

FS properties	Time domain	FS domain
<b>Basic properties</b>		
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X_k + \beta Y_k$
Shift in time	$x(t - t_0)$	$e^{-j(2\pi/T)kt_0} X_k$
Shift in frequency	$e^{j(2\pi/T)k_0 t} x(t)$	$X(k - k_0)$
Time reversal	$x(-t)$	$X_{-k}$
Differentiation	$d^n x(t)/dt^n$	$(j2\pi k/T)^n X_k$
Integration	$\int_{-T/2}^t x(\tau) d\tau$	$(T/(j2\pi k))X_k, X_0 = 0$
Circular convolution in time	$(h \otimes x)(t)$	$TH_k X_k$
Convolution in frequency	$h(t) x(t)$	$(H * X)_k$
Circular deterministic autocorrelation	$a(t) = \int_{-T/2}^{T/2} x(\tau) x^*(\tau - t) d\tau$	$A_k = T  X_k ^2$
Circular deterministic crosscorrelation	$c(t) = \int_{-T/2}^{T/2} x(\tau) y^*(\tau - t) d\tau$	$C_k = TX_k Y_k^*$
Parseval equality	$\ x\ ^2 = \int_{-T/2}^{T/2}  x(t) ^2 dt = T \sum_{k \in \mathbb{Z}}  X_k ^2 = T \ X\ ^2$	
<b>Related functions</b>		
Conjugate	$x^*(t)$	$X_{-k}^*$
Conjugate, time-reversed	$x^*(-t)$	$X_k^*$
Real part	$\Re(x(t))$	$(X_k + X_{-k}^*)/2$
Imaginary part	$\Im(x(t))$	$(X_k - X_{-k}^*)/(2j)$
Conjugate-symmetric part	$(x(t) + x^*(-t))/2$	$\Re(X_k)$
Conjugate-antisymmetric part	$(x(t) - x^*(-t))/(2j)$	$\Im(X_k)$
<b>Symmetries for real <math>x</math></b>		
$X$ conjugate symmetric		$X_k = X_{-k}^*$
Real part of $X$ even		$\Re(X_k) = \Re(X_{-k})$
Imaginary part of $X$ odd		$\Im(X_k) = -\Im(X_{-k})$
Magnitude of $X$ even		$ X_k  =  X_{-k} $
Phase of $X$ odd		$\arg X_k = -\arg X_{-k}$
<b>Common transform pairs</b>		
Dirac comb	$\sum_{n \in \mathbb{Z}} \delta(t - nT)$	$1/T$
Periodic sinc function (ideal lowpass filter)	$\sqrt{\frac{k_0}{T}} \frac{\text{sinc}(\pi k_0 t/T)}{\text{sinc}(\pi t/T)}$	$\begin{cases} 1/\sqrt{k_0 T}, &  k  \leq \frac{1}{2}(k_0 - 1); \\ 0, & \text{otherwise} \end{cases}$
Box function (one period)	$\begin{cases} 1/\sqrt{t_0}, &  t  \leq \frac{1}{2}t_0; \\ 0, & \frac{1}{2}t_0 <  t  \leq \frac{1}{2}T \end{cases}$	$\frac{\sqrt{t_0}}{T} \text{sinc}(\pi k t_0/T)$
Square wave (one period with $T = 1$ )	$\begin{cases} -1, & t \in [-\frac{1}{2}, 0); \\ 1, & t \in [0, \frac{1}{2}) \end{cases}$	$\begin{cases} -2j/(\pi k), & k \text{ odd}; \\ 0, & k \text{ even} \end{cases}$
Triangle wave (one period with $T = 1$ )	$\frac{1}{2} -  t , \quad  t  \leq \frac{1}{2}$	$\begin{cases} 1/4, & k = 0; \\ 1/(\pi k)^2, & k \text{ odd}; \\ 0, & k \neq 0 \text{ even} \end{cases}$
Sawtooth wave (one period with $T = 1$ )	$2t, \quad  t  \leq \frac{1}{2}$	$\begin{cases} 0, & k = 0; \\ j(-1)^k/(\pi k), & k \neq 0 \end{cases}$

Table 4.3 Properties of the Fourier series.