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Using the Analytic Hierarchy Process in House Selection

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Abstract

The house selection process involves an assessment by the buyer of a myriad of qualitative and quantitative factors before negotiating a purchase. The complex interplay among these factors along with the limitations of humans to systematically consider the complex information set, create the potential to induce serious inconsistencies in decisions. This paper presents a formal judgmental model of the house selection process using the Analytic Hierarchy Process (AHP). The model will allow the buyer to consistently evaluate property attributes. We also present a simple extension of the model to help the prospective buyer arrive at an attribute weighted price that can be used for comparative rankings.

Key Words: Analytic Hierarchy Process (AHP), residential brokerage, house selection

Most investment decisions involve the assessment of a myriad of both qualitative and quantitative decisions including evaluation of a complex set of interplay among these factors. Although normative finance theory has continued to become more sophisticated over the last two decades and more, there is increasing recognition (see e.g., Myers, 1984 and Srinivasan and Kim, 1987) that normative financial models provide little help to the decision maker in the estimation of the cash flow impact of qualitative factors. Hoffman, Schniederjans, and Sirmans (1990) found a similar problem with models in the area of corporate property evaluation and designed a multicriteria model to help fill this void. Since evaluation of qualitative factors is necessarily judgmental, two sets of issues deserve attention: i) the ability to generate consistent judgments, and ii) the need to explicitly integrate estimable cash flow with judgments. The Analytic Hierarchy Process (AHP) is an intuitive and robust multiattribute modeling methodology that can help address these issues.

Residential real estate brokers are often heard to say, "Buyers are liars," or "Buyers buy with emotion, not logic." The former statement results from practioners' observations that home buyers will often demand they be shown only properties reflecting an array of specific characteristics and meeting certain constraints yet wind up buying houses that abrogate those demands. The corollary is drawn from observations of many buyers who, upon finding a suitable house, reveal excitement and a form of attachment to the property and exclaim "This is it" or "it feels good," without reference to specific attributes.

As an alternative to concluding that buyers are bluffers, the statement "Buyers are liars," if true, could indicate that either buyers are not adept in articulating their preferences before the market search, or that buyers modify their preferences and constraints as they gather market information. Also, the emotional purchase could actually reflect an instant processing of many attributes that meet the buyer's preferences; however, the satisfaction may be short-lived if certain attributes were not given proper attention. In any case, perhaps what is needed is a formal model for rigorously defining and applying buyer preferences when entering the market to purchase a residence.

The house selection process exemplifies the type of investment decision where much of the evaluation is qualitative. The prospective house buyer faces the daunting task of evaluating a large and diverse array of information before being able to reach a rational decision. First, even limiting the search to, for example, 3-bedroom, 2-bath single family detached houses listing between $200,000 and $230,000 within one hour's commute to work, the resulting sample in many megalopoli could be very large. Second, the buyer needs to gather data on all such properties. Third, even if data on each house were meticulously gathered, the buyer needs to evaluate the myriad of characteristics associated with each property. This frustrating state of information overload has the potential to cause buyers to inconsistently or inadequately assess the various attributes. Besides, typically, such evaluations are made informally. Given human limitations in processing information, this raises the specter of inconsistent judgments. Fourth, the buyer needs to integrate the judgmental assessment of property attributes with the listed price both for purposes of ranking the final choices and for the negotiations process.

We envision a model of the house selection process to consist of two phases. The first phase is essentially where the large universe of houses are reduced by applying a buyer's extreme preferences. Essentially, such preferences will stochastically dominate other attributes. Examples of such preferences include a buyer wanting to buy a house (single-family detached). In the second phase, the buyer faces the more difficult choice of evaluating the remaining choices in detail and ranking them in terms of the attributes and perceived values. Not only does the buyer need to evaluate property attributes in this phase but such an evaluation needs to be integrated with the listed price to yield an estimate of perceived price. In this paper, we present a model of the second phase and assume that the buyer has already limited his or her set of properties by applying the extreme preferences. The AHP model presented in this paper will allow they buyer to consistently evaluate property attributes. A simple extension of the basic AHP model allows the buyer to explicitly integrate the judgmental evaluation of attributes with house prices to yield attribute weighted prices. Attribute weighted prices allow the user a simple mechanism to rank competing houses. Alternatively, the extended model can also be used to yield reservation prices in preparation for negotiation with the seller.

In the sections that follow, we will i) briefly review the Analytic Hierarchy Process, ii) outline an illustrative AHP model for house selection, iii) illustrate the use of the model with a simple example, iv) outline ways of combining judgments of multiple decision makers, and v) conclude with suggestions for further applications in real estate analysis.

1. The Analytic Hierarchy Process[1]

The AHP is an appealing methodology to systematically evaluate (often conflicting) qualitative criteria. It is a relatively new addition to the family of multiattribute decision models

(MADM) and has been pioneered by Saaty (1980). Like other MADM, AHP also attempts to resolve conflicts and analyze judgments through a process of determining the relative importance of a set of activities, players or criteria. The methodology is based on the concept of "tradeoff" and enables the decision maker to develop the tradeoff implicitly in the course of structuring and analyzing a series of reciprocal pairwise comparison matrices.

The AHP can be succinctly summarized in terms of its three principal components. First, the principal problem is decomposed into a hierarchy. Each level of the hierarchy consists of a set of elements and each element, in turn, is broken into subelements for the next level of the hierarchy. The final level consists of the specific courses of action that are being contemplated for adoption. Structuring any problem hierarchically is an efficient and intuitive way of dealing with complexity and identifying the relevant components of the problem. The AHP is very flexible in allowing the decision maker to structure the hierarchy to fit individual needs and preferences. Further, in a group decision setting, the use of the AHP to structure a problem may help in achieving consensus over critical elements or pinpoint areas of disagreement so that more attention can then be focused on these areas to achieve consensus.

Second, with each hierarchical level, relative weights for the various elements are established using a measurement methodology. Use of the methodology requires the decision maker to evaluate the elements in a particular level in a pairwise fashion using a 9-point scale shown in Table 1.[2] The pairwise comparison indicates the degree to which one element dominates another element of the same level, to respect with each element of the preceding level.

Third, the pairwise comparison matrices are evaluated using a measurement theory. The basic assumption underlying the measurement theory in the AHP is that relative dominance

Table 1. Analytic hierarchy measurement scale.

Reciprocal Measure of Intensity of Importance	Definition	Explanation
1	Equal importance.	Two activities contribute equally to the objective
3	Weak importance of one over another.	Experience and judgment slightly favor one activity over another.
5	Moderate importance.	Experience and judgment strongly favor one activity over another.
7	Strong importance.	An activity is strongly favored and its dominance is demonstrated in practice.
9	Absolute importance.	The evidence favoring one activity over another is of the highest possible order of affirmation.
2, 4, 6, 8	Intermediate values between two adjacent judgments.	When compromise is needed.
Reciprocal of above.	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.	

can be measured by pairwise comparisons. Assume that we wish to conduct pairwise comparisons of a set of n attributes and establish their relative weights. If we denote the attributes by $O_1, O_2, \ldots, O_n$ and their relative importance by $w_1, w_2, \ldots, w_n$, the pairwise comparison matrix **O** may be expressed as the reciprocal matrix shown below:

$$\mathbf{O} = \begin{array}{cccccc} & O_1 & O_2 & \cdots & O_n \\ O_1 & w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ O_2 & w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ O_n & w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{array}$$

where the w_i/w_j reflects the relative importance of element i over element j. If we multiply **O** by the transpose of the vector $w^T = (w_1, \ldots, w_n)$, we obtain the vector nw. The problem now takes the form

$$\mathbf{O}w = nw$$

We have implicitly started with the assumption that the weights, w, are known. But if we only had **O** and wanted to recover the weights, w, we would have to solve the system $(\mathbf{O} - n\mathbf{I})w = 0$, in the unknown w. This has a nonzero solution if and only if n is an eigenvalue of **O**, i.e., it is a root of the characteristic equation of **O**. But **O** has unit rank since every row of **O** is a constant multiple of the first row. Thus, all the eigenvalues, λ_i, $i = 1, \ldots, n$, of **O** are zero except one. It is also known that

$$\sum_{i=1}^{n} \lambda_i = \mathrm{tr}(O) \equiv \text{sum of the diagonal elements} = n$$

Therefore, only one of the λ_i, λ_{max}, equals n and all other $\lambda_i = 0$. The solution w of the above eigenvalue problem is any column of **O**. These solutions differ by a multiplicative constant. However, for operational reasons, it is desirable to normalize the solution so that the weights sum to unity. The result is a unique solution no matter which column is used.

The matrix **O** satisfies the cardinal consistency property $O_{ij}O_{jk} = O_{ik}$. Thus, given any row of **O**, we can determine the rest of the entries from this relation. However, the scale w is unknown and we may only have estimates of the ratios in the matrix. In this case, the cardinal consistency relation above need not hold, nor need an ordinal transitivity relation of the form: $O_i > O_j$, $O_j > O_k$ imply $O_i > O_k$ hold (where the O_i are the row of **O**). Since human judgments may remain inconsistent and intransitive, despite best efforts, we need to consider the inconsistency in the comparison data. It can be shown that in any matrix, small changes in coefficients imply small changes in eigenvalues, the problem $\mathbf{O}w = nw$ is transformed to $\mathbf{O}'w' = \lambda_{max}w'$. We know from the Perron-Frobenius theorem that a matrix of positive entries has a real positive eigenvalue whose modulus exceeds those of all other eigenvalues. The corresponding characteristic eigenvector solution is unique when normalized.

The question that comes up then is how close, in such cases, is λ_{max} to n and w' to w. It can be shown that $\lambda_{max} \geq n$ for all possible states and that $(\lambda_{max} - n)/(n - 1)$ serves as an index measure of consistency.[3] The index indicates the departure from consistency of the comparison ratios, w_i/w_j, and the ratios are deemed consistent, if $\lambda_{max} = n$. The consistency index is compared to a level of consistency that can be obtained merely by chance. Saaty and Mariano (1979) have established for different order random entry reciprocal matrices, an average consistency index which ranges from 0 for 1 to 2 element matrices, to 0.9 for 4 element matrices and to 1.49 for 10 element matrices. In general, a consistency ratio (consistency index/average random consistency) of 10% or less is considered very good. If consistency is poor, additional data or another round of comparisons may be required. Note that improving consistency does not necessarily imply getting an answer to the "real" solution to the problem being studied. It only implies that the ratio estimates in the comparison matrix are closer to being logically related than to being randomly chosen.

Thus, the pairwise comparison matrix **O** is used to elicit the judgment of experienced decision maker or a group of experts with regards to the relative importance of a set of attributes for each element of the preceding level. The principal (right) eigenvector of this matrix (corresponding to the largest eigenvalue) is then extracted and normalized to yield local priorities for the elements of the matrix.[4] Local priorities are then transformed to global priorities by weighting them with the global priorities of the elements of the preceding level. Continuing this process of eigenvector extraction and weighting through the levels of the hierarchy leads to a unidimensional priority (weight) scale for the elements in the final level. Since the final level of the matrix represents the alternatives being compared, global priorities for the final level reflect the decision maker's relative weight for the alternative course of action.

To illustrate the process of eigenvector weighting, consider any two adjacent levels in a hierarchy, i and i + 1. Each level has its own decision elements. Let the first element of the ith level be denoted as O_1 (i). If one compares the element of level i + 1 with respect to O_1 (i), a comparison matrix will result whose eigenvector describes the relative *local* priorities of elements in level i + 1 with respect to O_1 (i). Let us denote this vector by w_1. Similarly comparisons are made with respect to each of the n elements in level *i*. This process results in *n m*-dimensional local priority vectors, where *m* is the number of elements in level i + 1. To find the global priorities of elements in the i + 1 level:

$$w_g(i + 1) = [w_1, w_2, \ldots, w_n]w_g(i)$$

where $w_g(i)$ is the global priority vector for the elements of the *i*th level.

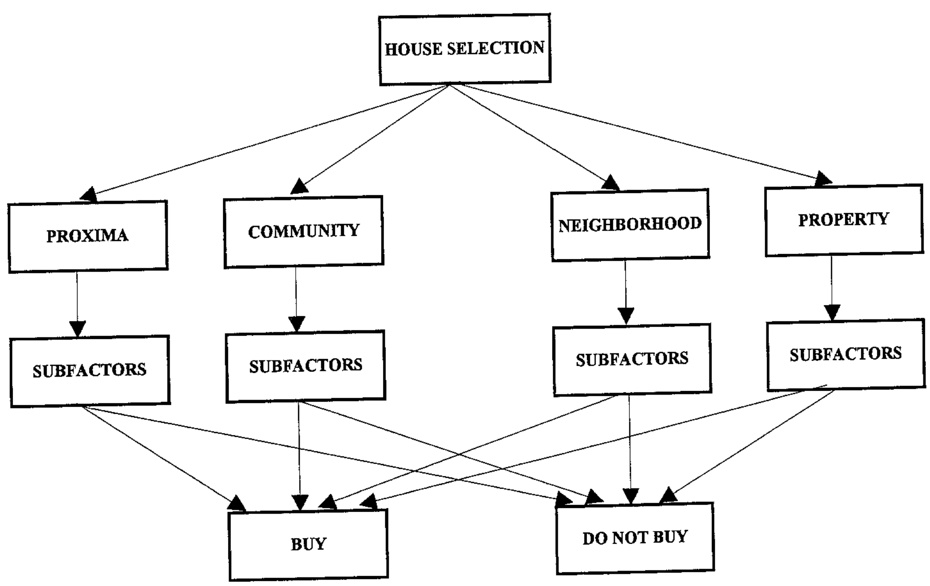
The AHP has been applied to many varied and complex situations ranging from predicted oil prices and planning for a national waterway (Saaty and Vargas, 1982), to a variety of marketing decisions (Wind and Saaty, 1980). Srinivasan and Kim (1986) illustrate the applicability of the AHP to the corporate credit granting problem. Further, Srinivasan and Bolster (1990) illustrate the potential for applying the AHP for determining investment quality ratings. For a comprehensive review of nonfinancial applications of the AHP, see Zahedi (1986). The major difference between the AHP and other MADM is that the AHP enables the systematic structuring of any complex multiplayer, multidimensional problem.[5] It differs from the classical multiattribute utility approach in that the deterministic nature of the problem permits direct value assessment, rather than resorting to an assessment of risk attitude. The latter approach yields multiattribute utility curves, while the AHP yields a single priority point in the multiattribute space.

2. The AHP and House Selection

2.1. A House Selection Model Based on AHP

House selection using the AHP requires the construction of a hierarchy of attributes such as that found in Appendix 1. The higher-order factors (noted by Roman numerals) are compared with each other in a series of pairs by the house purchaser as are each of the lower order factors (noted by uppercase letters) within each category. Further subdivisions of factors (noted by Arabic numerals and lowercase letters) are put forth merely for elaboration and will not be explicitly included. Even the mere process of constructing the hierarchy (figure 1) often helps decision makers focus and clearly articulate their preferences which otherwise may not have been done.

To evaluate the hierarchy, the buyer will first rate each attribute to others of that order in a series of pairwise comparisons using a scale from 1 to 9 (Table 1). The "1" represents no difference in importance between two factors. A "3" means the column factor is weakly



See Appendix I for a detailed listing of the subfactors

Figure 1. House selection hierarchy.

more important than the row factor. A "5" indicates the column factor is moderately more important, a "7" means that it is strongly more important, and a "9" indicates absolute domination of the column factor over the row factor. Likewise, the reciprocal of these numbers indicate the same magnitude in the opposite direction (row factor dominance over the column factor).

Next the buyer (with the aid of a broker or consultant) will find the set of properties for sale within the price range and locales of the buyer's extreme preferences. If this set is too large for timely showings, then a manageable subset can be formed by a driveby of each property. As the buyer tours the houses within the subset, he or she may revise the rankings and weights given to the top level elements of the hierarchy. A review at the end of the tour of notes, photographs will establish the final set of properties that the buyer wishes to consider. The buyer then ranks each of the houses in the final set by evaluating the house with respect to the second-order attributes independent of other properties and the hierarchy.[6] The measurement process allows the derivation of global weights for each of the second-order attributes for each of the properties being considered. A scheme of hierarchical weighting yields overall weights for the alternatives of interest. For each property, we can describe the final result as an index of the property's desirability or undesirability. The desirability weights can then be integrated with the property's price to yield an attribute weighted price as shown subsequently.

2.2. *Illustration*

We will now illustrate the use of AHP in selecting a house with a particular case. A buyer was seeking a house in the western suburbs of Boston, Massachusetts. We began by having our buyer rate the importance of the first-order attributes-proxima, community, neighborhood, and property, to one another in pairs using the scales in Table 1. The resulting matrix, Table 2 indicates the buyer finds a) community, neighborhood, and property attributes have weak, moderate and strong dominance (respectively) over proxima; b) neighborhood and property attributes have strong and moderate dominance over community; and c) neighborhood and property attributes are of equal importance.

Similarly, the buyer rated second-order attributes within each of the first-order attributes. As shown in Table 3, respecting proxima, the commute of member A of the buyer's household held weak and absolute dominance over the commute of member B and proximity to other amenities, respectively, but that member B's commute strongly dominated proximity

Table 2. Comparison of higher-order factors.

	Proxima	Community	Neighborhood	Property	Global Weights
Proxima	1.00	0.33	0.20	0.14	0.06
Community	3.00	1.00	0.20	0.33	0.13
Neighborhood	5.00	5.00	1.00	0.91	0.41
Property	7.00	3.00	1.10	1.00	0.40
Total	16.00	9.33	2.50	2.38	1.00

Table 3. Comparison of second-order factors.

Proxima	Commute A	Commute B	Other Proximity	Local Weights	Global Weights
Commute A	1.00	3.00	9.00	0.66	0.04
Commute B	0.33	1.00	6.00	0.28	0.02
Other Proximity	0.11	0.17	1.00	0.06	0.00
Total	1.44	4.17	16.00	1.00	0.06

Community	Schools	Government	Social	Entertainment	Local Weights	Global Weights
Schools	1.00	0.14	0.11	0.13	0.04	0.00
Government	7.00	1.00	0.11	0.33	0.14	0.02
Social	9.00	9.00	1.00	0.14	0.31	0.04
Entertainment	8.00	3.00	7.00	1.00	0.51	0.07
Total	25.00	13.14	8.22	1.60	1.00	0.13

Neighborhood	Aesthetics	Safety	Local Weights	Global Weights
Aesthetics	1.00	0.20	0.17	0.07
Safety	5.00	1.00	0.83	0.34
Total	6.00	1.20	1.00	0.41

Property	Lot	Exterior	Interior	Systems	Local Weights	Global Weights
Lot	1.00	5.00	0.33	7.00	0.30	0.12
Exterior	0.20	1.00	0.14	3.00	0.09	0.04
Interior	3.00	7.00	1.00	7.00	0.56	0.22
Systems	0.14	0.33	0.14	1.00	0.05	0.02
Total	4.34	13.33	1.61	18.00	1.00	0.40

to other amenities. It is not surprising that the buyer, who plans to remain without children, finds all other community attributes of government, social and entertainment, dominate the community attribute of schools. Social factors absolutely and strongly dominated government and entertainment though entertainment was weakly more important than government.

The buyer found safety moderately more important than aesthetics when thinking of suitable neighborhoods. Respecting a property, our buyer held systems within the house as relatively unimportant compared to the lot, building exterior, and building interior attributes. Interior considerations were strongly and weakly more important than the exterior and lot, but the lot was moderately more important than the exterior.

Next, we consulted with the buyer to establish global or extreme preferences. It was determined that our search would consist of properties that were a) single family detached dwellings; b) within a walk of the light rail system; c) in a high demand city; and d) for sale for less than $230,000. These constraints limited the search to villages in the city of Newton along the Riverside rail line. Classified advertising and the local multiple listing service revealed 25 properties meeting our buyer's extreme preferences. A drive by all of these properties resulted in an instant elimination of 14 properties although nearly half of those remaining would have to be perfect upon showing to remain in contention.

A tour through the houses quickly reduced the set to 6 of which photographs and notes were taken. Upon close comparisons, including first-order attributes, of these properties, two houses stood apart form the other four—20 Sol Street and 15 Boyls Road in Newton Highlands. A final preoffer tour was conducted of the properties with the buyer rating each second-order attribute for that property, independent of other attributes and of the other property. The results of the ratings appear in Tables 4 and 5. The overall global weights for 20 Sol Street is 0.89 and for 15 Boyls Road is 0.55. Purely based on these weights, the decision would be to buy 20 Sol Street.

2.3. Attribute Weighted Price

If the house selection process only required the evaluation of qualitative factors, or if the houses were all priced the same, decision could be reached on the basis of the global priorities for the alternatives. But, this is not likely to be the case. Therefore, we need to integrate judgmental weights for the various houses with the listed prices to realize an attribute weighted price. Fortunately, such an estimate can be found by:

$$\text{Attribute Weighted Price (AWP)} = \text{Price/Global Weight for Buying} \qquad (1.0)$$

The listed price for 20 Sol Street was $225,000 and for 15 Boyls road was $205,000. Attributed weighted prices for the two properties were $253,000 and $373,000, respectively. Aggregation is useful for establishing a common unit of comparison and for computing estimates of reservation prices to be used in the negotiations process. For example, a price of $140,000 is an estimate of a reservation price for the 15 Boyls road property using the list price of $225,000 for 20 Sol Street as a basis. The buyer was initially surprised with such a low value for the lesser alternative; however, upon further reflection on the hierarchy, he realized that he was placing too much value on the condition of the houses and not enough on neighborhood when touring the properties. The better alternative was purchased for $212,000.

3. Aggregation of AHP Judgments

In the previous sections, we have only considered a single decision maker or expert. In reality, there may be multiple experts thus giving rise to an aggregation problem of combining the multiple judgments, if the judgments show a significant degree of disagreement. In the context of financial decisions identified earlier, differences may arise in the relative priorities for the policy variables (objectives, criteria) or the global priorities for the alternative courses of action. Further, the aggregation issue has two dimensions. The first dimension is one where there is a need to build consensus among the experts. The second dimension is where the expert judgments need to be aggregated numerically using an appropriate statistical methodology. The second dimension may also be relevant where significant differences persist after attempts have been made to reach a consensus.

Table 4. Property attributes—20 Sol Street.

Commute A	Buy	Do Not Buy	Local Weights	Global Weights
Buy	1.00	5.00	0.83	0.03
Do Not Buy	0.20	1.00	0.17	0.01
Total	1.20	6.00	1.00	0.04
Commute B	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	3.00	0.75	0.01
Do Not Buy	0.33	1.00	0.25	0.00
Total	1.33	4.00	1.00	0.01
Other Proximity	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	1.00	0.50	0.00
Do Not Buy	1.00	1.00	0.50	0.00
Total	2.00	2.00	1.00	0.00
Schools	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	9.00	0.90	0.00
Do Not Buy	0.11	1.00	0.10	0.00
Total	1.11	10.00	1.00	0.00
Government	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	3.00	0.75	0.01
Do Not Buy	0.33	1.00	0.25	0.00
Total	1.33	4.00	1.00	0.01
Social	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	7.00	0.88	0.04
Do Not Buy	0.14	1.00	0.13	0.01
Total	1.14	8.00	1.01	0.05
Entertainment	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	1.00	0.50	0.03
Do Not Buy	1.00	1.00	0.50	0.03
Total	2.00	2.00	1.00	0.06
Aesthetics	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	8.00	0.89	0.06
Do Not Buy	0.13	1.00	0.11	0.01
Total	1.13	9.00	1.00	0.07

Table 4. (Continued).

Safety	Buy	Do Not Buy	Local Weights	Global Weights
Buy	1.00	9.00	0.90	0.31
Do Not Buy	0.11	1.00	0.10	0.03
Total	1.11	10.00	1.00	0.34
Lot	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	7.00	0.88	0.11
Do Not Buy	0.14	1.00	0.13	0.02
Total	1.14	8.00	1.01	0.13
Exterior	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	2.00	0.67	0.03
Do Not Buy	0.50	1.00	0.33	0.01
Total	1.50	3.00	1.00	0.04
Interior	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	4.00	0.80	0.18
Do Not Buy	0.25	1.00	0.20	0.04
Total	1.25	5.00	1.00	0.22
System	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	0.33	0.25	0.01
Do Not Buy	3.00	1.00	0.75	0.02
Total	4.00	1.33	1.00	0.03

Table 5. Property attributes—15 Boyls Road.

Commute A	Buy	Do Not Buy	Local Weights	Global Weights
Buy	1.00	7.00	0.88	0.03
Do Not Buy	0.14	1.00	0.13	0.00
Total	1.14	8.00	1.01	0.03
Commute B	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	3.00	0.75	0.01
Do Not Buy	0.33	1.00	0.25	0.00
Total	1.33	4.00	1.00	0.01

Table 5. (Continued).

Other Proximity	Buy	Do Not Buy	Local Weights	Global Weights
Buy	1.00	1.00	0.50	0.00
Do Not Buy	1.00	1.00	0.50	0.00
Total	2.00	2.00	1.00	0.00
Schools	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	8.00	0.89	0.00
Do Not Buy	0.13	1.00	0.11	0.00
Total	1.13	9.00	1.00	0.00
Government	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	3.00	0.75	0.01
Do Not Buy	0.33	1.00	0.25	0.00
Total	1.33	4.00	1.00	0.01
Social	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	5.00	0.83	0.03
Do Not Buy	0.20	1.00	0.17	0.01
Total	1.20	6.00	1.00	0.04
Entertainment	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	1.00	0.50	0.03
Do Not Buy	1.00	1.00	0.50	0.03
Total	2.00	2.00	1.00	0.06
Aesthetics	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	0.14	0.12	0.01
Do Not Buy	7.00	1.00	0.88	0.06
Total	8.00	1.14	1.00	0.07
Safety	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	0.50	0.33	0.11
Do Not Buy	2.00	1.00	0.67	0.23
Total	3.00	1.50	1.00	0.34
Lot	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	2.00	0.67	0.08
Do Not Buy	0.50	1.00	0.33	0.04
Total	1.50	3.00	1.00	0.12

Table 5. (Continued).

Exterior	Buy	Do Not Buy	Local Weights	Global Weights
Buy	1.00	3.00	0.75	0.03
Do Not Buy	0.33	1.00	0.25	0.01
Total	1.33	4.00	1.00	0.04
Interior	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	5.00	0.83	0.19
Do Not Buy	0.20	1.00	0.17	0.04
Total	1.20	6.00	1.00	0.23
System	**Buy**	**Do Not Buy**	**Local Weights**	**Global Weights**
Buy	1.00	3.00	0.75	0.02
Do Not Buy	0.33	1.00	0.25	0.01
Total	1.33	4.00	1.00	0.03

3.1. Consensus Formation

When there is great degree of divergence among the expert judgments, some of the differences may be eliminated by allowing the experts to interact. There are several methods that seek to facilitate such interaction. The most widely known being the Delphi method (Dalkey, 1969; Linstone and Turoff, 1975). The essence of the Delphi method is to allow the experts to revise their judgments in light of a group summary based on the initial estimates of the experts, such as the mean, median, and the quantiles of the initial estimates. The process of revision is continued until a consensus is reached or no further reduction in divergence can be achieved. A variation of the Delphi method is the Shang Inquirer (Ford, 1975). The Shang Inquirer relies on binary response alternatives to indicate an expert's judgment of the location of the true value of an estimate relative to some reference point. The reference point is determined using the modal response of the experts. Both the Delphi and the Shang Inquirer methods have been criticized on grounds that they have a built in bias towards consensus. Several authors have suggested alternative methods that provide more qualitative feedback to the experts (see, e.g., DeGroot, 1975; Chatterjee and Seneta, 1977; and Press, 1978). Most of these methods are theoretical schemes orginating from behavioral considerations and are difficult to apply.

3.2. Numerical Aggregation

In some cases, there may not be any desire to necessarily develop consensus. Besides, even after interaction among the experts, differences in judgments may persist. These cases require statistical methods for numerically aggregating the expert judgments. Several methods have been proposed in the literature (Chatterjee and Chatterjee, 1985). We first consider cases where there has been no interaction between experts. Assume that there are k experts and that θ_i, a vector, is the estimate of global priorities of interest of the ith expert. The most obvious estimate is θ, the average of the k-estimates given by

$$\theta = \left(\frac{1}{k}\right) \sum_{1}^{k} \theta_i$$

An alternative to θ is the trimmed mean. To use the trimming procedure, it is necessary to order the priority vectors. One way to order them would be to use the normed length of the vectors. The Euclidean norm of the vector θ is given by

$$\theta'\theta = \sum_{1}^{p} \theta_k^2$$

where θ' is the transpose of θ.

If $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_k$ denote the ordered estimates of the normed lengths in terms of magnitude, then the one-trimmed mean is obtained by leaving out the largest and the smallest corresponding priority vector, θ_i. The one-trimmed mean is given by:

$$\theta_T = \left[\frac{1}{(k-2)}\right] \sum_{2}^{k-1} \theta_i$$

By deleting the two largest and the two smallest, the two-trimmed mean is obtained. The merits of such a process are discussed in Huber (1979). Other methods are available if one assumes that the expert judgments are a realization from a known probability distribution function (see, e.g., Harter, 1975). If the decision maker can rank the various experts, the estimate θ can be modified to reflect the prior weights on the various experts:

$$\theta_w = \sum_{1}^{k} w_i \theta_i$$

where θ_w is the weighted estimate and w_i may be taken as

$$w_i = r_i / \sum r_i, \; w_i \geq 0, \; \sum w_i = 1$$

and r_i is the rank for the ith expert. The highest rank is given to the most well regarded expert, the next highest rank given to the next best expert and so on. There are other more complex methods for combining expert judgments if such judgments can be assumed to be realizations of some known probability distributions (Chatterjee and Chatterjee, 1985).

4. Conclusions

The analytic hierarchy process may be a powerful tool in helping make consistent, formalized decisions that are based upon multiple qualitative and quantitative attributes. The selection of a house is considered by most a very important decision yet the process usually employed is naive and not methodical. This paper showed how AHP can help buyers make a more methodical and consistent decision when ranking properties for possible purchase. It also extended the model to the weighting of prices to aid the buyer in negotiations.

This paper is being extended to ascertain the effects of small perturbations in judgments as well as assess the impact of allowing feedback relationships between various levels in the hierarchy. The model presented has potential application in the area of real estate portfolio management, site selection, and mortgage underwriting, to name a few.

Appendix 1

House Selection: Factors, Criteria, and Subcriteria

	Global Weights	
I. Proxima		**0.06**
A. Commute to Work, Member A	0.04	
1. Utility of primary route/mode		
2. Utility of alternative routes/modes		
B. Commute to Work, Member A	0.02	
1. Utility of primary route/mode		
2. Utility of alternative routes/modes		
C. Proximity to Other Locations	0.00	
1. Schools		
2. Shopping		
3. Church and other regularly frequented places		
II. Community		**0.13**
A. Quality of Schools	0.00	
B. Government Services/Taxes	0.02	
C. Social Considerations	0.04	
D. Entertainment	0.07	
III. Neighborhood		**0.41**
A. Aesthetics	0.07	
1. Quality of Infrastructure		
2. Landscaping, autos, etc.		
3. Infrastructure		

B. Safety, Noise, and Air Pollution	0.34	
1. Auto Traffic		
2. Rail, Air, and other traffic		
3. Children, neighbors, industry, and commerce		
IV. Property		**0.40**
A. Lot	0.12	
1. Utility of usable space		
2. Landscape, lawn, foliage, privacy, etc.		
3. Walkway, driveway, fences, outbuildings, etc.		
B. Building—Exterior	0.04	
1. Condition of walls		
2. Architectural design and garage		
C. Building—Interior	0.22	
1. Rooms		
a. Floor Plan		
b. Kitchen		
c. Bathrooms		
d. Living and Dining		
e. Utility and Recreation		
2. Systems	0.02	
a. HVAC		
b. Electrical		
c. Plumbing		

Notes

1. This section is based on Saaty (1986, 1980), Wind and Saaty (1980) and Saaty and Vargas (1982). Further, a number of recent articles have presented the methodology in detail, e.g., Zahedi (1986). To avoid redundancy, we only present a brief discussion of the major elements of the methodology.
2. According to Saaty (1980), the rationale for selecting the nine-point scale is that i) it was found to be the most reliable of a number of scales that were examined; and ii) it offers a wide range of levels while still being within the number of options respondents can handle (7 ± 2).
3. For a proof, see Saaty, 1980.
4. Several methods can be used for deriving priority vectors from pairwise reciprocal comparison matrices. Saaty (1980) advocates the use of the right eigenvector method. For a comparative analysis of the various methods, see also Fichtner (1986).
5. Ang, Chua, and Sellers (1979) report the application of the Delphi technique, an MADM, for estimating cash flows in the context of capital budgeting. By itself, the Delphi technique is quite restricted in its application potential. Saaty (1980) discusses situations in which the Delphi technique can be used in conjunction with the AHP.
6. This allows the buyer to modify the a priori judgments of the higher-order factors as per his or her learning during the market search. The buyer must take care, however, that such changes in the weights are not made simply to prejudice particular properties viewed.

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