

38-2: Addition of angular momenta

$$S_{\pm} |s m\rangle = \hbar \sqrt{s(s+1) - m(m \pm 1)} |s \ m \pm 1\rangle$$

$$S_- |s m\rangle = \hbar \sqrt{s(s+1) - m(m-1)} |s \ m-1\rangle$$

$\Rightarrow S_-^{(1)} \uparrow$: m is denoted as a positive quantum value

$$S_- |s m\rangle \text{ [overall spin state goes up]} \\ = \hbar \sqrt{s(s+1) - m(m-1)} |s \ m-1\rangle$$

* $S = \frac{1}{2}$ because of the positive quantum value

$$\begin{aligned} & \hbar \sqrt{\frac{1}{2}(\frac{3}{2}) - [\frac{1}{2}(-\frac{1}{2})]} | \frac{1}{2} \quad \frac{1}{2}-1 \rangle \\ & = \hbar \sqrt{(\frac{3}{4}) - (-\frac{1}{4})} | \frac{1}{2} \quad -\frac{1}{2} \rangle \\ & = \hbar \sqrt{(\frac{4}{4})} | \frac{1}{2} \quad -\frac{1}{2} \rangle \\ & = \hbar (1) \underbrace{| \chi_- \rangle}_{\downarrow} \\ & = \hbar \chi_- \downarrow \end{aligned}$$