uncoupled states

$|j_1 \; m_1\rangle$ or, $|j_2 \; m_2\rangle$

coupled states

$|jm\rangle$

$|j_1 \; m_1\rangle \otimes |j_2 \; m_2\rangle$ — product state

{ coupled state

CG coefficient

$|jM\rangle = \sum_{m_1 m_2} |j_1 m_1 j_2 m_2\rangle \times \langle j_1 m_1 j_2 m_2 | j \; M\rangle$

unitary matrix [identity matrix]

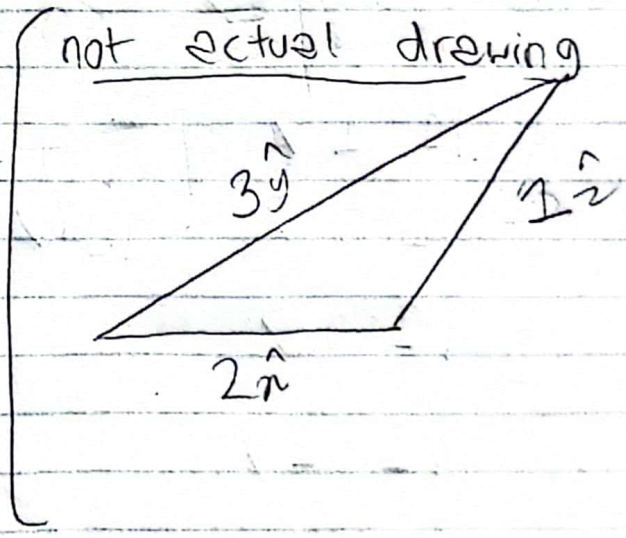
$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

↳ coupled

Example

$\vec{r} = 2\hat{x} + 3\hat{y} + \hat{z}$

not actual drawing



Uncoupled states

1 coupled state

$$|1\ 0\rangle = C_1|\uparrow\uparrow\rangle + C_2|\uparrow\downarrow\rangle + C_3|\downarrow\uparrow\rangle + C_4|\downarrow\downarrow\rangle$$

the uncoupled states can be derived from each coupled state with their coefficients

C_1, C_2, C_3, C_4 [CG coefficients]

how to find C_1

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ [orthonormal]

$\hat{y} \cdot \vec{r} = y$ where x and $z = 0$

$|\uparrow\uparrow\rangle|1\,0\rangle = C_1$ where $C_2, C_3, C_4 = 0$

[outputs]

$\frac{1}{2} \times \frac{1}{2}$		1		
	$+\frac{1}{2}$	1	0	
$+\frac{1}{2}$	$+\frac{1}{2}$	1	0	0

$+\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
$-\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1
		$-\frac{1}{2}$	$-\frac{1}{2}$	1

① $j_1 \cdot j_2$ ▼ Table

② m_1, m_2 ▼ row

③ J, M ▼ column

④ x ▼ $\sqrt{\pi}$ or $-x$ ▼ $-\sqrt{\pi}$

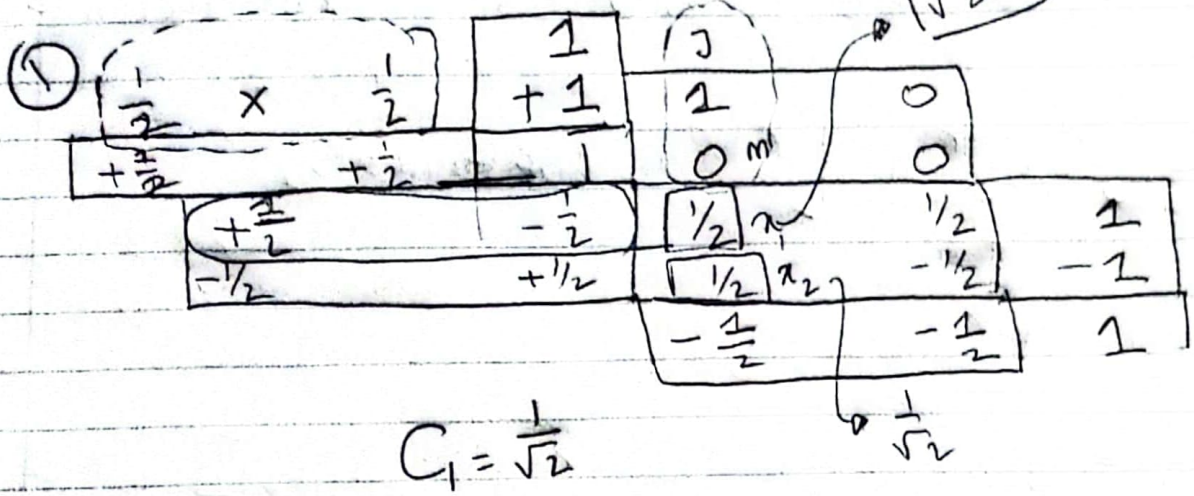
PROCEDURE

$j_1 = \frac{1}{2}$ $j_2 = \frac{1}{2}$

locate the table with $\frac{1}{2} \times \frac{1}{2}$

J M

$\therefore |1\ 0\rangle = \frac{1}{\sqrt{2}} \left|\frac{1}{2}\ \frac{1}{2}\right\rangle$ (2)



$C_1 = \frac{1}{\sqrt{2}}$

(1) j_1 j_2

(2) $|J\ m\rangle =$

$1\ 0$

I got

$n = \sqrt{n}$

$= \sqrt{\frac{1}{2}}$

$= \frac{1}{\sqrt{2}}$

!! from $\frac{1}{2}$ value, my two values are $\frac{1}{2}$ & $-\frac{1}{2}$ for m_1 & m_2

$$|1\ 0\rangle = \frac{1}{\sqrt{2}} \left|\frac{1}{2}\ \frac{1}{2}\ \ \frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$+ \frac{1}{\sqrt{2}} \left|\frac{1}{2}\ -\frac{1}{2}\ \ \frac{1}{2}\ \frac{1}{2}\right\rangle$$