

F'T properties	Time domain	F'T domain
Basic properties		
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\omega) + \beta Y(\omega)$
Shift in time	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Shift in frequency	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Scaling in time and frequency	$x(\alpha t)$	$(1/\alpha) X(\omega/\alpha)$
Time reversal	$x(-t)$	$X(-\omega)$
Differentiation in time	$d^n x(t)/dt^n$	$(j\omega)^n X(\omega)$
Differentiation in frequency	$(-jt)^n x(t)$	$d^n X(\omega)/d\omega^n$
Integration in time	$\int_{-\infty}^t x(\tau) d\tau$	$X(\omega)/j\omega, X(0) = 0$
Moments	$m_k = \int_{-\infty}^{\infty} t^k x(t) dt = (j)^k \frac{d^k X(\omega)}{d\omega^k} \Big _{\omega=0}$	
Convolution in time	$(h * x)(t)$	$H(\omega) X(\omega)$
Convolution in frequency	$h(t) x(t)$	$\frac{1}{2\pi} (H * X)(\omega)$
Deterministic autocorrelation	$a(t) = \int_{-\infty}^{\infty} x(\tau) x^*(\tau - t) d\tau$	$A(\omega) = X(\omega) ^2$
Deterministic crosscorrelation	$c(t) = \int_{-\infty}^{\infty} x(\tau) y^*(\tau - t) d\tau$	$C(\omega) = X(\omega) Y^*(\omega)$
Parseval equality	$\ x\ ^2 = \int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega = \frac{1}{2\pi} \ X\ ^2$	
Related functions		
Conjugate	$x^*(t)$	$X^*(-\omega)$
Conjugate, time-reversed	$x^*(-t)$	$X^*(\omega)$
Real part	$\Re(x(t))$	$(X(\omega) + X^*(-\omega))/2$
Imaginary part	$\Im(x(t))$	$(X(\omega) - X^*(-\omega))/(2j)$
Conjugate-symmetric part	$(x(t) + x^*(-t))/2$	$\Re(X(\omega))$
Conjugate-antisymmetric part	$(x(t) - x^*(-t))/(2j)$	$\Im(X(\omega))$
Symmetries for real x		
X conjugate symmetric		$X(\omega) = X^*(-\omega)$
Real part of X even		$\Re(X(\omega)) = \Re(X(-\omega))$
Imaginary part of X odd		$\Im(X(\omega)) = -\Im(X(-\omega))$
Magnitude of X even		$ X(\omega) = X(-\omega) $
Phase of X odd		$\arg X(\omega) = -\arg X(-\omega)$
Common transform pairs		
Dirac delta function	$\delta(t)$	1
Shifted Dirac delta function	$\delta(t - t_0)$	$e^{-j\omega t_0}$
Dirac comb	$\sum_{n \in \mathbb{Z}} \delta(t - nT)$	$(2\pi/T) \sum_{k \in \mathbb{Z}} \delta(\omega - (2\pi/T)k)$
Constant function	1	$2\pi \delta(\omega)$
Exponential function	$e^{-\alpha t }$	$(2\alpha)/(\omega^2 + \alpha^2)$
Gaussian function	$e^{-\alpha t^2}$	$\sqrt{\pi/\alpha} e^{-\omega^2/\alpha}$
Sinc function (ideal lowpass filter)	$\sqrt{\frac{\omega_0}{2\pi}} \text{sinc}(\frac{1}{2}\omega_0 t)$	$\begin{cases} \sqrt{2\pi/\omega_0}, & \omega \leq \frac{1}{2}\omega_0; \\ 0, & \text{otherwise.} \end{cases}$
Box function	$\begin{cases} 1/\sqrt{t_0}, & t \leq \frac{1}{2}t_0; \\ 0, & \text{otherwise.} \end{cases}$	$\sqrt{t_0} \text{sinc}(\frac{1}{2}t_0\omega)$
Triangle function	$\begin{cases} 1 - t , & t < 1; \\ 0, & \text{otherwise} \end{cases}$	$\text{sinc}^2(\frac{1}{2}\omega)$

Table 4.1 Properties of the Fourier transform.