Differential equation

$$\frac{d^2u}{d\rho^2} = \left(1 - \cancel{\frac{\rho_0}{\rho}}^{\,0} + \cancel{\frac{\ell(\ell+1)}{\rho^2}}^{\,0}\right)u$$

$$\Rightarrow \lim_{\rho\to\infty} : \frac{d^2u}{d\rho^2} = (1)u = u \qquad \Rightarrow u(\rho) = Ae^{-\rho} + Be^{-\rho}$$

$\Rightarrow u = rR \rightarrow$ wave function

$\delta = kr$

$\delta \to \infty \quad r \to \infty \quad R \to 0$ [wave function]

$\Rightarrow R = u(\delta)$

a) $\delta \to \infty$

$u(\infty) = Ae^{-\infty} + \cancel{Be^{\infty}}$ 0

$u(\delta) = Ae^{-\delta} \quad B = 0$

When δ goes to infinite

b) $\cancel{\delta \to -\infty}$

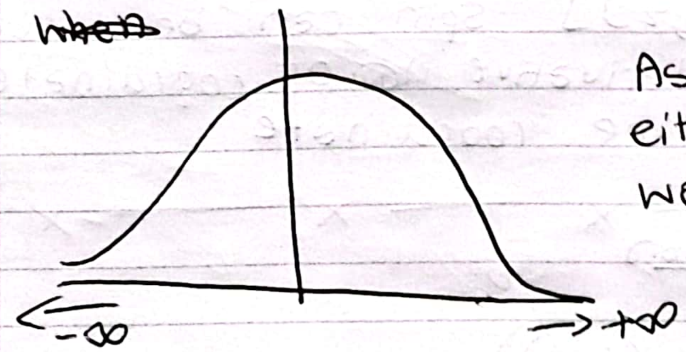
$\cancel{u(-\infty) = Ae^{\infty}} + Be^{-\infty}$ 0

$\cancel{u(-\infty) = Be^{-\infty}}$

$$\frac{d^2u}{d\delta^2} = \frac{l(l+1)}{\delta^2}u$$

$$u(\delta) = C\delta^{l+1} + D\delta^{-l}$$

~~when~~



As the δ goes to either $+\infty$, or $-\infty$, our wave function will deacrease

$|\psi| = 0 \qquad |\psi|^2 = 0$

$\Rightarrow$ Since our wave function will be 0 as the δ goes to $[+\infty]$ or $[-\infty]$ due to the gaussien distribution, the probability density becomes 0.

$u(\delta) = C\delta^{l+1} + \cancel{D\delta^{-l}}$ 0 when $\delta = 0$

$\Rightarrow u(\delta) = C\delta^{l+1} \qquad D = 0$

Two solutions for wave function

$$U(\rho) = Ae^{-\rho}$$
$$u(\rho) = C\rho^{l+1}$$

© $u(\rho) = Ae^{-\rho} C\rho^{l+1}$ A & C are constants

$AC = v(\rho)$

$$u(\rho) = \underbrace{e^{-\rho}}\ \underbrace{\rho^{l+1}}\ \underbrace{v(\rho)}_{\text{constants}}$$

product rule differentiation

[changing position of value doesn't change the solution in multiplication]

$$\Rightarrow \rho\frac{d^2v}{d\rho^2} + 2(l+1-\rho)\frac{dv}{d\rho} + [\rho_0 - 2(l+1)]v =$$

rho

power series solution of this equation

$$C_{j+1} = \left[\frac{2(j+l+1) - \rho_0}{(j+1)(j+2l+2)}\right] C_j \qquad v(\rho) = \sum_{j=0}^{\infty} C_j \rho^j$$

recursion relation

C_0 :- normalisation constant

$$C_j = \frac{2^j}{j!} C_0 \qquad v(\rho) = C_0 \sum_{j=0}^{\infty} \frac{2^j}{j!} \rho^j$$

$$= C_0 e^{2\rho}$$

$$u(\delta) = e^{-\delta}\rho^{l+1} C_0 e^{2\delta}$$

$$u(\delta) = C_0 \delta^{l+1} e^{\delta}$$

$$\lim_{\delta \to \infty} U(\delta) \to \infty$$

$\Rightarrow C_{j_{max}+1} = 0$

$\uparrow$

when

[numeretor $(2(j+l+1)) - \delta) = \underline{0}$]

$\therefore n \equiv j_{max} + l + 1 \equiv$ principle quantum number