

## Ampere's law

Equation of Ampere's law =  $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}}$   
 $B L = \mu_0 I_{\text{enclosed}}$

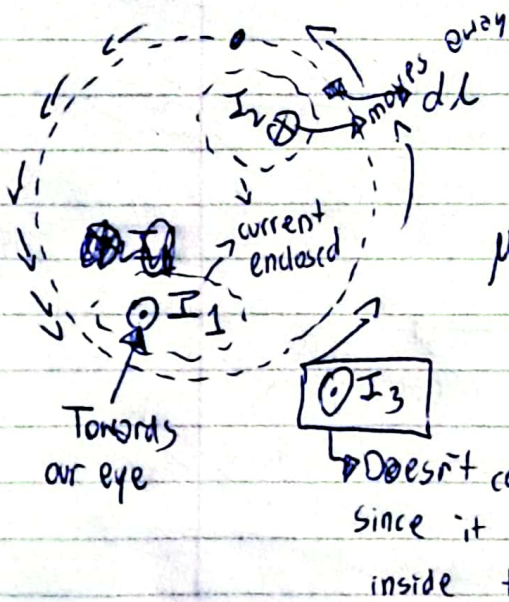
Ampere's law in detail

IMPORTANT 1

$\oint$  : closed integration

Main objective

Find the total magnetic field with the total integrated from the chunk length =  $\mu_0 I_{\text{enclosed}}$



need to find the total magnetic field within the total length

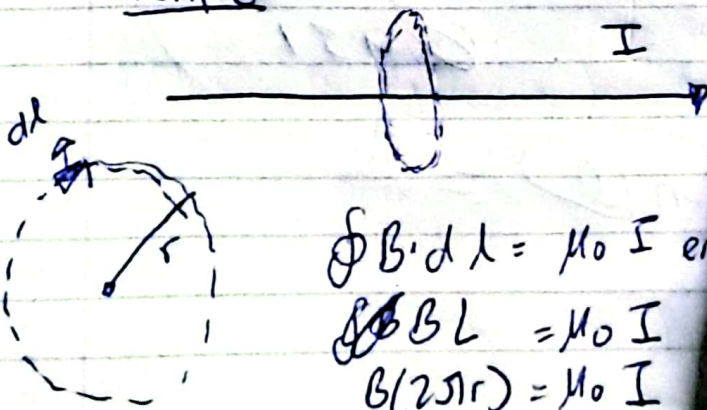
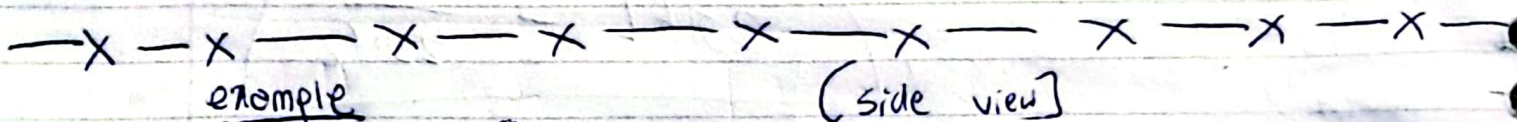
$\mu_0 I$  [indicates the current inside the loop]

Ampere's law

⇒ Right hand clamp rule

thumb ⇒ current

hands clamped ⇒ magnetic field



$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

$$B L = \mu_0 I$$

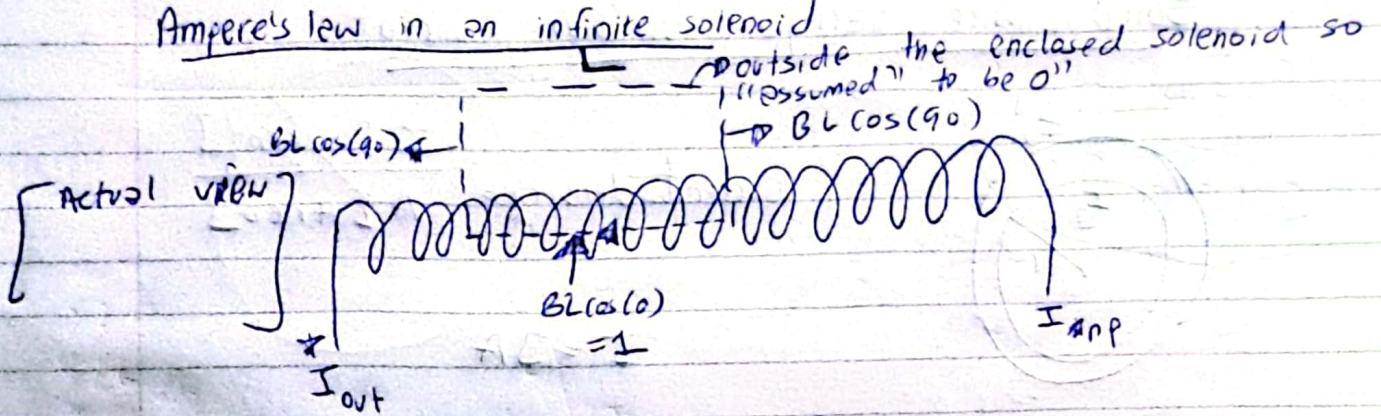
$$B(2\pi r) = \mu_0 I$$

IMPORTANT 2





## Ampere's law in an infinite solenoid



## Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

$$BL = \mu_0 (NI)_{\text{enclosed}}$$

$$B = \frac{\mu_0 (NI)}{L}$$

$$B = \mu_0 \frac{N}{L} I_{\text{enc}}$$

$$B = \mu_0 n I_{\text{enc}}$$

turns per length

Then number of magnetic field points the length has "pierced" through

## Toroid with N total windings



[Toroid]

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}}$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI)_{\text{enclosed}}$$

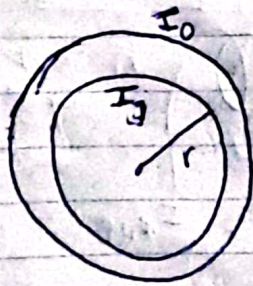
$$BL = \mu_0 (NI)$$

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$



## Solid wire carrying uniform current



$$\text{current density} = \frac{I [\text{current}]}{A [\text{Area}]}$$

$$I = JA$$

$$\oint B \cdot dl = \mu_0 I_{\text{enc}}$$

$$B(2\pi r) = \mu_0 (J)(A) \quad r < R$$

$$B(2\pi r) = \mu_0 I_0 \quad r > R$$

$$B(2\pi r) = \mu_0 \left( \frac{I}{\pi R^2} \right) (\pi r^2) \quad r < R$$

Area within the enclosed wire

$$J = \frac{\text{Total current}}{\text{Total Radius}}$$

$$B(2\pi r) = \mu_0 \left( \frac{I_0}{\pi R^2} \right) \pi R^2$$

$$= \mu_0 I_0$$

## Solid wire with uniform current density

$$\oint B \cdot dl = \mu_0 I \quad \left[ \text{Assume } \right] \text{current}$$

density is given

$$B(2\pi r) = \mu_0 [J] \times (\pi r^2) \quad \rightarrow \text{Inside the wire}$$

Given current density

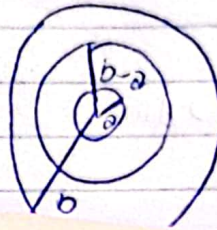
$$= \mu_0 (J) (\pi R^2) \quad \rightarrow \text{outside the loop}$$



Solid wire with hollow wire uniform current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Inside the hollow



Just hollow  
no ~~current~~ enclosed



$$(r < a) \rightarrow B(2\pi r) = \mu_0(0) \text{ [not enclosed]}$$

Just hollow  
and enclosed

$$\Rightarrow B(2\pi r) = \mu_0 I_0$$

$$= \mu_0 \left( \frac{I_0}{(\pi b^2 - \pi a^2)} \right) (\pi r^2 - \pi a^2) \quad r > a \quad r < b$$

outside  
big sphere  
enclosed

$$\Rightarrow B(2\pi r) = \mu_0 I_0$$

$$= \mu_0 \left( \frac{I_0}{(\pi b^2)} \right) (\pi b^2)$$

$$= \mu_0 I_0$$

Same with current density  $\mu_0 \left( \frac{I_0}{\pi b^2} \right)$  with the given current density & Area depending on the shape

—X—X—X—X—X—X—X—X—X—  
Solid wire with non uniform current density

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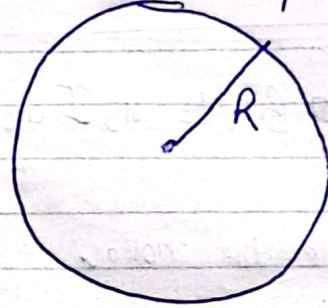
Hilroy



In a non-uniform current density, the current density itself depends on the small radius of the wire

$$I = \int J(r) dA$$

$$dA = 2\pi r^2 dr$$



$$B(2\pi r) = \mu_0 \int_0^r J(r) 2\pi r dr$$

given  
charge density  
with the radius  
dependent

$$r < R$$

$$r > R$$

$$B(2\pi r) = \mu_0 \int_0^R J(r) 2\pi r dr$$

hollow wire

$$B(2\pi r) = \mu_0 \int_0^a (0) [no magnetic field]$$

$$= \mu_0 \int_0^r J(r) 2\pi r dr$$

$$= \mu_0 \int_0^b J(r) 2\pi r dr$$