EJERCICIOS CAPITULO 3- MODELADO Y SIMULACION.

3.1

3.1.1 $\dot{x} = 1 + rx + \frac{a}{x}$

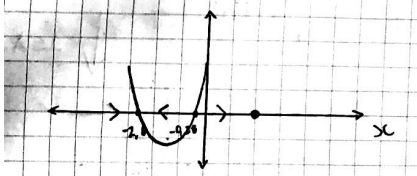
$\dot{x} = 0 \longrightarrow$

$x_{1,2} = \frac{-r \pm \sqrt{r^2-4}}{2}$

$= -\frac{r}{2} \pm \frac{\sqrt{r^2-4}}{2}$

$\left(x + \frac{r}{2} + \frac{\sqrt{r^2-4}}{2}\right)\left(x + \frac{r}{2} + \frac{\sqrt{r^2-4}}{2}\right) = 0$

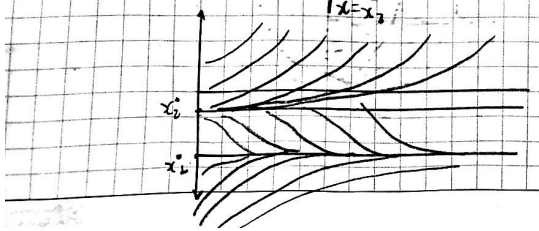
$r^2 - 4 > 0 \longrightarrow r > 2$



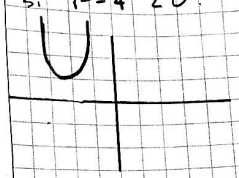
$x_1^* = \frac{-3-\sqrt{5}}{2}$, $x_2^* = \frac{-3+\sqrt{5}}{2}$

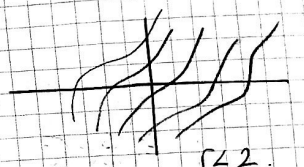
$fx = 2x + r \rightarrow fx\big|_{x=x_1^*} = -3 - \sqrt{5} + 3 = -\sqrt{5}$

$fx\big|_{x=x_2^*} = -3 + \sqrt{5} + 3 = \sqrt{5}$



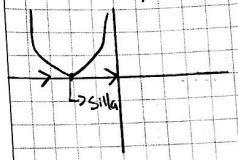
si $r^2 - 4 < 0$

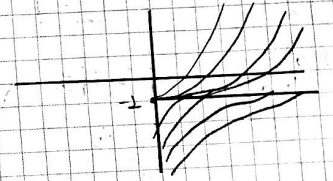




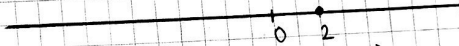
$r < 2$.

$r^2 - 4 = 0 \longrightarrow r = 2$

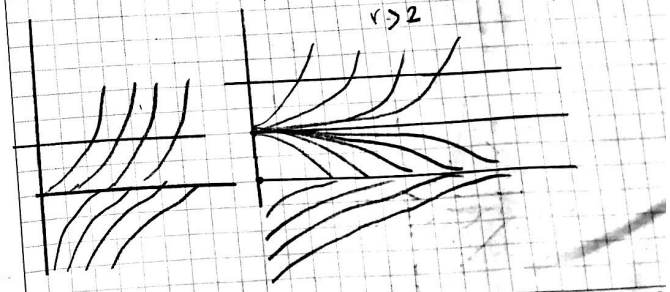


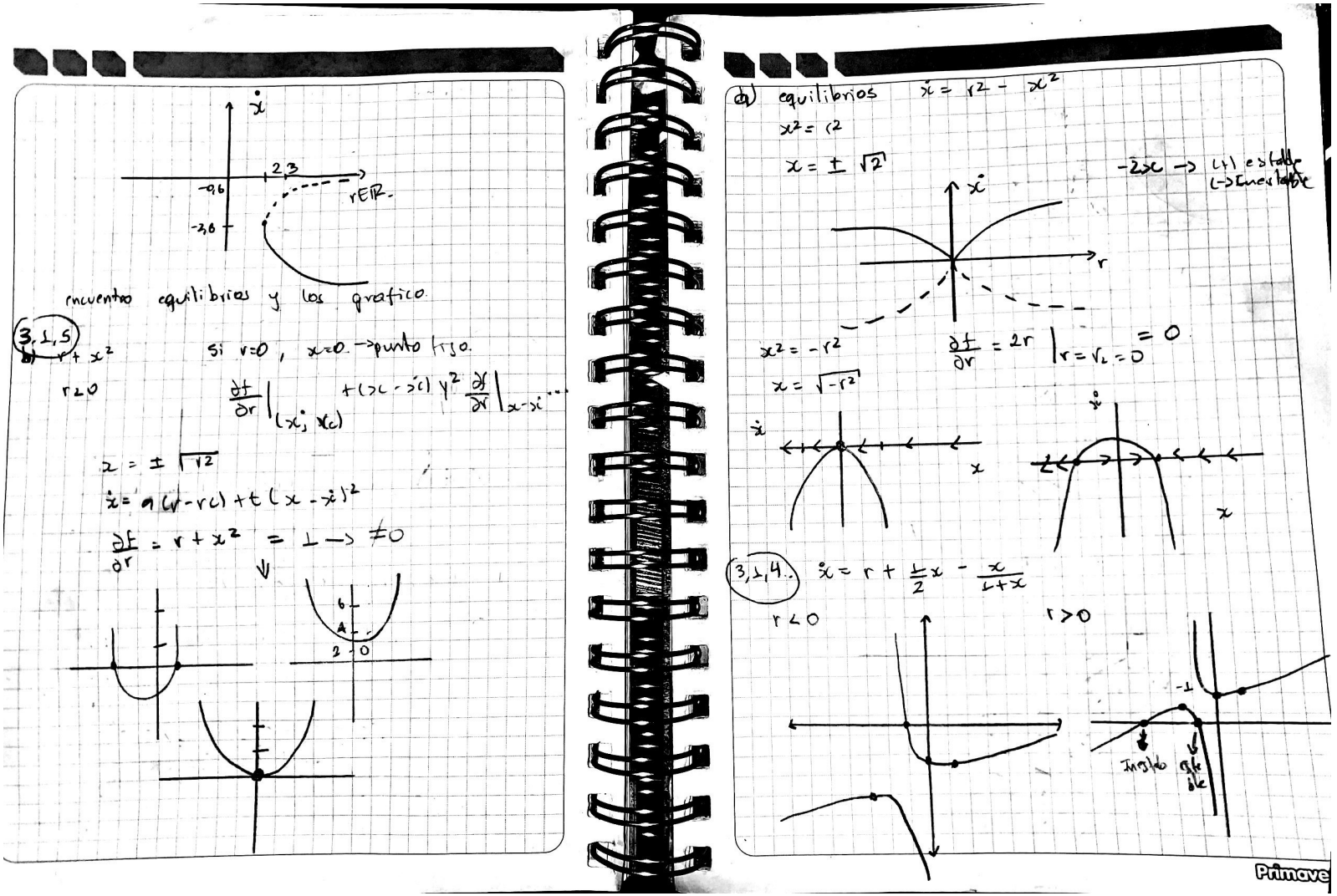


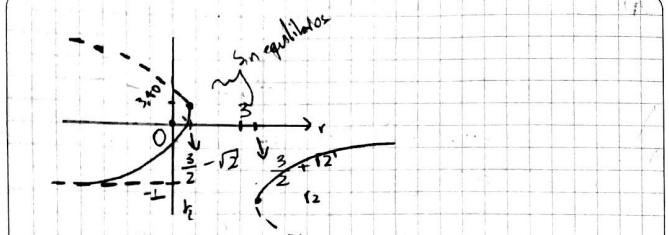
$r \in (-\infty, 2)$, $r = 2$, $r \in (2, \infty)$



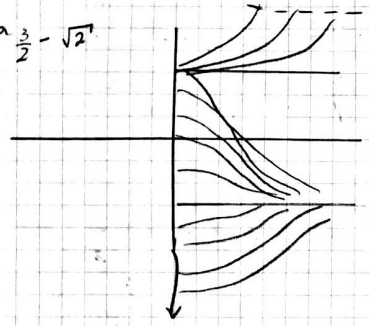
$P = \{(-\infty, 2), \{2\}, (2, \infty)\}$



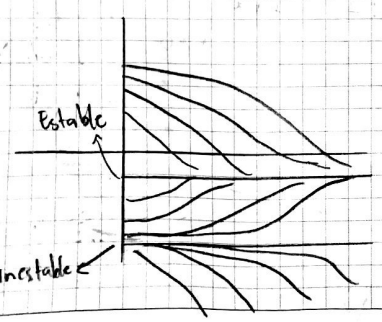




Para $\frac{3}{2}-\sqrt{2}$



Para $r > 3/2 + \sqrt{2}$



3,1,3 $\dot{x} = r + x - \ln(1+x)$

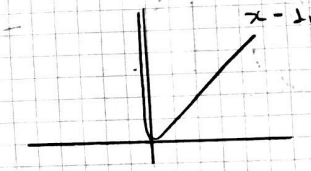
$\dot{x} = r + x - \ln(1-x)$

$r + x - \ln(1+x) = 0 \rightarrow x=1,\ x=2$

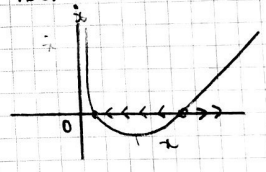
$r = \ln(1+x) - x \rightarrow$ dominio $\rightarrow x > -1$ $(-1, \infty)$

punto en $x = -1$ $\rightarrow$ estable

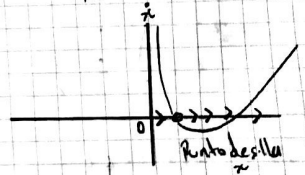
r no puede ser mayor a cero, no tiene puntos de equilibrio



Para $r=-1$



Para $r=0$



Para $r=1$

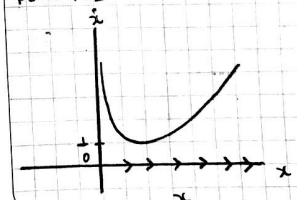


Diagrama de Bifurcación

3, 1, 2

$\dot{x} = r - \cosh x$ → nunca será menor que 1, entonces $r \geq 1$

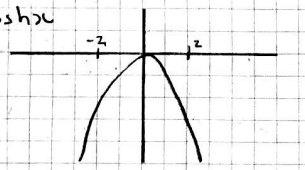
$r - \cosh x = 0$

$x_1 = \arccos(r)$

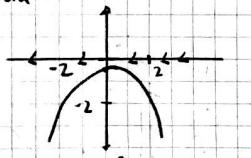
$x_2 = -\arccos(r)$ → punto inestable

entonces, $r = 0$, $r = 1$, $r = 2$.

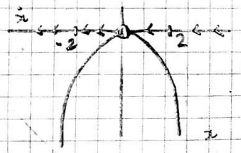
$r - \cosh x$



Para $r = 0$



Para $r = 1$



Para $r = 2$

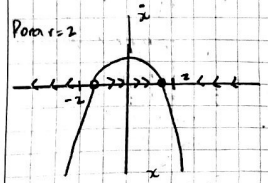
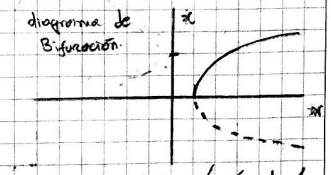
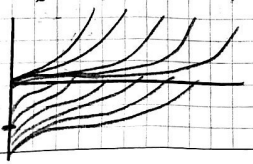


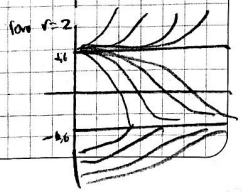
diagrama de Bifurcación



Para $r = 1$



Para $r = 2$



3, 2

3, 2, 1

$\dot{x} = rx + x^2$

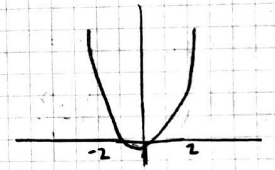
$rx + x^2 = 0$

$x_1 = \frac{-r + \sqrt{r^2 - 4(1)(0)}}{2} = \frac{-r + \sqrt{r^2}}{2} = \frac{0}{2} = 0$

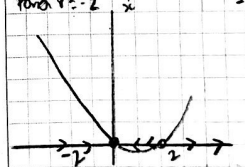
$x_2 = \frac{-r - \sqrt{r^2 - 4(1)(0)}}{2} = -r - \sqrt{r^2 - 4(1)(0)} = -r - \sqrt{r^2}$

$= \frac{-2r}{2} = -r$

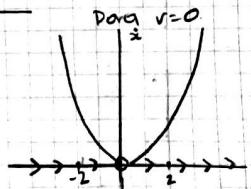
Si $r < 0$ → punto estable, $(x_1) \to (x_2)$ será punto inestable



Para $r = -2$ $\dot{x}$



Para $r = 0$



Para $r = 2$ $\dot{x}$

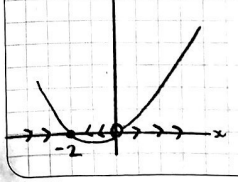
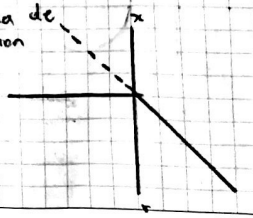


diagrama de Bifurcación



Primavera

3,2,2 $\dot{x} = rx - \ln(1+x)$

$rx - \ln(1+x) = 0$

$x(r - \frac{\ln(1+x)}{x}) = 0$

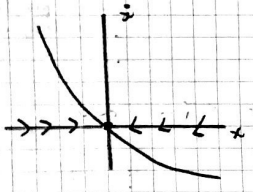
$x = 0$ con $r < 1$

$\dot{x} = x - \ln(1-x) \rightarrow x = 0 \rightarrow$ inestable

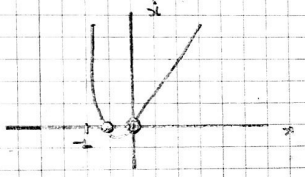
$r = \frac{\ln(1+x)}{x} \rightarrow x = -1$

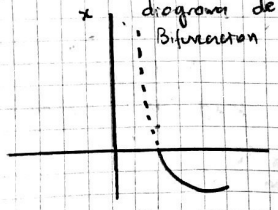
Grafica en anteriores ejercicios

Para $r = 0$.



Para $r = 1$

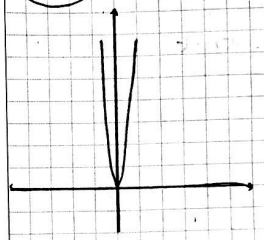


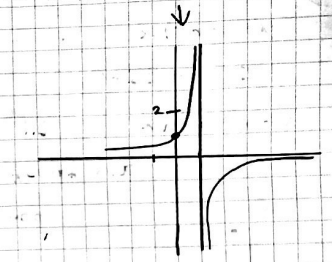


para

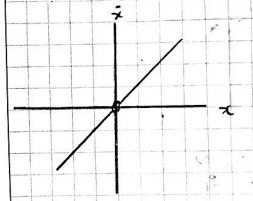
$\dot{x} = rx - \ln(1+x)$

3,2,3 $\dot{x} = x - rx(1-x) \rightarrow x - rx + rx^2$





Para $r = 0$



$x - rx + rx^2 = 0$

$rx^2 + (1-r)x = 0$

$x_1 = \frac{-(1-r) + \sqrt{(1-r)^2 - 4r0}}{2r} = 0$

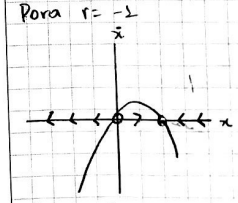
$x_2 = \frac{-(1-r) - \sqrt{(1-r)^2 - 4r0}}{2r} = -$

$= -(1-r) - \sqrt{(1-r)^2 - 0} = -2(1-r)$

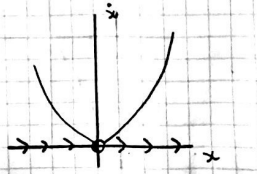
$= \frac{-2(-r+1)}{2r} = -\frac{r-1}{r}$ //

con $r \neq 0$.

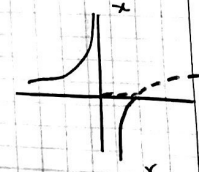
Para $r = -1$



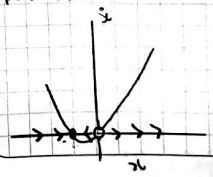
Punto de silla



Diograma de Bifurcación



Para $r = 1$



Primavera

3,2,4) $\dot{x} = x(r - e^x)$ $\dot{x} = 0$ si $r - e^x = 0$ → $r = e^x$

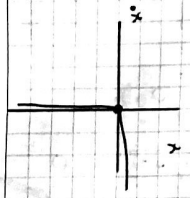
$x(r - e^x) = 0$. $xr - e^x x = 0$.

$\dot{x} = 0 \rightarrow x_1 = 0$, $\ln x = r - \ln e^x$

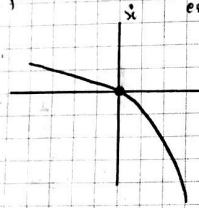
$\ln r = x \rightarrow \dot{x} = \ln r$ indefinido si $r < 0$ en $0 < r < 1$, $\ln r < 0 \rightarrow r = 1$, $\ln r = 0 \rightarrow r > 1$

$\dot{x} = f(x) \rightarrow f'(x) = r - e^x + 1 e^x$ con $x = 0 \rightarrow f(x) = r - 1$ | $(+) \rightarrow r > 1$, $(-) \rightarrow r < 1$

$\dot{x}$ $r = 0$.



para $r = 1$ → sólo un punto estable con $x = 0$



para $r = 1 \rightarrow (x - e^x)$ si r crece, $r < 0$, se acerca más a $\dot{x}$.

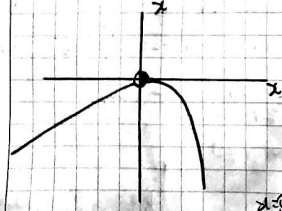
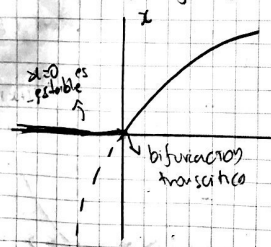


diagrama de Bifurcación



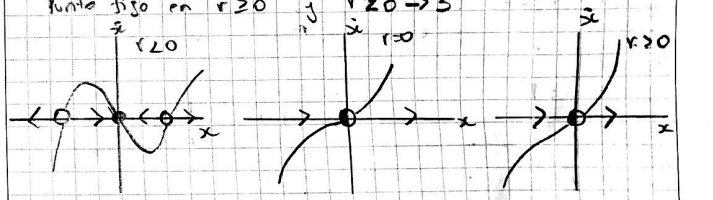
3,4,1) $\dot{x} = rx + 4x^3$

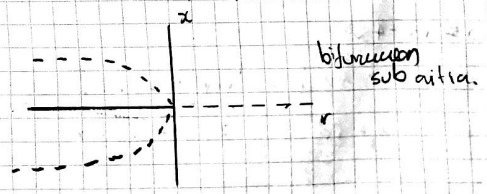
$f(x) = rx + 4x^3 = x(r + 4x^2) = 0$

$\dot{x} = 0$, $x_1 = \frac{-r + \sqrt{r^2 - 4 \cdot 0}}{2} = + \sqrt{\frac{-r}{2}}$ } sólo existe para $r < 1$

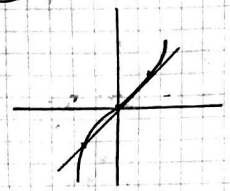
Inestable para $r > 0$. $x_2 = \frac{-r - \sqrt{r^2}}{2} = -\sqrt{\frac{-r}{2}}$

Punto fijo en $r \geq 0$ y $r \geq 0 \rightarrow 3$





3,4,2 $\dot{x} = rx - \operatorname{senh} x \rightarrow r = \frac{\operatorname{senh} x}{x}$

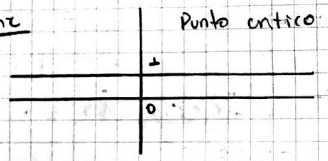


3 puntos fijos para $r > 1$

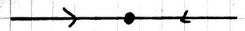
1 punto fijo para $r < 1$

$x = 0 \rightarrow$ metastable para $r > 1$

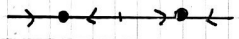
$r = \frac{\operatorname{senh} x}{x}$ Punto crítico $r = 1$

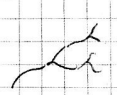


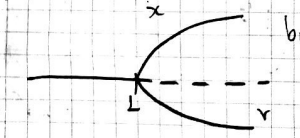
Para $r > 1$



Para $r < 1$







bifurcación supercrítica

3,4,3 $\dot{x} = rx - 4x^3$

$f(x) = x(r - 4x^2) = x(r - 4x^2) = 0$

$x = 0, \quad x_1 = \frac{\sqrt{r}}{2}, \quad x_2 = -\frac{\sqrt{r}}{2}$

$r < 0$, $r = 0$, $r > 0$

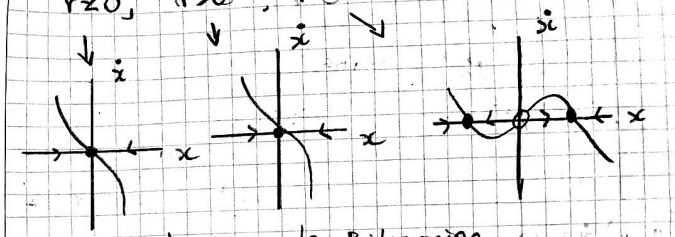
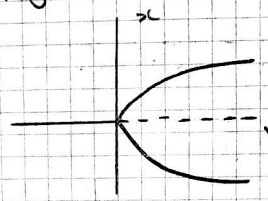


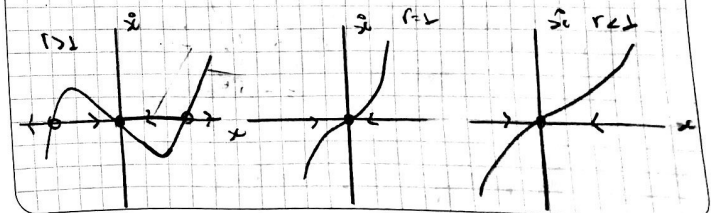
diagrama de Bifurcación



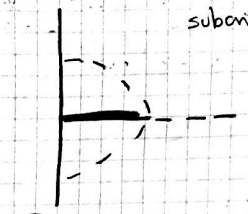
3,4,4 $\dot{x} = x + \frac{rx}{1+x^2}$ $x = 0 \rightarrow$ Punto fijo $\rightarrow$ estable

$\dot{x} = x + rx + 0x^3 \approx (r+1)x$ estable para $(r+1) < 0$

$-1 = \frac{r}{1+x^2} \rightarrow 1 + x^2 = r, \quad x = \pm\sqrt{-(r+1)}$ sole existe para $r + 1 < 0$, $r < -1$ } subcrítico



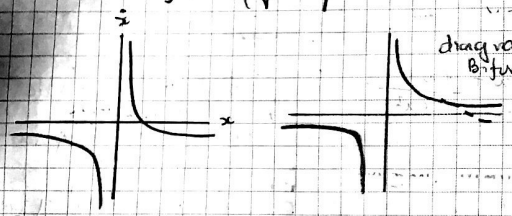
subcrítico.



3,4,6 $\dot{x} = rx - \frac{x}{1+x}$

$\dot{x} = rx - \frac{x}{1+x} = x\left(\frac{r-1}{1+x}\right)$ $r>1$ $r<1$?

$x=0$, $x = \left(\frac{1}{r} - 1\right)$ con $r \neq 0$



3,4,8 $\dot{x} = rx - \frac{x}{1+x^2}$

$x=0$ → punto fijo para todo r.

Sin puntos fijos para $x = \left(\frac{r-1}{1+x^2}\right)$

$x_{1,2} = \pm\sqrt{\frac{1}{r} - 1}$

existe para $\frac{1}{r} - 1 > 0$, $0 < r < 1$

$x=0$ → estable para $r<1$ y $\}$ Subcrítica en $r=1$ y $x=0$
inestable para $r>1$

$f(x,r) = rx - x(1+x^2)^{-1} = (r-1)x + x^3 + O(x^5)$

3,4,10 $\dot{x} = rx + \frac{x^3}{1+x^2}$

$x=0$ → punto fijo.

$-r = \frac{x^2}{(1+x^2)}$ → $-r(1+x^2) = x^2$

$x^2(1+r) = -r$

$x_{1,2} = \pm\sqrt{\frac{-r}{1+r}}$

$\frac{-r}{1+r} > 0$ para tener 3 puntos fijos

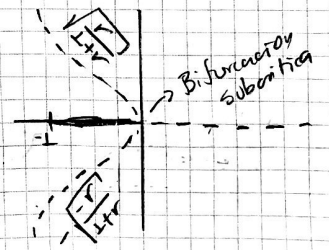
" < 0 para $r>0$

" < 0 para $r<-1$

" para $-1 < r < 0$ → 3 puntos fijos

estable en $x=0$ para $r \begin{cases} >0 & \text{inestable} \\ <0 & \text{estable} \end{cases}$

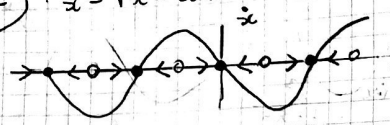
Por lo tanto, horquilla para $r=0$, pero no para $r=-1$



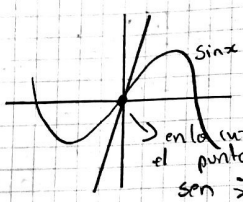
Primav

(3, 4, 11) $\dot{x} = rx - \operatorname{sen} x$.

a.



b. $r > 1$



en la intersección solapado > 1
el punto fijo es inestable
sen $\dot{x} \approx (r-1)x$.

c. a $r = 1$, hay tipo orquilla

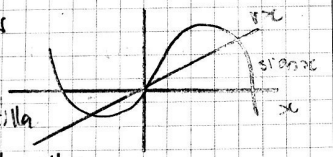
$r > 1 \rightarrow 1$ punto fijo (inestable)

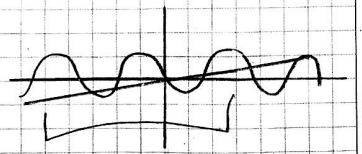
$r \downarrow 1 \rightarrow 3$ puntos fijos

bifurcación subcrítica

cada vez hay más puntos de silla

como $r \rightarrow 0$, 00 es punto de silla





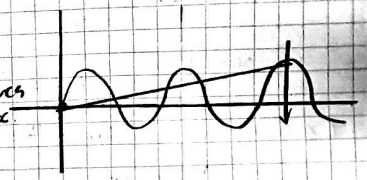
d. para $r > 0$,

$rx \approx \operatorname{sen} x \approx 1 \rightarrow r \approx \frac{1}{x}$

$r(4m+1)\frac{\pi}{2} \approx 1$

$r \approx [(4m+1)\frac{\pi}{2}]^{-1}$

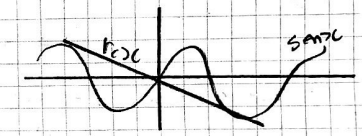
la bifurcación ocurre cerca al valor máximo de sen x



$x \approx \frac{\pi}{2} + 2m\pi$

$= (4m+1)\frac{\pi}{2}$ con $m \in \mathbb{Z}$

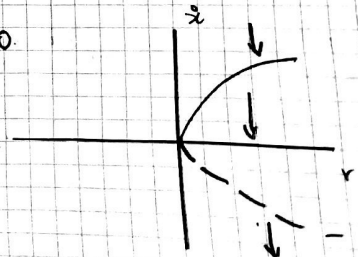
e.



(3, 4, 5) $\dot{x} = r - 3x^2 \rightarrow$ puntos fijos

$x = \pm\sqrt{\frac{r}{3}}$ para $r > 0$.

Punto de silla en $r = 0$.



3.4.7 $\dot{x} = 5 + re^{-x^2}$ punto fijo

$e^{-x^2} = \frac{5}{r}$

$-x^2 = \ln\left(\frac{5}{r}\right)$

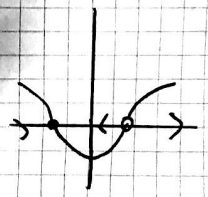
$x = \pm\sqrt{\ln(r/5)}$

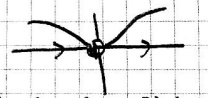
$r \leq 0$ → sin puntos fijos ($\ln(-1)$)

$0 < r < 5$ sin puntos fijos

$r = 5$; $x = 0$ es un punto fijo , $\dot{x} = 5(1 - e^{-x^2})$

$r > 5$ → 2 puntos fijos

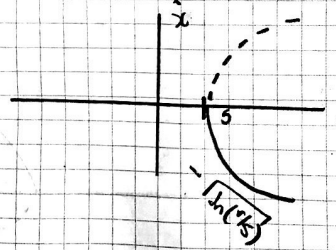




cerca a $x=0$, $\dot{x} = 5 - r(1 - x^2) + O(x^4)$

$= (5-r) + rx^2 + O(x^4)$

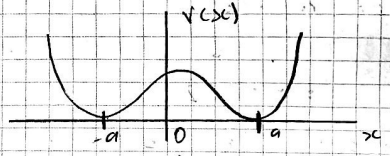
punto de silla en $r=5$



3.4.25 $\dot{x} = rx + x^3 - x^5 = \frac{-dV}{dt}$

$V(x) = -\frac{1}{2}rx^2 - \frac{1}{4}x^4 + \frac{1}{6}x^6$

con $V=0$



$V(\pm a) = 0$, $V'(\pm a) = 0$ → $(a \neq 0)$ $a > 0$.

$-\frac{1}{2}ra^2 - \frac{1}{4}a^{\cancel{2}4} + \frac{1}{6}a^4$

$-\frac{1}{2}r = \frac{1}{4}a^2 - \frac{1}{6}a^4$

$r = -\frac{1}{2}a^2 + \frac{1}{3}a^4$

$V'(\pm a) = 0$ → $r + a^2 - a^4 = 0$ → $r = a^4 - a^2$

$a^4 - a^2 = -\frac{1}{2}a^2 + \frac{1}{3}a^4$ → $\frac{2}{3}a^{\cancel{4}2} - \frac{1}{2}a^2 = 0$

$a^2 = \frac{3}{4}$ → $a = \frac{\sqrt{3}}{2}$

$r + a^2 - a^4 = 0$ → $r = a^4 - a^2$

$r = \left(\frac{-\sqrt{3}}{2}\right)^4 - \left(\frac{\sqrt{3}}{2}\right)^2$

$r = -\frac{3}{16}$ //

3.5

(3.5.3)

$$\frac{d\phi}{dt} = \operatorname{sen}\phi(r\cos\phi - 1)$$

$$= f(\phi)$$

$$f'(\phi) = \cos\phi(r\cos\phi - 1) + \operatorname{sen}\phi(-r\operatorname{sen}\phi)$$

$$= -\cos\phi + r(\cos^2\phi - \operatorname{sen}^2\phi)$$

$$= -\cos\phi + r(2\cos^2\phi - 1)$$

$$f'(\pi) = 1 + r(2-1) = r + 1 > 0.$$

$\phi^* = \pi \longrightarrow$ inestable

$$f'(0) = -1 + r(2-1) = r - 1 \begin{cases} \text{estable } r < 1 \\ \text{inestable } r > 1 \end{cases}$$

$$f'(\pm\cos^{-1}(r^{-1})) =$$