Embedding Prospect Theory in Recurrent Neural Networks for Financial Timeseries

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Introduction

For decades financial market forecasting has been modeled on rational expectations of agents under the Efficient Market Hypothesis (Fama, 1970). These rigid assumptions are the building blocks of traditional economic theory where conclusions are sensitive to the nature of assumptions. More recently, research in behavioral responses of agents, crowd psychology, and cognitive biases (Kahneman, 2011) have given reason to challenge traditional economic theory. In particular, since the 2008 Global Financial Crisis approaches stressing the impact of human psychology have been popularized to explain market price action (Shiller et al., 2009). With advances in Behavioral Finance and Machine Learning (ML), combined with the increasing availability of historical financial data, one can construct an appropriate hypothesis space to explain markets, amalgamating both the traditional economic and psychological influences on financial market price action. The goal of the proposed research is to extend Deep Learning Algorithms to consider a temporal behavioral component in addition to the normative optimisation performed in human-centred predictive models.

Background and Literature

The seminal work on Prospect Theory (Kahneman and Tversky, 1979) and further advancements in modeling real-life choices in Cumulative Prospect Theory (Kahneman and Tversky, 1992) have shed significant light on behavioral principles of loss aversion and uncertainty in risk perception. The psychological phenomenon of diminishing and asymmetric sensitivity to loss and gains can particularly be observed in professional trading decisions as Haigh and List (2005) analyze in their experiments. The paper establishes evidence of framing where traders act differently depending on the frequency of decision or outcome evaluation, and outcomes are evaluated relative to a reference point rather than underlying fundamental value. Similarly, Zhang and Semmler (2009) demonstrate evidence of the impact of prior stock market losses or gains affecting investment behavior. Further, Prospect Theory and the predictability of stock returns that may arise from it has been validated previously (Barberis, 2014). Through these works, it is evident that what may be considered as irrational behavior in industry parlance, is a deep seated part of financial markets.

Given the presence of an irrational component driving price action, we seek to better understand agent behaviour to improve financial timeseries forecasting. For this purpose, historic financial data, intermarket and intramarket dependencies of highly liquid and high volume traded instruments can be explored using ML to enhance our understanding of market behavior. Such an approach refrains from making assumptions about the rationality of agents and focuses completely on the interaction between traded timeseries to infer collective market preferences and outcomes.

It has been shown that behavioral scientists can benefit from incorporating ML techniques in their daily practice (Peysakhovich and Naecker, 2017). The focus of this proposal is to turn this idea around and explore how Behavioral Economics can benefit human-centred ML algorithms. To this extent, the quantifiable aspects of Prospect Theory can be embedded in supervised ML algorithms where agent utility differs from fundamental value. Such consideration has become particularly feasible with developments in Cumulative Prospect Theory (CPT) where mathematical curves represent cognitive biases. There is little work done in such multi-disciplinary approaches using Decision Sciences and ML for financial time series. This would seem to be the natural evolution of ML for human-centred learning problems when framing, loss aversion, and reference dependence is pervasive.

For financial markets, valuations of assets are driven by two expectation regimes - a fundamentalist regime and a trend following regime (Boswijk et al., 2007). The former is often associated as a rational feature, while the latter considered to be irrational. Asset Managers, Traders, and Investors tend to weigh Fundamental Fair Value factors, the rational, disproportionately simply due to the absence of an accepted method to quantify behavioral biases. There also remains an intrinsic aversion to deviate too far from the Efficient Market Hypothesis which assumes rationality; investors attempt to forecast prices as they *should* be with relative disregard to collective investment behaviour. However, by combining cognitive evaluation with data on market positioning, technicals, mathematical features that best capture price movement, economic/econometric principles, and advances in machine learning, this may provide a more robust understanding of patterns and behaviour derived from price action. This may not only lend more credence to the irrational risk premia in financial assets, but also help identify states of crowd psychology using supervised classification algorithms.

Methodology

The proposed research aims to adjust for the irrational component of agent behaviour using machine learning and utility functions offered by Cumulative

Prospect Theory.

Table 1: CPT Utility Functions

$$P = w(p) * v(x) \tag{1}$$

$$v(x) = \begin{cases} \mathrm{x}^{\alpha}, \text{ for } x \geq 1 \\ \text{-}\lambda(-x)^{\alpha}, \text{ for } x < 1 \end{cases} \tag{2}$$

$$w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}} \tag{3}$$

$w(p)$ = decision weight
$v(x)$ = value of the potential outcome
α = risk attitude
λ = a loss aversion parameter
γ = degree of curvature in the decision weighting function.

Empirical estimates by Tversky and Kahneman (1992):
$\alpha = 0.88$, $\lambda = 2.25$, $\gamma = 0.61$ for gains, $\gamma = 0.68$ for losses

Modeling of risk and uncertainty has been investigated in Reinforcement Learning where the Value Function in a Markov Decision Process is evaluated using CPT (Prashanth et al., 2016). For this, an arbitrary reference point is chosen to separate gains and losses. Given the importance of reference dependency, this proposal intends to focus on Recurrent Neural Networks (RNN) to preserve temporal context, establish reference points and calibrate the loss function over time in the learning phase of the algorithm. Within Deep Learning architectures, RNNs are considered state of the art for predicting sequential data. Using Long-Short term memory networks (LSTM) these are more reliably able to maintain temporal dependencies (Hochreiter and Schmidhuber, 1997). Such an RNN-LSTM network will be used to optimise weights in consideration of both the fundamental features and behavioral influences that may cause deviation from an assumed rational, predicted output. A novel approach of using CPT to calibrate network losses over time would allow for more accurate valuation of financial products, capturing both the rational and irrational pricing of assets as perceived by agents.

For the supervised RNN-LSTM, a liquid and high-volume traded asset such as a major Equity Index or Sovereign Bond yield can be modeled using a selection of commonly accepted fundamental features. This serves as a baseline classification model for predicting *rational* pricing. In such an approach, the implicit economic assumption is that optimisation by backward propagation through time (BPTT) will aim to capture just the fundamental pricing. When predicting next day returns for the asset, we know that in addition to expected Fundamental Fair Value, asset valuation is also a function of perceived asymmetric utility where payoffs are generally steeper/convex for losses and more gradual/concave for gains (Kahneman and Tversky, 1979); this assymmetric behavior or *irrational* component can then be modeled using CPT Utility functions as an adjusting term added to the categorical cross-entropy loss. While losses themselves can be scaled to the utility function, this may place explicit assumptions around the noise in the data as Cobb, Roberts, and Gal (2018) note; To avoid this, a penalty term is added to a standard dropout Bayesian neural network for task-specific approximation of assymmetric utility:

$$\underbrace{-\sum_{i}\left(\log \sum_{c \in C} u(h_i, c)\, \rho(y_i = c \mid x_i, \hat{\omega}_i)\right)}_{\text{Additional utility-dependent penalty term}} \tag{4}$$

$\hat{\omega}_i$ = dropped out weights
h_i = optimal prediction
c = class labels
u = CPT utility function

The augmentation of a utility dependent penalty term allows for inclusion of a behavioral process in the learning phase. This research proposal aims to extend this framework to an RNN-LSTM model with such a loss function evaluated at each time-step and averaged, potentially using reference dependent weighting, to arrive at a single global loss for BPTT.

In CPT, gains or losses are evaluated relative to a reference point rather than total wealth. Inferring asset positioning and the magnitude of change from such a reference point is a challenge in itself; below are two approaches that may satisfy reference dependency:

- An extrinsic solution which captures market data on collective positioning, thereby being able to frame gains or losses over time, within a sequence of training.

- A more generalised intrinsic solution where the gain/loss $\varepsilon^{(k)}$ is related to past prediction error measured up to the kth time step as follows (Young Choi and Lee, 2018):

$$\varepsilon^{(k)} = \sum_{t=k-v+1}^{k} \gamma^{k-t} e^{(t)} \tag{5}$$

  where $0 < \gamma \leq 1, 1 \leq v \leq k$ and $e^{(t)}$ is the prediction error at each time step t over a sliding window. γ is devised for reducing the influence of old prediction errors. Overall, ϵ provides a reference for gains or losses over a time sequence during training. A basic technical momentum indicator such as

$$RSI = 100 - \left(\frac{100}{1 + (\frac{\text{Ave } \epsilon \text{ gain}_{window}}{\text{Ave } \epsilon \text{ loss}_{window}})}\right) \tag{6}$$

  applied to these errors can also provide framing for the potential outcome.

The proposed loss term [7] contains a rational component and a utility dependent irrational component. This is broadly equivalent to multi-task classification with a dependency on relative weighting derived from uncertainty around the two components (Kendall et al., 2017). The uncertainty can be modeled as aleatoric homoscedastic uncertainty, information that our data cannot explain and is task dependent rather than input dependent.

$$\underbrace{\frac{1}{\sigma_1^2}\,\mathcal{L}_1(\mathrm{W})}_{\text{rational}} + \underbrace{\frac{1}{\sigma_2^2}\mathcal{L}_2(\mathrm{W})}_{\text{irrational}} + \log\sigma_1 + \log\sigma_2 \qquad (7)$$

$\mathcal{L}_n(\mathrm{W}) = -\log \mathrm{Softmax}(y_n, \mathrm{f}^{\mathrm{w}}(x))$
where $f^{\mathrm{w}}(\mathrm{x})$ = the output of an RNN with weights W on input x

Such multi task weightings of losses can be learnt with the cross entropy loss for y_n optimised with respect to W as well as observation noise $\sigma_{1,2}$. In an RNN-LSTM, these weightings can be averaged across time-steps. The outcome of this algorithm would particularly be interesting to quantify the relative weighting of fundamental and behavioral drivers in financial markets, a problem that the industry has perennially struggled with.

References

[1] Nicholas Barberis. Prospect theory and stock returns : An empirical test. 2014.

[2] H. Peter Boswijk, Cars Hommes, and Sebastiano Manzan. Behavioral heterogeneity in stock prices. *Journal of Economic Dynamics and Control*, 31(6):1938–1970, 2007. URL https://EconPapers.repec.org/RePEc:eee:dyncon:v:31:y:2007:i:6:p:1938-1970.

[3] Adam D. Cobb, Stephen J. Roberts, and Yarin Gal. Loss-calibrated approximate inference in bayesian neural networks. *CoRR*, abs/1805.03901, 2018.

[4] E Fama. Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance*, 25(2):383, 1970.

[5] Michael S. Haigh and John A. List. Do professional traders exhibit myopic loss aversion? an experimental analysis. *The Journal of Finance*, 60(1):523–534, 2005. doi: 10.1111/j.1540-6261.2005.00737.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2005.00737.x.

[6] Sepp Hochreiter and Jürgen Schmidhuber. Long short-term memory. *Neural Comput.*, 9(8):1735–1780, November 1997. ISSN 0899-7667. doi: 10.1162/neco.1997.9.8.1735. URL http://dx.doi.org/10.1162/neco.1997.9.8.1735.

[7] D Kahneman. Thinking, fast and slow. *New York: Farrar, Straus and Giroux*, 2011.

[8] D Kahneman and A Tversky. A. prospect theory: An analysis of decision under risk. *47(2)*, pages 263–291, 1979.

[9] D Kahneman and A Tversky. Advances in prospect theory: Cumulative representation of uncertainty. 5(4):297, 1992.

[10] Alex Kendall, Yarin Gal, and Roberto Cipolla. Multi-task learning using uncertainty to weigh losses for scene geometry and semantics. *CoRR*, abs/1705.07115, 2017.

[11] Alexander Peysakhovich and Jeffrey Naecker. Using methods from machine learning to evaluate behavioral models of choice under risk and ambiguity. *Journal of Economic Behavior Organization*, 133(C):373–384, 2017. URL https://EconPapers.repec.org/RePEc:eee:jeborg:v:133:y:2017:i:c:p:373-384.

[12] L. A. Prashanth, Cheng Jie, Michael Fu, Steve Marcus, and Csaba Szepesvári. Cumulative prospect theory meets reinforcement learning: Prediction and control. In *Proceedings of the 33rd International Conference on International Conference on Machine Learning - Volume 48*, ICML'16, pages 1406–1415. JMLR.org, 2016. URL http://dl.acm.org/citation.cfm?id=3045390.3045539.

[13] R Shiller et al. Animal spirits: How human psychology drives the economy, and why it matters for global capitalism. page 264, 2009.

[14] Willi Sr and Wenlang Zhang. Prospect theory for stock markets: Empirical evidence with time-series data. *Journal of Economic Behavior Organization*, 72:835–849, 12 2009. doi: 10.1016/j.jebo.2009.08.003.

[15] Jae Young Choi and Bumshik Lee. Combining lstm network ensemble via adaptive weighting for improved time series forecasting. *Mathematical Problems in Engineering*, 2018:1–8, 08 2018. doi: 10.1155/2018/2470171.