$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2} = n^2 K$$

Case ①

Distinguishable particles case

total wave function

$$\psi_{n_1 n_2}(x_1, x_2) = \psi_{n_1}(x_1)\, \psi_{n_2}(x_2)$$

$$E_{n_1 n_2} = (n_1^2 + n_2^2) K$$

$$\therefore E_{n_1 n_2} = K n_1^2 + K n_2^2$$

$$E_{n_1} = (n_1)^2 \left[\frac{\pi^2 \hbar^2}{2ma^2}\right]$$

$$E_{n_2} = (n_2)^2 \left[\frac{\pi^2 \hbar^2}{2ma^2}\right]$$

$$E_{n_1} = K(n_1)^2$$

$$E_{n_2} = K(n_2)^2$$

$$E_{n_1 n_2} = K(n_1)^2 + K(n_2)^2$$

ground state $\rightarrow \psi_{(1,1)}$

$$\psi^{(1)}_{n_1}(x_1) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$\psi^{(1)}_{n_2}(x_2) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$\psi_{(1)}(x_1) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$$

$$\psi_{(1)}(x_2) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$$

$$\psi_{(1,1)} = \left(\sqrt{\frac{2}{a}} \sin\left(\frac{\pi x_1}{a}\right)\right)\left(\sqrt{\frac{2}{a}} \sin\left(\frac{\pi x_2}{a}\right)\right)$$

$$E_{11} = 2K$$

1st excited state:

(1,1)

$1,1 \to 2,1$ $\quad$ $1,2 \to 1,2$ — same thing

$$E_{12} = (2^2 + 1^2)K = 5K$$

$$E_{21} = (1^2 + 2^2)K = 5K$$

doubling degenerate

Why degenerate

1) $[P, H]$ assume $[1, 2]$ or $[2, 1]$ eigen values aren't the same but the energy output is the same

$$\psi_{(1,2)} = \left(\frac{2}{a} \sin\left(\frac{\pi x}{a}\right)\right)\left(\frac{2}{a} \sin\left(\frac{2\pi x}{a}\right)\right)$$

$$\psi_{(2,1)} = \left(\frac{2}{a} \sin\left(\frac{2\pi x}{a}\right)\right)\left(\frac{2}{a} \sin\left(\frac{\pi x}{a}\right)\right)$$