Expressing the state as a linear combination

(*) vector of χ [chi] $= a\chi_+ + b\chi_-$ can be expressed as a linear combination of eigen functions $\chi_\pm$ χ

Main objective

→ Determine the vector quantity χ, in terms of $\chi_\pm$ χ

Derivation of χ

$$\chi = c\chi_+ + d\chi_-$$

→ whatever the derivation that we will retrieve for ~~c~~ and ~~c~~ d ~~as~~, that will come in correlation for variable derived as a and b

(*) [direction] $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

[Based on the above equation, we can derive ~~the~~ some value derived as x in the x axis, y in the y axis and so on

In short

Some specific value, as x, moving in some component $\hat{x}$

" " " " y, " " " " $\hat{y}$

" " " " z, " " " " $\hat{z}$

to get only one of the componetnt

$$(\hat{x}) = (x)(\vec{r})$$
$$(x) = (\hat{x})(\vec{r})$$

[moving] or [direction] = [direction] or [moving] $\vec{r}$

x or $\hat{x}$ = $\hat{x}$ or x $\vec{r}$

Back to the x equation

$$x = c x_+ + d x_-$$

to get c, use Bra Ket notation

$$\underbrace{\hat{x}_+ \cdot x}_{\text{how to derive it}} = c$$

$$\langle \hat{x}_+ | x \rangle = c$$

to ~~get~~ get d same thing

$$\underbrace{\langle \hat{x}_- | x \rangle}_{\text{dot product of } (\hat{x}_-) \cdot (x)} = d$$

Our given information

$$|\hat{x}_+\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$c = \langle \hat{x}_+ | x \rangle$$

$$x = a x_+ + b x_-$$

$$c = \langle \hat{x}_+ | a x_+ + b x_- \rangle$$

multiply

multply

we are deriving $\hat{x}_+$ in bra notation instead of ket notation so we transpose the matrix that represents the $\hat{x}_+$

$$(1,1)\left[(a)\left(\frac{1}{\sqrt{2}}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] + (1,1)\left[(b)\left(\frac{1}{\sqrt{2}}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$$

$$(1,1)\left[\frac{a}{\sqrt{2}}\underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\hat{x}_+}\right] + (1,1)\left[\frac{b}{\sqrt{2}}\underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\hat{x}_-}\right]$$

multiply

$$= \frac{a}{\sqrt{2}} (1, 1) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{b}{\sqrt{2}} (1, 1) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

multiply then add two values

$$= \frac{a}{\sqrt{2}} \left((1 \times 1) + (1 \times 0) \right) + \frac{b}{\sqrt{2}} \left[(1 \times 0) + (1 \times 1) \right]$$

$$= \frac{a}{\sqrt{2}} (1) + \frac{b}{\sqrt{2}} (1)$$

$$c = \frac{a}{\sqrt{2}} + \frac{b}{\sqrt{2}}$$

$$= \frac{a+b}{\sqrt{2}}$$

$$d = \langle x_-^{\dagger} | x \rangle = \langle x_-^{\dagger} | a x_+ + b x_- \rangle$$

$$= \frac{a}{\sqrt{2}} (1 \quad -1) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{b}{\sqrt{2}} (1 \quad -1) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{a-b}{\sqrt{2}}$$

<u>why $-x$</u>

Because any negative x value determines x_-

any positive x value determines x_+