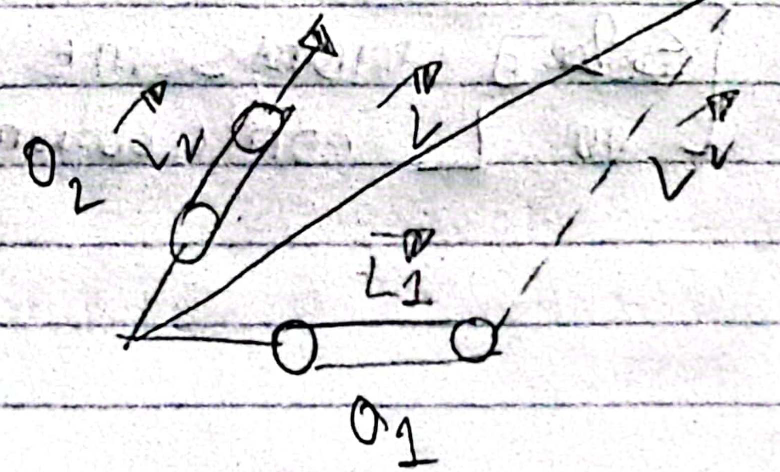
Lesson 40.1

Clebsch Gordon coefficient

- coupling

1) orbital angular momentum with itself
2) orbital angular momentum with spin angular momentum

example



$\vec{L} = \vec{L_1} + \vec{L_2}$

3 possibilites from O_1 & O_2

* Both orbital angular momentum
* Both spin angular momentum
* either or are spin and orbit angular momentum

- spin and orbital angular momentum ⇒ spin orbital coupling
  " and spin : total angular momentum

—X——X——X——X——X—

Quantized values

- $L^2\psi = \ell(\ell+1)\hbar^2\psi$

$L_z\psi = m_\ell \hbar \psi$

$[-\ell \leq m_\ell \leq \ell]$

spin $S^2\psi = s(s+1)\hbar^2\psi$

$S_z\psi = m_s \hbar \psi$

$-s \leq m_s \leq s$

<u>MAIN</u>

$\vec{J} = \vec{L} \pm \vec{S}$

orbital angular momentum — spin angular momentum

[if any s-orbital atom, no L since the ℓ quantum number is 0, so L is 0 and has no orbital angular momentum]

$\vec{J} = 0 + \vec{S}$ or $\vec{J} = \vec{S}$

$\boxed{\text{s-orbital ONLY!!!!}}$

$|s\, m\rangle = |J\, m\rangle$ [S & J not same, no L contribution]

Clebsch gordon coefficients

- CG coefficents, = expansion coefficients of total angular momentum eigen states in uncoupled product basis

Example

Product basis

$J_1 = 1/2 \qquad J_2 = 1/2$

$$|J_1\, m_1\rangle \otimes |J_2\, m_2\rangle = \begin{cases} |\tfrac{1}{2}\ \tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\ \tfrac{1}{2}\rangle \\ |\tfrac{1}{2}\ \tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \\ |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\ \tfrac{1}{2}\rangle \\ |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \end{cases}$$

J remains 1/2 throughout

$$\begin{array}{c} \quad m_1 \quad m_2 \\ \sim |\uparrow\ \ \uparrow\rangle \\ |\uparrow\ \ \downarrow\rangle \\ |\downarrow\ \ \uparrow\rangle \\ |\downarrow\ \ \downarrow\rangle \end{array} \longrightarrow |JM\rangle$$

$\tfrac{1}{2}-\tfrac{1}{2} \qquad \tfrac{1}{2}+\tfrac{1}{2}$

$$\underbrace{|J_1 - J_2|}_{\text{J is only positive so must have only absolute value}} \le J \le J_1 + J_2$$

$\boxed{J = 0, 1}$

$m_1 + m_2 = M$

$M = \{1, 0, 0, -1\}$

$$|JM\rangle = \begin{cases} |1\ 1\rangle \\ |1\ 0\rangle \\ |1\ 0\rangle \\ |1\ -1\rangle \end{cases}$$