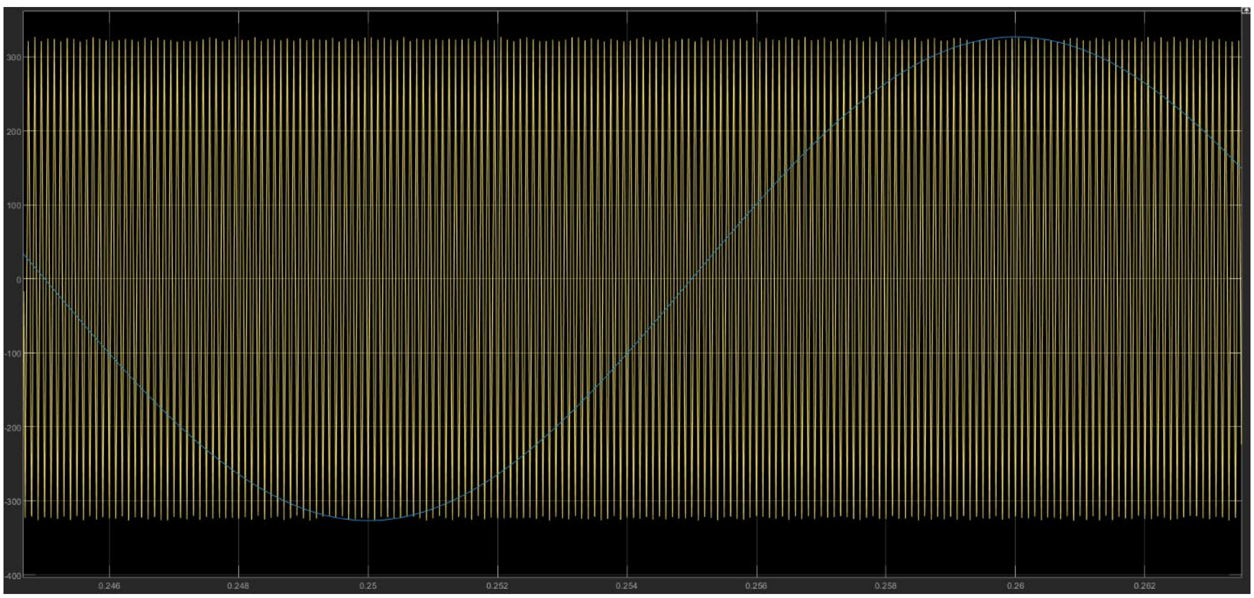


Figure 18. Implementation of modulation signal

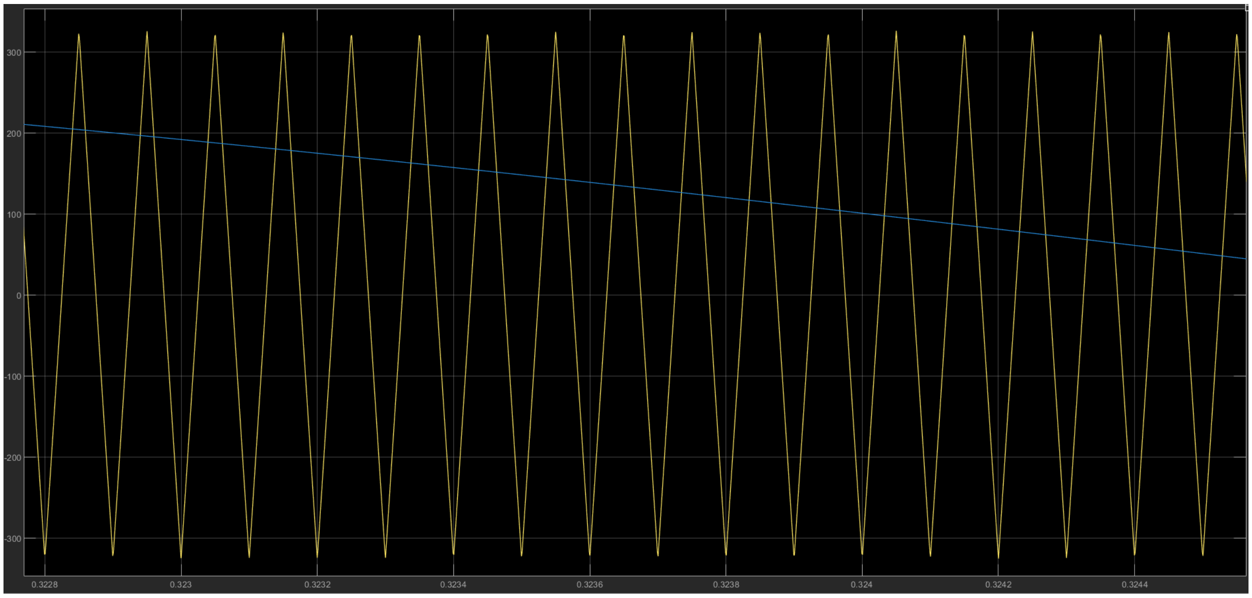
From the plot, we can observe that the carrier frequency (20 kHz) is significantly higher than the modulation frequency (50 Hz). This is a key characteristic of Pulse Width Modulation (*PWM*), where a high-frequency carrier wave is used to control the switching of power electronics based on a lower-frequency modulation signal.



400
300
200
100
0
-100
-200
-300
-400
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



300
200
100
0
-100
-200
-300
-400
0.246
0.248
0.25
0.252
0.254
0.256
0.258
0.26
0.262



In Simulink, we use the relational operator >= (greater than or equal to) to compare the modulating signal and the carrier signal. This operator outputs True (1) when the first input (modulating signal) is greater than or equal to the second input (carrier signal) and False (0) otherwise.

This is a fundamental part of PWM generation, where:

1. When the modulating signal is greater than or equal to the carrier signal, the switch S1,S2,S3 is turned ON (logic 1).
2. When the modulating signal is less than the carrier signal, the switch is turned OFF (logic 0).

The **NOT block** is used to control **complementary switches** in power electronic circuits like a full-bridge or half-bridge converter.

Here's how it works:

- If the input to the NOT block is **TRUE (1)**, the output will be **FALSE (0)**.
- If the input to the NOT block is **FALSE (0)**, the output will be **TRUE (1)**.

This ensures that the two complementary switches (S1 and S4, S2 and S5, S4 and S6) do not turn on at the same time, which would result in a short circuit. By using the NOT block, we can automatically and safely alternate the switching states between complementary switches.

Question 2: Observe the voltage applied by the converter (without filtering it)

Answer 2:

When PHASE SHIFT is equal to 0 there is no current flow (it averages around 0) or power flow:

Converter voltage before low pass-filter. Power flow is from DC side to AC side. All three phases.

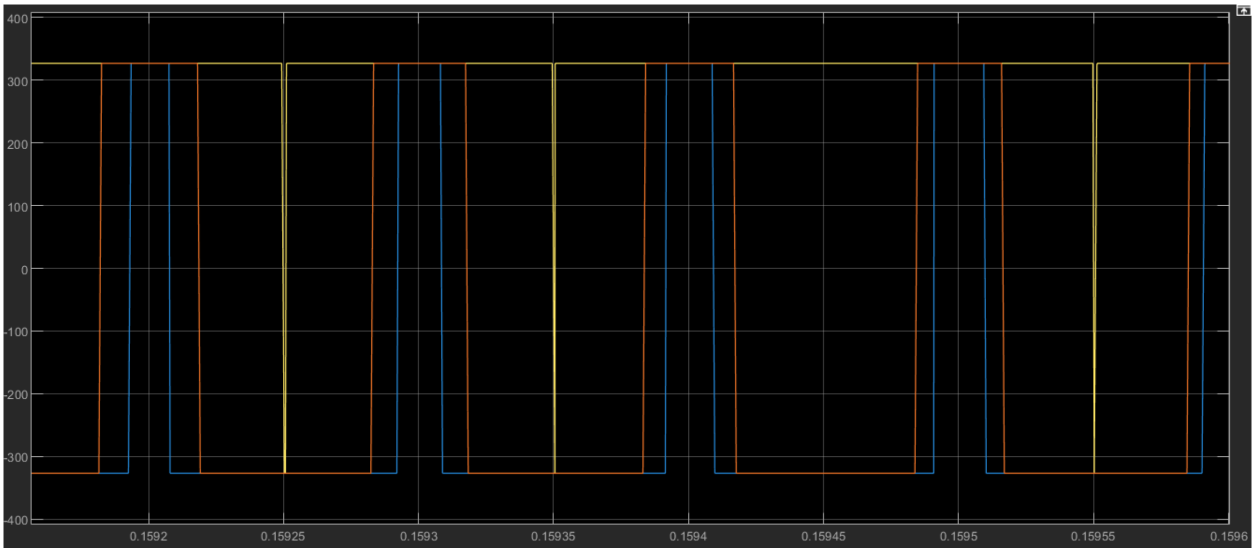


Figure 19. Voltage Applied by the Converter (before filter)

Voltage applied by the converter after low pass filter, in comparison with grid voltage (perfect sinus without ripple – orange line in the middle and difficult to see):

Question 3: Observe the voltage applied by the converter (filtering it)

Answer 3:

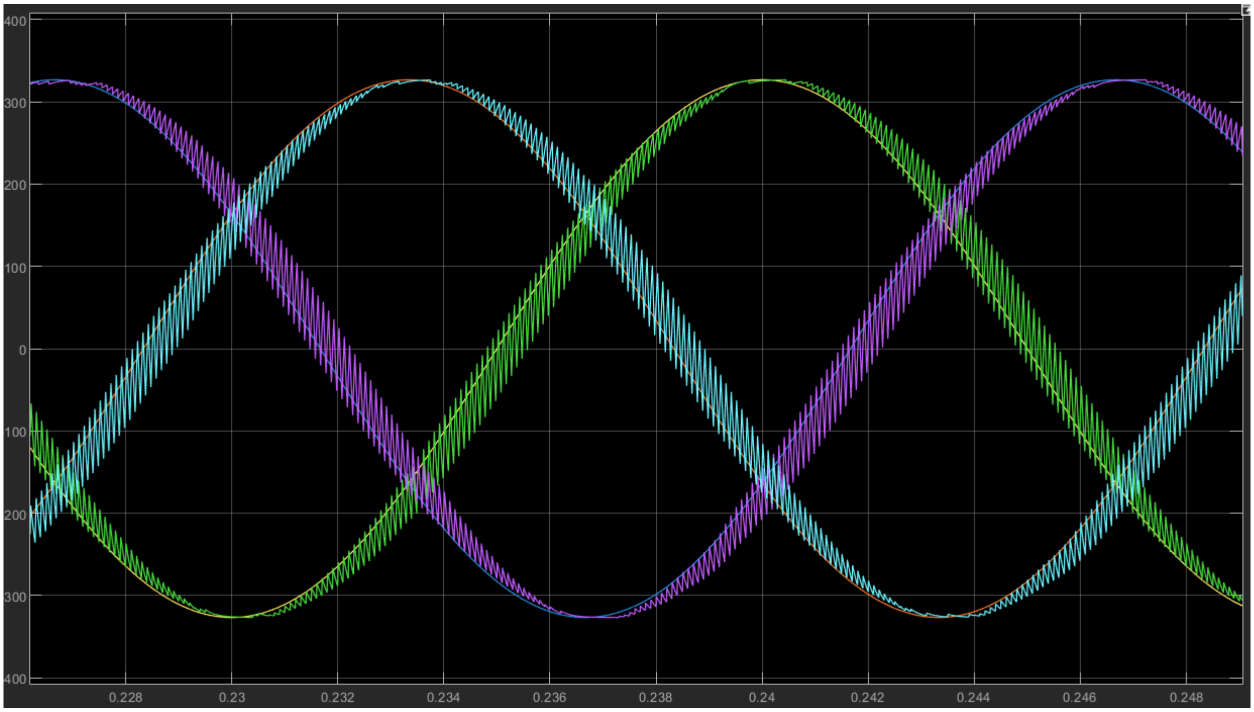


Figure 20. Voltage Applied by the Converter (after filter)

We use transfer function of a first-order in its standard form as a low pass filter:

$$H(s) = \frac{1}{\frac{1}{(2\pi\frac{8e3}{10})}s + 1}$$

Where the time constant τ is:

$$\tau = \frac{1}{2\pi\frac{8e3}{10}} = \frac{10}{2\pi 8e3} = 0.00019894$$

Since the ripple from rectification mainly contains higher-frequency components, in order to filter them more effectively we need to lower the cutoff frequency $f_c = \frac{1}{2\pi\tau}$. Which translates into increasing the τ.

Doubling the τ we obtain:

$$\tau = \frac{2}{2\pi\frac{8e3}{10}} = \frac{20}{2\pi 8e3} = 0.00039789$$

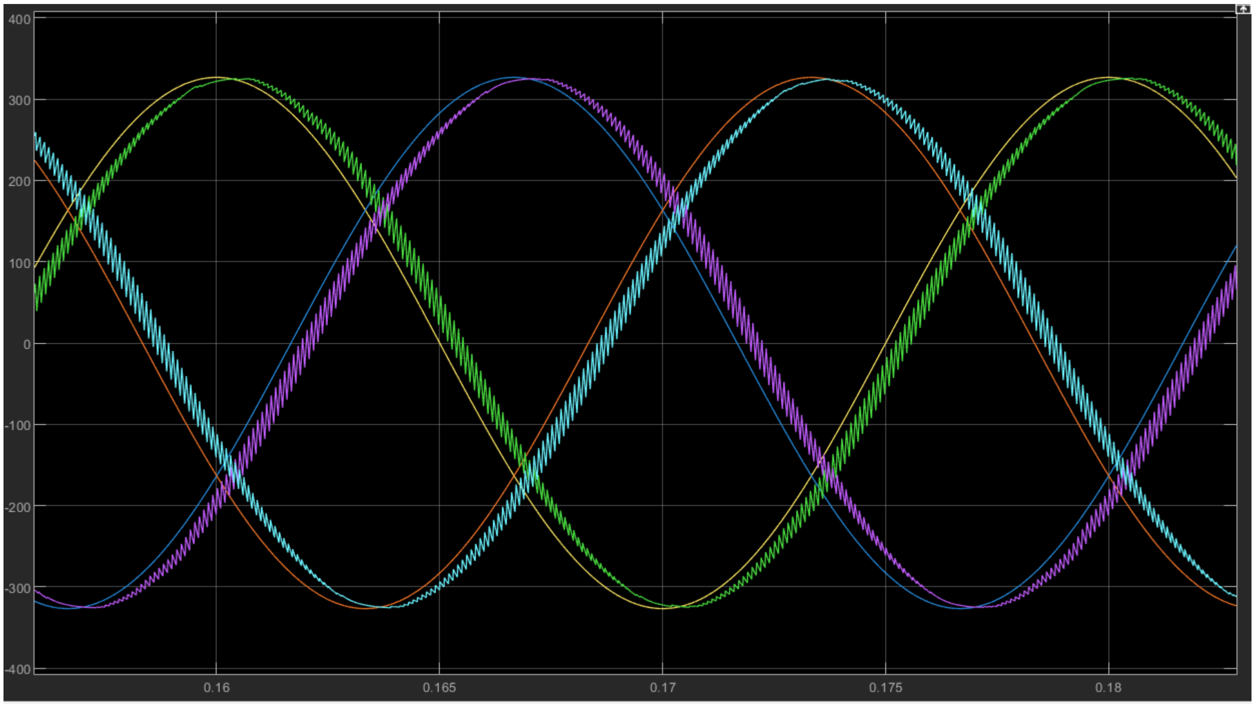


Figure 21. Reduced Ripple with increased time constant

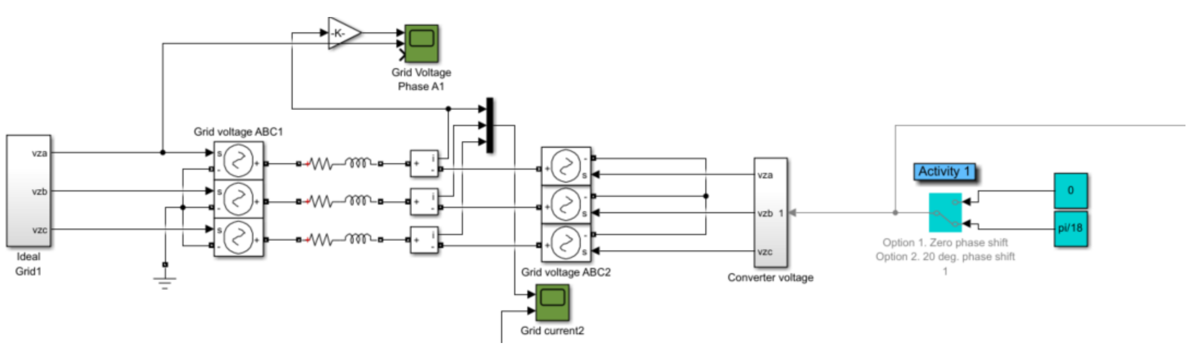
Doubling τ lowers the cutoff frequency, improving ripple reduction. However, it also increases the filter's response time, which causes a phase delay, shifting the signal to the right in time. A first order low pass filter introduces a phase shift that increases with frequency. The phase shift is given by:

$$\phi(f) = -tan^{-1}(2\pi f\tau)$$

In order to reduce filter even further without delay, we should use a higher order filter (with more than one reactive component (capacitor or inductor), LC filter).

A Voltage Source Controller can be represented as a VOLTAGE SOURCE (in a simplified model – without feedback or control loops)!

If we look on Class Discussion model:



Which proves why phase shift is used in the control of a Voltage Source Controller to manage and regulate power flow. In alternating current (AC) systems, power flow is closely related to both the magnitude and phase angle of the voltages and currents. The power delivered to a load (or grid) depends not only on the voltage amplitude but also on the phase difference between the voltage and current waveforms.

The active power (P) and reactive power (Q) in an AC system can be expressed as:

$$P = V_s I_s \cos\theta$$

$$Q = V_s I_s \sin\theta$$

To control the power flow, we need to control either the Voltage Amplitude or the phase difference (θ) between the voltage and current. This is where phase shift comes into play.

Question 4: Apply the phase different and observe the impact on the: grid voltage, converter voltage and network current

Answer 4:

Let's change the phase shift to $\frac{\pi}{18}$:

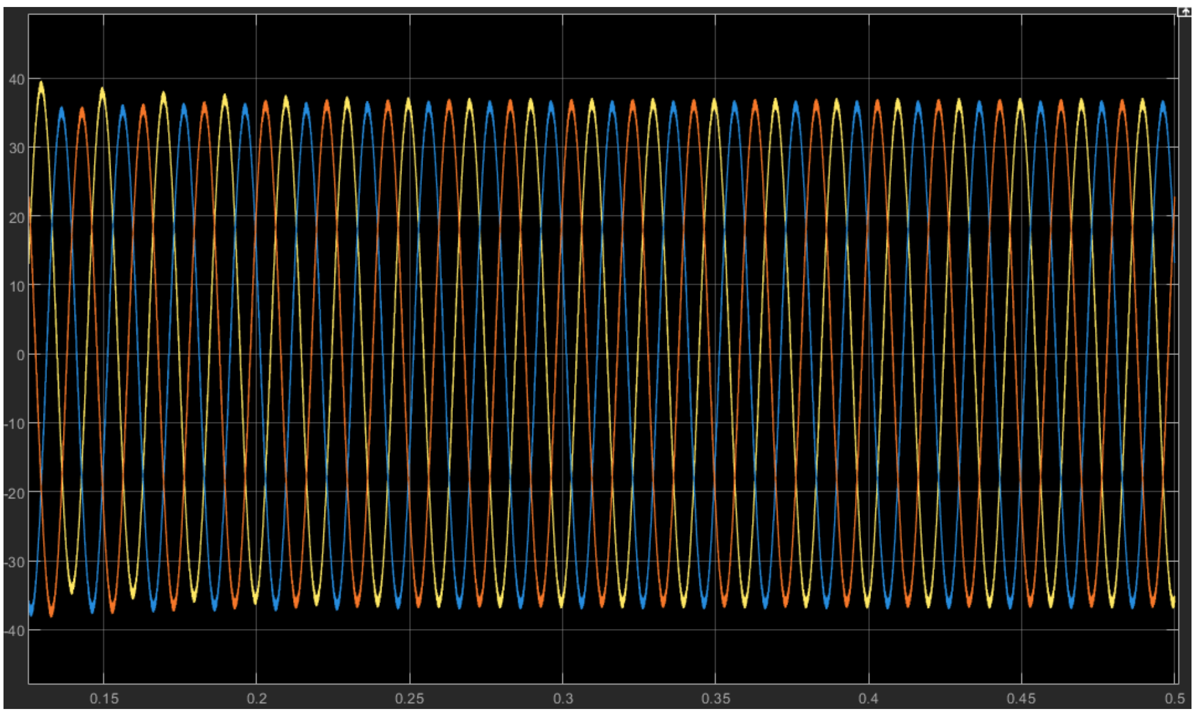


Figure 22. Grid Current

Since grid current increased to almost 40 amperes, while it was averaging around 0 when phase shift was 0, it's obvious that power is flowing from one side to the other!

Problem 3: Model1_Clarke_Park

Problem 3
Open the file 'Model1_Clarke_Park.slx' and proceed with the following tasks:

- Observe the Clarke transformation application in time domain ($\alpha\beta 0$)
- Observe the Park transformation application in time domain ($qd0$)
- Check that Clark is a Park transformation without rotation
- Apply an angle shift and observe the impact

Once again, starting from the basics and trying to understand simple sinusoidal waveform:

$$v(t) = V_{max}\cos(2\pi f t + \theta)$$

Phase shift is another measurement of the sine wave, and it indicates where the wave is in its cycle. It is measured in degrees (0°-360°) or radians (0-2π) and is denoted with the Greek symbol Phi (ϕ). Or in other words, a phase shift tells us how much the wave has been horizontally displaced from its standard starting point, which is usually at 0 degrees or 0 radians.

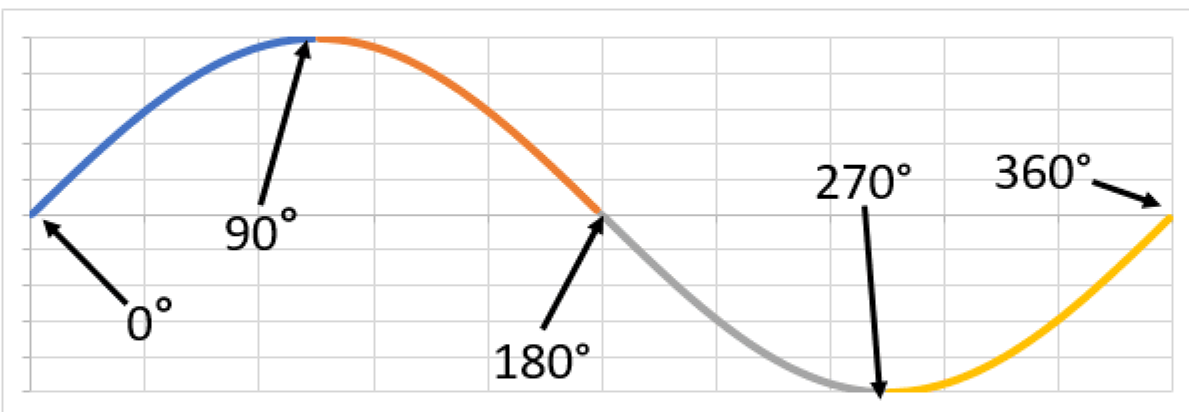


Figure 23. Different phases of a sine wave

A positive phase shift would shift the wave to the right, while a negative phase shift would shift it to the left. Two sine waves are out of phase when they are not at the same point in their cycle at the same time.

"90 degrees out of phase" means when one wave is at zero, the other will be at its peak. In other words, when the green wave is at 0° phase, the blue wave is at 90°. "180 degrees out of phase" means the zero points remain the same, but when one signal is at its peak, the other is at its trough. In other words, when the green wave is at 0° phase, the blue wave is at 180°.

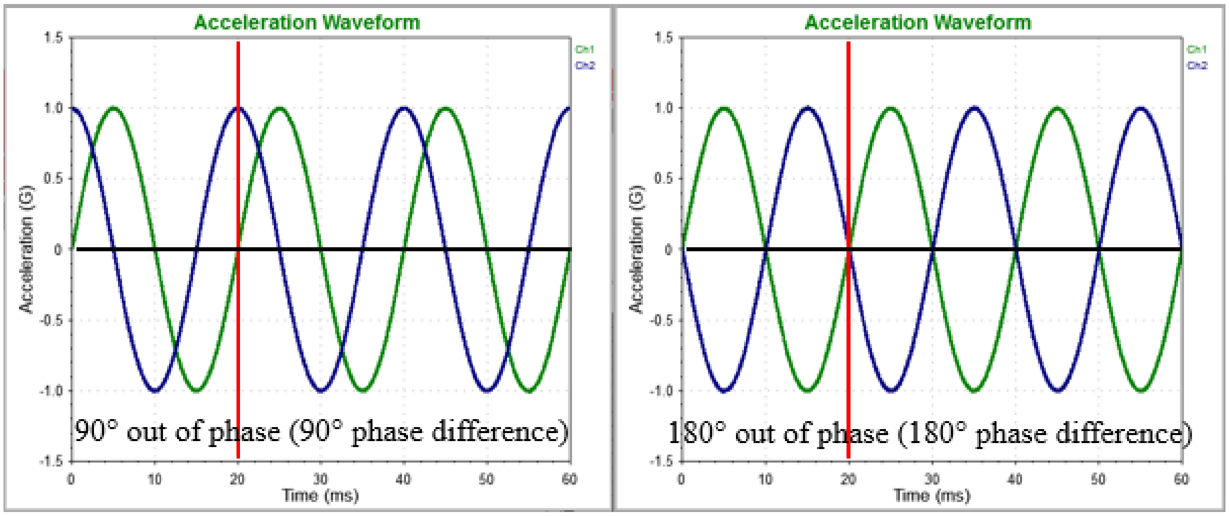
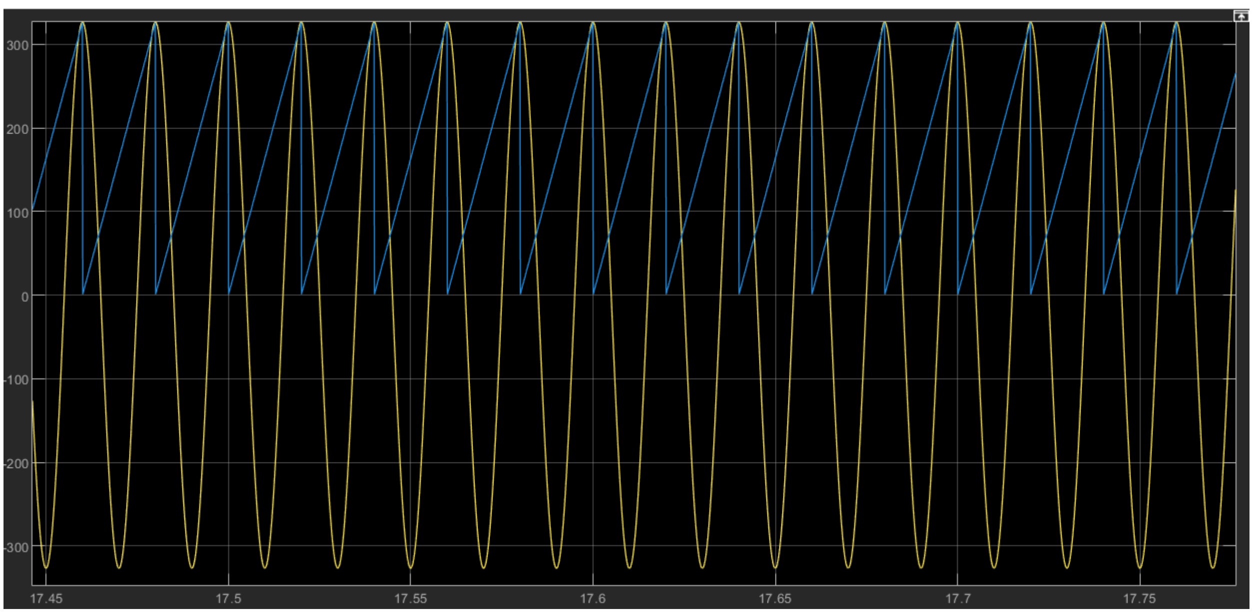


Figure 24. The phase difference between two sine waves. The left is a 90° phase difference; the right is a 180° difference.

As can be seen on our graph the phase angle will complete one full cycle ($360^{\circ} or\ 2\pi$ radians) in one period T, where $T = \frac{1}{f}$. As time increases, the phase angle increases from 0° to 360° and repeats.



Phase angle is important because it determines how much power is delivered from the source to the load in an AC circuit. Power is the product of voltage and current, but if they are not in sync, the power will be less than the maximum possible.

The phase angle increases as time progresses:

$$\theta(t) = 360 \cdot f \cdot t$$

If we use 50 Hz signal, the period T:

$$T = \frac{1}{f} = \frac{1}{50} = 0.02\ seconds$$

Now, the phase (grid) angle will increase over time:

At $t = 0\ seconds, \theta(0) = 0^{\circ}$

At $t = 0.01\ seconds\ (half\ period), \theta(0.01) = 180^{\circ}$

At $t = 0.02\ seconds\ (one\ full\ period), \theta(0.02) = 360^{\circ}$

Since time is infinite, and we want our grid angle to go from 0 to 360, we are trying to reset the phase angle back to zero after each period.

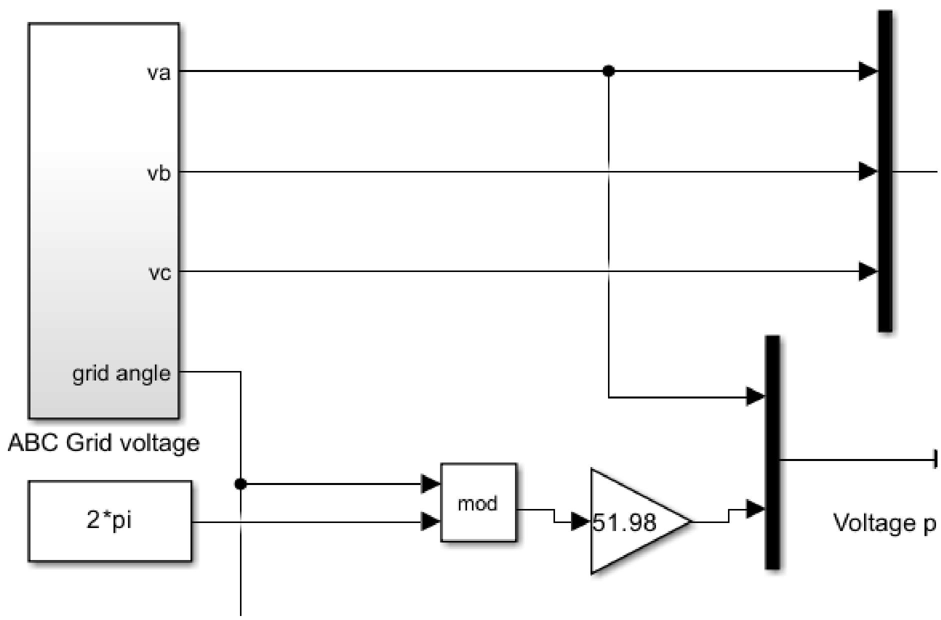


Figure 25. Approach used to keep phase angle from going beyond 360

To ensure that the phase angle stays within one cycle (0 to 2π radians or 0° to 360°), we can use the MOD function to "wrap" the angle after each full period T.

Question 1: Observe the Clarke transformation application in time domain ($\alpha\beta0$)

Answer 1:

Clarke Transformation:

From balanced three-phased system, using Clarke transformation we obtain two signals in time domain scaled to alpha-beta axis:

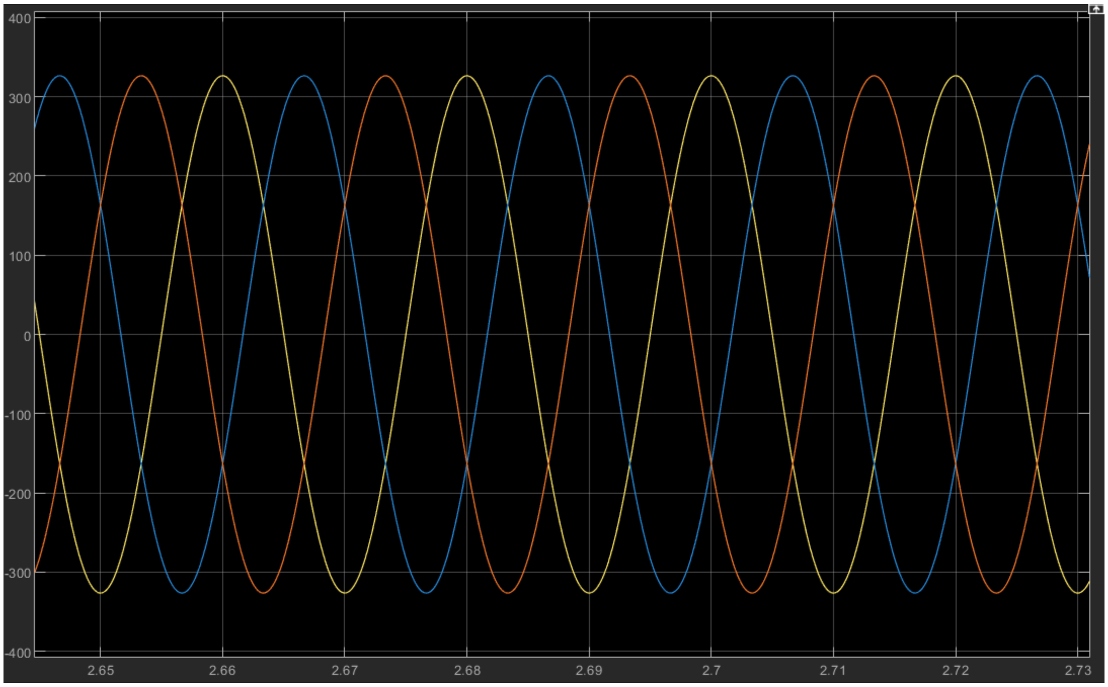


Figure 26. Three-Phased Voltages

Scaled to alpha-beta frame using Clarke transformation:

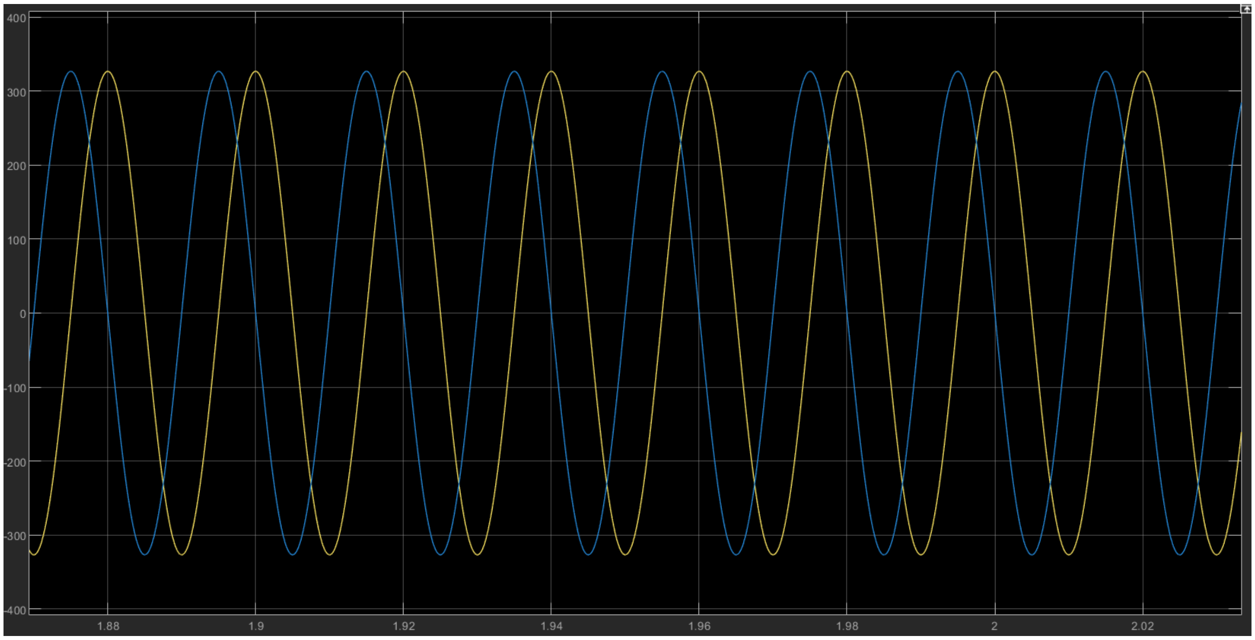


Figure 27. Clarke Transformation

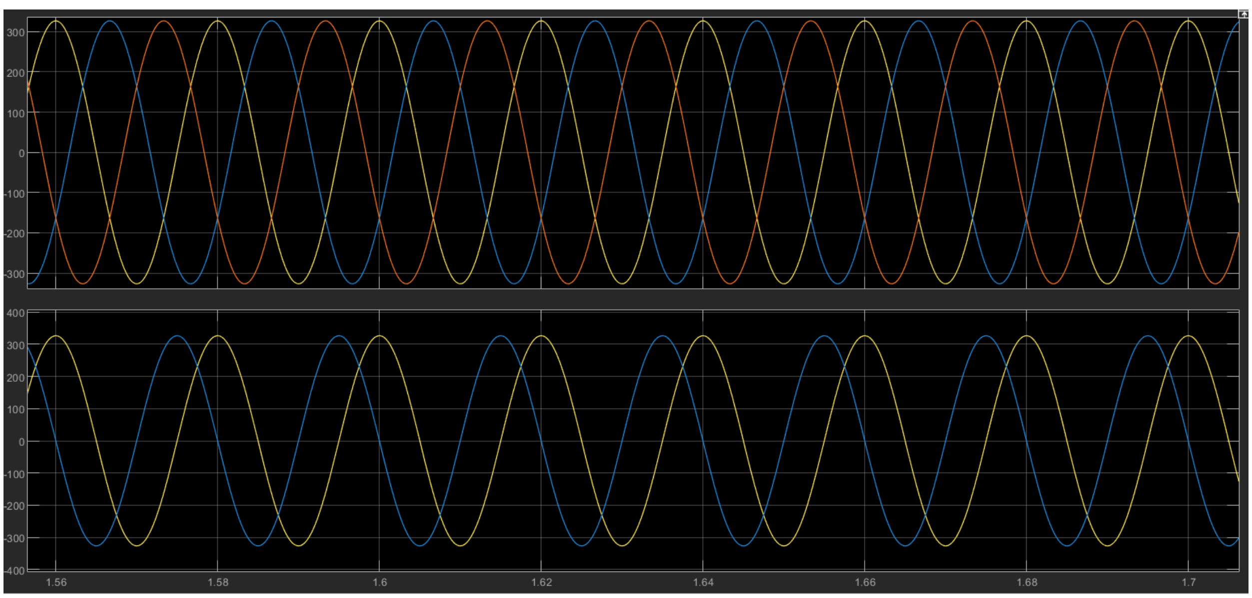


Figure 28. Same time values

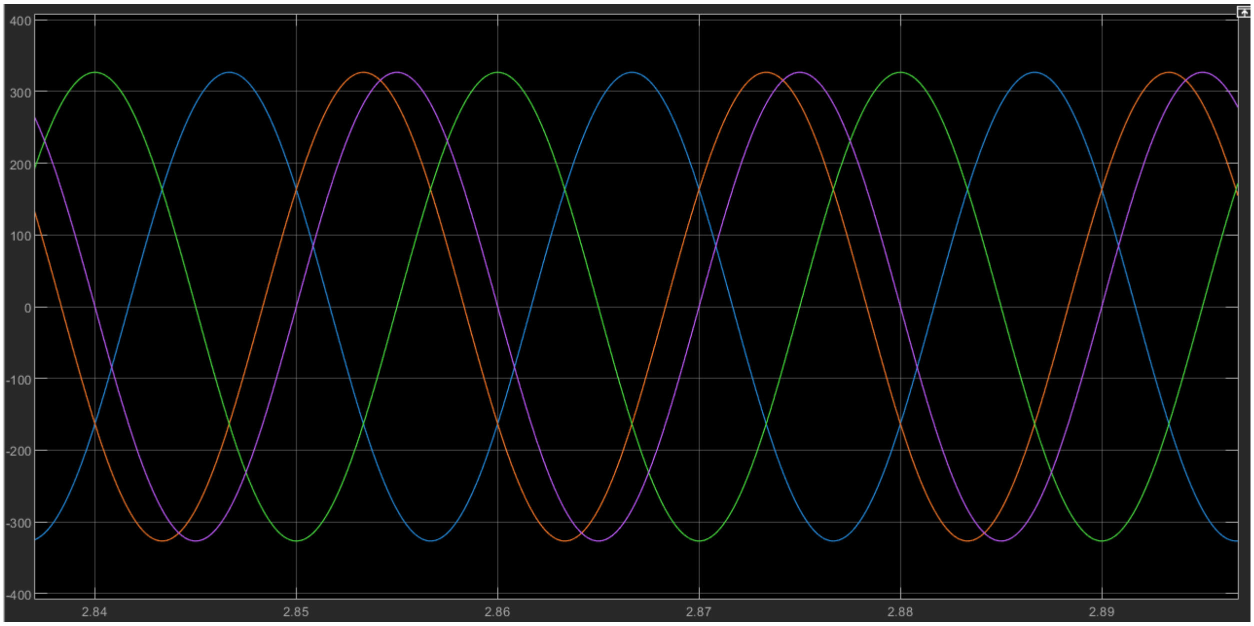


Figure 29. Three-phased ABC signals with alpha beta on same graph

Question 3: Check that Clark is a Park transformation without rotation

Answer 3:

The control in the αβ-frame reduces the number of control loops needed from three to two. However, the reference, feedback, and feed-forward signals are typically sinusoidal functions of time. Also, the design of the control compensator in αβ- frame is not an easy task. The dq-frame based control offers a solution to these problems because the signals are in DC waveforms under steady-state conditions. Which in return allows use of compensators with a simpler structure.

A dq-frame representation of a three-phase system is often more effective for analysis and control design. For instance, in the abc-frame, the equations for a salient-pole synchronous machine involve time-varying self- and mutual inductances. However, when the machine's equations are transformed into the appropriate dq-frame, these time-varying inductances become constant parameters.

For the space phasor $\vec{f} = f_\alpha + jf_\beta$, (the j in this case is added in order to tell us that $\alpha\ and\ \beta$, frames are at 90 degrees angle), the $\alpha\beta\ to\ dq -$ frame transformation is defined as:

$$f_d + jf_q = (f_\alpha + jf_\beta)e^{-j\varepsilon(t)}$$

Which is equivalent to a phase shift of $-\varepsilon(t)$ of the space phasor.

If we want to go the other way, from $dq\ to\ \alpha\beta$- frame we multiply both sides of the formula above with $e^{j\varepsilon(t)}$.

$$(f_d + jf_q)e^{j\varepsilon(t)} = (f_\alpha + jf_\beta)e^{-j\varepsilon(t)}e^{j\varepsilon(t)}$$

We obtain:

$$f_\alpha + jf_\beta = (f_d + jf_q)e^{j\varepsilon(t)}$$

Space phasor can be considered in the following form:

$$\vec{f}(t) = f_\alpha + jf_\beta = f_m e^{j(\theta_0 + \int w(t)dt)}$$

Space phasor is rotating with varying speed $w(t)$ and with θ_0 as the initial phase angle of the three-phase signal.

Now going back to Park transformation, in order to highlight its most useful feature, if $\varepsilon(t)$ is chosen as:

$$\varepsilon(t) = \varepsilon_0 + \int w(t)dt$$

Then based on:

$$f_d + jf_q = (f_\alpha + jf_\beta)e^{-j\varepsilon(t)}$$

We obtain:

$$f_d + jf_q = (f_m e^{j(\theta_0 + \int w(t)dt)})e^{-j(\varepsilon_0 + \int w(t)dt)}$$

$$f_d + jf_q = f_m e^{j(\theta_0 - \varepsilon_0)}$$

WHICH IS STATIONARY! Therefore, the three-phased signal from the beginning became DC signals.

θ_0 and ε_0 are not necessarily equal, but it is advisable for control and in that purpose PLL is used.

Concluding, we can consider our space phasor $\vec{f}$ represented with f_d and f_q in a coordinate system that is rotated by $\varepsilon(t)$ with respect to the $\alpha\beta$ – frame. This rotated coordinate system is referred to as dq – frame or *rotating reference frame*.

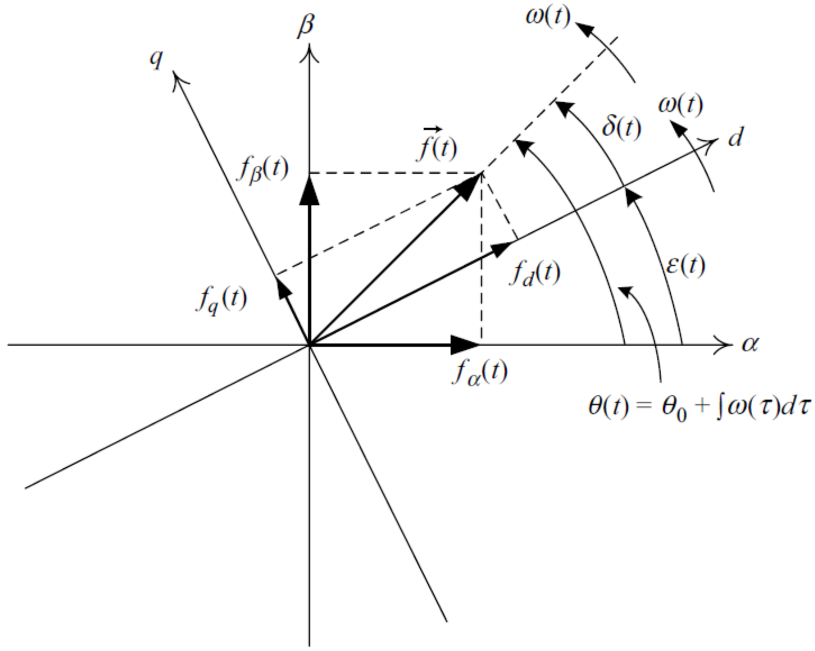


Figure 30. Rotating reference frame

Again, based on Euler's identity discussed at the beginning of the report:

$$\begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix} = \mathbf{R}\left[\varepsilon(t)\right] \begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \end{bmatrix}$$

Where,

$$\mathbf{R}\left[\varepsilon(t)\right] = \begin{bmatrix} \cos\varepsilon(t) & \sin\varepsilon(t) \\ -\sin\varepsilon(t) & \cos\varepsilon(t) \end{bmatrix}$$

In the same way, the dq to $\alpha\beta$ frame transformation:

$$\begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \end{bmatrix} = \mathbf{R}^{-1}\left[\varepsilon(t)\right] \begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix}$$
$$= \mathbf{R}\left[-\varepsilon(t)\right] \begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix}$$

Where,

$$\mathbf{R}^{-1}\left[\varepsilon(t)\right] = \mathbf{R}\left[-\varepsilon(t)\right] = \begin{bmatrix} \cos\varepsilon(t) & -\sin\varepsilon(t) \\ \sin\varepsilon(t) & \cos\varepsilon(t) \end{bmatrix}$$

Verified,

$$\mathbf{R}^{-1}\left[\varepsilon(t)\right] = \mathbf{R}^{T}\left[\varepsilon(t)\right]$$

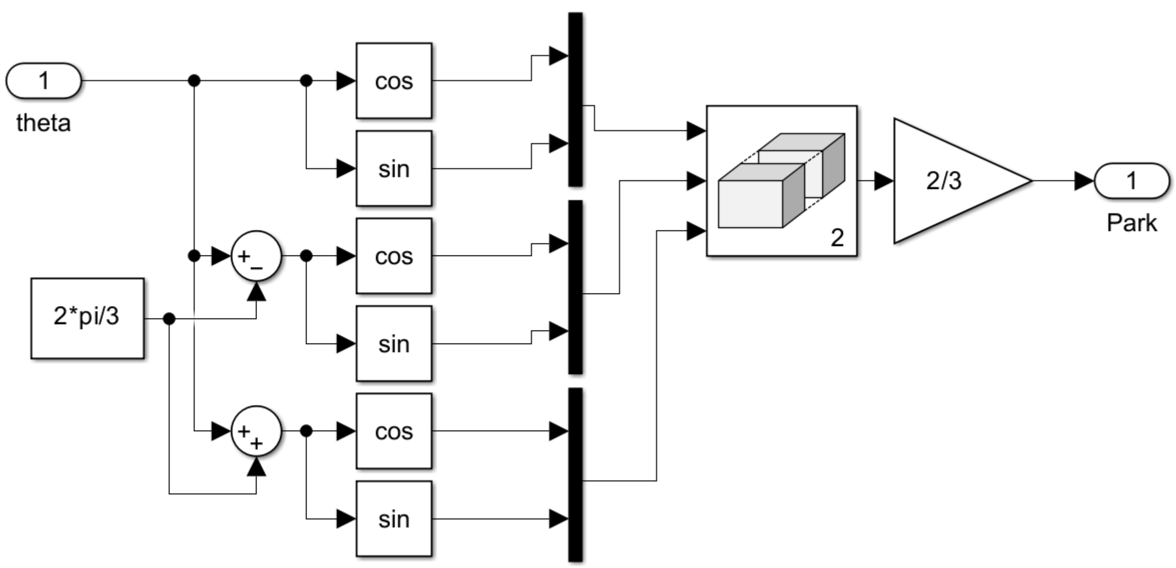
A direct transformation from *abc*-frame to the *dq*-frame

$$\begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix} = \frac{2}{3}\mathbf{T}[\varepsilon(t)] \begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix}$$

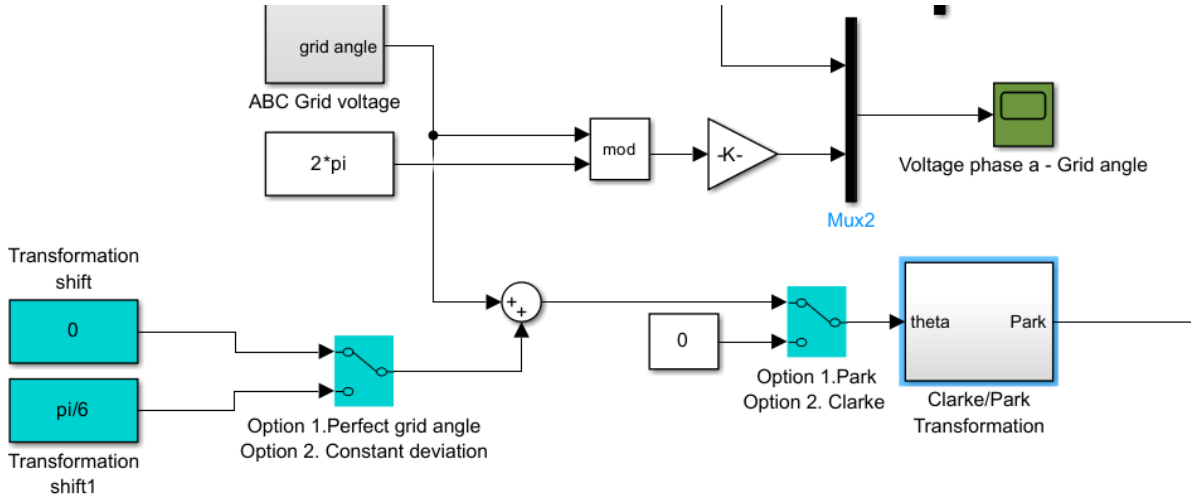
Where,

$$\mathbf{T}[\varepsilon(t)] = \mathbf{R}[\varepsilon(t)]\mathbf{C} = \begin{bmatrix} \cos\left[\varepsilon(t)\right] & \cos\left[\varepsilon(t) - \frac{2\pi}{3}\right] & \cos\left[\varepsilon(t) - \frac{4\pi}{3}\right] \\ \sin\left[\varepsilon(t)\right] & \sin\left[\varepsilon(t) - \frac{2\pi}{3}\right] & \sin\left[\varepsilon(t) - \frac{4\pi}{3}\right] \end{bmatrix}$$

In our SIMULINK model, the above matrix is achieved by following:



If theta is qual to zero, we have a Clarke transformation, and not a rotating reference frame.



But if we measure the grid angle, and rotate the $\alpha\beta$ with that angle (plus angle shift θ_0) we get a rotating reference frame or PARK transformation.

Since $\varepsilon(t)$ is equal to a grid angle we only have d component of the signal and q is equal to 0. The reasons behind this will be explained in PLL part of the problem.

Question 2: Observe the Park transformation application in time domain ($qd0$)

Answer 2:

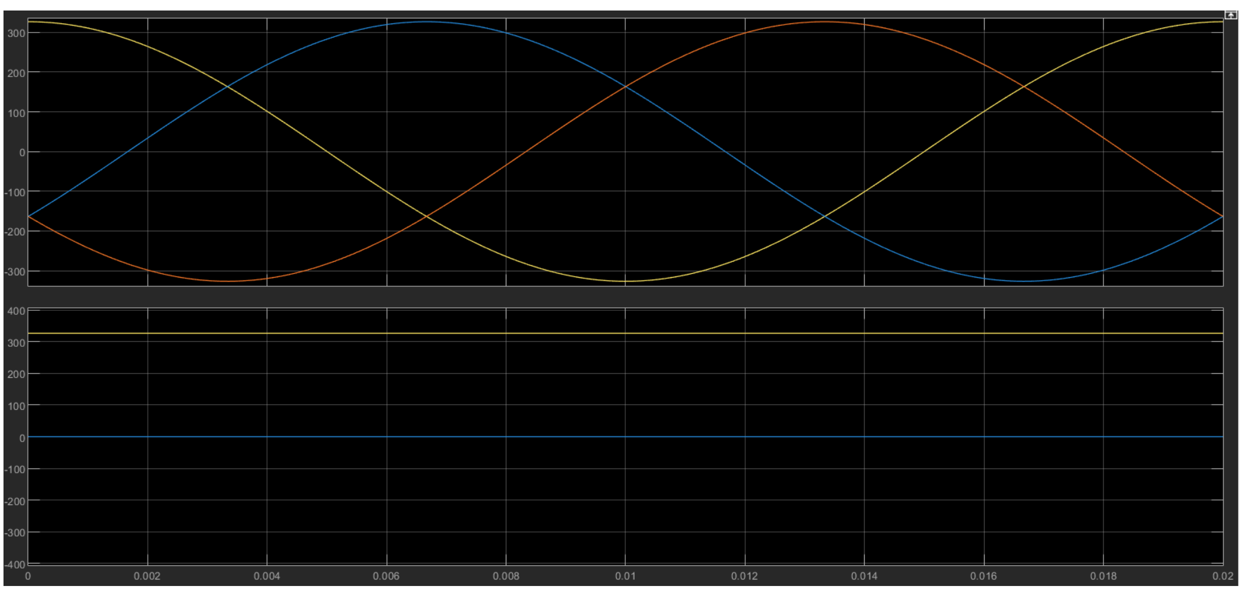


Figure 31. Three-phased signal converted to two d and q signals

Question 4: Apply an angle shift and observe the impact

Answer 4:

But if we add phase shift to the grid angle, the phasor is not aligned with d axis (90 degrees with q axis), so we are able to scale phasor to both axis, and q component of the signal is not 0!

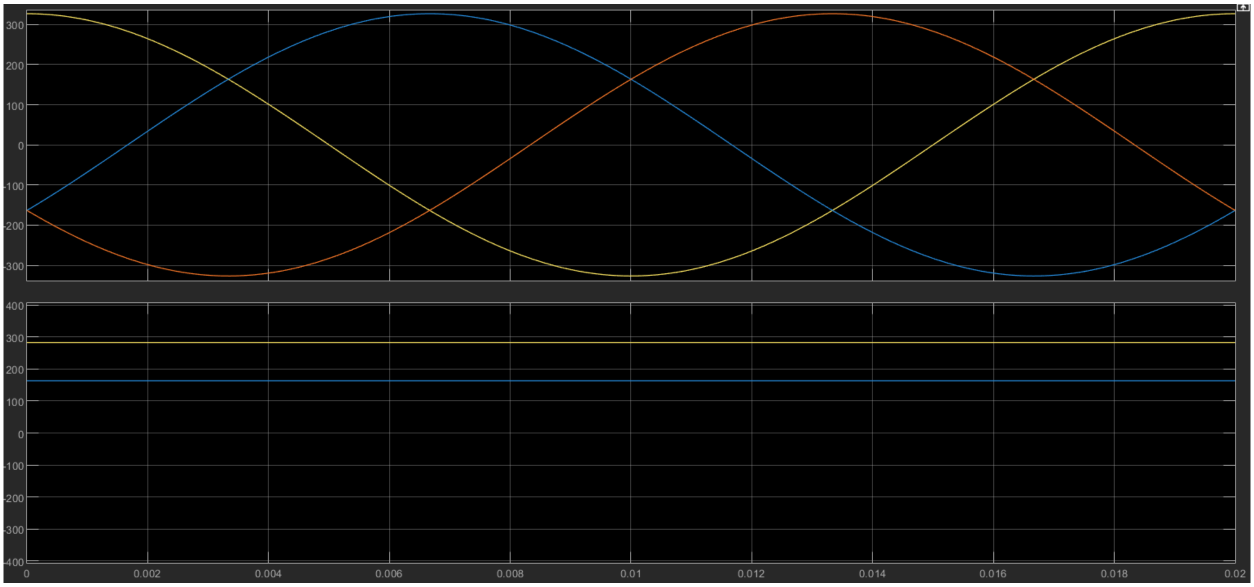


Figure 32. Case when Park transformation angle is different from grid angle

Problem 4: Model2_PLL

Problem 4
Open the file 'Model2_PLL.slx' and proceed with the following tasks:

- Change the phase and amplitude of the grid voltage
- Apply a phase jump to the *abc* voltage generator and observe how the PLL is tracking the angle
- Apply a phase jump changing the PI controller parameters
- Change the amplitude of one of the phases individually and observe the impact on the $qd0$ magnitudes

Question 1: Change the phase and amplitude of the grid voltage

Answer 1:

Unlike the ABC-frame control, the $\alpha\beta$-frame control in a grid-connected VSC system simplifies the control by reducing the number of control loops from three to two. Additionally, in the $\alpha\beta$-frame, it becomes possible to instantly decouple and control the real and reactive power exchanged between the VSC system and the AC grid. However, the control variables and control signals are sinusoidal functions of time.

On the other hand, the dq-frame control of a grid-connected VSC system offers all the benefits of the $\alpha\beta$-frame control, along with the added advantage that, in steady state, the control variables become DC quantities.

In contrast to the $\alpha\beta$-frame control, dq-frame control needs a synchronization mechanism, typically provided by the phase-locked loop (PLL). This requirement can be seen as a drawback of the dq-frame control.

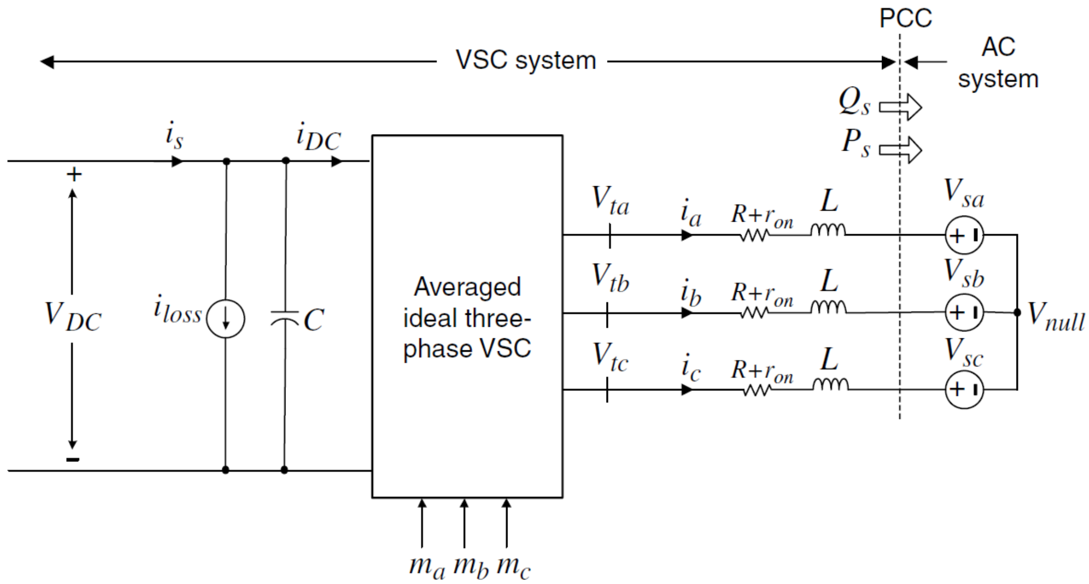


Figure 33. Schematic diagram of a grid-imposed frequency VSC system

VSC is modeled as a lossless power processor, which includes an equivalent *DC*-bus capacitor, a current source that represents the *VSC's* switching power loss, and series on-state resistances at the *AC* side to account for the *VSC's* conduction power loss. The DC side of the *VSC* can be connected to either a *DC* voltage source or a DC power source. Each phase of the *VSC* connects to the AC system through a series *RL* branch.

We consider an "*infinitely stiff*" AC system (as an approximation), which refers to an idealized concept in electrical engineering where the AC system is assumed to have an unlimited capacity to maintain its voltage and frequency, regardless of the amount of power injected or withdrawn. Or in other words, the voltage and frequency of the system do not fluctuate or vary, no matter what load or generation changes occur.

Therefore, the AC system is represented by an ideal three-phase voltage source, V_{abc}, which is assumed to be balanced, sinusoidal, and operate at a constant frequency. The *VSC* system, as shown, exchanges both real and reactive power with the AC system.

The grid-imposed frequency *VSC* system can be used as a real or reactive power controller. In this setup, the VSC's DC side is connected in parallel with a DC voltage source, and the goal is to control the instantaneous real and reactive power exchanged between the VSC system and the AC system, that is $P_s(t) and\ Q_s(t)$.

Control is performed in *dq*- frame. Using the knowledge explained in the last problem, we express space phasor:

$$\vec{f}(t) = f_\alpha + j f_\beta$$

The $\alpha\beta$ to *dq,*

$$f_d + jf_q = \vec{f}(t)e^{-\varepsilon(t)} = (f_\alpha + jf_\beta)e^{-\varepsilon(t)}$$

The angle $\varepsilon(t)$ can be chosen freely. But, if the $\vec{f}(t) = f_m e^{j(wt+\theta_0)}$, then choosing $\varepsilon(t) = wt$, or in other words we rotate the dq frame by speed of the space phasor, we obtain,

$$f_d + jf_q = f_m e^{j(wt+\theta_0)} e^{-jwt} = f_m e^{j\theta_0}$$

Which is not time varying, therefore f_d, f_q are DC components.

The inverse transformation,

$$\vec{f}(t) = f_\alpha + jf_\beta = (f_d + jf_q)e^{j\varepsilon(t)} = f_m e^{j\theta_0} e^{jwt} = f_m e^{j(wt+\theta_0)}$$

In our SIMULINK model our grid voltages are represented like,

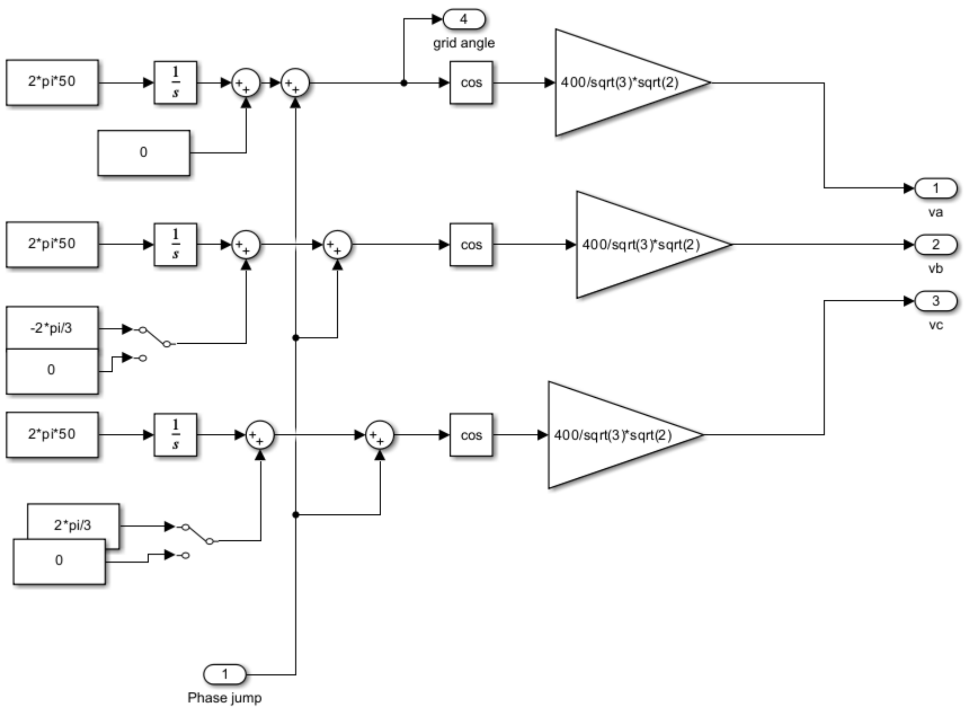


Figure 34. Grid Voltages

AC system voltages:

$$v_a(t) = V_m \cos(w_o t + \theta_0)$$

$$v_b(t) = V_m \cos\left(w_o t + \theta_0 - \frac{2\pi}{3}\right)$$

$$v_c(t) = V_m \cos\left(w_o t + \theta_0 - \frac{4\pi}{3}\right)$$

Where, $V_m = \frac{\sqrt{2}}{\sqrt{3}}400 - peak\ value\ line\ to\ neutral$, and $\theta_0 = \left[0\ or\ \frac{\pi}{9}\right] - initial\ phase\ shift.$

Space phasor of these three voltages is:

$$\overrightarrow{V_{abc}}(t) = V_m e^{j(w_o t + \theta_0)}$$

Based on schematic diagram of a grid-imposed frequency VSC system, using second Kirchoff's law,

$$L\frac{d\vec{\imath}}{dt} = -(R + r_{on})\vec{\imath} + \overrightarrow{V_t} - \overrightarrow{V_{abc}}$$

Substituting,

$$L\frac{d\vec{\imath}}{dt} = -(R + r_{on})\vec{\imath} + \overrightarrow{V_t} - V_m e^{j(w_o t + \theta_0)}$$

Then we use the inverse version of Park transformation,

$$\vec{f}(t) = f_\alpha + jf_\beta = (f_d + jf_q)e^{j\varepsilon(t)}$$

to express in *dq* frame,

$$\vec{\imath} = i_{dq}e^{j\varepsilon}$$

$$\overrightarrow{V_t} = V_{tdq}e^{j\varepsilon}$$

We deduce,

$$L\frac{d}{dt}(i_{dq}e^{j\varepsilon}) = -(R + r_{on})i_{dq}e^{j\varepsilon} + V_{tdq}e^{j\varepsilon} - V_m e^{j(w_o t + \theta_0)}$$

We integrate,

$$L(\frac{di_{dq}}{dt}e^{j\varepsilon} + ji_{dq}e^{j\varepsilon}\frac{d\varepsilon}{dt}) = -(R + r_{on})i_{dq}e^{j\varepsilon} + V_{tdq}e^{j\varepsilon} - V_m e^{j(w_o t + \theta_0)}$$

Then,

$$Le^{j\varepsilon}(\frac{di_{dq}}{dt} + ji_{dq}\frac{d\varepsilon}{dt}) = -(R + r_{on})i_{dq}e^{j\varepsilon} + V_{tdq}e^{j\varepsilon} - V_m e^{j(w_o t + \theta_0)}$$

We divide the right side by $e^{j\varepsilon}$,

$$L\left(\frac{di_{dq}}{dt}\right) = -ji_{dq}\frac{d\varepsilon}{dt}L - (R + r_{on})i_{dq} + V_{tdq} - V_m e^{j(w_o t + \theta_0 - \varepsilon)}$$

Decomposing into real and imaginary components,

$$L\left(\frac{di_d}{dt}\right) = +i_q\frac{d\varepsilon}{dt}L - (R + r_{on})i_d + V_{td} - V_m\cos(w_o t + \theta_0 - \varepsilon)$$

$$L\left(\frac{di_q}{dt}\right) = -i_d\frac{d\varepsilon}{dt}L - (R + r_{on})i_q + V_{tq} - V_m\sin(w_o t + \theta_0 - \varepsilon)$$

When decomposing into real part we need to pick ji_q(imaginary component of current), because we already have j in the starting equation.

Then we introduce,

$$\frac{d\varepsilon}{dt} = w(t)$$

This produces,

$$L\left(\frac{di_d}{dt}\right) = +i_q w(t)L - (R + r_{on})i_d + V_{td} - V_m \cos(w_o t + \theta_0 - \varepsilon)$$

$$L\left(\frac{di_q}{dt}\right) = -i_d w(t)L - (R + r_{on})i_q + V_{tq} - V_m \sin(w_o t + \theta_0 - \varepsilon)$$

$$\frac{d\varepsilon}{dt} = w(t)$$

Where,

State variables: i_d, i_q, ε

Control inputs: V_{td}, V_{tq}, w

The system is nonlinear.

USEFULNESS OF THE *dq* – FRAME DEPENDS ON THE PROPER SELECTION OF w and ε.

If we choose $w = w_o \; and \; \varepsilon = w_o t + \theta_0$ and put it in the above equations we obtain:

$$L\left(\frac{di_d}{dt}\right) = +i_q w_o L - (R + r_{on})i_d + V_{td} - V_m \cos(w_o t + \theta_0 - w_o t - \theta_0)$$

$$L\left(\frac{di_q}{dt}\right) = -i_d w_o L - (R + r_{on})i_q + V_{tq} - V_m \sin(w_o t + \theta_0 - w_o t - \theta_0)$$

Next,

$$L\left(\frac{di_d}{dt}\right) = +i_q w_o L - (R + r_{on})i_d + V_{td} - V_m \cos(0)$$

$$L\left(\frac{di_q}{dt}\right) = -i_d w_o L - (R + r_{on})i_q + V_{tq} - V_m \sin(0)$$

We obtain,

$$L\left(\frac{di_d}{dt}\right) = +i_q w_o L - (R + r_{on})i_d + V_{td} - V_m$$

$$L\left(\frac{di_q}{dt}\right) = -i_d w_o L - (R + r_{on})i_q + V_{tq}$$

The mechanism that ensures that $\varepsilon = w_o t + \theta_0$ is called PHASE LOCKED LOOP or PLL.

This mechanism ensures that $V_{abcq} = 0$. (We don't have q component). Also $V_{abcd} = V_m$.

Which can be achieved by the following feedback law:

$$w(t) = H(p)V_{sq}(t)$$

Where, $H(p)$ is a linear transfer function (compensator), while p is differentiation operator.

Substituting,

$$\frac{d\varepsilon}{dt} = H(p)V_m \sin(w_o t + \theta_0 - \varepsilon)$$

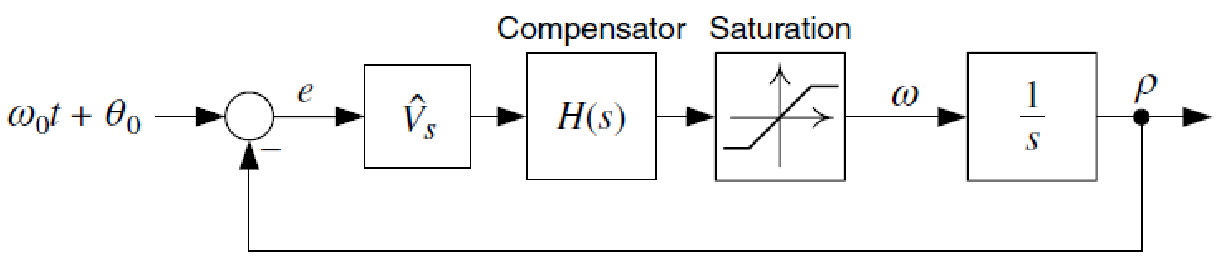
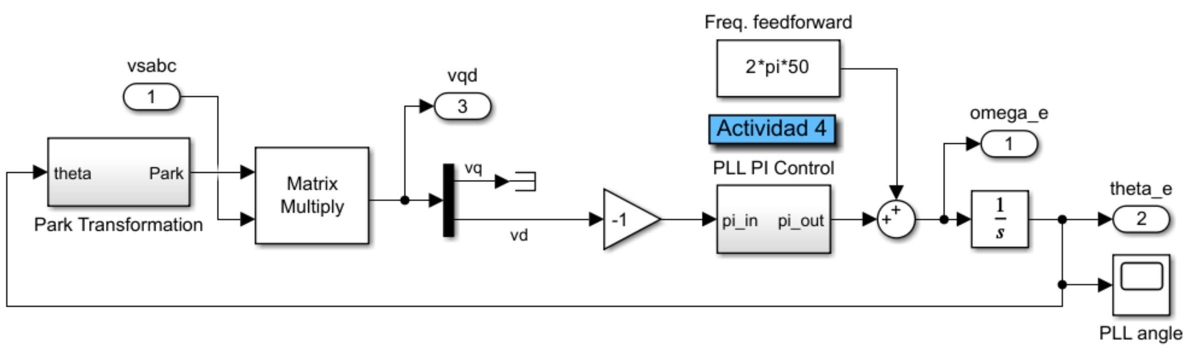


Figure 35. Control diagram of PLL

In our SIMULINK model,



Question 2: Apply a phase jump to the abc voltage generator and observe how the PLL is tracking the angle

Answer 2:

We can notice that PLL angle and grid angle are align perfectly:

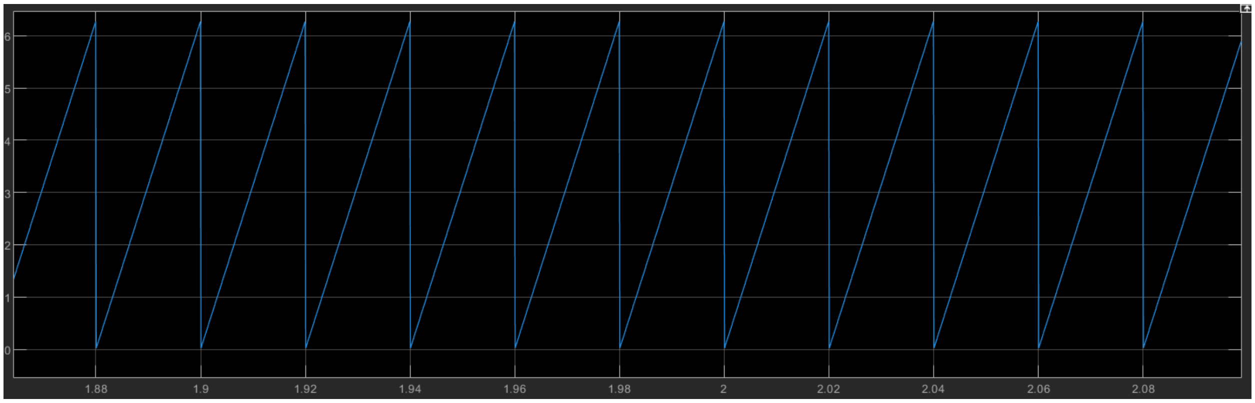
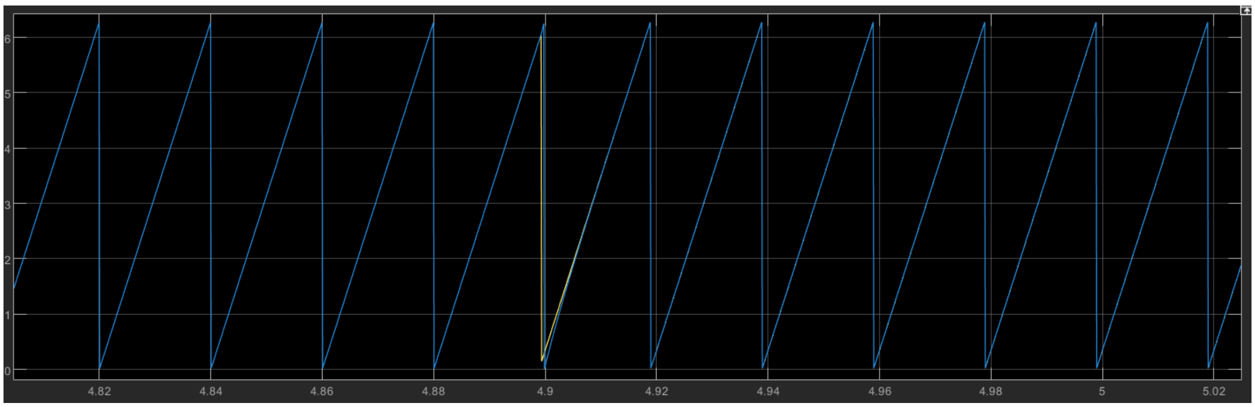
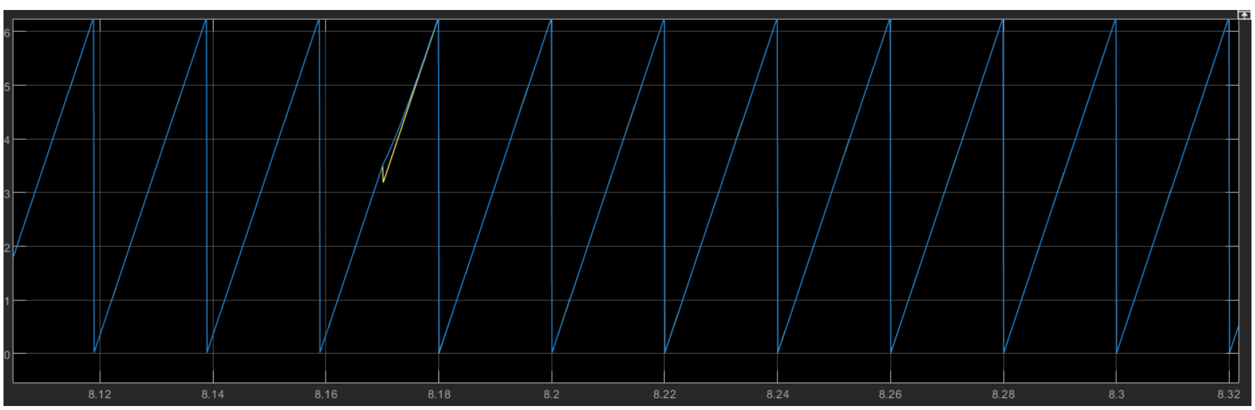


Figure 36. Grid angle and PLL angle on the same graph

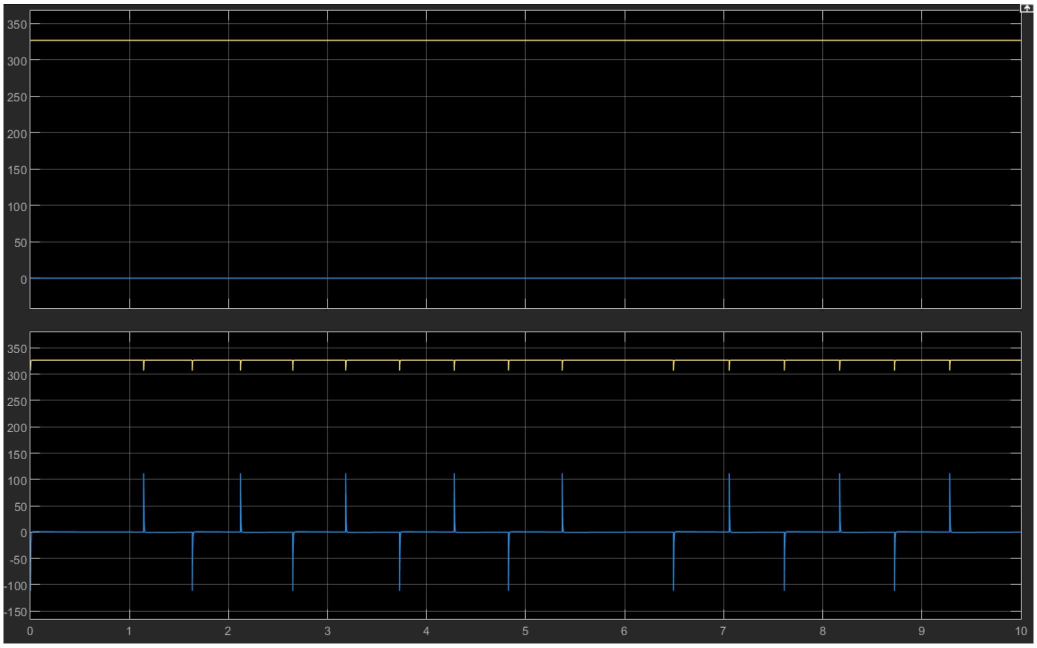
But if we suddenly add a phase shift to the grid voltage, we can observe the deviation:



This process is extremely fast:



On the next graph we can compare ideal tracking with PLL tracking:

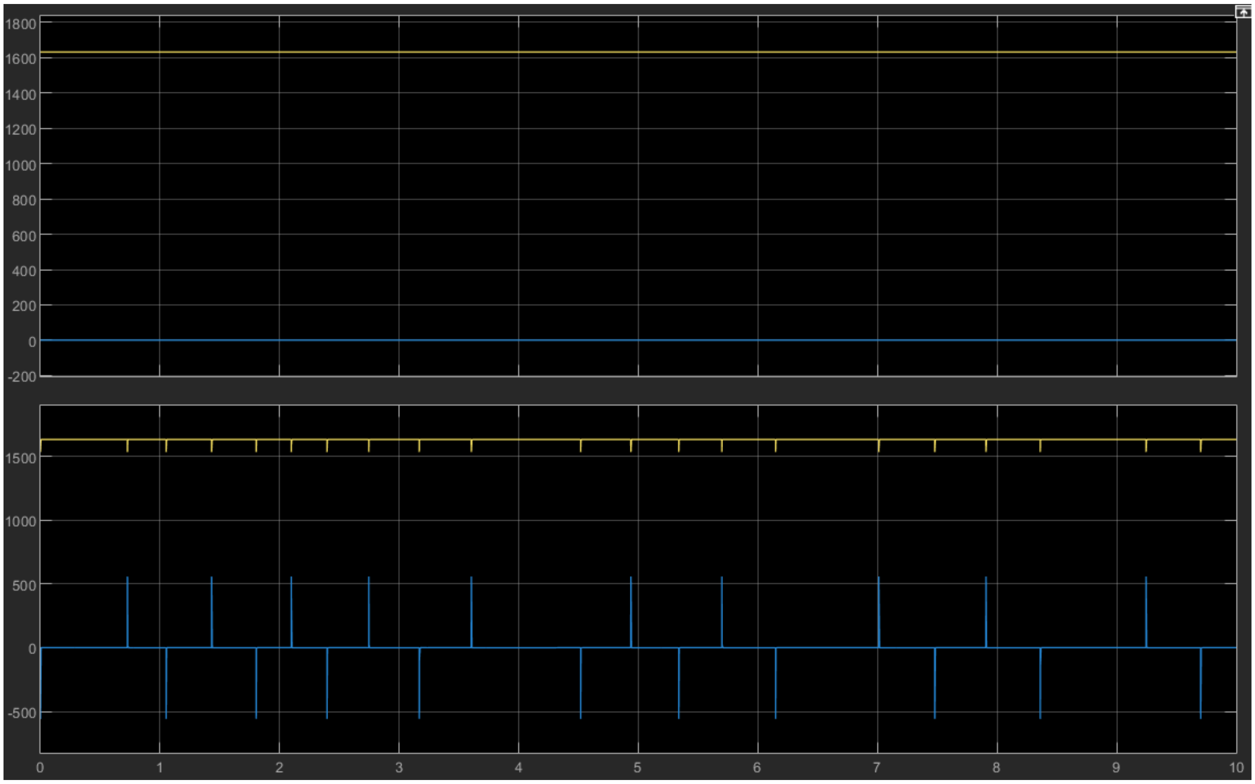


As can be noticed with ideal tracking (when angle of dq frame rotation is same as grid angle) dq component is always 0. But in real world applications when we add phase shift, we can decompose space phasor on d and q components. As can be seen PLL is tracking the new angle, in order for q component to always be zero. This process is extremely fast.

Question 1: Change the phase and amplitude of the grid voltage

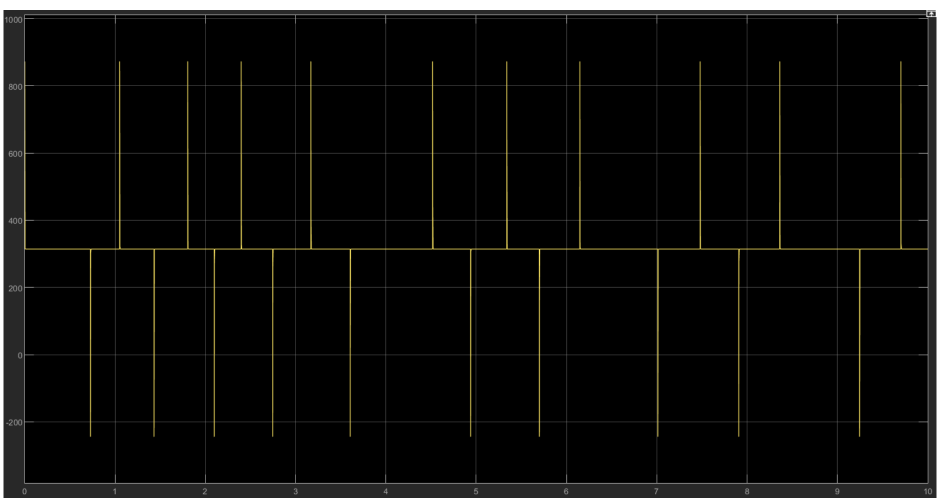
Answer 1:

When amplitude of the grid voltage is multiplied by 5:



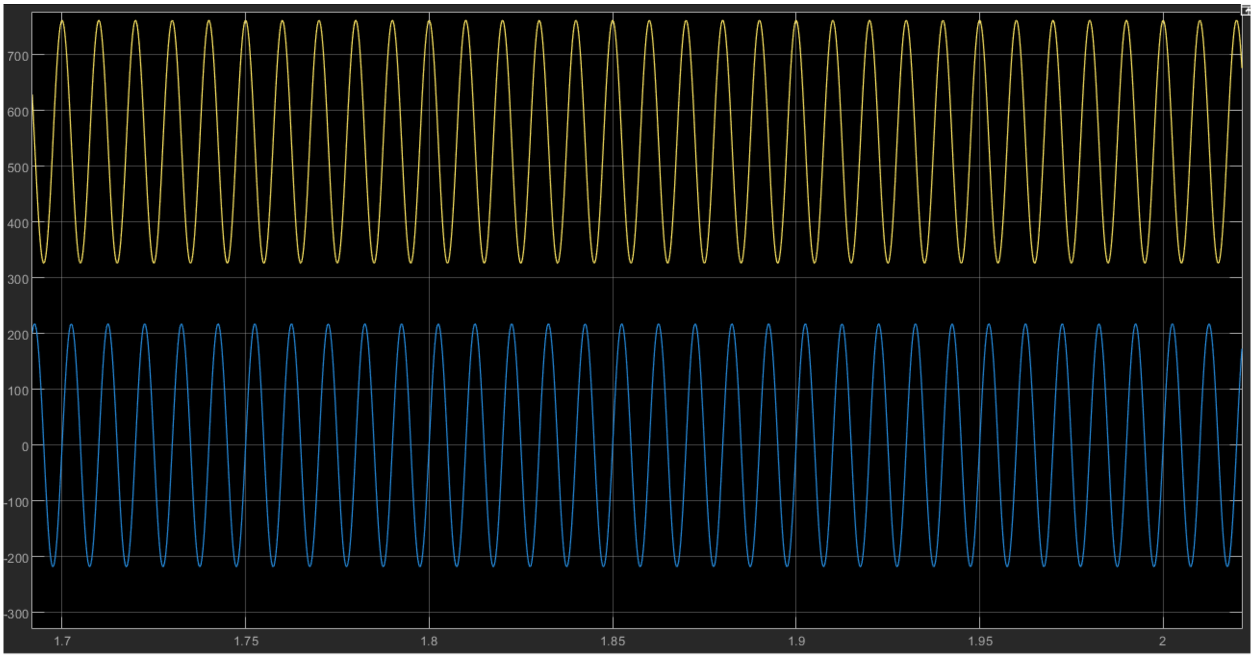
We can also notice that d component is also 5 times higher.

When we change phase shift, we can notice instantaneous change in speed, because the grid angle is trying to “catch”:



Question 4: Change the amplitude of one of the phases individually and observe the impact on the $qd0$ magnitudes

Answer 4:



Problem 5: Model3_CurrentLoop

Problem 4
Open the file 'Model3_CurrentLoop.slx' and proceed with the following tasks:

- Revise how the current control operates
- Change the current step changes in qd
- Change the parameters of the current loop (PI controllers)
- Disconnect the decoupling and observe the impact on the currents

Question 1: Revise how the current control operates

Answer 1:

Current-Mode and Voltage-Mode Control of P and Q

There are two main methods for controlling P_s *and* Q_s in the VSC system. The first approach is known as voltage-mode control.

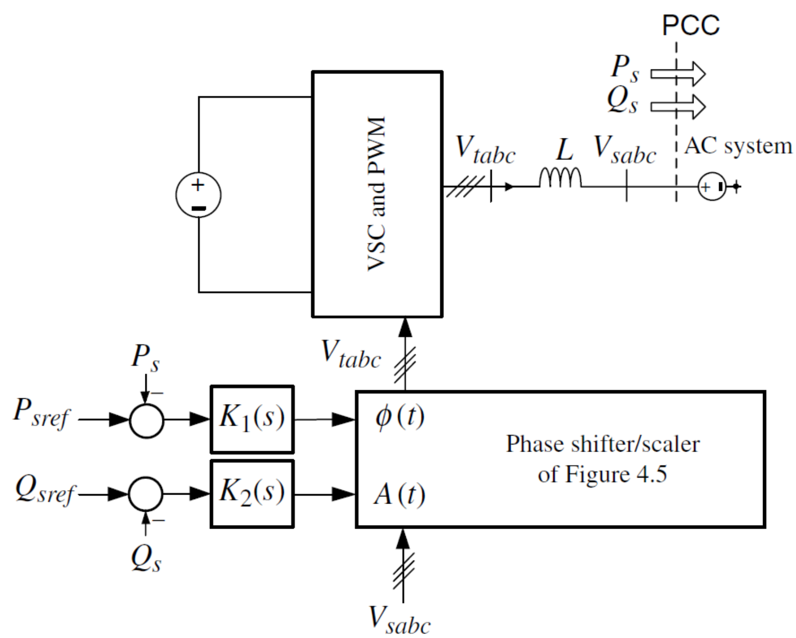


Figure 37. Schematic diagram of a voltage-controlled real/reactive power controller

The figure above shows that in a voltage-controlled VSC system, the active and reactive power are controlled by the phase angle and amplitude of the VSC's AC-side terminal voltage, relative to the grid (PCC) voltage.

Voltage-mode control is straightforward and involves low number of control loops. However, its main drawback is the lack of a control loop for the VSC line current. As a result, the VSC is not protected against overcurrents, and the current can experience large fluctuations if power commands are changed quickly or if faults occur in the AC system.

The second approach of control of active and reactive power in VSC system is referred to as *current-mode control*. In this method, the VSC line current is tightly regulated using a dedicated current-control scheme, through the VSC's AC-side terminal voltage. Real and reactive power are then controlled by the phase angle and amplitude of the VSC line current relative to the PCC voltage. Because of the current regulation, the VSC is protected from overcurrent conditions. Other benefits of current-mode control include increased robustness against changes in the VSC and AC system parameters, better dynamic performance, and improved control accuracy.

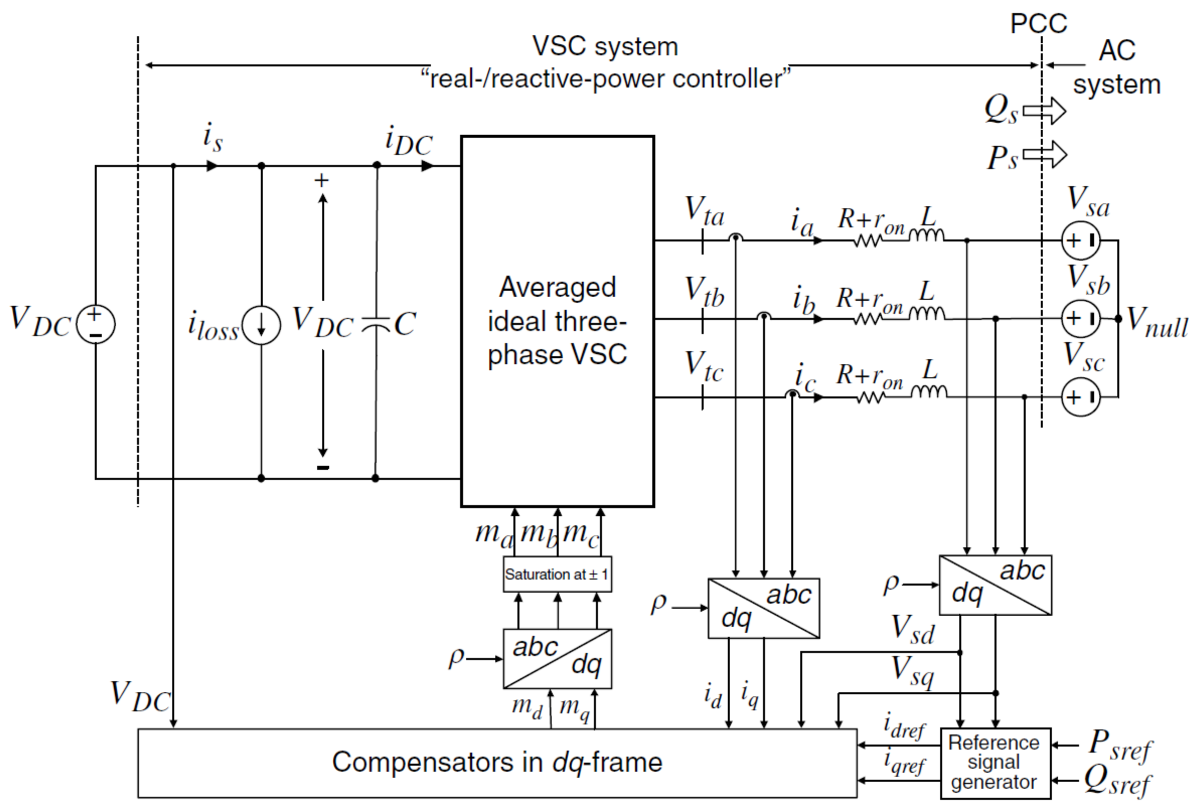


Figure 38. Schematic diagram of a current-controlled real/reactive power controller in dq frame

The real and reactive power delivered to the AC system:

$$P_s(t) = \frac{3}{2}[V_{abcd}(t)i_d(t) + V_{abcq}(t)i_q(t)]$$

$$Q_s(t) = \frac{3}{2}[-V_{abcd}(t)i_q(t) + V_{abcq}(t)i_d(t)]$$

Where, V_{abcd} *and* V_{abcq}, are the AC system dq-frame voltage components and CAN NOT be controlled by VSC system. As described above, if the PLL angle is same as grid angle, $V_{abcq} = 0$. Equations can be rewritten as:

$$P_s(t) = \frac{3}{2} V_{abcd}(t) i_d(t)$$

$$Q_s(t) = -\frac{3}{2} V_{abcd}(t) i_q(t)$$

Therefore, P_s and Q_s can be controlled with i_d and i_q respectively.

We can introduce,

$$i_{dref}(t) = \frac{2}{3V_{sd}} P_{sref}(t)$$

$$i_{qref}(t) = \frac{2}{3V_{sd}} Q_{sref}(t)$$

If the control system can provide fast reference tracking, that is:

$$i_d \approx i_{dref} \; and \; i_q \approx i_{qref}$$

Then,

$$P_s \approx P_{sref} \; and \; Q_s \approx Q_{sref}$$

That means that $P_s(t) and\ Q_s(t)$ can be INDEPENDENTLY CONTROLLED by their respective reference controls.

In the steady state, since V_{abcd} is a DC variable, both i_{dref} and i_{qref} will also be DC variables, assuming that P_{sref} and Q_{sref} are constant signals. Therefore, as anticipated, the control system in the *dq*-frame operates with DC variables, in contrast to the control system in the $\alpha\beta$-frame, which handles sinusoidal signals.

Question 1: Change the current step changes in *qd*

Answer 1:

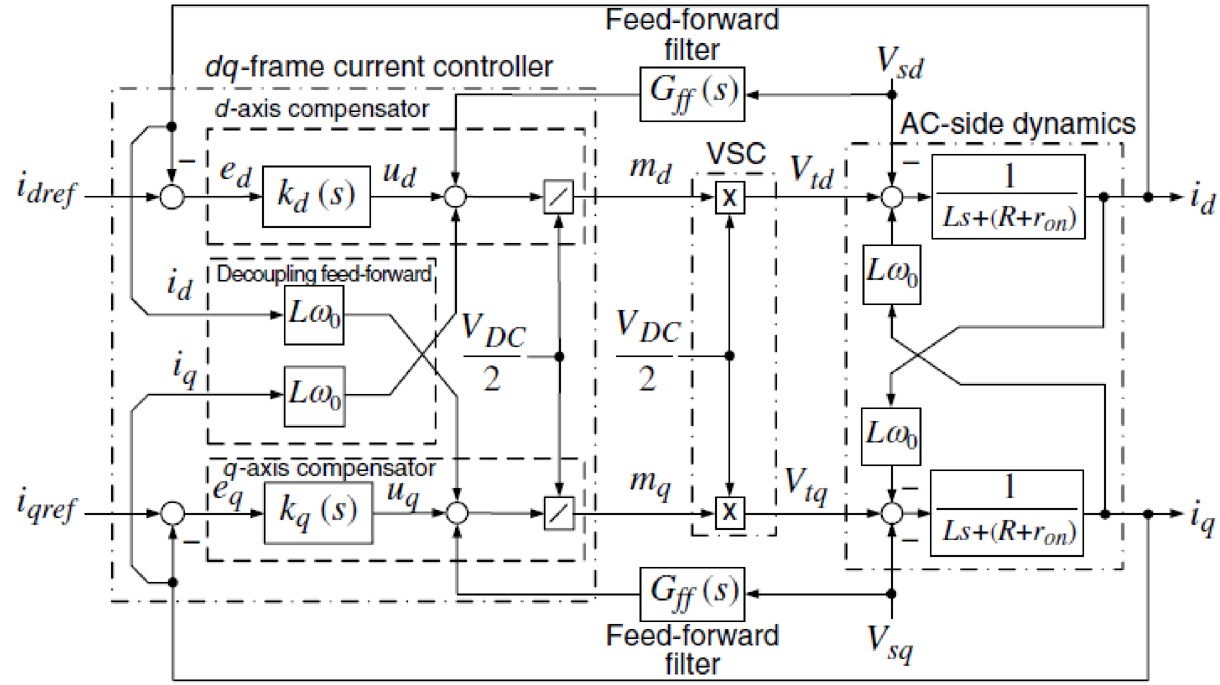


Figure 39. Control diagram of a current-controlled VSC system

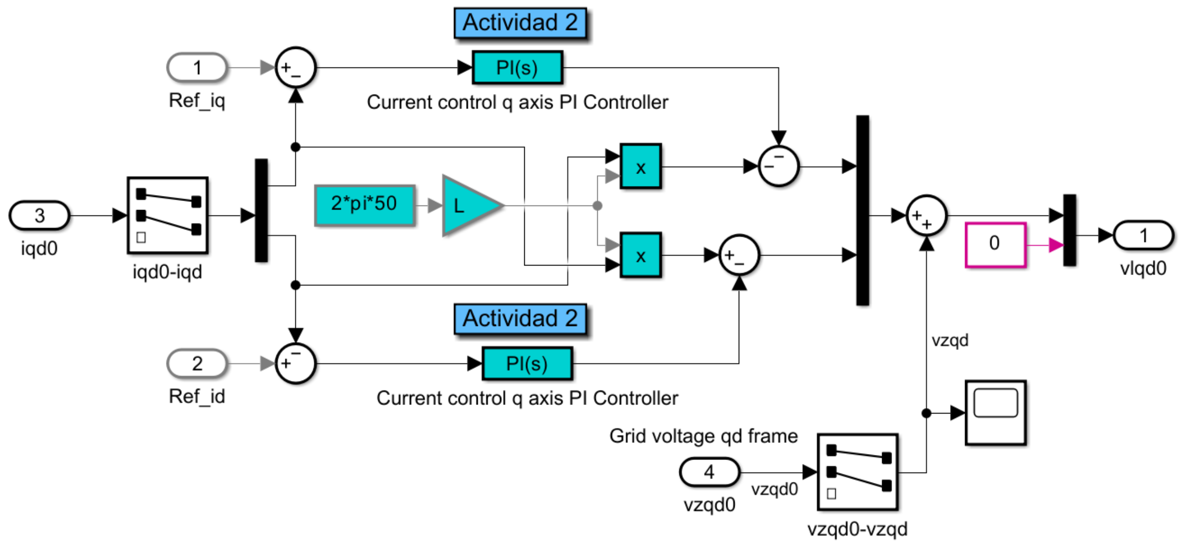
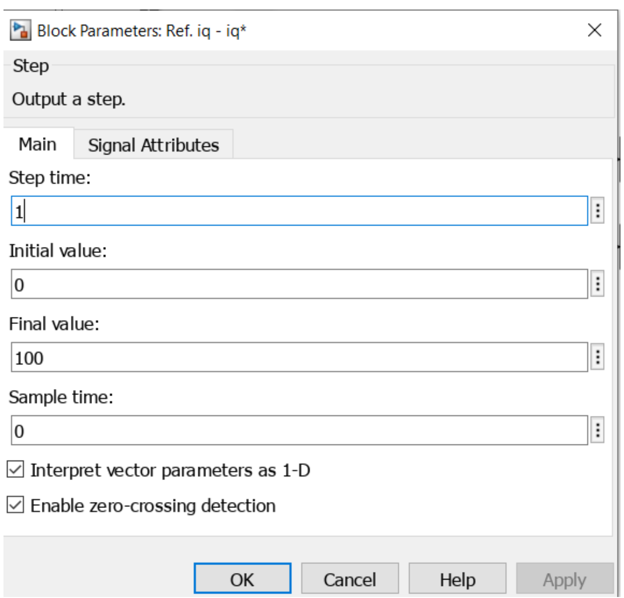
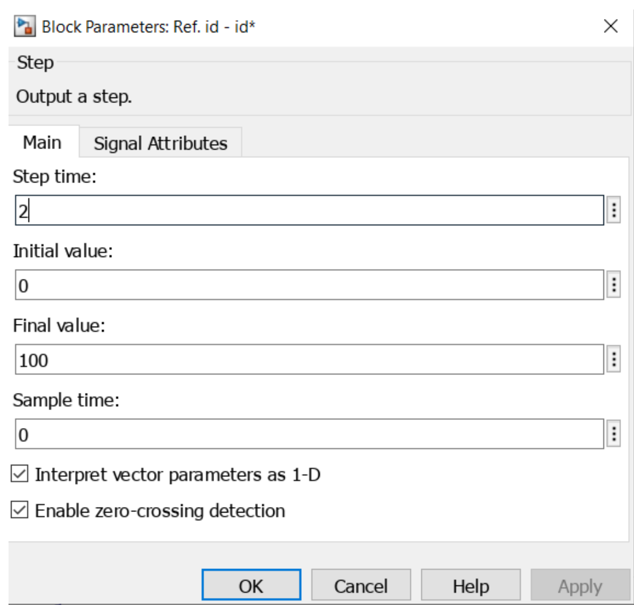


Figure 40. Control Block Diagram in our SIMULINK model

HERE IS CONTROLLED CURRENT AXIS:

With initial values of i_{dref} and i_{qref}:





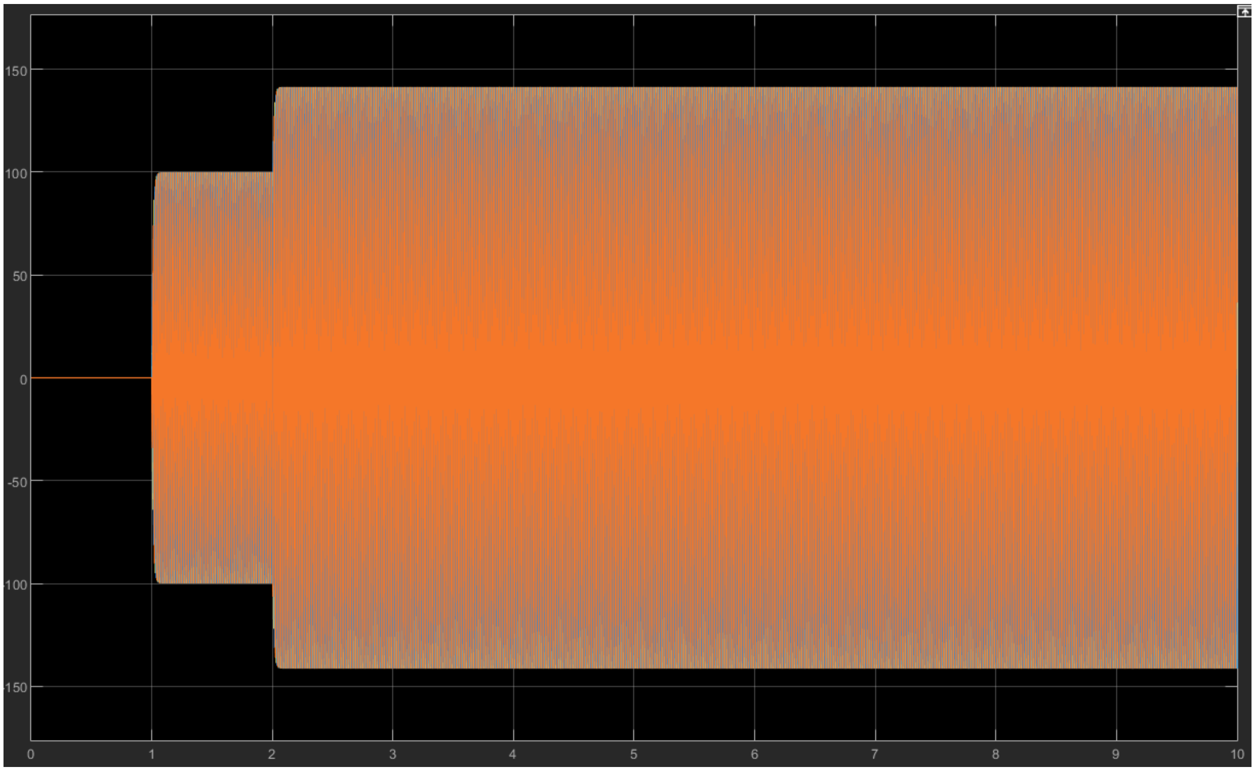


Figure 41. ABC grid currents

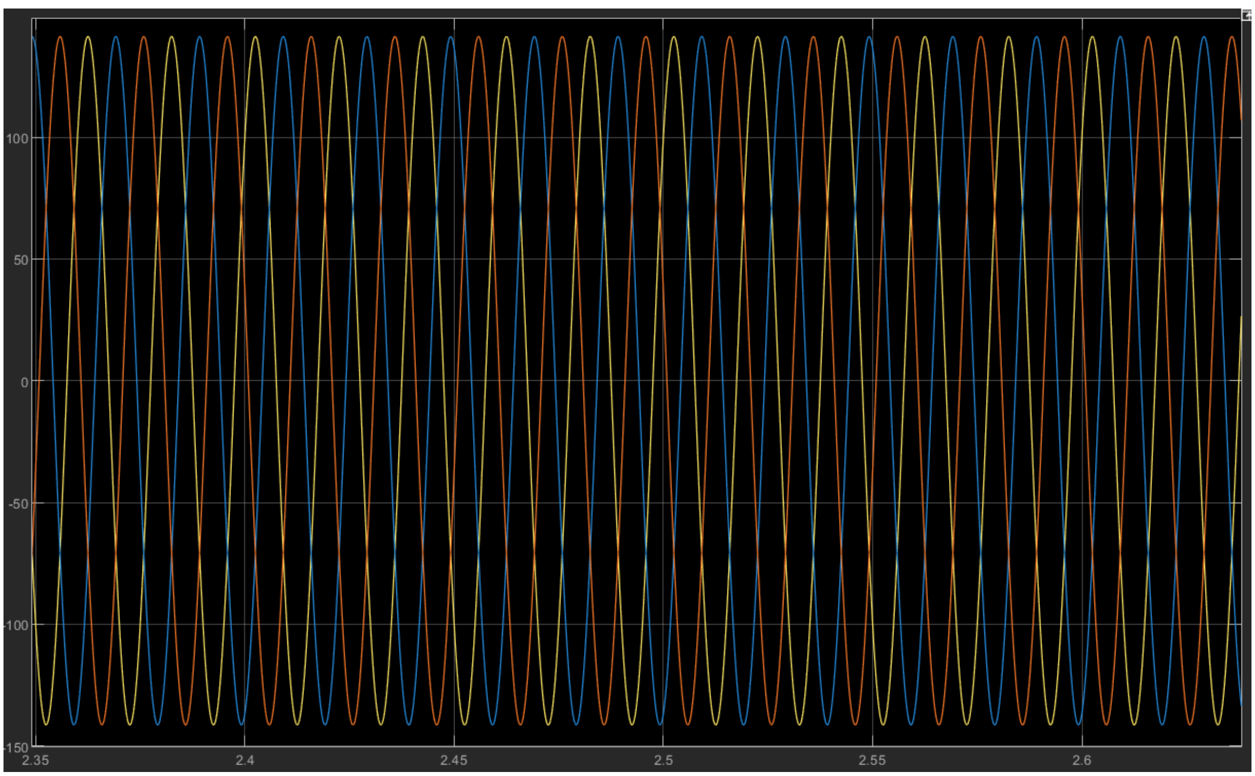
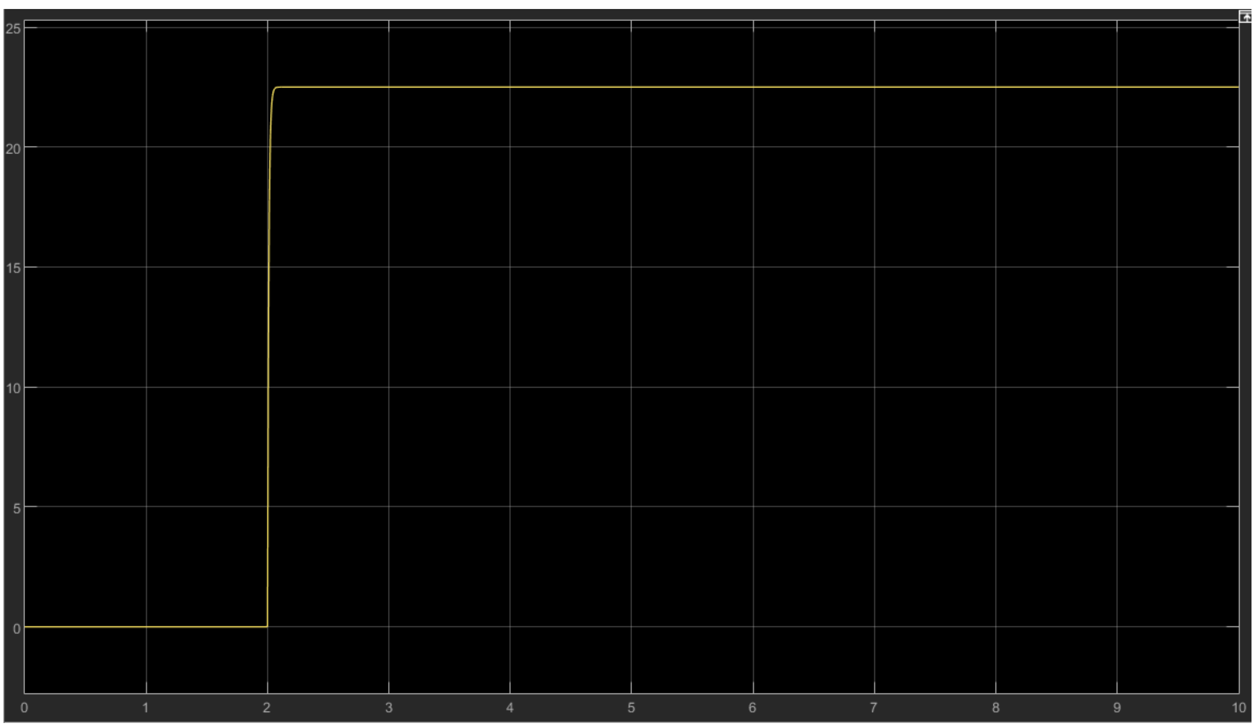


Figure 42. Zoomed

Active Power:

Reactive Power:



Question 3: Change the parameters of the current loop (PI controllers)

Answer 3:

Changed Parameters in current loop:

Active power:

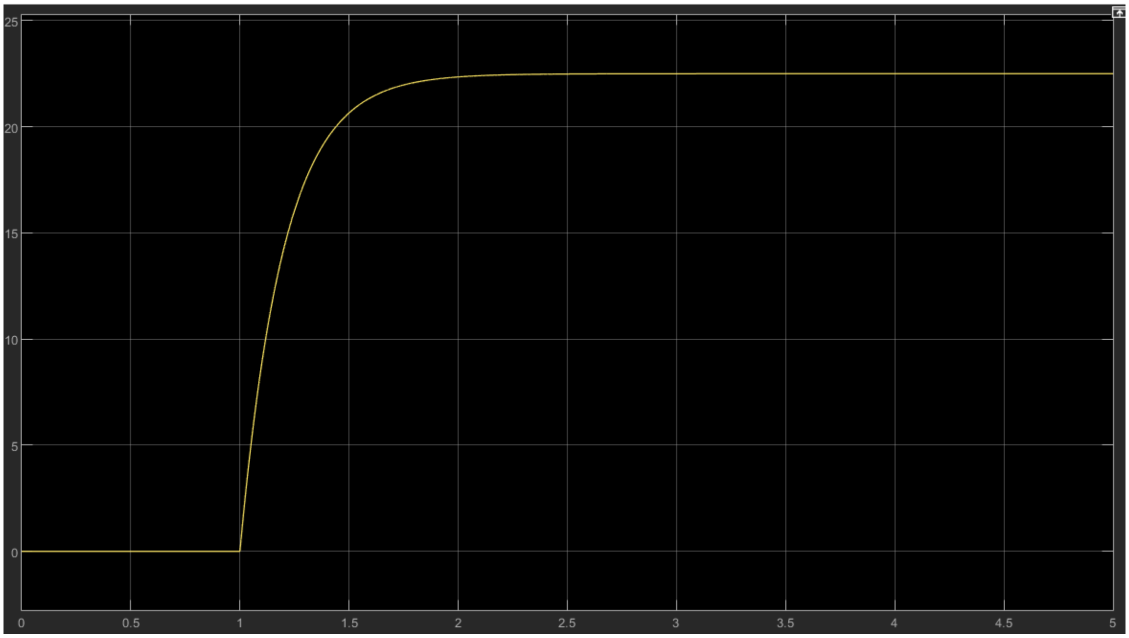


Figure 43. Active power

Reactive power:

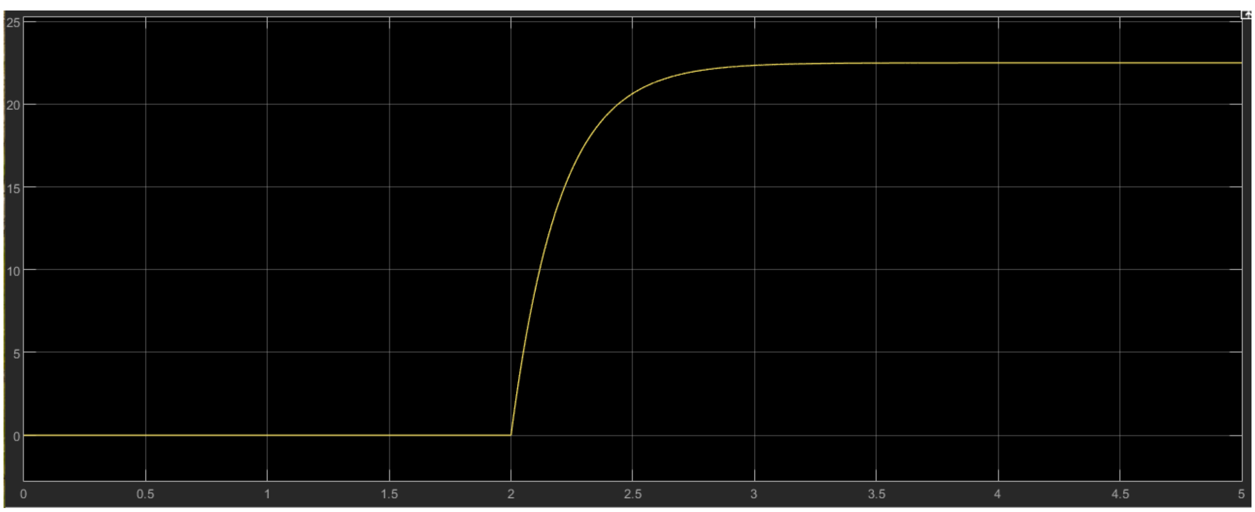


Figure 44. Reactive power

Problem 6: Model4_Refcalculation1 and Model4_Refcalculation2

Problem 5
Open the file 'Model4_Refcalculation1.slx' and 'Model4_Refcalculation2.slx' and proceed with the following tasks:

- Change the active and reactive power set-points
- Change the control parameters of the PQ controllers

Question 1: Change the active and reactive power set-points

Answer 1:

Initial configuration of setpoints:

Active power (Initial value = 0, Final value = 100 MW after 1 second):

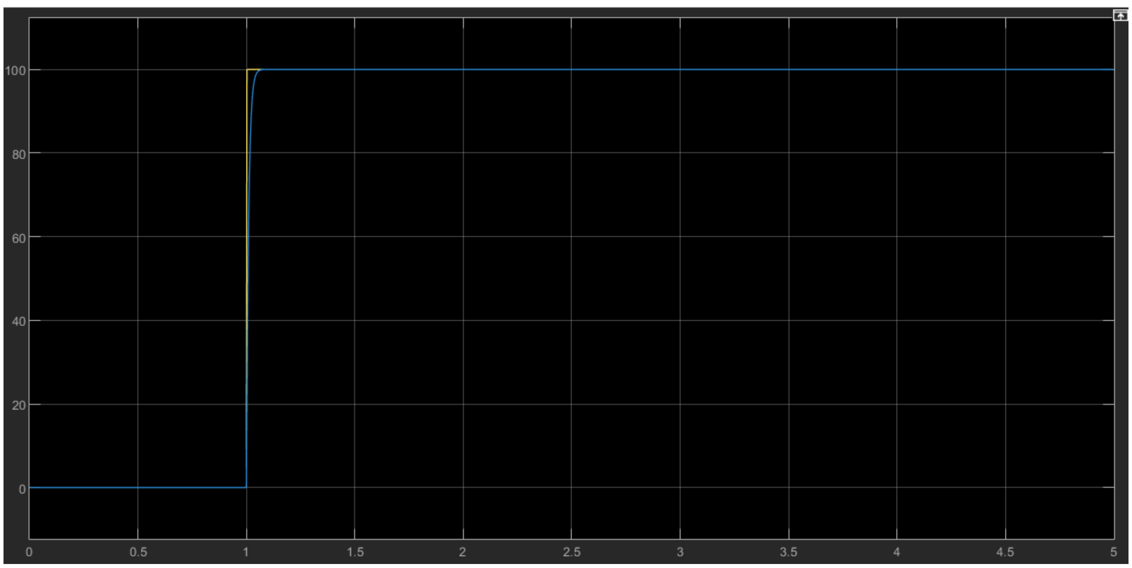


Figure 45. Real Active Power (blue) vs Reference Active Power (yellow)

Reactive power (Initial value = 0, Final value = 20 MVAr after 2 second):

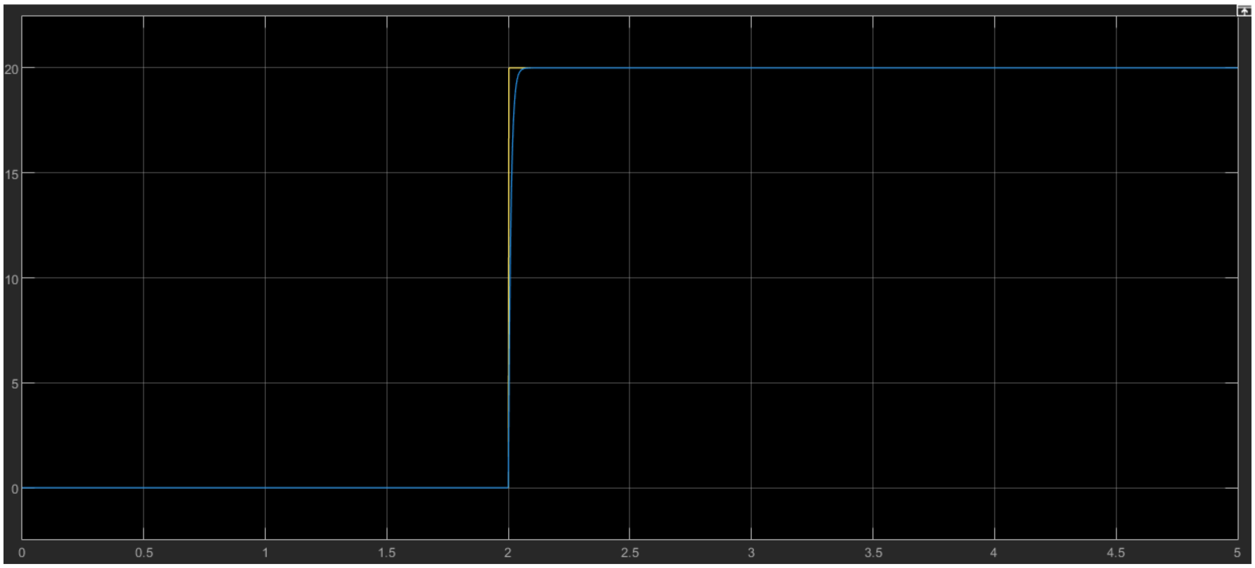


Figure 46. Real Reactive Power (blue) vs Reference Reactive Power (yellow)

Changing active and reactive setpoints:

Active power (Initial value = 0, Final value = 50 MW after 3 second):

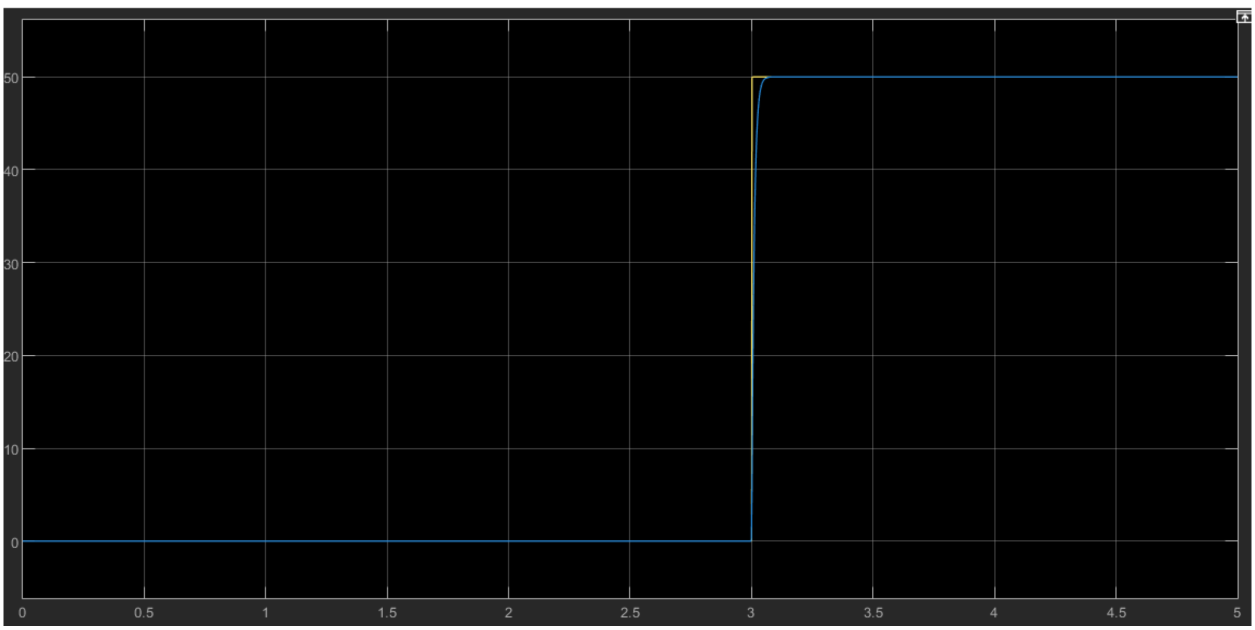


Figure 47. New Reference Point for Active power

Reactive power (Initial value = 0, Final value = 10 MVAr after 3 second):

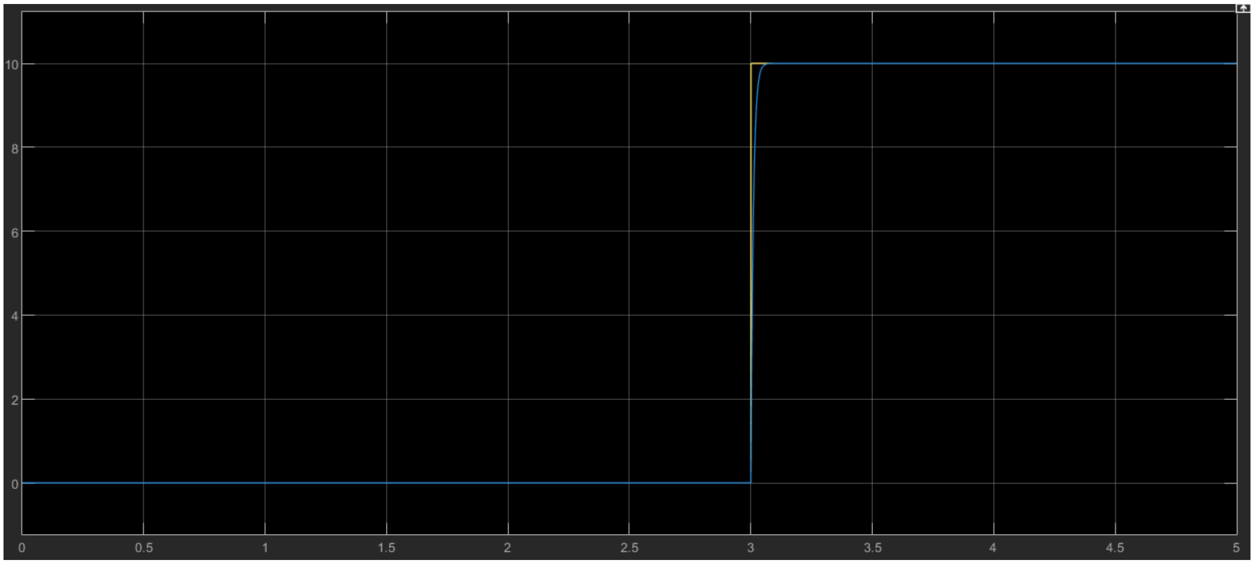


Figure 48. New Reactive Power Reference Point

Question 2: Change the control parameters of the PQ controllers

Answer 2:

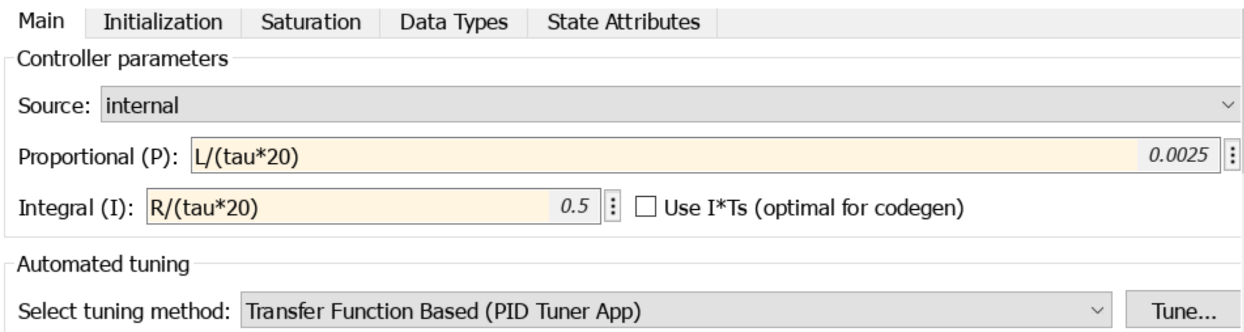


Figure 49. Changing Control Parameters to Observe effects

We can observe much slower response time:

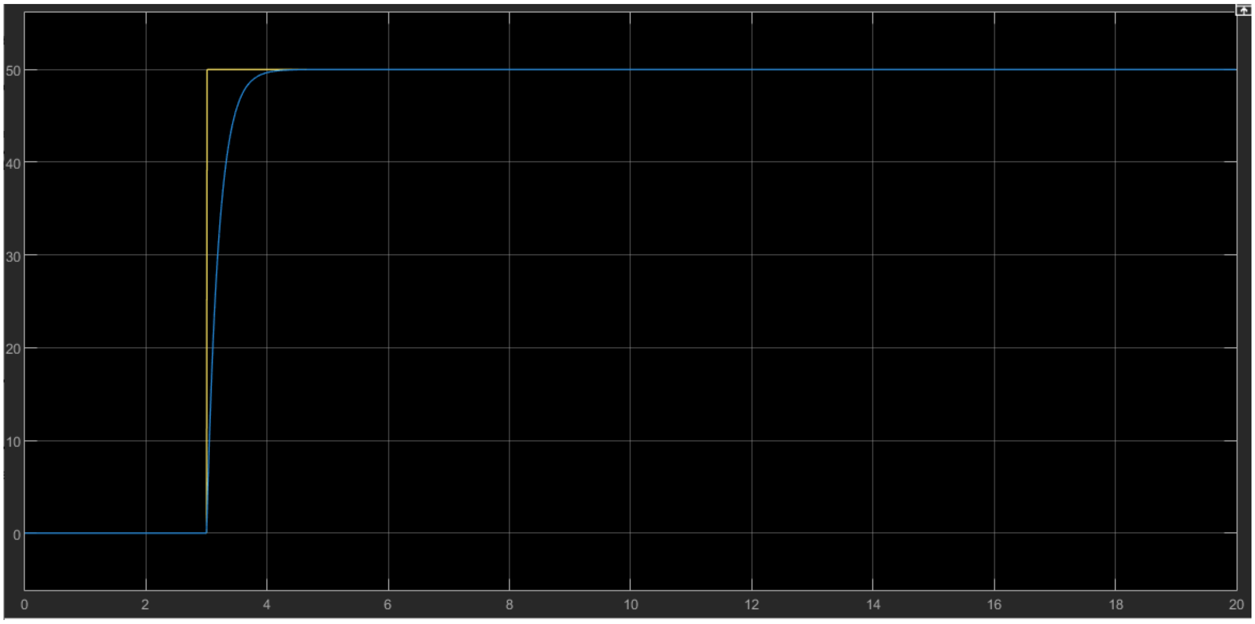


Figure 50. Active Power

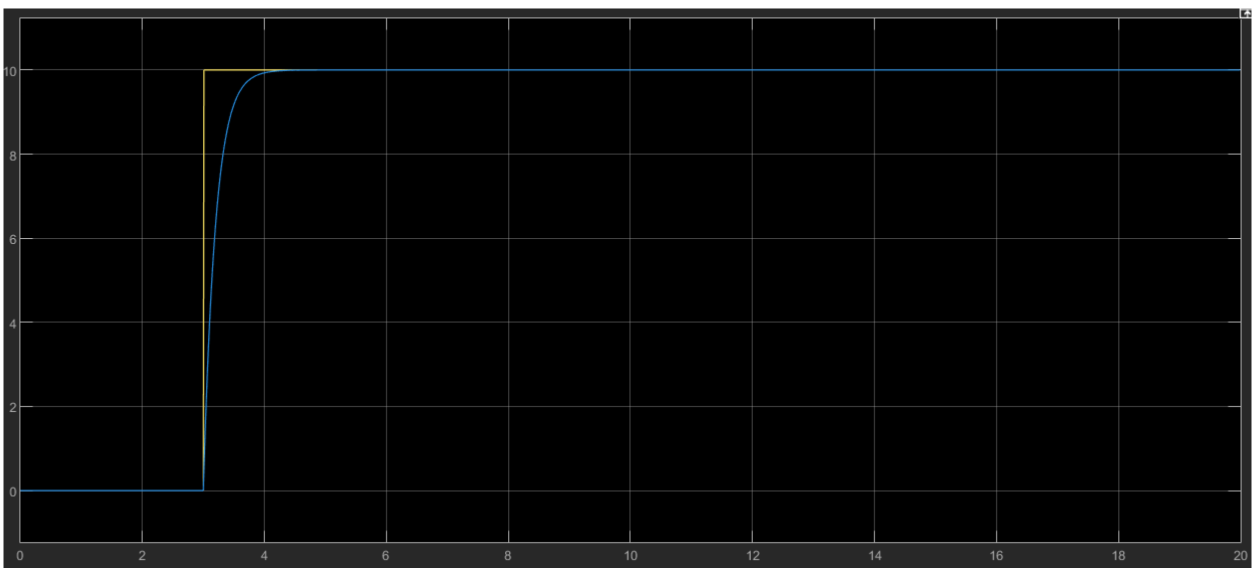


Figure 51. Reactive Power