Eigen functions of total spin

Component of spin angular momentum

S^2 :- Total spin value

S

S_z

[magnetic spin]

m_s :- Orientation for spin angular momentum

L^2

l

L_z [z component of orbital angular momentum]

[magnetic orbital]

m_l :- orientation for the orbital angular momentum

eigen functions of S^2

$S^2 \cdot 2\hbar^2$

$S_x^2 \quad S_y^2 \quad S_z^2$

$S_x + S_y \cdot S_y + S_z \cdot S_z$

two different spin state values

Keep in mind

$$|1\ 1\rangle = |\overset{e^-}{\uparrow}\ \overset{p}{\uparrow}\rangle$$

one s [spin] value   another s [spin] value

$$S^2 = (\underset{\text{electron}}{S^{(1)}} + \underset{\text{proton}}{S^{(2)}})^2$$

$$= (\vec{S}^{(1)} + \vec{S}^{(2)}) \cdot (S^{(1)} + S^{(2)})$$

$$(S^{(1)})^2 + 2S^{(1)}S^{(2)} + (S^{(2)})^2$$

$$\Rightarrow \underset{\text{Total spin value}}{S^{(1)}} = S_x^{(1)} + S_y^{(1)} + S_z^{(1)}$$

same goes for $S^{(2)} = S_x^{(2)} + S_y^{(2)} + S_z^{(2)}$

back to the S^2 equation

$$S^2 = (S^{(1)})^2 + \underline{2S^{(1)}S^{(2)}} + (S^{(2)})^2$$

⊛ $S^{(1)}S^{(2)}$ derived from the $|1\ 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

Because $|1\ 1\rangle$ and $|1\ -1\rangle$ are
$\uparrow\uparrow$ $\downarrow\downarrow$

$$\Rightarrow S_-^{(1)}S_-^{(2)}|\uparrow\downarrow\rangle = \underbrace{(S_x^{(1)}|\uparrow\rangle)(S_x^{(2)}|\downarrow\rangle)}$$

⊛ Because χ_1 and χ_2 are independent, the state spin χ_1 has nothing to do with spin state χ_2

$$S^{+(1)} S^{+(2)} |\uparrow\downarrow\rangle = \left(S_x^{(1)} |\uparrow\rangle\right)\left(S_x^{(2)} |\downarrow\rangle\right) + \left(S_y^{(1)} |\uparrow\rangle\right)\left(S_y^{(2)} |\downarrow\rangle\right)$$

(*) axis x, y, z and the main objective is that the core reason for $S_x + S_y + S_z = S$

(*) $S^{(1)} S^{(2)}$ $\qquad S = S_x + S_y + S_z$

$+ S_x^{(1)} S_x^{(2)}$

$+ S_y^{(1)} S_y^{(2)}$

$+ S_z^{(1)} S_z^{(2)}$

$$S^2 = (S^{(1)})^2 + (S^{(2)})^2 + 2 S^{(1)} S^{(2)}$$

$$= 2\left[\left(S_x^{(1)} |\uparrow\rangle\right)\left(S_x^{(2)} |\downarrow\rangle\right) + \left(S_y^{(1)} |\uparrow\rangle\right)\left(S_y^{(2)} |\downarrow\rangle\right) + \left(S_z^{(1)} |\uparrow\rangle\right)\left(S_z^{(2)} |\downarrow\rangle\right)\right]$$

$$S_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{\hbar}{2}$$

$$S_y = \frac{i\hbar}{2} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_-^{(1)} S_-^{(2)} |\uparrow\downarrow\rangle$$

$$\Rightarrow S_x = \frac{\hbar}{2}$$

$$= \left(\frac{\hbar}{2} |\downarrow\rangle\right)\left(\hbar/2 |\uparrow\rangle\right) + \left(\frac{\hbar i}{2} (|\downarrow\rangle)\right)\left(-\frac{i\hbar}{2} (|\uparrow\rangle)\right)$$

$$+ \left(\left(\frac{\hbar}{2}\right) |\uparrow\rangle\right)\left(-\left(\frac{\hbar}{2}\right)(|\downarrow\rangle)\right)$$

for S_x $\uparrow$ notation converts to down

$\downarrow$ notation converts to up

for S_y $\uparrow$ notation converts to down

$\downarrow$ notation converts to up and $(-)$

Trick to understand 5:32

$\Rightarrow S_x \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

0 1 is the first value focused so ↑ becomes ↓
1 0 is the second value focused so ↓ becomes ↑

$S_x \begin{pmatrix} 1 & 0 \\ & 1 \end{pmatrix} \begin{matrix} \uparrow \to \downarrow \\ \downarrow \to \uparrow \end{matrix}$

$\Rightarrow S_y$ same thing

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ same as S_x but negative value present

$\Rightarrow$ same rule but multiply with (−1)

$\Rightarrow S_z \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

1 0 → doesn't change
0 −1 → remains as it is but add (−) to it