$$S_- \chi_+ = \hbar \chi_-$$
$$S_+ \chi_- = \hbar \chi_+$$
$$S_+ \chi_+ = 0$$ [no other states higher than maximum state]
$$S_- \chi_- = 0$$ [no other states lower than the minimum state]

[these spin chi values are single component spin states]

m_1		m_2		m
$\frac{1}{2}$	$\uparrow$	$\frac{1}{2}$	$\uparrow$	1
$\frac{1}{2}$		$-\frac{1}{2}$		0
$-\frac{1}{2}$		$\frac{1}{2}$		0
$-\frac{1}{2}$		$-\frac{1}{2}$		-1

[Double component spin states :- m_1 has one spin state & m_2 has one spin state]

$S_{\pm} |s\ m\rangle$

↳ only m which derives the only spin state

Derivation of double composite state

$$\underset{(\chi_1)}{S_-}\ \overset{\chi_1}{\uparrow}\ \overset{\chi_2}{\uparrow} = \left(S_-^{(1)} + S_-^{(2)}\right)$$

electron spin state   proton spin state

Note: χ_1 & χ_2 are independent of each other so χ_1 has no correlation with χ_2 and χ_2 has no correlation with χ_1

from 38.1 derivation of $S_z \chi_1 \chi_2$

$$S_z \chi_1 \chi_2 = \left(S_z^{(1)} \chi_1\right)\chi_2 + \chi_1 \left(S_z^{(2)} \chi_2\right)$$

$$S_- \uparrow \uparrow = \underset{(\chi_1)}{\left(S_-^{(1)} \uparrow\right)} \underset{(\chi_2)}{\uparrow} + \underset{(\chi_1)}{\uparrow} \left(S_-^{(2)} \underset{(\chi_2)}{\uparrow}\right)$$

$$S_z(\chi_1\chi_2) = (S_z^{(1)}\chi_1)\chi_2 + \chi_1(S_z^{(2)}\chi_2)$$

$$S_-(\uparrow\uparrow) = \underset{(\chi_1)}{(S_-^{(1)}\uparrow)}\underset{(\chi_2)}{\uparrow} + \underset{(\chi_1)}{\uparrow}\underset{(2)}{(S_-^{(2)}\uparrow)}$$

$$= \boxed{\hbar}\downarrow\uparrow + \uparrow\boxed{\hbar}\downarrow$$

$$= \hbar(\downarrow\uparrow + \uparrow\downarrow)$$

[non normalized equation]

$\frac{1}{\sqrt{2}}$

$\therefore$ out of 4 states : 3 states $|s\ m\rangle$

$s = 1$, $\boxed{m = 1, 0, -1}$

$s \quad m$

$|1\ 1\rangle = \uparrow\uparrow$

$|1\ 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$

$|1\ {-1}\rangle = \downarrow\downarrow$

Spin $s = 0$ $m = 0$: $|0\quad 0\rangle$

$$|0\,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$