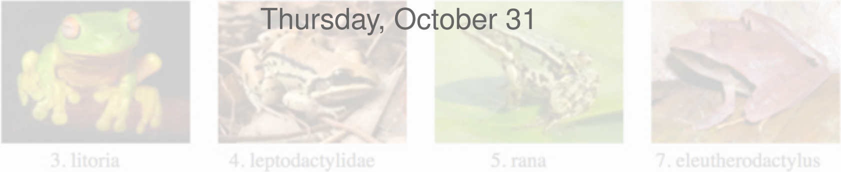
S&DS 355 / 555

Introductory Machine Learning

Word Embeddings

Thursday, October 31



Yale

For Today

- Quiz ✓
- Class-based LMs (redux)
- Embeddings
- Embedding embeddings
- Next: Neural language models

Class-based bigram model

- Model takes form

$$
\begin{aligned}
p(w_2 \mid w_1) &= p(\text{class}(w_2) \mid \text{class}(w_1))\, p(w_2 \mid \text{class}(w_2)) \\
&= p(c_2 \mid c_1)\, p(w_2 \mid c_2)
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Brown et al., "Class-based *n*-gram models of natural language"

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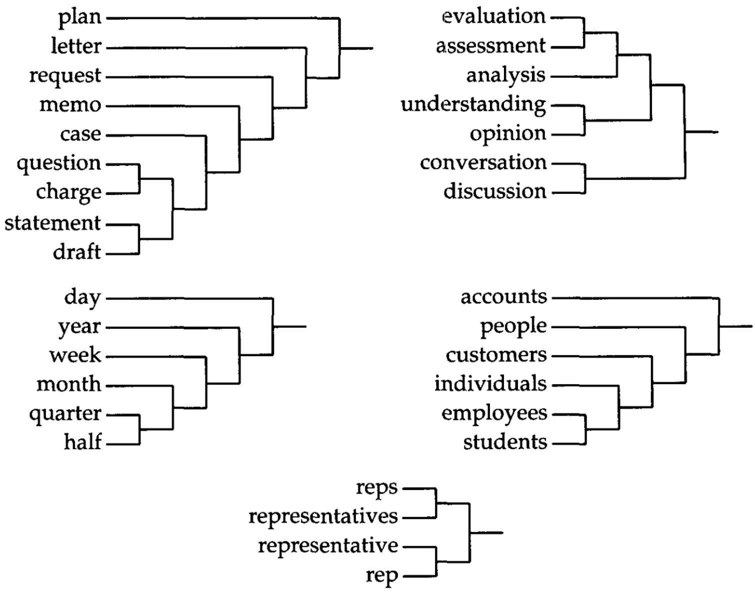
- Use bottom-up agglomerative clustering to group the words.
- In each step, merge the pair of classes that gives the smallest reduction in likelihood of the data. (The MLE bigram model has the greatest likelihood.)

Brown et al., "Class-based *n*-gram models of natural language"

Sample merges



Sample clusters

Friday Monday Thursday Wednesday Tuesday Saturday Sunday weekends Sundays Saturdays

June March July April January December October November September August

people guys folks fellows CEOs chaps doubters commies unfortunates blokes

down backwards ashore sideways southward northward overboard aloft downwards adrift

water gas coal liquid acid sand carbon steam shale iron

great big vast sudden mere sheer gigantic lifelong scant colossal

man woman boy girl lawyer doctor guy farmer teacher citizen

American Indian European Japanese German African Catholic Israeli Italian Arab

pressure temperature permeability density porosity stress velocity viscosity gravity tension

mother wife father son husband brother daughter sister boss uncle

machine device controller processor CPU printer spindle subsystem compiler plotter

John George James Bob Robert Paul William Jim David Mike

anyone someone anybody somebody

feet miles pounds degrees inches barrels tons acres meters bytes

director chief professor commissioner commander treasurer founder superintendent dean custodian

liberal conservative parliamentary royal progressive Tory provisional separatist federalist PQ

had hadn't hath would've could've should've must've might've

asking telling wondering instructing informing kidding reminding bothering thanking deposing

that tha theat

head body hands eyes voice arm seat eye hair mouth

Group globally, compute locally

- Clusters contain syntactic and semantic elements
- Surprising, since use local statistics only
- “A word is known by the company it keeps”
- Two words are similar if they appear with similar words

Perplexity

Perplexity is a simple transformation of the likelihood:

$$\text{Perplexity}(\theta) = \left(\prod_{i=1}^{N} p_\theta(w_i \mid w_{1:i-1}) \right)^{-1/N}$$

Inverse of geometric mean.

If perplexity is 100, then the model predicts, on average, as if there were 100 equally likely words to follow.

Class-based bigram model

- Clusters contain a lot of information
- Each word in a single class; partition of vocabulary
- Number of parameters: $O(C^2 + C \cdot V)$ where C is number of classes.
- Reduces size of model with relatively small increase in perplexity, $244 \mapsto 271$.

Pointwise mutual information (PMI)

Average mutual information

$$I(W_1, W_2) = \sum_{w_1, w_2} p(w_1, w_2) \log \frac{p(w_1, w_2)}{p(w_1)p(w_2)}$$

Pointwise mutual information (PMI)

$$\log \left(\frac{p_{\text{near}}(w_1, w_2)}{p(w_1)p(w_2)} \right)$$

- How likely are specific words/clusters to co-occur together within some window, compared to if they were independent?

Example clusters from PMI

we our us ourselves ours
question questions asking answer answers answering
performance performed perform performs performing
tie jacket suit
write writes writing written wrote pen
morning noon evening night nights midnight bed
attorney counsel trial court judge
problems problem solution solve analyzed solved solving
letter addressed enclosed letters correspondence
large size small larger smaller
operations operations operating operate operated
school classroom teaching grade math
street block avenue corner blocks
table tables dining chairs plate
published publication author publish writer titled
wall ceiling walls enclosure roof
sell buy selling buying sold

Shortcomings of word clusters

- Can't use vector space operations
- Doesn't give "features" of words
- These are addressed with "distributed representations" (next)

Core idea of embeddings

- Form a language model but replace classes by vectors, one for each word
- Use PMI-like scores to fit the vectors
- Can be applied whenever have cooccurrence data.

Embedding LM

Model is

$$p(w_2 \mid w_1) = \frac{\exp(\phi(w_2)^T \phi(w_1)}{\sum_w \exp(\phi(w)^T \phi(w_1))}.$$

As before,

$$\begin{aligned}\ell(\phi) &= \sum_{w_1, w_2} p(w_1, w_2) \log p(\phi_2 \mid \phi_1) p(w_2 \mid \phi_2) \\ &= I(\Phi_1, \Phi_2) - H(W)\end{aligned}$$

Thus, we want embedding vectors with high mutual information.

Constructing embeddings

Carry out stochastic gradient descent over the embedding vectors $\phi \in \mathbb{R}^d$ (where $d \approx 50$–100 is chosen by trial and error)

This is what Mikolov et al. (2014, 2015) did at Google. With a couple of heuristics:

"Distributed representations of words," (2014) "Efficient representations of words in vector space" (2015)

Constructing embeddings

Heuristics used:

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- Use "negative sampling": Approximation

$$\sum_{w} \exp(\phi(w)^T \phi(w_1))$$

$$\approx \exp(\phi(w_2)^T \phi(w_1)) + \sum_{\text{random } w} \exp(\phi(w)^T \phi(w_1))$$

Analogies

These heuristics enable training on very large text collections. Leads to vector representations of words with interesting properties.

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`Paris` is to `France` as ? is to `Germany`

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For example, analogies:

$\texttt{king}$ is to $\texttt{man}$ as ? is to $\texttt{woman}$

$\texttt{Paris}$ is to $\texttt{France}$ as ? is to $\texttt{Germany}$

$$\phi(\texttt{king}) - \phi(\texttt{man}) \overset{?}{\approx} \phi(\texttt{queen}) - \phi(\texttt{woman})$$

$$\widehat{w} = \arg\min_{w} \|\phi(\texttt{king}) - \phi(\texttt{man}) + \phi(\texttt{woman}) - \phi(w)\|^2$$

Does $\widehat{w} = \texttt{queen}$?

Learned Analogies

Table 8: *Examples of the word pair relationships, using the best word vectors from Table 4 (Skip-gram model trained on 783M words with 300 dimensionality).*

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza

Mikolov et al., "Distributed representations of words," (2014); "Efficient representations of words in vector space" (2015)

Evaluation Analogies

Type of relationship	Word Pair 1		Word Pair 2	
Common capital city	Athens	Greece	Oslo	Norway
All capital cities	Astana	Kazakhstan	Harare	Zimbabwe
Currency	Angola	kwanza	Iran	rial
City-in-state	Chicago	Illinois	Stockton	California
Man-Woman	brother	sister	grandson	granddaughter
Adjective to adverb	apparent	apparently	rapid	rapidly
Opposite	possibly	impossibly	ethical	unethical
Comparative	great	greater	tough	tougher
Superlative	easy	easiest	lucky	luckiest
Present Participle	think	thinking	read	reading
Nationality adjective	Switzerland	Swiss	Cambodia	Cambodian
Past tense	walking	walked	swimming	swam
Plural nouns	mouse	mice	dollar	dollars
Plural verbs	work	works	speak	speaks

Mikolov et al., "Distributed representations of words," (2014); "Efficient representations of words in vector space" (2015)

GloVe

Shortly after: Stanford group introduced a computational expedient (with attempt to give a "principled" motivation)

$$\mathcal{O}(\phi) = \sum_{w_1, w_2} f(c_{w_1,w_2}) \left(\phi(w_1)^T \phi(w_2) - \log c_{w_1,w_2} \right)^2$$

where $c_{w,w'}$ are cooccurrence counts.

- A type of regression estimator. Can interpret/relate this to other objectives.
- Main advantage is that SGD can be carried out much more efficiently

Pennington et al., "GloVe: Global vectors for word representation," (2015)

GloVe

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where $c_{w,w'}$ are cooccurrence counts.

- Heuristic weighting function

$$f(x) = \left(\frac{x}{x_{\text{max}}} \right)^{\alpha}$$

  where $\alpha = 3/4$ set empirically.

- So $10^{-4} \mapsto 10^{-3}$. Each order of magnitude down gets "boosted" by 1/4-magnitude.

Pennington et al., "GloVe: Global vectors for word representation," (2015)

GloVe site and code

GloVe: Global Vectors for Word Representation

Jeffrey Pennington, Richard Socher, Christopher D. Manning

Introduction

GloVe is an unsupervised learning algorithm for obtaining vector representations for words. Training is performed on aggregated global word-word co-occurrence statistics from a corpus, and the resulting representations showcase interesting linear substructures of the word vector space.

Getting started (Code download)

- Download the code (licensed under the Apache License, Version 2.0)
- Unpack the files: unzip GloVe-1.2.zip
- Compile the source: cd GloVe-1.2 && make
- Run the demo script: ./demo.sh
- Consult the included README for further usage details, or ask a question
- The code is also available on GitHub

Download pre-trained word vectors

- Pre-trained word vectors. This data is made available under the Public Domain Dedication and License v1.0 whose full text can be found at: http://www.opendatacommons.org/licenses/pddl/1.0/.
  - Wikipedia 2014 + Gigaword 5 (6B tokens, 400K vocab, uncased, 50d, 100d, 200d, & 300d vectors, 822 MB download): glove.6B.zip
  - Common Crawl (42B tokens, 1.9M vocab, uncased, 300d vectors, 1.75 GB download): glove.42B.300d.zip
  - Common Crawl (840B tokens, 2.2M vocab, cased, 300d vectors, 2.03 GB download): glove.840B.300d.zip
  - Twitter (2B tweets, 27B tokens, 1.2M vocab, uncased, 25d, 50d, 100d, & 200d vectors, 1.42 GB download): glove.twitter.27B.zip
- Ruby script for preprocessing Twitter data

Citing GloVe

Jeffrey Pennington, Richard Socher, and Christopher D. Manning. 2014. GloVe: Global Vectors for Word Representation. [pdf] [bib]

Highlights

1. **Nearest neighbors**
   The Euclidean distance (or cosine similarity) between two word vectors provides an effective method for measuring the linguistic or semantic similarity of the corresponding words. Sometimes, the nearest neighbors according to this metric reveal rare but relevant words that lie outside an average human's vocabulary. For example, here are the closest words to the target word *frog*:

   0. *frog*
   1. frogs
   2. toad
   3. litoria
   4. leptodactylidae
   5. rana
   6. lizard
   7. eleutherodactylus

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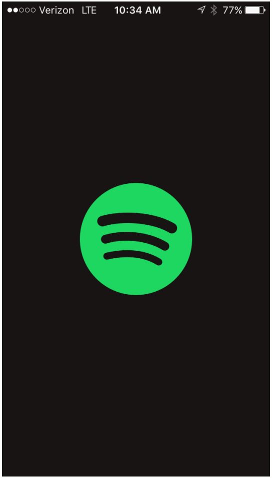
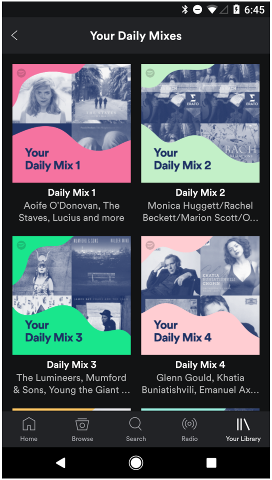
2. **Linear substructures**
   The similarity metrics used for nearest neighbor evaluations produce a single scalar that quantifies the relatedness of two words. This simplicity can be problematic since two given words almost always exhibit more intricate relationships than can be captured by a single number. For example, *man* may be regarded as similar to *woman* in that both words describe human beings; on the other hand, the two words are often considered opposites since they highlight a primary axis along which humans differ from one another.

Recommendation via Embedding





Notebook

Let's go to the Python notebook!

Embedding embeddings: t-SNE

- How can we visualize the embeddings?

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- How can we visualize the embeddings?
- We’re in a very high dimensional space
- Can do PCA, but this will introduce an additional projection/approximation step
- Many visualization techniques exist. A currently popular one is t-SNE: “Student-t Stochastic Neighborhood Embedding”

t-SNE

Here's the idea behind t-SNE:

- Form a language model using the embeddings

Pronounced: **tee**-snee

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- Represent word i by $y_i \in \mathbb{R}^2$. Use a heavy-tailed distribution (Student-t with one degree of freedom)
- Select y_i using stochastic gradient descent

Pronounced: **tee**-snee

t-SNE: Detailed algorithm

For each word w_i compute a language model

$$P_{j\,|\,i} \propto \exp\left(-\frac{\|\phi(w_i) - \phi(w_j)\|^2}{2h_i^2}\right)$$

That is:

$$P_{j\,|\,i} = \frac{\exp\left(-\frac{\|\phi(w_i) - \phi(w_j)\|^2}{2h_i^2}\right)}{\sum_k \exp\left(-\frac{\|\phi(w_i) - \phi(w_k)\|^2}{2h_i^2}\right)}$$

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Choose the bandwith h_i so that the perplexity is, say, 10. This puts the probabilities all on the same scale.

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Now form

$$P_{ij} = \frac{1}{2}\left(P_{j\,|\,i} + P_{i\,|\,j}\right)$$

as a simple way of symmetrizing.

t-SNE: Detailed algorithm

Now form Student-t distribution depending on the visualization vectors $y_i \in \mathbb{R}^2$:

$$Q_{ij} \propto \left(1 + \|y_i - y_j\|^2\right)^{-1}$$

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That is:

$$Q_{ij} = \frac{\left(1 + \|y_i - y_j\|^2\right)^{-1}}{\sum_{k \neq \ell} \left(1 + \|y_k - y_\ell\|^2\right)^{-1}}$$

This has fatter tails than a Gaussian

t-SNE: Detailed algorithm

Finally, run stochastic gradient descent (SGD) over the vectors y_i to optimize:

$$\widehat{y} = \arg\min \sum_{ij} P_{ij} \log P_{ij}/Q_{ij}$$
$$= \arg\max \sum_{ij} P_{ij} \log Q_{ij}$$

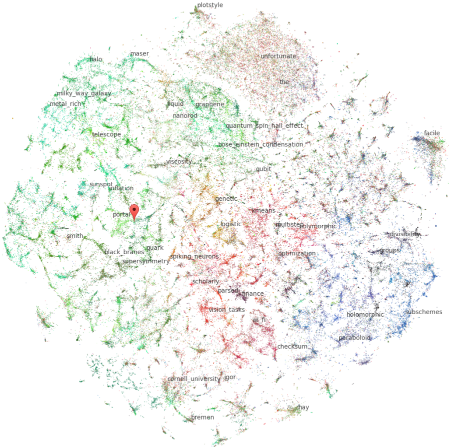
Interpretation: if $\phi(w_i)$ is very close to $\phi(w_j)$ then y_i will be close to y_j.
(long distances may be stretched further...)

t-SNE: More info and examples

`https://lvdmaaten.github.io/tsne/`
`http://cs.stanford.edu/people/karpathy/tsnejs/`

Note: This is just a visualization technique, to give intuition for the high dimensional embedding

t-SNE: Examples



http://www.cs.cornell.edu/~ginsparg/arxiv/gmaps2.html

Summary: Word embeddings

- Word embeddings are vector representations of words, learned from cooccurrence statistics
- The models can be viewed in terms of logistic regression and class-based bigram models
- Surprising semantic relations are encoded in linear relations
- Various heuristics have been introduced to get scalability
- Embeddings improve with more data
- t-SNE is an algorithm for visualizing embeddings