Granular Gravity: Equity-Bond Returns and Correlation

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ABSTRACT

I find strong empirical evidence that the correlation between firms bond and equity returns increases significantly when the distribution of firms in the economy becomes more granular than atomistic. Data supports the hypothesis that this arises from granularity being a priced factor in the cross-sections of both equity and bond returns. I construct a theoretical framework in which risk inherent in very large firms grows systematic. This granular channel brings about two predictions validated by the data: First, bond and equity returns co-move in the same direction with respect to granularity shocks. Second, this co-movement is due to a mutually and similarly priced factor in the cross section of equity and bond returns.

JEL Classification Numbers: G12, G14

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I. Introduction

As of February 2021, the five biggest high tech firms, GAFAM [1], are worth \$7.8 trillion; the NYSE \$26 trillion. In this granular economy, one comprised of few gigantic, and numerous atomic firms, some key assumption in portfolio theory, diversification of firm-specific risk barely holds. We know that risk inherent in very large firms (grains) is not compressible (Gabaix, 2011). Moreover, systematic versus idiosyncratic components of risk share a common component (Herskovic, Kelly, Lustig and Van Nieuwerburgh, 2016). I propose a theoretical framework, alongside ample empirical evidence, of how granularity drives correlations and why it is a priced factor in the cross section of bond and equity returns.

The extent of influence of incompressibility of firm specific risk has been subject of a growing strand of literature in macroeconomics (Gabaix, 2011; di Giovanni and Levchenko, 2012; di Giovanni, Levchenko, and Mejean, 2014; Barrot and Sauvagnat, 2016; Carvalho and Grassi, 2019; Gabaix and Koijen, 2020). Yet, there is little known about whether some key determinants of relevance in asset pricing theory, risk premia and correlations, behave differently in a granular financial market, and if so, how.

In this paper, I investigate correlations and risk premia in a granular financial market. While bond and equity carry different risk profiles, I show that they behave similarly in response to granular (discount rate) shocks. Such impact is observed in how bond and equity returns co-move. In other words, I show that granularity is a channel for equity-bond correlations (henceforth EBC). I model this channel by using the immediate consequence of a granular market: Risk inherent in grains does not diversify away, hence it is systematic.

Commonly priced or correlated risk factors in equity and bond markets are potential drivers of EBC (Elton, Gruber, Agrawal, and Mann, 2001; Bethke, Gehde-Trapp, and Kempf, 2017). I first provide evidence that conditional correlations are significantly related to granularity. Then, I test the implication of my theoretical framework and show that granularity is mutually priced in the cross section of bond as well as equity markets. Admittedly, granularity is far from being the sole driver of EBC. However, the empirical evidence herein documents that it is an important channel, and these finding are supported by my theoretical contribution.

Granularity is measured similar to market concentration. Gabaix and Koijen (2020) use such measure to identify exogenous shocks stemming from grains. I slightly modify the idea and consider such measure the likelihood of propagation of granular shocks. This approach facilitates an empirical investigation of the cross section of security returns rather than identification of a specific exogenous shock. When the economy is more granular, it is more likely that

[1]Google, Apple, Facebook, Amazon, and Microsoft

diversification does not fully fulfills its role, and it is more likely to trace the impact of granularity in financial markets. A measure of granularity is meant to assess market concentration. As a spectrum, the higher end of this measure corresponds to an economy where grains exert a larger influence. On the other hand, lower granularity signals a relatively more size-homogenous market, hence a lesser impact. That said, I show that both correlations and risk premia behave significantly differently in high versus low granularity environments.

Throughout this paper I consider *changes* in granularity in my empirical analysis. The main reason is the persistence of granularity at level when all other variables of interest, returns, are least persistence. Also, over the 2002-2019 period, months of high versus low granularity coincide with macroeconomic conditions and cluster around specific months. Changes in granularity, on the other hand, are not clustered and high versus low months are fairly spread across all months.

In the remainder of this paper, I provide a brief review of the literature where I found this study in Section II. I introduce the theoretical framework in Section III, and describe my data in Section IV. I then proceed by the empirical approach in Section V, and concluding remarks in Section VI.

II. Literature Review

This paper contributes to three strands of literature: granularity, the drivers of the bond-equity conditional correlation, and cross sectional predictors of corporate bond returns. I provide a brief review of either one in the following sections.

Like many economic variables, the size of the firms is power law, and not normally distributed. As a consequence, firm-specific risk of relatively large firms becomes incompressible (Gabaix, 2011; Gabaix and Koijen, 2020). This phenomenon also implies that country size and trade affect macroeconomic volatility (di Giovanni and Levchenko, 2012), that large firms drive business cycles (Carvalho and Grassi, 2019), and that firm-specific shocks propagate through production networks and affect firms' sales growth and stock prices (Barrot and Sauvagnat, 2016).

From an asset pricing perspective, Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) find that systematic versus firm-specific risk are not easily distinguished, suggesting that the two are fundamentally linked and driven by a common component. Herskovic, Kelly, Lustig, and Nieuwerburgh (2020) explore the channels through which firm size distribution determines how firm-specific shocks propagate and affect firm volatilities. Abolghasemi, Bhamra, Dorion, and Jeanneret (2020) show that the slope of the SML is flat even negative, when granularity is high, hence evaluating the conditional CAPM and the low-risk anomaly in a granular economy.

Traditional predictors of corporate bond returns, the term and default spreads, reflect the economic conditions. The term spread captures unexpected changes in interest rates, and the default spread the likelihood of default (Fama and French, 1993). Other variables such as inflation (Elton, Gruber, and Blake, 1995; Kang and Pflueger, 2015), or uncertainty about the macroeconomy (Bali, Subrahmanyam, and Wen, 2020b), are shown to affect corporate bond returns.

Elton et al. (2001) provide evidence that excess corporate bond returns are explained by common risk factors in the equity market. Subsequently, the literature substantiates the relevance of factors such as (il)liquidity (Chen, Lesmond, and Wei, 2007; Covitz and Downing, 2007; Bao, Pan, and Wang, 2011; Lin, Wang, and Wu, 2011; Friewald, Jankowitsch, and Subrahmanyam, 2012; Acharya, Amihud, and Bharath, 2013), volatility (Chung, Wang, and Wu, 2019), investor sentiment (Guo, Lin, Wu, and Zhou, 2019), momentum (Jostova, Nikolova, Philipov, and Stahel, 2013), and reversal (Bali, Subrahmanyam, and Wen, 2020a) to corporate bond returns.

Inspired by Daniel and Titman (1997), Gebhardt, Hvidkjaer, and Swaminathan (2005) compare the pricing implications of systematic risk (default and term) versus characteristics (e.g. ratings and duration) in the bond market. Corroborating the relevance of either approach, they find that systematic risk matters in the cross section of bond returns. Israel, Palharesa, and Richardson (2018) consider, in tandem, the carry, quality, momentum, and value factors in the bond market. Israelov (2019) investigates the capacity of the option markets to explain corporate bond returns.

Mostly through a default channel, factors exclusive to bonds are also shown to claim a premium in the cross section of corporate bond returns. Driessen (2004) shows that default event risk is priced in the cross section of corporate bonds. Bai, Bali, and Wen (2019) develop three bond-market-specific measures of down-side risk that are priced in the cross section of corporate bonds. Not only default events, but also ambiguity about the credit rating, an indicator for default, predicts higher premium for the corporate bonds (Kim, Kim, and Park, 2018).

Correlation between bond and equity returns, regardless of its sign, is examined in the literature via some information-based models. Kwan (1996) argues that information flows from the equity to the bond market, and Acharya and Johnson (2007) provide evidence that the equity market reacts to innovations in the CDS market, derivatives written on corporate bonds. In a similar investigation, Bao and Pan (2013) show that corporate-bond CSD returns volatilities exceed equity return volatilities and what the Merton model predicts. When it comes to the sign of the correlation, Back and Crotty (2014) use a Kyle model and argue that when information is about the asset means, the correlation is positive, and when it is about the asset risk, it is negative.

Aretz and Yang (2019) use a disinvestment model to explain the negative relation between bond returns and firm distress risk, measured by Campbell's measure of distress. The bond-equity correlation literature is richer for government issued rather than corporate bonds. Baele, Bekaert, and Inghelbrecht (2010) develop a structural model form the Treasury bonds and equity returns correlation, estimated via the component model of dynamic correlations of Colacito, Engle, and Ghysels (2011).

In a more recent work, Baele, Bekaert, Inghelbrecht, and Wei (2019) propose a regime-switching model to identify Flight-to-Safety (FTS) episodes where in distressed times, large and positive bond returns are accompanied by large and negative equity returns. They show that FTS represents a flight to quality, but not liquidity, in the corporate bond market, hence a potential explanation for equity-corporate-bond correlation.

Bethke et al. (2017) argue that bond-equity correlation stems from correlated risk factors. They show that investor sentiment is the main driver of bond-equity (positive and negative) correlation. Bektić (2018a) shows that high beta equity corresponds to low return in the equity market for European firms. Bektić (2018b) also documents momentum spill-over from the equity market to the global bond market.

III. Model

A. Setting

There are 'N' firms in the economy, represented by a cashflow dynamics that is governed, under the $\mathbb{P}$ measure, by

$$\frac{dX_i}{X_i} = \mu_i dt + \sigma_i dB_i. \tag{1}$$

where μ_i and σ_i are constant parameters, and dB_i is the incremental change of a standard Brownian motion defined on the probability space $(\Omega,\mathcal{F},\mathbb{P})$. The parameters of the model are common knowledge. The firm cashflow process is composed of a market-wide systematic as well as an orthogonal firm-specific component,

$$\frac{dX_i}{X_i} = \mu_i dt + \sigma_i \Big(\rho_i dZ_M + \sqrt{1-\rho_i^2} dZ_i\Big), \tag{2}$$

where $\rho_i = Corr(dB_i, dZ_M)$. I assume $\rho_i > 0$. In words, I exclude negative beta firms.

There is a representative agent with log utility,

$$U_t = \mathbb{E}\Big[\int_0^\infty e^{-\delta\tau}\ln(C_{t+\tau})\Big], \tag{3}$$

who consumes the aggregate firms cashflow,

$$C_t = \sum_{i=1}^{N} X_i. \tag{4}$$

With log-utility, the CCAMP is equivalent to the CAPM, and risk premiums are determined by how cashflows covary with consumption growth,

$$\begin{aligned}\frac{dC_t}{C_t} &= \sum_{i=1}^{N} w_i \frac{dX_i}{X_i}\\ &= \sum_{i=1}^{N} w_i \Big[\mu_i dt + \sigma_i\Big(\rho_i dZ_{_M} + \sqrt{1-\rho_i^2}dZ_i\Big)\Big],\\ &= \mu_{_M} dt + \sigma_{_M} dZ_{_M} + \sum_{i=1}^{N} w_i\sigma_i\sqrt{1-\rho_i^2}dZ_i,\end{aligned} \tag{5}$$

where w_i is the share of firm i in the consumption basket or $w_i = \frac{X_i}{C}$, $\mu_{_M}$ is the consumption growth, $\sigma_{_M} = \sum_{i=1}^{N} w_i\sigma_i\rho_i$ is the consumption volatility of systematic innovations. Notice that the stochastic discount factor is

$$\frac{d\pi_t}{\pi_t} = -r_f dt + \frac{dC_t}{C_t}, \tag{6}$$

and r_f is the instantaneous interest rate such that

$$\begin{aligned}r_f dt =& \delta dt + \mathbb{E}\Big[\frac{dC}{C}\Big] - Var\Big[\frac{dC}{C}\Big]\\ =& \Big(\delta + \mu_{_M} - \sigma_{_M}^2 - \sum_{i=1}^{N} w_i^2\sigma_i^2(1-\rho_i^2)\Big)dt\end{aligned} \tag{7}$$

I further assume that all firms are similarly productive, that is for any firm i, $\mu_i = \mu$. Notice that there there are two extreme cases nested in this specification. If all firms are perfectly correlated with the common systematic component, the SDF is reduced to

$$\frac{d\pi_t}{\pi_t} = -r_f dt + \sigma_{_M} dZ_{_M}, \tag{8}$$

and when $\rho_i = 0$ for all firms,

$$\frac{d\pi_t}{\pi_t} = -r_f dt + \sum_{i=1}^{N} w_i \sigma_i dZ_i. \tag{9}$$

B. Firm Risk Premia

Cochrane, Longstaff, and Santa Clara (2008) solve this model with two firms in an endowment economy where they consider w_i a stochastic process. Martin (2013) solves a three-firm economy model and provides semi-closed form solutions. To simplify the model and to attain a closed-form solution, I assume that the representative agent, at any given point of time, reveals her preferences about the share of the firms in her consumption basket.

A larger w_i signifies that the consumption basket is more dominated by a specific firm. Also, by granularity, relatively (very) large firms can bring about systematic impact through their firm-specific shocks. These shocks grow incompressible because granular economies do not yield full diversification of firm-specific risk. Incompressibility of firm-specific risk makes it indistinct from systematic risk. I model this key property of the grains by assuming that $\rho_g = w_g$, the larger the grains grow, in relative terms, the more correlated they become with the common risk component. In an extreme scenario, when there is only one firm in the economy, $\rho_g = w_g = 1$. The cashflow risk premium for any firm $'i'$ is

I am interested in how changes in the share of the grains in the consumption basket, all else equal, affects any firm $'i'$ in the economy. Hence, I examine the risk premium channel,

$$\begin{aligned} &E\Big[\frac{d\pi}{\pi}, \frac{dX_i}{X_i}\Big] = \\ &E\Big[\Big(\sigma_M dZ_M + \sum_{i=1}^{N} w_i \sqrt{(1-\rho_i^2)}\sigma_i dZ_i\Big), \Big(\rho_i \sigma_i dZ_M + \sqrt{(1-\rho_i^2)}\sigma_i dZ_i\Big)\Big] = \\ &\rho_i \sigma_i \sigma_M + w_i (1-\rho_i^2)\sigma_i^2. \end{aligned} \tag{10}$$

Cashflow risk premium is determined by two main components, one proportionate to systematic risk, second to firm-specific risk. Both components are functions of granularity. The overall effect is what I am interested in.

For the first component, it is straightforward to show that the volatility of the systematic component changes the share of the grains in the economy,

$$\begin{aligned}\frac{\partial \sigma_M}{\partial w_g} = \frac{\partial \sum_{i=1}^{N} w_i \rho_i \sigma_i}{\partial w_g} = \frac{\partial \left(w_g^2 \sigma_g + \sum_{i=1}^{N-1} w_i \rho_i \sigma_i \right)}{\partial w_g} \\ = 2 w_g \sigma_g + \sum_{i=1}^{N-1} \frac{w_i}{w_g} \rho_i \sigma_i.\end{aligned} \tag{11}$$

Accordingly,

$$\begin{aligned}\frac{\partial E\left[\frac{d\pi}{\pi}, \frac{dX_i}{X_i}\right]}{\partial w_g} &= \rho_i \sigma_i \frac{\partial \sigma_M}{\partial w_g} + \frac{\partial w_i}{\partial w_g}(1-\rho_i^2)\sigma_i^2 \\ &= \rho_i \sigma_i \Big[2 w_g \sigma_g + \sum_{i=1}^{N-1} \frac{\partial w_i}{\partial w_g} \rho_i \sigma_i \Big] + \frac{\partial w_i}{\partial w_g}(1-\rho_i^2)\sigma_i^2 \\ &= \underbrace{2\rho_i \sigma_i w_g \sigma_g}_{>0} + \underbrace{\rho_i \sigma_i \sum_{i=1}^{N-1} \frac{\partial w_i}{\partial w_g} \rho_i \sigma_i}_{<0} + \underbrace{\frac{\partial w_i}{\partial w_g}(1-\rho_i^2)\sigma_i^2}_{<0}\end{aligned} \tag{12}$$

The risk premium is affected via three channels. Since ρ_i, σ_i, and σ_g are constant parameters, the first out of three terms in equation (12) is always positively related to the risk premium. By excluding negative bate firms from the analysis, changes in w_g in the second and third terms are negatively related to firm risk premia. To find out which effect offsets the other, I solve the model for different values for the share of the grain in the consumption basket, and show the results in Figure (1).

I build a three-firm economy, small, big, and grains. I also consider three different scenarios: all firms are equally volatile, grains are less volatile than big and small firms, and grains are more volatile than big and small firms. The model is quite flexible in producing different patters. However, the most reasonable parametrization for the model is when grains tend to be more volatile than the rest of the economy, hence Panel B where the market as well as small and big firm risk premia are negatively related to granularity but positively related to grains risk premia. I show in the empirical section that this patter support what I observe in the data.

Since the focus of this paper is on equity and bond risk premia, I need to show that this relation holds in the equity and bond dimensions too. To this end, I examine bond and equity risk premia in the next section. I focus on the second (more realistic) scenario.

C. *Firms Security Risk Premia*

To investigate the behaviour of risk premia for firm equity and bond, I assume firms make an optimal debt mix decision. As I provide details in the Appendix, I use the framework of Leland (1994) to solve the firm problem. By applying Ito's lemma, equity and bond dynamics of the firm are governed by

$$\frac{dE_t}{E_t} = \mu_e dt + \rho_i \sigma_e dZ_{_M} + \sqrt{1-\rho_i^2}\sigma_e dZ_i, \tag{13}$$

and

$$\frac{dB_t}{B_t} = \mu_b dt + \rho_i \sigma_b dZ_{_M} + \sqrt{1-\rho_i^2}\sigma_b dZ_i, \tag{14}$$

and equity and bond risk premia are

$$\mathbb{E}\left[\frac{d\pi_t}{\pi_t}, \frac{dE_t}{E_t}\right] = \rho_i \sigma_e \sigma_{_M} + w_i(1-\rho_i^2)\sigma_e^2, \tag{15}$$

and

$$\mathbb{E}\left[\frac{d\pi_t}{\pi_t}, \frac{dB_t}{B_t}\right] = \rho_i \sigma_b \sigma_{_M} + w_i(1-\rho_i^2)\sigma_b^2, \tag{16}$$

respectively. $\sigma_e = \frac{\partial E}{\partial X}\frac{X}{E}$ is the levered equity volatility, and $\sigma_b = \frac{\partial B}{\partial X}\frac{X}{B}$ is the levered bond volatility. I solve the model for a range of cashflow values for the grain. Results are displayed in Figure (2). Panel A shows the equity risk premium. Similar to the patterns observed in Figure (1), the equity risk premium, for firms as well as the market, is negatively related to granularity. This observation is very well supported by empirical evidence.

Panel B shows that bond risk premia behave similarly. However, the bond risk premium for the grains decreases with granularity. Since the current theoretical framework is meant to examine cross sectional effects, hence a static structural model, increasing the cashflow of the grains means that they de-lever as they grow, driving the levered bond volatility considerably low. Thus, bonds issued by grains grow less risky with granularity.

IV. Data

A. *Corporate Securities*

I construct my sample by matching monthly equity and corporate bond data, respectively from CRSP and Enhanced TRACE daily OTC transactions. The latter is widely used in recent corporate bond pricing studies, and is a more reliable source (Bessembinder, Maxwell, and Venkataraman, 2006). I retrieve bond characteristics such as coupon information, time to maturity, and ratings from the Mergent FISD dataset. Data span July 2002 to June 2019. To compute bond returns, I follow the methodology of Bai et al. (2019). I apply the procedure recommended by Nielsen (2014) to clean the Enhanced TRACE transaction data. I then exclude all bonds with special features, and with less than one year to maturity. For an exhaustive list of filters applied to the dataset see Appendix B.

The cross section of monthly bond returns in the final dataset contains between 2,600 to 8,000 observations every month. The final sample is comprised of about 1.4 million month-return observations, based on 47,900 unique bonds issued by 5,588 firms. Due to their peculiar properties, I exclude bonds with less than a year to maturity. Panel A in Table II provides the summary statistics of the bond returns dataset.

B. Matching Bond and Equity Returns

I use NCUSIP from CRSP and the Issuer CUSIP and Issue Id from TRACE and FISD and match securities for 1,797 firms. While these firms issue only one type of equity, they can issue more than one bond (on average 6 and maximum 167 issues per firm). The final subset of matched securities contains 10,900 bonds, 23% of the initial bonds dataset. Table II presents descriptive statistics about the number of firms and securities underpinning this study. Table III shows descriptive statistics of the final bond sample.

C. Measure of Granularity

Granularity is a measure of market concentration. Gabaix and Koijen (2019) introduce the excess Herfindahl–Hirschman Index (exHHI), an adjustment to the Herfindahl–Hirschman Index (HHI), as a measure for granularity,

$$exHHI = \sqrt{\sum_{i=1}^{N} -\frac{1}{N} + w_i^2}, \tag{17}$$

where $w_i = \frac{V_i}{V_M}$, V_i is the market value of firm $'i'$, and V_M is the total market capitalization of firms. Firm value is obtained by adding the market value of firm equity and bond(s). I construct a different measure, the market value of the top $'g'$ firms (V_g) as fraction of total market capitalization of firms equity and debt (V_M),

$$Top_g = \frac{V_g}{V_M}. \tag{18}$$

This measure is more aligned with the theoretical framework of this paper, it is close to perfectly correlated with exHHI, and is more intuitive as it measures granularity in terms of a specific number of firms in the economy.

In Panel A and B of Figure (3) I plot exHHI and Top_{20}, and in Panel C and D I display changes in either measure. Throughout the upcoming sections of the paper I use ΔTop_{20} as measure of change granularity. I use changes in granularity for two main reasons. First, granularity is quite persistent and I need to address this issue in my regression analysis. Second, while conditioning on the level of granularity also produces favourable results in terms of the conditional distribution of equity-bond correlation, high (low) granularity months happen to concentrate around recession (recovery) periods, making it difficult to tell whether macroeconomic conditions versus granularity govern conditional correlations.

I further observe that changes in granularity is significantly negatively correlated with market return, both equity and bond. The immediate question is whether this measure is a nonlinear proxy of the market. I investigate this issue in Figure (4), where I compute return on the top $'i'$ firms as well as the bottom $'100-i'$ firms in my sample. I then compute the correlation between different measures of granularity with either return and plot the result. It is, as expected, evident that return on Top_i firms is positively correlated with return on the same firms. After all, when Top_i firms occupy a bigger slice of the market pie, it should be the case that they have, on average, appreciated in value, hence offering positive returns (blue lines).

On the other hand, changes in Top_i is always negatively correlated with return on bottom firms. I also produce similar figures based on equity and bond market valuation data only (Figure 5-6).

V. Empirical Methodology

I present two sets of results in this section. First, I show conditional correlations between firm bond and equity return is higher when the economy is more granular (either level or shocks). I further show that changes in the correlation coefficient are predicted by changes in granularity. Then, I present a more formal set of results about whether this co-movement is due to a commonly priced factor in the cross-section of equity and bond returns.

A. Conditional Correlations

I follow a two-step procedure to examine the conditional EBC. I compute conditional EBCs as

$$\begin{aligned} \rho_h &= Corr(r_{b,t}, r_{e,t}) | \Delta G_t > Q_{\Delta G,70}, \\ \rho_l &= Corr(r_{b,t}, r_{e,t}) | \Delta G_t < Q_{\Delta G,30}. \end{aligned} \tag{19}$$

$\rho_h(\rho_l)$ is the correlation between firm bond and equity returns when granularity is high (low). I consider the 70^{th} and 30^{th} percentiles of granularity shocks, $\Delta\mathrm{G}_t$, at time t as thresholds for high or low granularity, respectively. I measure granularity by the market value of the 20 largest firms as fraction of total market value of the firms. The latter is the combined value of firm equity and bond(s). The histogram of the bootstrapped averages of conditional versus unconditional correlations is illustrative for the purpose of this section.

I form equity and bond portfolios comprised of 20 randomly chosen firms. I estimate as many EBC coefficients as the number of these portfolios. A formal portfolio construction based on specific characteristics is not favourable for my objective for two reasons. First, the main purpose of portfolio formation in this section is to make sure there are enough observations in the equity versus bond returns time series to have a reliable estimate of conditional EBCs. Notice that with a balanced panel of firm equity and bond return observations the conditioning criterion yields estimates based on 60 months for high and 60 months for low granularity shocks.

By aggregating 20 random firms together I approach this number. Second, portfolio construction procedures usually require an estimation window, hence a less inclusive final sample. Besides, investigating correlations based on sorted securities forces the reader to wonder whether the underlying characteristic is crucial to the results.

In order to construct the distribution of the mean for the EBC, I draw 50,000 samples from the estimated EBCs, and record the mean of the corresponding sample for each iteration. I plot the histogram of the bootstrapped means for both conditional correlations and the difference between them in Figure (7) where returns are conditioned on ΔexHHI. I also present the histograms in Figure (8) with respect to $\Delta\mathrm{Top}_{20}$.

In either figure, the average EBC is higher when granularity is higher. Panel A and C do not overlap, suggestive of a significant difference of conditional correlations. In percentage points, conditional correlations are more than 50% higher in high granularity months. The results are robust to including as few as five firms in the $\Delta\mathrm{Top}_i$ measure. It is hard to claim that only five firms among thousands are representative of the market return. In other words, the $\Delta\mathrm{Top}_i$ measure picks a different financial force. Still, I provide other tests.

A more formal empirical test should substantiate what the histograms show. I use firm level data and compute a rolling correlation measure (36-month window) for each firm such that

$$\hat{\rho}_{e,b,t} = \frac{\sum_{s=t-n-1}^{t-1} r_{e,s}, r_{b,s}}{\left(\sum_{s=t-n-1}^{t-1} r_{e,s}^2\right)\left(\sum_{s=t-n-1}^{t-1} r_{b,s}^2\right)}. \tag{20}$$

where n is the length of the estimation window, 36 months. This procedure yields a panel of EBCs. I then test if changes in the rolling correlations relate to granularity shocks. Due to a relatively short span of my corporate bond returns (about 200 months), applying a more sophisticated methodology such as the Dynamic Conditional Correlation would not be reliable (Engel & Shepherd, 2001).

I estimate

$$\Delta\hat{\rho}_{f,t} = \beta_g \Delta G_t + \sum_{c=1}^{C} \beta_c Control_c + e_{f,t}, \tag{21}$$

where the evolution of the cross section of changes in firms EBC is regressed on changes in granularity as well as control variables. In Table (IV), I report fixed effect results, and take into account, incrementally, potential predictors of EBC in the specification. I provide a detailed list of control variables underpinning the empirical analysis of this paper in Table (I). I compute t-statistics by clustering standard deviations at firm level.

The first row in Table (IV) reports the granularity coefficient. It is always positive and significantly different from zero. Hence, these results support the hypothesis that granularity shocks lead to co-movement between firm securities.

Among other control variables, I include excess return on the top 20 firms, r_{top}-r_f, same firms used to construct the granularity measure. Unlike granularity, return on top firms could be positively, negatively, or insignificantly related to changes in correlations. This finding highlights the distinct nature of changes in granularity and return on top firms.

B. Asset Pricing Implications

The results in the previous section show that granularity is significantly related to the conditional co-movement of securities issued by the same firm, but traded in different markets. It is natural to investigate whether this channel is paved by a similarly priced factor in both markets. In other words, a commonly priced risk factor in the cross section of financial securities, in different markets, shows in co-movement between security returns.

In this section, I investigate if granularity is a risk factor mutually priced in both equity and bond markets. I conduct two analyses, a standard two-pass Fama-Macbeth regression as well as panel regressions. Also, I focus the asset pricing implication of granularity to the bond market.

B.1. Fama-MacBeth Regressions

I follow the standard two-step procedure and for every month estimate

$$r_{b,t+1} - r_f = \gamma_{0,b}^{t+1} + \gamma_{g,b}^{t+1}\hat{\beta}_{g,t} + \sum_{c=1}^{C}\gamma_{c,e}^{t+1}\hat{\beta}_{c,b,t} + \epsilon_{1,t+1}, \tag{22}$$

for the bond market, and

$$r_{e,t+1} - r_f = \gamma_{0,e}^{t+1} + \gamma_{g,e}^{t+1}\hat{\beta}_{g,t} + \sum_{c=1}^{C}\gamma_{c,e}^{t+1}\hat{\beta}_{c,e,t} + \epsilon_{2,t+1}. \tag{23}$$

In either specification excess returns on firms bond or equity in month $t+1$ are regressed on a constant and βs with respect to different risk factors, including granularity shocks, from month t. Granularity betas, β_gs are obtained from a rolling window regression (36 months) where bond or equity returns are regressed on a constant and granularity shocks. I control for $'C'$ common cross sectional predictors of bond and equity returns (Table (I)). This procedure yields a time series of monthly prices of risk associated with each risk factor, $\hat{\gamma}_{c,t=1}^{T}$. I test whether the time series average of the prices of risk, $\bar{\hat{\gamma}}_c = \frac{\sum_{t=1}^{T}\hat{\gamma}_{c,t}}{T}$, is significantly different from zero.

I show estimation results for individual bond returns in Table (V), and for firm equity returns in Table (VI). Granularity shocks are always negatively and significantly related to the cross section of bond and equity returns. Newey-West HAC robust t-statistics are in parentheses. The sample size for the bond market is relatively limited due to availability of quality data. I cannot wait for a longer sample of bond data, but I can address this issue by constructing an equity-only measure of granularity (see Abolghasemi, Bhamra, Dorion, and Jeanneret, 2020), and estimating equation (23) for a much longer sample, 1973-2019. As long as granularity is a priced factor in both equity and bond markets, the estimated price of risk associated with granularity shocks should be significant in over a longer horizon in the equity market. I find (reported in the online Appendix) very similar results, both in magnitude and significance, for the equity market over 1973-2019.

B.2. Panel Regressions

In this section I further investigate whether granularity shocks matter in the cross section of bond returns. I use an alternative estimation methodology, namely fixed effect regressions. By using a fixed effect estimation, I avoid the common criticism regarding estimation errors in the Fama-Macbeth two-pass regressions.

I use rolling betas with respect to granularity shocks as well as all the bond market predictors mentioned Table (I). I then estimate

$$r_{b,t+1} - r_f = \gamma_g \beta_{g,t} + \sum_{c=1}^{C} \gamma_c \beta_{b,t} + \eta_{b,t+1}, \tag{24}$$

where excess bond returns in month $t+1$ are regressed on betas from month t. I show estimation results for all firms and only firms with actively traded equity in Table (VII) and Table (VIII), respectively. I standardize all variables for ease of comparison. All standard errors are clustered at firm level. Results in these two tables concur with those of the Fama-Macbeth regressions. Granularity shocks are always negatively related to the cross section of corporate bond returns.

Lastly, I estimate fixed effect regressions for value and equal weighted bond returns of bond portfolios. I include beta-sorted, credit rating sorted, and maturity sorted portfolios. I follow the standard procedure to form beta portfolios. I use 36-month rolling window betas obtained from regressing excess bond returns on excess bond marker return and a constant. For credit rating portfolios I combine Moody's and S&P ratings. Where there is an overlap,I consider the average of the two. Each row in Table(IX) corresponds to estimation results for a portfolio. The last column shows results for all portfolios in tandem. The first row reports the granularity coefficient. It is always negative and significantly different from zero.

VI. Conclusion

In this paper I argue that the immense difference in relative size of the firms introduces a channel such that security returns are driven by the gravity towards the grains. This granular channel shows up in conditional EBCs in high versus low granularity months.

I construct a theoretical framework in which risk specific to firms grows more systematic when they are relatively larger than the rest of the firms in the economy. I test two distinct predictions of the model. First, I show that granularity brings about discount rate shocks, hence co-movement between firm securities.

When grains grow larger, the volatility of the systematic component of market return gravitates towards that of the grains. Since the correlation with the market return for other firms is time invariant, this granular shock results in a smaller systematic compensation for the non-grains; they grow less systematic. The idiosyncratic component of security returns intensifies this relation. Larger grains means smaller non-grains, and as a consequence an even smaller premium for exposure to non-systematic risk. This shows up in equity and bond returns, both negatively related to granularity.

Further, I show that granularity is a mutually priced factor in the cross section of equity and bond returns. I concur the results of Abolghasemi, Bhamra, Dorion, and Jeanneret (2020) that market and security risk premia are negatively related to changes in granularity. The results are robust to including an extensive set of control variables.

Table I
Definition of Variables

This table defines the variables underpinning this study and the corresponding data sources. All series are retrieved monthly.

Variable	Definition	Source
G	Degree of granularity measured as the total market capitalization value of the largest firms in the U.S. in percentage of the total market capitalization of the CRSP universe. We use either the top 50, 100, or 1% of the firms with the largest market capitalization. Alternatively, we consider the excess Herfindahl-Hirschman Index proposed by Gabaix and Koijen (2020), defined as exHHI $= \sqrt{-\frac{1}{N} + \sum_1^N w_i^2}$, where w_i the market value of firm i to total market capitalization and N is the total number of firms.	Wharton Research Data Services
R_b - R_f	Bond market excess return, computed as the value-weighted return on the Enhanced TRACE universe.	Enhanced TRACE & Mergent IFSD
Term	Term spread, defined as the difference between the yield of long and short term government bonds.	Federal Reserve Bank of Saint Louis
Def	Default spread.	Federal Reserve Bank of Saint Louis
BBW	Bond market risk factors of Bai et al. (2019), including Credit risk factor (CRF), downside risk factor (DRF), and liquidity risk factor (LRF).	Author's Website
Uncertainty	Measures of economic and financial uncertainty à la Jurado, Ludvigson, and Ng (2015).	Author's Website
Fama French Factors	Fama and French five factor model as control for the cross section of equity returns: Excess market return, SMB, HML, RMW, and CMA.	Kenneth French data library

Table II
Summary Statistics: Number of Bonds and Firms

This table presents the descriptive statistics for the number of firms or the number of securities issued by firms. Each month I record the number bonds / firms based on availability of trading data. Therefore, this table does not account for securities or firms that have not traded in a given month. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

Measure	Min	Max	Mean	Median	StD	1%	25%	75%	99%
Bond & Equity	376	834	670.32	681	113.33	404.32	580	780	826
Bonds	1019	1669	1484.1	1518	132.8	1090.8	1426	1581	1667.5
Individual Bonds	3489	7046	5527.9	5577.5	769.5	3689.5	5018	6091	6924.3

Table III
Summary Statistics: Individual Bonds and Bond Portfolios

This table presents the summary statistics for individual bonds. Bond returns are computed from daily transaction data. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Data span July 2002 to June 2019.

Measure	Mean	Med	Std	1%	5%	25%	75%	95%	99%
r_b	0.64	0.40	6.98	-11.86	-3.96	-0.53	1.59	5.30	14.67
β_{bond}	1.20	0.90	1.67	-0.40	0.10	0.46	1.54	3.03	7.39
TTM	9.84	6.73	9.06	1.12	1.56	3.69	13.20	27.62	30.26
Credit Rating	8.21	8.00	3.13	1.00	4.00	6.00	10.00	14	17.50

Table IV
Fixed Effect Regressions, Equity-Bond Correlation (2002 - 2016)

This table presents estimates of fixed effect regressions where the dependent variable is value weighted return on 15 (market) beta-sorted portfolios. Granularity is the market value of total assets for the 20 largest firms to total market value of assets. Independent variables include return on top 20 firms, returns on bottom firms, bond market return, default and term spreads, and the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and three measures of financial uncertainty ofJurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are clustered at portfolio level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
ΔG_t	0.035***	0.035***	0.033***	0.034***	0.032***	0.043***	0.044***	0.043***	0.052***	0.061***	0.058***
	(9.38)	(9.42)	(8.90)	(8.96)	(8.23)	(10.68)	(10.91)	(10.69)	(11.69)	(13.51)	(13.31)
R_{top} - R_f		0.002	0.007	0.009**	0.008*	−0.015***	−0.016***	−0.015***	−0.004	−0.010**	−0.013***
		(0.59)	(1.64)	(2.23)	(1.81)	(−2.86)	(−2.88)	(−2.84)	(−1.00)	(−2.22)	(−2.94)
Unc_1			0.025***	−0.006	−0.015**	−0.024***	−0.019*	−0.018*	−0.035***	−0.030**	−0.033***
			(3.86)	(−0.93)	(−2.21)	(−3.61)	(−1.93)	(−1.76)	(−2.81)	(−2.54)	(−2.69)
Unc_2				0.041***	−0.011	−0.015	−0.014	−0.022**	0.010	0.012	0.020
				(4.50)	(−1.20)	(−1.63)	(−1.56)	(−2.00)	(0.66)	(0.82)	(1.33)
Unc_3					0.064***	0.061***	0.060***	0.067***	0.044***	0.039***	0.037***
					(5.71)	(5.94)	(5.94)	(5.92)	(3.38)	(3.13)	(3.04)
R_b - R_f						0.071***	0.072***	0.071***	0.075***	0.062***	0.079***
						(8.60)	(8.45)	(8.46)	(9.03)	(8.19)	(8.72)
Def							−0.007	−0.003	0.005	−0.006	−0.006
							(−0.84)	(−0.33)	(0.53)	(−0.73)	(−0.72)
Term								−0.007	0.007	0.012	0.019**
								(−1.55)	(0.95)	(1.55)	(2.53)
DRF									−0.026***	−0.036***	−0.029***
									(−4.20)	(−5.36)	(−4.54)
CRF										0.037***	0.035***
										(6.49)	(6.46)
LRF											−0.032***
											(−5.40)
adj-R^2(%)	0.122	0.121	0.180	0.254	0.311	0.740	0.741	0.743	1.009	1.080	1.136

Table V
Fama-Macbeth Regressions, Cross Section of Bond Returns (2002 - 2016)

This table presents estimates of Fama-Macbeth two-pass regressions where the dependent variable is individual bond excess returns. Granularity is the market value of total assets for the 20 largest firms to total market value of assets. Independent variables include return on top 20 firms, returns on bottom firms, bond market return, default and term spreads, and the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and three measures of financial uncertainty of Jurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ΔG_t	-0.110^{**}	-0.105^{*}	-0.097^{*}	-0.151^{***}	-0.137^{***}	-0.122^{***}	-0.151^{***}	-0.159^{***}	-0.198^{***}	-0.169^{***}	-0.158^{***}	
	(-2.55)	(-1.85)	(-1.71)	(-2.87)	(-2.84)	(-2.79)	(-3.58)	(-4.00)	(-3.70)	(-2.96)	(-2.77)	
R_{top} - R_f		0.125^{***}	0.127^{***}	0.125^{***}	0.144^{***}	0.116^{***}	0.117^{***}	0.138^{***}	0.173^{***}	0.159^{***}	0.155^{***}	
		(3.42)	(3.42)	(3.44)	(3.64)	(3.19)	(3.25)	(3.77)	(3.17)	(2.96)	(2.90)	
Unc_1			-0.200^{***}	-0.189^{**}	-0.198^{**}	-0.241^{***}	-0.264^{***}	-0.254^{***}	-0.336^{***}	-0.340^{***}	-0.366^{***}	
			(-3.28)	(-2.50)	(-2.45)	(-3.14)	(-3.36)	(-3.25)	(-2.89)	(-2.67)	(-2.87)	
Unc_2				-0.217^{***}	-0.213^{***}	-0.263^{***}	-0.266^{***}	-0.242^{***}	-0.338^{***}	-0.334^{***}	-0.331^{***}	
				(-3.88)	(-3.72)	(-4.42)	(-4.35)	(-4.00)	(-3.87)	(-3.48)	(-3.45)	
Unc_3					-0.192^{***}	-0.235^{***}	-0.225^{***}	-0.210^{***}	-0.286^{***}	-0.275^{***}	-0.263^{***}	
					(-3.00)	(-3.71)	(-3.59)	(-3.32)	(-3.30)	(-2.90)	(-2.75)	
R_b - R_f						0.153^{***}	0.165^{***}	0.175^{***}	0.226^{***}	0.227^{***}	0.232^{***}	
						(5.64)	(6.04)	(6.14)	(5.63)	(5.64)	(5.59)	
Def							-0.240^{***}	-0.226^{***}	-0.317^{***}	-0.310^{***}	-0.331^{***}	
							(-3.31)	(-2.93)	(-2.88)	(-2.64)	(-2.86)	
Term								-0.106	-0.107	-0.105	-0.129	
								(-1.47)	(-1.10)	(-0.92)	(-1.13)	
DRF									0.139^{**}	0.155^{**}	0.162^{**}	
									(2.09)	(2.26)	(2.47)	
CRF										0.196^{***}	0.195^{***}	
										(4.18)	(4.18)	
LRF											0.124^{***}	
											(2.68)	

Table VI
Fama-Macbeth Regressions, Equity Returns (2002 - 2019)

This table presents estimates of Fama-Macbeth two-pass regressions where the dependent variable is individual equity excess returns. Granularity is the market value of total assets for the 20 largest firms to total market value of assets. Independent variables include return on top 20 firms, returns on bottom firms, bond market return, Fama and French five-factor model, three measures of financial uncertainty of Jurado et al. (2015), and momentum. Equity data is from CRSP. I obtain betas by rolling a 60-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
ΔG_t	−0.114***	−0.082**	−0.089**	−0.087**	−0.098***	−0.093***	−0.094***	−0.093***	−0.094***	−0.098***	−0.098***
	(−2.81)	(−2.09)	(−2.30)	(−2.29)	(−2.62)	(−2.69)	(−2.70)	(−2.73)	(−2.80)	(−3.01)	(−2.95)
R_{top} - R_f		0.065	0.069	0.071	0.057	0.054	0.040	0.040	0.040	0.038	0.039
		(1.22)	(1.32)	(1.37)	(1.14)	(1.19)	(0.90)	(0.91)	(0.91)	(0.85)	(0.89)
Unc_1			−0.127***	−0.125***	−0.137***	−0.137***	−0.123***	−0.123***	−0.122***	−0.120***	−0.117***
			(−3.30)	(−3.29)	(−3.75)	(−3.76)	(−3.58)	(−3.57)	(−3.54)	(−3.51)	(−3.47)
Unc_2				−0.059*	−0.063**	−0.062**	−0.060**	−0.062**	−0.068**	−0.062**	−0.057**
				(−1.92)	(−2.06)	(−2.06)	(−2.04)	(−2.13)	(−2.44)	(−2.23)	(−2.03)
Unc_3					−0.055*	−0.055*	−0.056*	−0.056*	−0.063**	−0.060**	−0.054*
					(−1.77)	(−1.80)	(−1.83)	(−1.87)	(−2.19)	(−2.09)	(−1.95)
R_e - R_f						0.099*	0.093*	0.090*	0.091*	0.093*	0.091*
						(1.88)	(1.84)	(1.79)	(1.81)	(1.85)	(1.83)
SMB							0.059*	0.065*	0.064*	0.067*	0.066*
							(1.65)	(1.85)	(1.83)	(1.90)	(1.86)
HML								0.028	0.027	0.027	0.027
								(0.68)	(0.65)	(0.66)	(0.67)
RMW									−0.010	−0.016	−0.015
									(−0.28)	(−0.45)	(−0.44)
CAM										−0.015	−0.015
										(−0.40)	(−0.39)
Mom											−0.026
											(−0.85)

Table VII
Fixed effect Regressions, Bond Returns, Firm Level (2002 - 2016)

This table presents estimates of fixed effect regressions where the dependent variable is excess return on firms bond. Granularity is the market value of total assets for the 20 largest firms to total market value of assets. Independent variables include return on top 20 firms, bond market return, default and term spreads, and the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and three measures of financial uncertainty of Jurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are clustered at firm level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
ΔG_t	−0.028***	−0.013***	−0.012***	−0.012***	−0.013***	−0.012***	−0.012***	−0.013***	−0.017***	−0.019***	−0.019***
	(−9.92)	(−4.61)	(−3.78)	(−3.84)	(−3.90)	(−3.10)	(−3.26)	(−3.84)	(−5.71)	(−6.04)	(−6.09)
R_{top} - R_f		0.024***	0.024***	0.024***	0.024***	0.023***	0.023***	0.022***	0.024***	0.024***	0.024***
		(7.77)	(7.56)	(7.59)	(7.50)	(5.76)	(5.71)	(5.76)	(6.51)	(6.93)	(6.81)
Unc_1			−0.004	−0.004	−0.005	−0.004	−0.003	−0.003	−0.003	−0.004	−0.004
			(−1.44)	(−1.47)	(−1.59)	(−1.60)	(−1.10)	(−1.33)	(−1.33)	(−1.51)	(−1.41)
Unc_2				−0.001	−0.007***	−0.007***	−0.007***	−0.007***	−0.005*	−0.005*	−0.005*
				(−0.31)	(−2.85)	(−3.03)	(−2.88)	(−2.78)	(−1.90)	(−1.95)	(−1.88)
Unc_3					0.009***	0.009***	0.009***	0.007**	0.006**	0.005*	0.004
					(3.23)	(3.41)	(3.10)	(2.56)	(2.06)	(1.86)	(1.44)
R_b - R_f						0.004	0.005	0.006	0.012***	0.013***	0.015***
						(1.20)	(1.27)	(1.57)	(3.22)	(3.32)	(3.86)
Def							−0.003	−0.002	0.000	0.000	0.000
							(−0.91)	(−0.84)	(0.02)	(0.18)	(0.06)
Term								0.006**	0.005**	0.005**	0.006**
								(2.21)	(2.12)	(2.13)	(2.45)
DRF									−0.016***	−0.015***	−0.015***
									(−5.38)	(−5.12)	(−4.86)
CRF										−0.004	−0.003
										(−1.36)	(−1.04)
LRF											−0.005
											(−1.14)
adj-R^2(%)	0.098	0.147	0.148	0.148	0.152	0.153	0.154	0.157	0.175	0.176	0.177

Table VIII

Fixed effect Regressions, Bond Returns, Matched Firms (2002 - 2016)

This table presents estimates of fixed effect regressions where the dependent variable is excess return on corporate bonds issued by matched firm, those with actively traded equity. Granularity is the market value of total assets for the 20 largest firms to total market value of assets. Independent variables include return on top 20 firms, bond market return, default and term spreads, and the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and three measures of financial uncertainty of Jurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are clustered at portfolio level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
ΔG_t	−0.059***	−0.031***	−0.025***	−0.025***	−0.026***	−0.053***	−0.054***	−0.055***	−0.057***	−0.066***	−0.066***
	(−7.21)	(−3.58)	(−2.74)	(−2.73)	(−2.93)	(−5.60)	(−5.80)	(−5.83)	(−6.33)	(−7.57)	(−7.72)
R_{top} - R_f		0.045***	0.045***	0.045***	0.048***	0.068***	0.068***	0.065***	0.065***	0.070***	0.067***
		(6.53)	(7.03)	(7.03)	(7.27)	(6.54)	(7.03)	(6.58)	(6.58)	(7.23)	(6.87)
Unc_1			−0.019***	−0.020***	−0.023***	−0.028***	−0.005	−0.007	−0.007	−0.011	−0.009
			(−2.93)	(−3.01)	(−3.31)	(−3.59)	(−0.68)	(−0.85)	(−0.85)	(−1.37)	(−1.05)
Unc_2				0.001	−0.016**	−0.004	0.006	0.004	0.005	0.005	0.006
				(0.19)	(−2.58)	(−0.69)	(0.94)	(0.67)	(0.76)	(0.82)	(0.87)
Unc_3					0.028***	0.017***	0.011*	0.007	0.006	0.003	−0.002
					(4.37)	(2.68)	(1.66)	(1.00)	(0.82)	(0.43)	(−0.25)
R_b - R_f						−0.064***	−0.065***	−0.060***	−0.058***	−0.054***	−0.045***
						(−6.78)	(−7.15)	(−6.29)	(−5.47)	(−5.11)	(−4.07)
Def							−0.039***	−0.038***	−0.036***	−0.033***	−0.034***
							(−4.57)	(−4.40)	(−4.22)	(−3.83)	(−3.95)
Term								0.016**	0.016**	0.017**	0.020***
								(2.28)	(2.31)	(2.45)	(2.80)
DRF									−0.006	−0.005	−0.003
									(−0.75)	(−0.73)	(−0.43)
CRF										−0.023***	−0.017**
										(−2.97)	(−2.21)
LRF											−0.020***
											(−2.85)
adj-R^2(%)	0.266	0.358	0.372	0.371	0.392	0.559	0.610	0.620	0.620	0.633	0.648

Table IX
Fixed Effect Regressions, Bond Returns, Various Portfolios (2002 - 2016)

This table presents estimates of fixed effect regressions where the dependent variable is excess return on various portfolios of corporate bonds, These portfolios, with respectively value and equal weighted returns, are sorted by market beta (P1 and P2), credit ratings (P3 and P4), and time to maturity (P5 and P6). The last column included all these portfolios in tandem (P7). Granularity is the market value of total assets for the 20 largest firms to total market value of assets. Independent variables include return on top 20 firms, bond market return, default and term spreads, and the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and three measures of financial uncertainty of Jurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are clustered at portfolio level.

	(P1)	(P2)	(P3)	(P4)	(P5)	(P6)	(P7)
ΔG_t	-0.450^{***}	-0.611^{***}	-0.403^{***}	-0.480^{***}	-0.139^{***}	-0.339^{***}	-0.400^{***}
	(-4.96)	(-5.12)	(-4.44)	(-4.63)	(-3.01)	(-4.86)	(-7.61)
R_{top} - R_f	-0.015	-0.072	-0.236^{***}	-0.203^{***}	-0.006	-0.057	-0.066^{**}
	(-0.25)	(-1.05)	(-3.57)	(-2.98)	(-0.16)	(-1.08)	(-2.02)
R_b - R_f	-0.395^{***}	-0.252^{***}	-0.203^{***}	-0.176^{***}	-0.161^{***}	-0.052	-0.256^{***}
	(-6.99)	(-3.56)	(-4.99)	(-4.12)	(-3.35)	(-1.13)	(-9.54)
Def	-0.382^{***}	-0.303^{***}	-0.322^{***}	-0.305^{***}	-0.205^{***}	-0.286^{***}	-0.301^{***}
	(-6.33)	(-7.00)	(-6.58)	(-7.40)	(-3.52)	(-6.46)	(-10.95)
Term	-0.095^{**}	-0.049	-0.079^{**}	-0.051	-0.104^{***}	-0.014	-0.072^{***}
	(-2.39)	(-0.79)	(-2.42)	(-1.35)	(-2.91)	(-0.36)	(-2.96)
DRF	-0.145^{**}	-0.097	-0.126^{***}	-0.088^{*}	-0.126^{***}	-0.105^{***}	-0.081^{**}
	(-2.31)	(-1.49)	(-2.62)	(-1.92)	(-3.30)	(-2.68)	(-2.27)
CRF	0.024	0.141^{*}	0.177^{***}	0.203^{***}	0.114^{***}	0.097^{***}	0.127^{***}
	(0.47)	(1.93)	(3.49)	(3.94)	(2.84)	(2.64)	(4.67)
LRF	0.117	-0.278^{***}	-0.131^{***}	-0.201^{***}	0.072^{*}	-0.147^{***}	-0.077^{**}
	(1.40)	(-3.14)	(-3.33)	(-4.07)	(1.69)	(-3.91)	(-2.29)
Unc_1	0.082^{*}	0.121^{**}	0.093^{***}	0.130^{***}	0.052	0.162^{***}	0.117^{***}
	(1.75)	(2.40)	(2.59)	(3.21)	(1.47)	(4.94)	(4.80)
Unc_2	0.227^{***}	0.176^{***}	0.241^{***}	0.211^{***}	0.212^{***}	0.169^{***}	0.200^{***}
	(3.99)	(2.92)	(4.32)	(4.22)	(3.38)	(3.42)	(6.66)
Unc_3	0.073	0.042	0.061	0.047	0.113^{**}	0.034	0.059^{**}
	(1.16)	(0.59)	(1.21)	(0.81)	(1.98)	(0.75)	(2.18)
adj-R^2(%)	7.113	9.159	9.189	10.211	7.377	10.085	7.802

VII. Figures

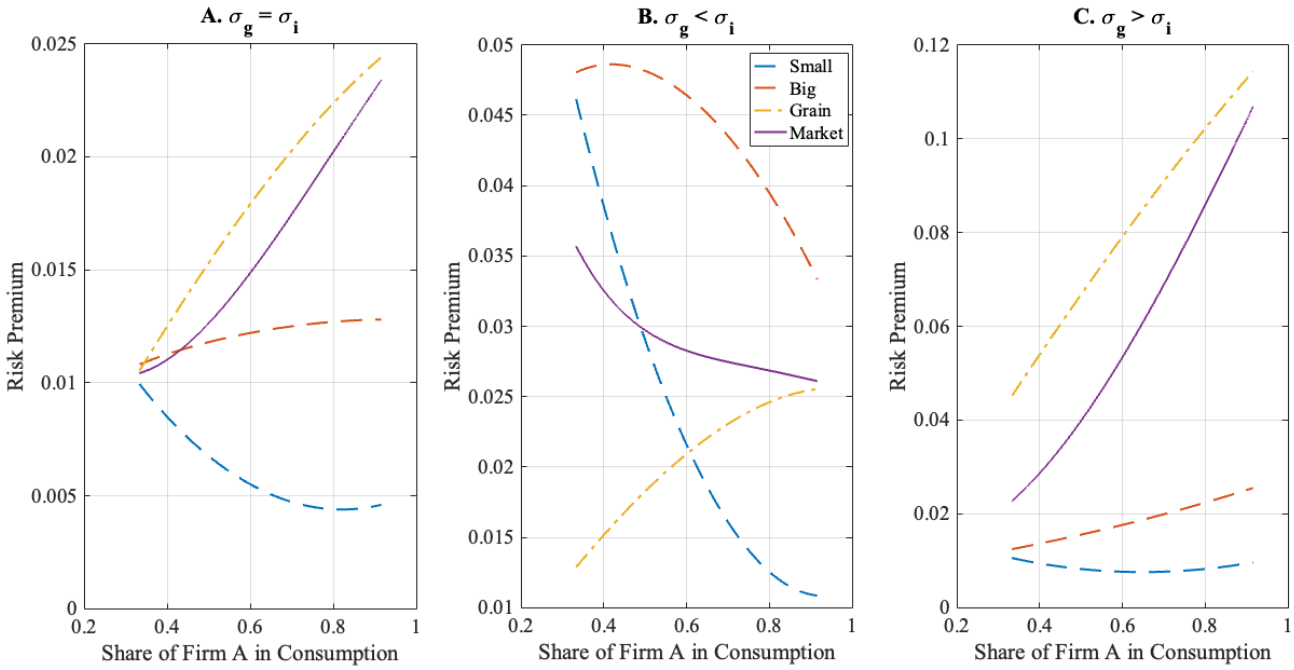

Figure 1. Model implied Risk Premia, Firm. This figure plots the cashflow risk premium for small, big, and grain firms in a three-firm economy. All else equal, only the level of cashflow for the grain varies. Panel A represents a scenario where all firms are equally volatile. Panel B and C show risk premia when grains are less or more volatile than the rest of the economy, respectively.

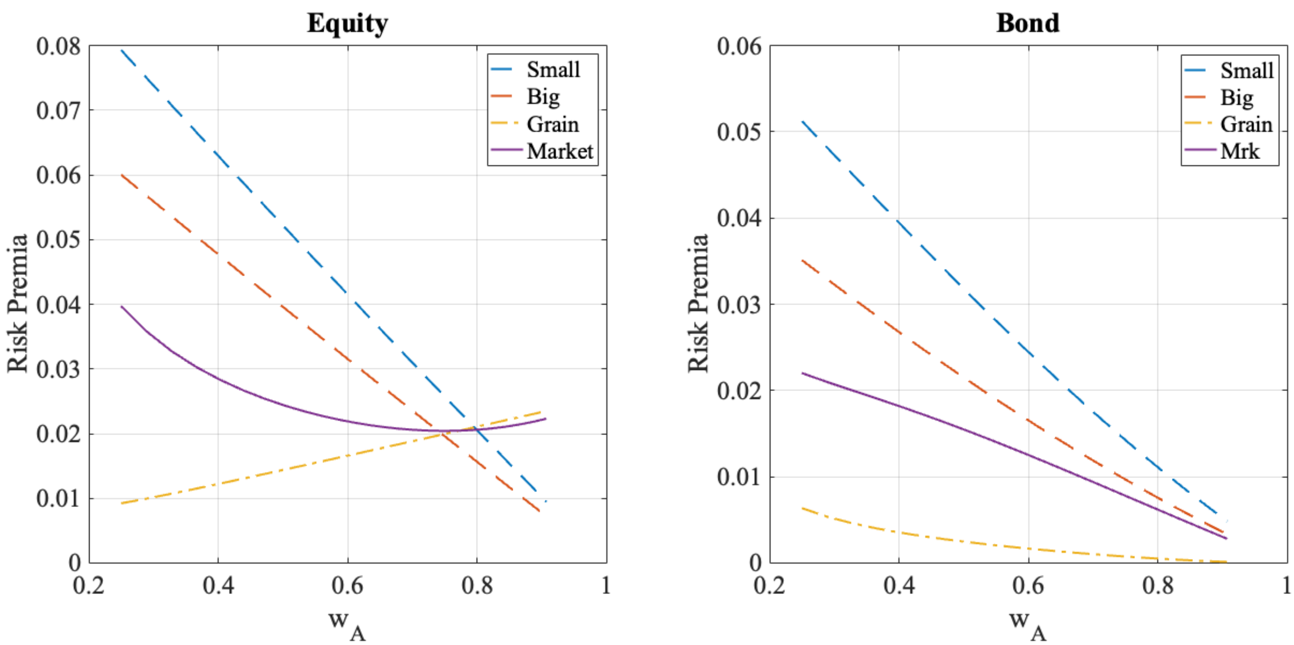

Figure 2. Model implied Risk Premia, Equity and Bond. This figure plots the bond and equity risk premia corresponding to the scenario where grains are less volatile than the rest of the economy (Panel B in the figure above). To obtain bond and equity risk premia, I solve the firm debt-mix problem à la Leland (1994) and plot security risk premia accordingly.

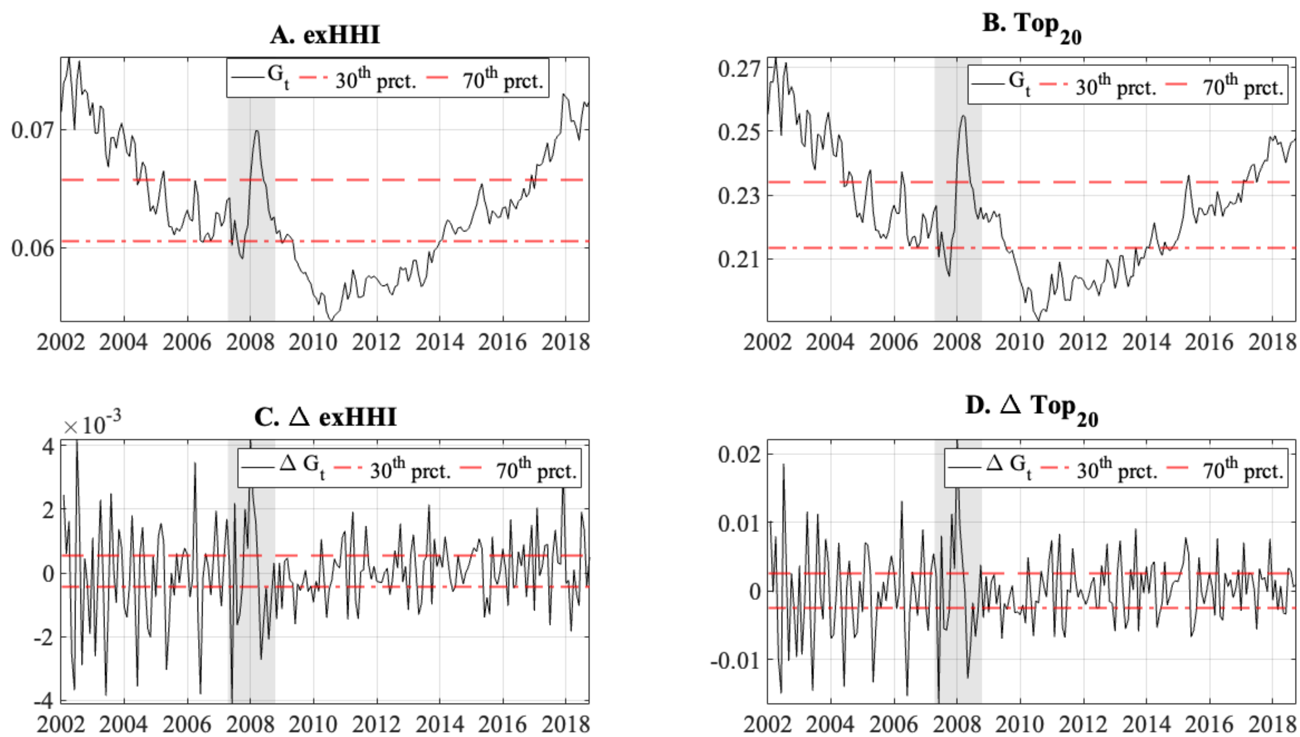

Figure 3. Model implied Risk Premia, Equity and Bond. This figure plots two different measures of granularity. The excess HHI measure of Gabaix and Koijen (2020) versus Top_{20}, the market value of the largest 20 firms as fraction of total market capitalization. The top panels show the measures at level, the bottom panel plot the corresponding change (first difference). The red lines indicate the 30_{th} versus 70_{th} percentile of each measure. Both measures are computed at firm level. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

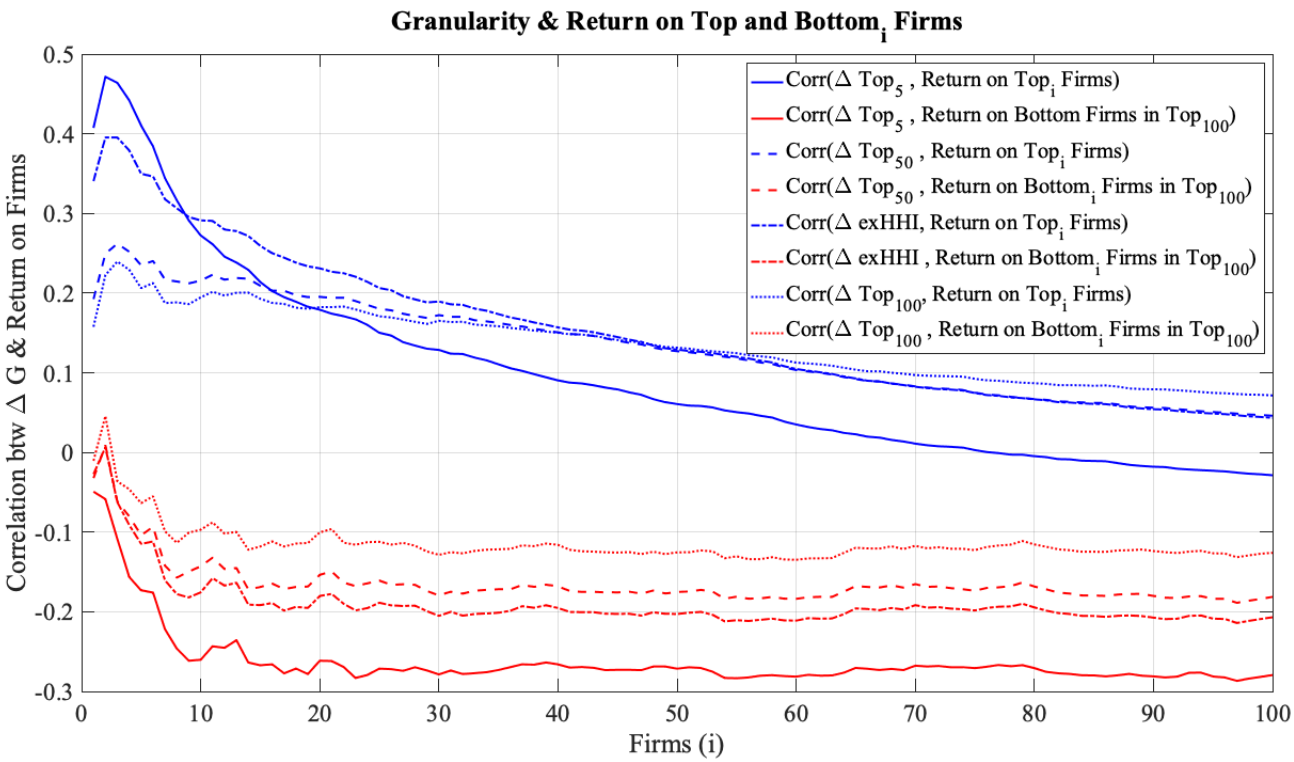

Figure 4. Changes in Firm Granularity versus Market Return. This figure plots the correlation between changes in a granularity measure, and returns on the top i (blue lines) as well as the bottom 100-i firms (red lines) in the market. Granularity is measured at firm level. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

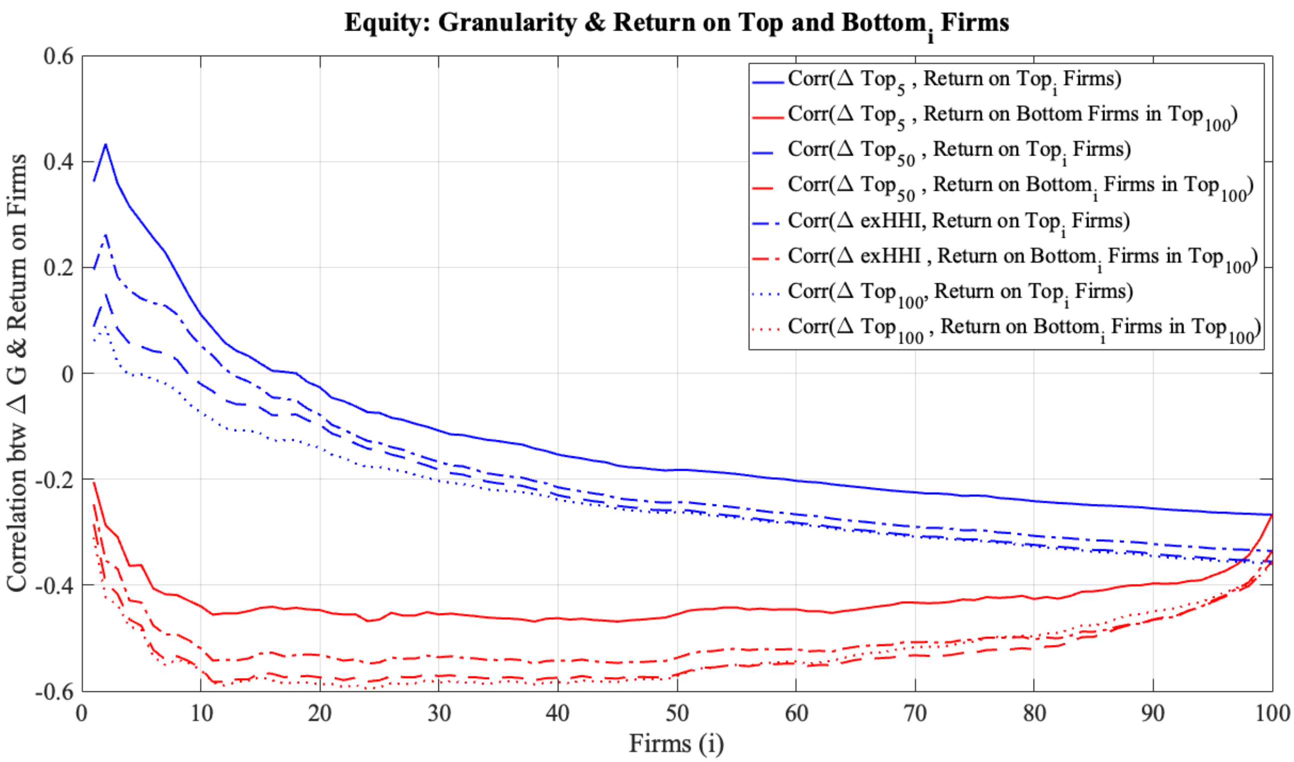

Figure 5. Changes in Equity Granularity versus Market Return. This figure plots the correlation between changes in a granularity measure, and returns on the top i (blue lines) as well as the bottom 100-i firms (red lines) in the market. Granularity is measured at equity level. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

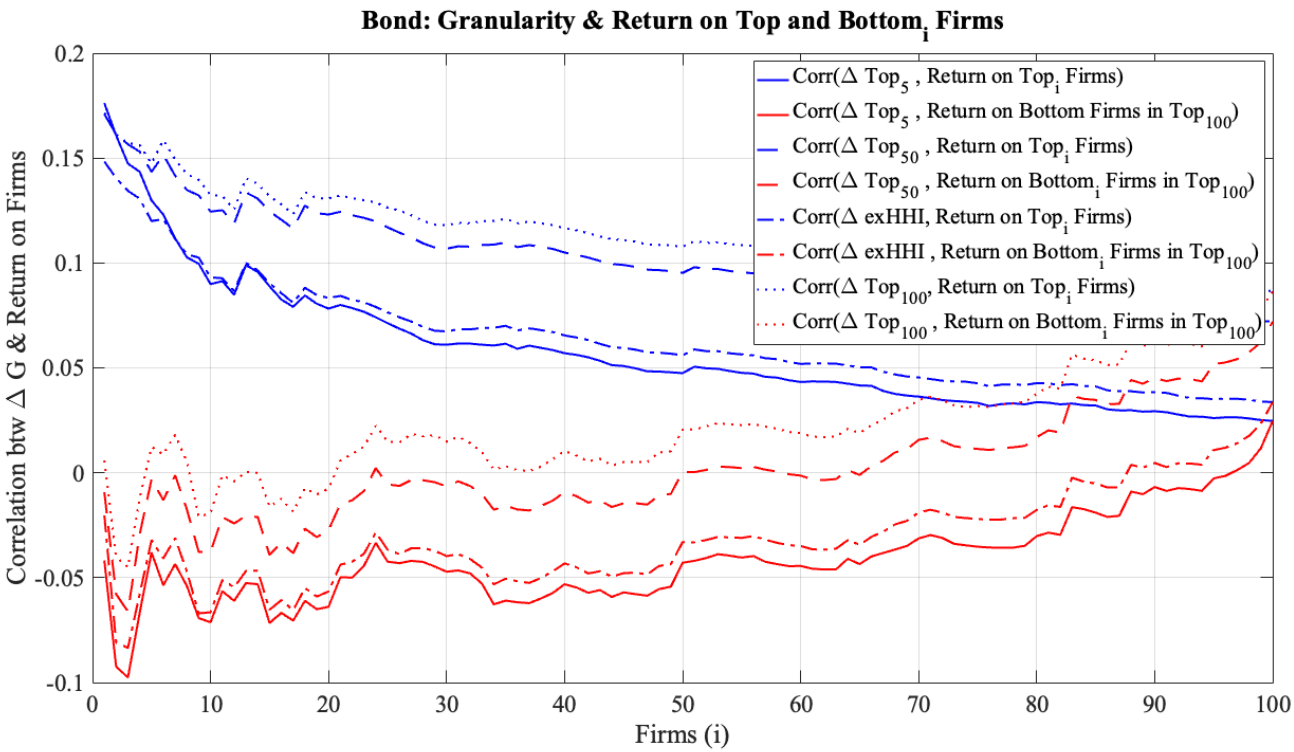

Figure 6. Changes in Bond Granularity versus Market Return. This figure plots the correlation between changes in a granularity measure, and returns on the top i (blue lines) as well as the bottom 100-i firms (red lines) in the market. Granularity is measured at bond level. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

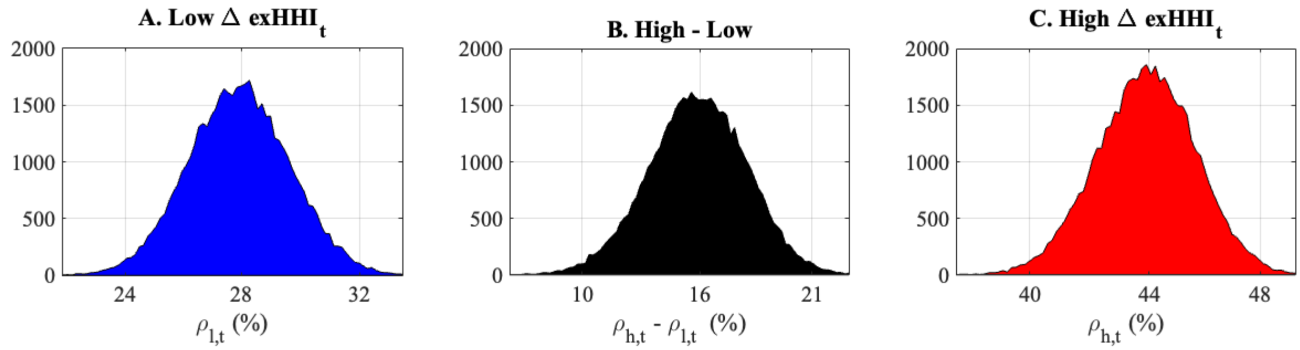

Figure 7. Conditional Distribution of Equity-Bond Correlations, ΔexHHI. This figure plots the conditional EBC when granularity shocks are below the 30_{th} percentile (Panel A), above the 70_{th} percentile (Panel C), and the corresponding difference (Panel B). First, correlations are computed based on 90 random portfolios (each portfolio 20 firms). These correlations are then re-sampled 50,000 times and the the distribution of the bootstrapped mean of conditional correlations is plotted. Granularity shocks are measured based on changes in the excess HHI measure. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

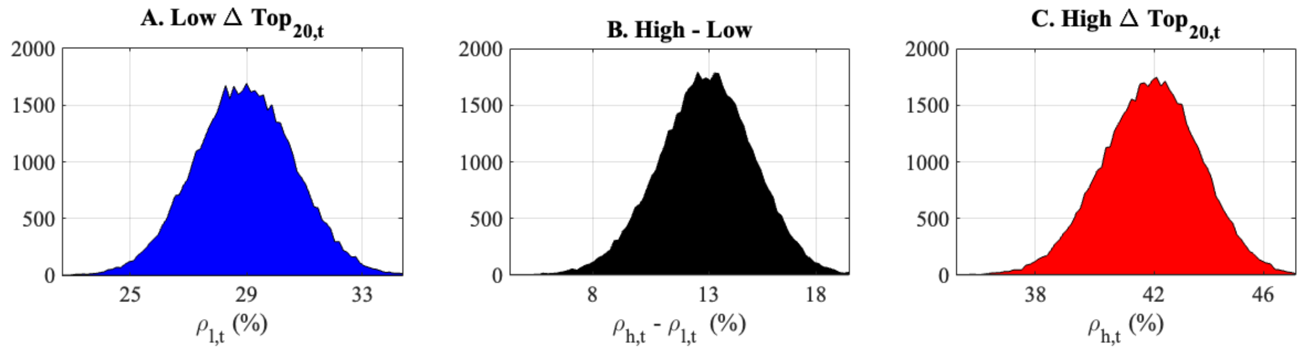

Figure 8. Conditional Distribution of Equity-Bond Correlations, ΔTop$_{20}$. This figure plots the conditional EBC when granularity shocks are below the 30_{th} percentile (Panel A), above the 70_{th} percentile (Panel C), and the corresponding difference (Panel B). First, correlations are computed based on 90 random portfolios (each portfolio 20 firms). These correlations are then re-sampled 50,000 times and the the distribution of the bootstrapped mean of conditional correlations is plotted. Granularity shocks are measured based on changes in the Top$_{20}$ measure. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

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Appendix A.I

To clean the Enhanced TRACE transaction data I first apply the procedure offered by Nielsen (2014), where 35% of the raw transaction data is deleted due to various problematic issues. Then, I use the following criterion, à la Bail, Bali, and Wen (2019), to filter the corporate bond dataset.

- Remove bonds that are not listed or traded in the US public market, which include bonds issued through private placement, bonds issued under the 144A rule, bonds that do not trade in US dollars, and bond issuers not in the jurisdiction of the United States.

- Remove bonds that are structured notes, mortgage backed or asset backed, agency backed, or equity linked.

- Remove convertible bonds since this option feature dis- torts the return calculation and makes it impossible to compare the returns of convertible and nonconvertible bonds

- Remove bonds that trade under $5 or above $1,000.

- Remove bonds that have a floating coupon rate, which means the sample comprises only bonds with a fixed or zero coupon.

- Remove bonds that have less than one year to maturity.

- Eliminate bond transactions that are labeled as when-issued, locked-in, or have special sales conditions, and that have more than a two- day settlement.

- Remove transaction records that are canceled and ad- just records that are subsequently corrected or re- versed.

- Remove transaction records that have trading volume less than $10,000.

Appendix A.II

This Appendix provides the framework sed in the paper to risk neutralize firm cashflows and solve for the optimal equity-debt mix.

A. Risk Neutral Dynamics of Revenue

To value firm securities, we need to write the firm revenue dynamics under the $\mathbb{Q}$ measure. Regardless of the SDF, the following adjustment presents output dynamics under the $\mathbb{Q}$ measure,

$$\frac{dX_i}{X_i} = \tilde{\mu}_{X,i} dt + \sigma_{X,i} d\tilde{Z}_i, \tag{25}$$

where $\tilde{\mu}_{X,i} = \mu_{X,i} + \mathbb{E}_t\left(\frac{d\pi}{\pi}\frac{dX_i}{X_i}\right)$ is the growth rate of firm i under the probability measure $\mathbb{Q}$.

B. Unlevered Firm Value

The unlevered firm value is a claim on the total earnings of the firm,

$$V_t^u(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)}(1-\tau)X_s ds\right] = \frac{(1-\tau)}{r-\tilde{\mu}_{X,i}} X_t. \tag{26}$$

C. Debt Value

The value of debt for firm i comprises two components, the claim on a coupon until bankruptcy and the claim on the assets at bankruptcy,

$$D_t(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D} ce^{-r(s-t)} ds\right] + \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D} (1-\alpha)(1-\tau)X_s ce^{-r(s-t)} ds\right] \tag{27}$$

in which $T_D = inf\,\{t \geq 0, |\; X_t \leq X_D\}$ is the first hitting time. We can write the first part in Equation (27) as

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D} ce^{-r(s-t)} ds\right] = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} ce^{-r(s-t)} ds\right] - \mathbb{E}^{\mathbb{Q}}\left[\int_{T_D}^{\infty} ce^{-r(s-t)} ds\right] \tag{28}$$

The first integral gives the value of a perpetuity,

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} ce^{-r(s-t)} ds\right] = \frac{c}{r} \tag{29}$$

The second integral is over a random domain. By Karatzas & Shreve (1991)

$$\mathbb{E}^{\mathbb{Q}}\left[\int_{T_D}^{\infty} ce^{-r(s-t)}ds\right] = -\frac{c}{r}\left(\frac{X_t}{X_D}\right)^{\omega} \tag{30}$$

where $\left(\frac{X_t}{X_D}\right)^{\omega}$ is the Arrow-Debreu price bankruptcy and ω is the negative root of the characteristic equation $\frac{1}{2}\sigma_{X,i}^2\omega(1-\omega)+\mu_{X,i}\omega - r = 0$. Put differently, the value of a claim, ν that pays a one unit of cashflow at bankruptcy and zero anywhere else, yields the risk free rate by no arbitrage,

$$\mathbb{E}^{\mathbb{Q}}\left[d\nu\right] = r\nu \tag{31}$$

and since ν is a function of the firm earnings, it must satisfy the following PDE

$$\frac{\partial \nu}{\partial X}\tilde{\mu}_{X,i}X + \frac{1}{2}\frac{\partial^2 \nu}{\partial X^2}\tilde{\sigma}_{X,i}^2 X^2 = r\nu \tag{32}$$

The general solution to the PDE is

$$\nu(X) = a\nu^{\alpha_1} + b\nu^{\alpha_2} \tag{33}$$

By applying the boundary conditions the specific solution is derived,

$$\nu(X) = \left(\frac{X}{X_D}\right)^{\omega} \tag{34}$$

and

$$\omega = \frac{1}{2} - \frac{\mu_X}{\sigma_X^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu_X}{\sigma_X^2}\right)^2 + \frac{2r}{\sigma_X^2}} \tag{35}$$

The Arrow-Debreu price of bankruptcy is used in other claims in a similar manner. The second part in Equation (27) is solved by the strong Markov property for Brownian motions by which

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}}\left[\int_{t}^{T_D}(1-\alpha)(1-\tau)X_s ce^{-r(s-t)}ds\right] &= \\
\mathbb{E}^{\mathbb{Q}}\left[(1-\alpha)(1-\tau)X_D\int_{t}^{\infty} e^{\sigma X_s - (r-\mu_X+\frac{\sigma^2}{2})}ds\right] &= \\
(1-\alpha)(1-\tau)\frac{X_D}{r-\tilde{\mu}_X}\left(\frac{X}{X_D}\right)^{\omega}
\end{aligned} \tag{36}$$

Therefore, the value of debt is given by what we derived in Equation (29), (30) and (36).

$$
\begin{aligned}
D_t(X) =& \frac{c}{r}\left[1-\left(\frac{X_t}{X_D}\right)^{\omega}\right] \\
&+(1-\alpha)(1-\tau)\frac{X_D}{r-\tilde{\mu}_X}\left(\frac{X}{X_D}\right)^{\omega} \\
=& \frac{c}{r}-\left[\frac{c}{r}-(1-\alpha)(1-\tau)\frac{X_D}{r-\tilde{\mu}_{X,i}}\right]\left(\frac{X_t}{X_D}\right)^{\omega},
\end{aligned} \tag{37}
$$

D. Levered Firm Value

By issuing debt, shareholders enjoy tax benefits on one hand and are exposed to costs of distress on the other hand. Tax benefits and costs of distress are both claims contingent on the earnings of the firm,

$$T_t(X) = E^{\mathbb{Q}}\left[\int_t^{T_D} \tau c e^{-r(s-t)} ds\right] \tag{38}$$

$$L_t(X) = E^{\mathbb{Q}}\left[\int_{T_D}^{\infty} \alpha(1-\tau) X_s e^{-r(s-T_D)} ds\right] \tag{39}$$

where Equations (40) and (41) are expected tax benefits and expected distress costs respectively. Following a similar procedure as in deriving debt value, the value of each claim is obtained as follows

$$T_t(X) = \frac{\tau c}{r}\left[1-\left(\frac{X_t}{X_D}\right)^{\omega}\right] \tag{40}$$

$$L_t(X) = \frac{\alpha(1-\tau)X_D}{r-\tilde{\mu}_{X,i}}\left(\frac{X}{X_D}\right)^{\omega} \tag{41}$$

$$
\begin{aligned}
V_t^l(X) &= V_t^u(X) + T_t(X) - L_t(X) \\
&= \frac{(1-\tau)}{r-\tilde{\mu}_{X,i}}X_t + \frac{\tau c}{r}\left[1-\left(\frac{X_t}{X_D}\right)^{\omega}\right] - \frac{\alpha(1-\tau)X_D}{r-\tilde{\mu}_{X,i}}\left(\frac{X}{X_D}\right)^{\omega}
\end{aligned} \tag{42}
$$

E. *Equity*

Shareholders have a claim on the earnings of the firm net of coupon payments until bankruptcy and the value of firm equity is then equal to

$$\begin{aligned} E_t(X) &= (1-\tau)\mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D}(X_s - c)e^{-r(s-t)}ds\right] \\ &= (1-\tau)\left[\frac{X_t}{r-\tilde{\mu}_{X,i}} - \frac{c}{r} - \left(\frac{X_D}{r-\tilde{\mu}_{X,i}} - \frac{c}{r}\right)\left(\frac{X_t}{X_D}\right)^{\omega}\right]. \end{aligned} \tag{43}$$

F. *Optimal Capital Structure*

The optimality of the default boundary is ensured by the smooth pasting condition

$$\frac{\partial E_t}{\partial X_t}|_{X_t = X_D} = 0, \tag{44}$$

that yields the optimal default boundary as

$$X_D = c\frac{\omega}{1-\omega}\frac{r-\tilde{\mu}_{X,i}}{r}. \tag{45}$$

By Equation (45), the default boundary is directly related to the value of the coupon. More leverage induces a higher default boundary, hence proximity to default.

Issuing debt is not costless, it increases the probability of default and the expected cost of distress. Thus, shareholders choose a level of leverage for which the marginal benefit of issuing and extra unit of debt is equal to the marginal cost of distress. Formally, firms solve the following problem

$$c^* = argmax\ V_0(c), \tag{46}$$

therefore,

$$c^* = X_0 c\frac{\omega}{1-\omega}\frac{r}{r-\tilde{\mu}_{X,i}}\left(1-\omega-\frac{\omega\alpha(1-\tau)}{\tau}\right)^{\frac{1}{\omega}}. \tag{47}$$