Outline of Foundations of Signal Processing

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Chapter 6 – Approximation and Compression

Various approximation techniques, including linear polynomial approx with primitive, Legandre and Lagrange polynomials; non-linear approximation and approximation with splines.

Of particular interest is non-linear approx (although not treated very thoroughly), minimax approximation which guarantees minimal error, and approximation with splines.

Approximation with Polynomials

- The common problem with polynomial approximation is that we fit a global function, and this turns out to be producing large errors near the edges. This is the main motivation of the minimax approximation, which prevents large errors around the edges by spreading the error throughout the interval evenly.
- Some common orthonormal polynomial bases:
  Legendre polynomials (#515), Lagrange polys (#518), Taylor series around point (#521) and Hermite interpolation (#522).
- Minimax approximation: $||x - \hat{x}||_\infty = \max_{t \in [a,b]} |x(t) - \hat{x}(t)|$
  The idea is to *minimise* the *maximal* error throughout an interval, so we can avoid the typical problem of polynomial approx (*i.e.* large errors near interval edges).
  - Minimax approx builds up based on a few theorems.
  - Thm 6.6 – Weierstrass approx thm (#524) (pointwise error can be arbitrarily minimised)
    We can always find a polynomial p s.t. $|e(t)| = |x(t) - p(t)| < \epsilon$ for any $\epsilon > 0$.
  - Thm 6.7 (De la Vallee-Poussin Alternation thm)
    Let's partition t into K subintervals I_k. Denote supremum error of minimax approx as $\epsilon_{p,K} = ||e_{p,K}||_\infty$. Suppose that there exists polynomial q_K of degree at most K such that $e_{q,K} = x(t) - q_K(t)$ alternates sign at each interval. Theorem says that $\epsilon_{p,K}$ *cannot* be less than the minimum of the interval-wise error of the sign-alternating polynomial q_K:
    $\min_{k=0,1,\ldots,K+1} |e_{q,K}(s_K)| \leq \epsilon_{p,K} \leq \epsilon_{q,K}$
    The left inequality is the interesting one (#525).
  - Thm 6.8 Chebyshev equioscillation Thm