

Addition of angular momenta

Example used

⇒ hydrogen atom $\left[\text{orbiting } S\text{-orbital } 1s^1 \right]$ $l=0$

What's inside the hydrogen atom

- 1) electron
 - 2) proton
 - 3) neutron
- ⇒ fermions with spin $1/2$

⇒ hydrogen atom: S-orbital

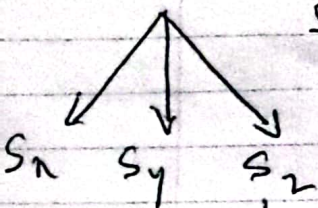
⊗ $l=0$

⊗ no orbital angular momentum

⇒ Add the spin value for electron and proton to get the angular momentum for hydrogen atom

⇒ $S = S_1 + S_2$

[Total spin] = electron spin + proton spin [or nuclear spin]



$S_x 1/2, 1/2$ → proton spin state

↳ electron spin state

[you can go for S_x or S_y as each spin axis condemns one individual spin component]

$$\Rightarrow S_z \chi_1 \chi_2 = (S_z^{(1)} + S_z^{(2)}) \chi_1 \chi_2$$

$$= (S_z^{(1)} \chi_1) \chi_2 + \chi_1 (S_z^{(2)} \chi_2)$$

$S_z^{(1)}$ \swarrow
 [first spin state]

$S_z^{(2)}$ \searrow
 [second spin state]

Note:- while you can write

$$(S_z^{(1)} \chi_1) \chi_2 + (S_z^{(2)} \chi_1) \chi_2$$

its best to keep like states on one side while the unlike states on the other side

$$\Rightarrow S_z = m\hbar \quad S_z(m\hbar) \text{ [eigen value]}$$

$$(S_z) \chi = (m\hbar) \chi$$

$$\Rightarrow S_z \chi_1 \chi_2 = m_1 \hbar \chi_1 \chi_2 + m_2 \hbar \chi_1 \chi_2$$

$$S_z \chi_1 \chi_2 = \hbar (\chi_1 \chi_2) [m_1 + m_2]$$

$$\Rightarrow m = m_1 + m_2$$