

examples

Example

⇒ If you measure S_x on a particle in general state

$\chi = a\chi_+ + b\chi_-$, what result will you get?

⇒ Find the eigenvalues and eigen functions of S_n

$$S_n = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

⊗ To solve only eigen values

$$\det(S_x - \lambda I) = 0$$

$$S_n = \begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{\hbar}{2} & 0 \end{pmatrix}$$

$I \vdash$ identity matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \det \left(\begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{\hbar}{2} & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right) = 0$$

$$\Rightarrow \det \left(\begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{\hbar}{2} & 0 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) = 0$$

$$\Rightarrow \det \left(\begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{\hbar}{2} & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right) = 0$$

$$\det \left\{ \begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{\hbar}{2} & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right\} = 0$$

$$\begin{vmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - \frac{\hbar^2}{4} = 0$$

$$\lambda^2 = \frac{\hbar^2}{4}$$

$$\lambda = \pm \frac{\hbar}{2}$$