hydrogen atom R_{20} and R_{21} derivation

Recap wave function

$$\boxed{\psi(_{n\ell m}) = R(r)_{n\ell} \times Y_{\ell}^{m}(\theta, \phi)}$$

$$R_{n\ell} = \frac{u}{r}$$

$$= \frac{(e^{-\delta})(\delta^{\ell+1})\, v(\delta)}{r}$$

$$v(\delta) = \sum_{j=0}^{\infty} C_j \delta^j$$

$\boxed{C_0}$

$$C_{j+1} = \left[\frac{2(j+\ell+1)-n}{(j+1)(j+2\ell+2)} C_j\right]$$

$j_{max} = n-\ell-$

$\Psi_{100} := (r, \theta, \phi) = R_{10} Y_0^0 \to \frac{1}{\sqrt{4\pi}}$

$\Psi_{100} := \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$

$\cancel{\Psi_{110}}$ and $\cancel{\Psi_{111}}$ $\quad l < n$

$l = n$

$\boxed{l = 1 \quad n = 1}$ $\quad f(l) = n$

$1 = 1 - 1 - J_{max}$

$[J_{max} = \text{negative}]$ ✗

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$\boxed{R_{20}, R_{21}}$: not normalizing method

↳ complicated

R_{20} : $n = 2, l = 0$: $\boxed{\begin{aligned} J_{max} &= n - l - 1 \\ &= 2 - 0 - 1 \\ J_{max} &= 1 \end{aligned}}$ ✱

$J_{max} =$

$\Rightarrow \sum_0^1 C_j S^j$

$= C_0(S^0) + C_1(S^1)$

$= C_0 + C_1 S$

$\boxed{C_1 = -C_0}$

$C_{j+1} = \frac{2(j + l + 1 - n)}{(j+1)(j+2l+2)} C_j$

$C_1 = \frac{2[0 + 0 + 1 - 2]}{[1+1][1 + 2(0) + 2]} C_0$

$C_1 = \frac{2[-1]}{[1][2]}$

$C_1 = \frac{-1}{2} C_0$

$\boxed{C_1 = -\frac{1}{2} C_0}$

$R_{n l}$ $n=2$, $l=0$

$C_1 = -C_0$

$\therefore v(\delta) = C_0 - C_0\delta$

$= C_0(1-\delta)$

$\therefore R_{nl} = \left[\frac{1}{r} e^{-\delta} \delta^{(l+1)} v(\delta)\right]$

$R_{nl} = \left[\frac{1}{r} e^{-\delta} \delta^{(l+1)}\right]\left[C_0(1-\delta)\right]$

$\delta = \frac{r}{a_n} \quad \frac{r}{2a}$

$R_{nl} = \left[\frac{1}{r}\delta e^{-\delta}\right]\left[C_0(1-\delta)\right]$

$R_{nl} = \left[\frac{1}{r}\frac{r}{2a} e^{-\delta}\right]\left[C_0(1-\delta)\right]$

$R_{nl} = \left[\frac{\cancel{r}\,1}{2a\cancel{r}} e^{-\delta}\right]\left[C_0(1-\delta)\right]$

$R_{20} = \frac{C}{2a}\left(1-\frac{r}{2a}\right) e^{-r/2a}$

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Derivation of R_{21}

$n=2 \quad l=1$

$J_{max} = n-l-1$

$= 2-1-1$

$J_{max} = 0$

J_m

$v(\delta) = C_0$

$\sum_{0}^{0} C_j \delta^j \quad C_0 \delta^0 = C_0(1) = C_0$

$= \frac{1}{r} \cdot \delta^2 e^{-\delta} C_0 = \frac{C_0}{\cancel{r}} \frac{r^{\cancel{2}}}{4z^2} e^{-r/2z}$

$$\boxed{= \frac{C_0 r}{4z^2} e^{-r/2z}}$$