Introduction

My notes from the legendary book of Papoulis and Pillai [1]. The book assumes that the reader does already have fairly strong background in calculus and linear algebra. I used two books [2, 3] as assistant for evaluating some of the mathematical identities or integrals.

This book made me realize that a good technical book is not necessarily one that's easy to follow – one that shows you very easily how each identity or result is arrived at. Some of the results in the book are presented without much explanation, and this required me to show a lot of effort to understand them; this effort seems to lead to a staying power.

Chapter 1

Sequences of Random Variables

1.1 Notes

- Multivariate Transformation
- How to integrate some RVs from a multi-variate distro to obtain ... (Sec 7.2)
- How to compute conditional mean considering only a subset of RVs
- Characteristic function leads to so much interesting applications, such as:
  - Computing the PDF of Bernoulli from Binomials (#256)
  - Computing the PDF of Poisson from Binomials (#256)
  - The sum of jointly normal RVs is normal (#257)
  - The sum of the squares of independent normal variables is chi-square (#259)
  - The sum of two chi-square distros is also chi-square (#260)
- The optimal single-value estimation (in the MS sense), c, of a future value of a RV $\mathbf{y}$ is $c = E(\mathbf{y})$.
- The optimal functional estimation of $\mathbf{y}$ *i.t.o.* a (dependent) RV $\mathbf{x}$ is $c(x) = E(\mathbf{y}|x)$. In general, this estimation is *non-linear*.
- The optimal linear f. estimation of $\mathbf{y}$ *i.t.o.* $\mathbf{x}$ is $c(x) = Ax+B$ where $B = \eta_y - A\eta_x$ and $A = r\sigma_x/\sigma_y$.
- *For Gaussian RVs, linear and non-linear estimation is identical (#264).*

1.2 Interesting identities/lemmas/theorems

- The unbiased linear estimator with minimum variance is the one shown in (7-17).
- Theorem 7.1: The correlation matrix is nonnegative definite
- Sum of jointly normal RVs is normal
- Sample mean and variance of n RVs $\mathbf{x}_1, \ldots, \mathbf{x}_n$ are $\bar{\mathbf{x}} = \frac{1}{n}\sum_{i=1}^{n}$ and $\bar{\mathbf{v}} = \frac{1}{n-1}\sum_{i=1}^{n}(\mathbf{x} - \bar{\mathbf{x}})^2$.
- Goodman's theorem: The statistics of a real zero-mean n-dimensional normal RV are completely determined i.t.o. the n^2 parameters—the elements of its covariance matrix. However, a similar *complex* RV requires $2n^2 + n$ elements. Goodman's theorem (#259) gives a special class of normal complex RVs that are determined completely with n^2 parameters only.

1.3 Important concepts

- Group independence
- Correlation and covariance matrices
- The orthogonality principle
- Mean square estimation (Linear and non-linear)

1.4 Some terminology

- Homogeneous linear estimation: Linear estimation without a bias term
- Nonhomogeneous linear estimation: Linear estimation with a bias term

1.5 Redo in future

- Poisson
- Chi square
- Show why sample variance is divided by $n-1$.
- Prove why the generalized orthogonality principle (7-92) leads to optimal MS estimators (linear c non-linear).

Towards a motivation guide

1.6 Why transformations are useful

- By applying an orthonormal transformation (*a.k.a.* whitening) to a set of RVs we can easily compute the optimal

Towards a cheatsheet

1.7 Interesting RV Transformations

- If $\mathbf{x}_1, \dots, \mathbf{x}_n$ are jointly normal, then $\mathbf{z} = \mathbf{x}_1 + \dots \mathbf{x}_n$ is also normal (#257)
- If $\mathbf{x}_1, \dots, \mathbf{x}_n$ are independent and each $\mathbf{x}_i$ is $N(0, 1)$, then $\mathbf{z} = \mathbf{x}_1 + \dots \mathbf{x}_n$ is $\chi^2(n)$ (#259)
- If $\mathbf{x}$ is $\chi^2(n)$ and $\mathbf{y}$ is $\chi^2(m)$ and $\mathbf{x}, \mathbf{y}$ independent, then $\mathbf{z} = \mathbf{x} + \mathbf{y}$ is $\chi^2(n + m)$ (#260)

Bibliography

[1] A. P. anb S. Unnikrishna Pillai, *Probability, Random Variables and Stochastic Processes.* McGraw - Hill, 2002.

[2] I. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products.* Academic Press, 1980.

[3] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions.* Dover, 1970.