0:00 to 9:53

1) Matrix representation of S^2

We know

(L^2, L_z)

$L^2 := L_x^2 + L_y^2 + L_z^2$

$\therefore (L^2, L_z)$

$\rightarrow (L_x^2 + L_y^2 + L_x^2, L_z)$

Note :-

[→ commutes]

$L_x^2 \rightarrow L_x \qquad L_z^2 \rightarrow L_z$

$L_y^2 \rightarrow L_y$

→ Similarly just like L, s also has the same equation but instead of l, you replace with s

equation of spin

$\boxed{S(s+1)\hbar^2} = S^2$

→ Two types of representations in regards to S^2

[positive chi] $\chi_+ \qquad \chi_-$ [negative]

$S^2\chi_+ = [S(S+1)\hbar^2]\chi_+ \qquad S := \frac{1}{2}$

$[\frac{1}{2}(\frac{1}{2}+1)\hbar^2]\chi_+$

$[\frac{1}{2}(\frac{3}{2})\hbar^2]\chi_+$

$S^2\chi_+ := [\frac{3}{4}\hbar^2]\chi_+$

$S^2\chi_- :=$ exact same but χ_-

All spin operators are in 2 X 2 matrices

spin operators 2x2 matrix $[S^2 \chi_+] = \frac{3}{4}\hbar^2 \chi_+$

$$\begin{pmatrix} c & d \\ e & f \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{3}{4}\hbar^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\chi_+ \; [|0\rangle]$ 1 present at position 0

$\chi_- \; [|1\rangle]$ 1 present at position 1

$c \times 1 \quad d \times 0$

$e \times 1 \quad f \times 0$

$$\begin{pmatrix} c & 0 \\ e & 0 \end{pmatrix} = \frac{3}{4}\hbar^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} c \\ e \end{pmatrix} = \frac{3}{4}\hbar^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$c = \frac{3}{4}\hbar^2$

$e = 0$

$$S^2 = \begin{pmatrix} c & d \\ e & f \end{pmatrix}$$

$[S^2 \chi_-] = \frac{3}{4}\hbar^2 \chi_-$

$$\begin{pmatrix} c & d \\ e & f \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{3}{4}\hbar^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} d \\ f \end{pmatrix} = \frac{3}{4}\hbar^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$f = \frac{3}{4}\hbar^2 (1)$

$\boxed{d = 0}$

$$S^2 = \begin{pmatrix} \frac{3}{4}\hbar^2 & 0 \\ 0 & \frac{3}{4}\hbar^2 \end{pmatrix}$$

$$S^2 = \frac{3}{4}\hbar^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\left[\frac{3}{4}\hbar^2\right.$ taken as common factor