② $\lambda = -\frac{\hbar}{2}$ $S_x \chi_-^x = -\frac{\hbar}{2} \chi_-^x$ (-1)

$\cancel{\frac{\hbar}{2}}$ $\cancel{\frac{\hbar}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \cancel{-\frac{\hbar}{2}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

(1) $\begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ $\therefore \alpha = -\beta$

$\therefore \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = -\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ so $\therefore \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} -\alpha \\ -\alpha \end{pmatrix}$

② $\begin{pmatrix} \alpha \\ \alpha \end{pmatrix}^* \begin{pmatrix} \alpha \\ \alpha \end{pmatrix} = 1$ ③ $(\alpha ; \alpha) \begin{pmatrix} -\alpha \\ \alpha \end{pmatrix} = 1$

$\alpha^2 + \alpha^2 = 1$

$2\alpha^2 = 1$

$\alpha^2 = \frac{1}{2}$ $\alpha = \frac{1}{\sqrt{2}}$

$$n_-^{\eta} = \frac{1}{\sqrt{2}}\begin{pmatrix} \alpha \\ -\alpha \end{pmatrix}$$

due to normalised α function we can conclude that

$\frac{\alpha}{\alpha}$ is $\frac{\gamma}{1}$ so $\begin{pmatrix} \alpha \\ -\alpha \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$\therefore n_-^{\eta} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$