hydrogen atom

$\Rightarrow$ Force $= K \frac{q_1 q_2}{r^2}$

$K = \frac{1}{4\pi\varepsilon_0 r^2}$

$\Rightarrow V(r) = -\vec{\nabla} F$

$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$

$= -\frac{e^2}{4\pi\varepsilon_0 r}$

Radial equation

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[-\frac{e^2}{4\pi\varepsilon_0 r} \cancel{+\frac{\hbar^2}{2m}} + \frac{\hbar^2}{2m}\left[\left(\frac{l(l+1)}{r^2}\right)u\right]\right] = Eu$$

(+) convert the radial equation into a more simpler form
$\Rightarrow$ asympdtotic solution

Radial equation cleaner solution

- $\Rightarrow$ asymptopic solution [hard to solve]
- $\Rightarrow$ Bound states : [electron remains bound to proton or neutron]
  - : $E < 0$: [remains negative]

$$\therefore -\frac{2m}{\hbar^2}\left[RE\right] = \frac{d^2u}{dr^2} - \frac{2m}{\hbar^2}\left[\frac{-e^2}{4\pi\varepsilon_0 r} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u = -\frac{2mE}{\hbar^2}u$$

$K = \frac{\sqrt{-2mE}}{\hbar}$ $\quad E < 0,\ K > 0$ real & positive

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$$\frac{me^2 u}{2\hbar^2 \pi \epsilon_0 r}$$

$$\Rightarrow \frac{d^2u}{dr^2} + \frac{\cancel{2}m\, e^2 u}{\hbar^2 \cancel{4}\pi \epsilon_0 r} - \frac{l(l+1)}{r^2} u = K^2 u$$

$$\Rightarrow \delta = Kr \Rightarrow \boxed{r = \frac{\delta}{K}}$$

$$K = \frac{\sqrt{-2mE}}{\hbar}$$

$$= dr = \frac{d\delta}{K\,[\text{constant}]} \Rightarrow dr^2 = \frac{d\delta^2}{K^2}$$

$$\Rightarrow \frac{d^2u}{dr^2} \text{ or } \frac{d^2u}{\frac{d\delta^2}{K^2}} \text{ or, } \frac{d^2u\,K^2}{d\delta^2}$$

$$\Rightarrow \frac{d^2u\,K^2}{d\delta^2} + \frac{me^2u}{\hbar^2 2\pi\epsilon_0 \left(\frac{\delta}{K}\right)} - \frac{l(l+1)}{\left(\frac{\delta^2}{K^2}\right)} u = K^2 u$$

$$\Rightarrow \frac{d^2u\,K^2}{d\delta^2} + \frac{me^2u\;K}{\hbar^2 2\pi\epsilon_0 \delta} - \frac{l(l+1)\,K^2u}{\delta^2} = K^2 u$$

After dividing by K^2

$$\frac{d^2u}{d\delta^2} + \frac{me^2u\,\frac{1}{K}}{\hbar^2 2\pi\epsilon_0 \delta} - \frac{l(l+1)u}{\delta^2} = u$$

constant δ_0

~~M=me~~

$$\frac{me^2u}{\hbar^2 2\pi\epsilon_0 \delta K} \boxed{\delta}$$

[Its asymptopic so the solution will be approximate]

① move δ to the right

$$\boxed{\frac{d^2u}{d\delta^2} = u\left[\left(1 - \frac{\delta_0}{\delta}\right) + \left(\frac{l(l+1)}{\delta^2}\right)u\right]}$$

~~differ~~ double differential equation

⇒ $m = m_e$ [the electron ~~mass~~ is moving around the nucleus so we find the reduced mass]

$$m^* = \frac{[m_e][m_p]}{m_e + m_p}$$

⇒ $\psi(r)$ [we are solving time independant shrodinger equation]

$\psi(\vec{r}) = \underbrace{\psi(r,t)}_{\text{spin cannot be derived}}$ [time dependant meaning we have to use time coordinate too but won't derive the spin of electron

⇒ non physical spin

⇒ $\psi(r)$ [$r = (x, y, z)$] spin can be visualised in only space derivative ~~too~~ or coordinate but not space and time coordinate

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<u>Differential equation</u>

$$\frac{d^2u}{d\rho^2} = \left(1 - \cancel{\frac{\rho_0}{\rho}}^{\,0} + \cancel{\frac{\ell(\ell+1)}{\rho^2}}^{\,0}\right)u$$

⇒ $\lim_{\rho \to \infty} : \frac{d^2u}{d\rho^2} = (1)u$
$= u$

⇒ $u(\rho) = Ae^{-\rho} + Be^{-\rho}$