

example

$$J_1 = 3/2$$

$$J_2 = 1/2$$

[illegible]

$$|2, 2\rangle = \sqrt{1} \begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix} \frac{1}{2} \frac{1}{2} \rangle$$

$$|21\rangle = \frac{1}{2} \left| \frac{3}{2} \frac{3}{2} \frac{1}{2} -\frac{1}{2} \right\rangle + \sqrt{\frac{3}{4}} \left| \frac{3}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right\rangle$$

$$|2-1\rangle = \sqrt{\frac{3}{4}} \left| \frac{3}{2} \quad -\frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \right\rangle + \sqrt{\frac{1}{4}} \left| \frac{3}{2} \quad -\frac{3}{2} \quad \frac{1}{2} \quad \frac{1}{2} \right\rangle$$

**Note**: ANY EMPTY ROWS ARE 0



# Derivation of Clebsch Gordon

## Derivations

$$J_1 = \frac{1}{2} \quad J_2 = \frac{1}{2}$$

$$m_1, m_2 \in \left\{ -\frac{1}{2}, +\frac{1}{2} \right\}$$

$$|j_1 - j_2\rangle \leq J \leq j_1 + j_2$$

$$0 \leq J \leq 1$$

$$M = \{1, 0, 0, -1\}$$

$$|1, 0\rangle = |\uparrow\downarrow\rangle \text{ \& \; } |\downarrow\uparrow\rangle$$

$$\begin{aligned} |0, 0\rangle &= \alpha_1 \left| \frac{1}{2}, \frac{1}{2} \right\rangle + \alpha_2 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle + \alpha_3 \left| -\frac{1}{2}, \frac{1}{2} \right\rangle + \alpha_4 \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle \\ |1, -1\rangle &= \beta_1 \left| \frac{1}{2}, \frac{1}{2} \right\rangle + \beta_2 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle + \beta_3 \left| -\frac{1}{2}, \frac{1}{2} \right\rangle + \beta_4 \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle \\ |1, 0\rangle &= \delta_1 \left| \frac{1}{2}, \frac{1}{2} \right\rangle + \delta_2 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle + \delta_3 \left| -\frac{1}{2}, \frac{1}{2} \right\rangle + \delta_4 \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle \\ |1, 1\rangle &= \epsilon_1 \left| \frac{1}{2}, \frac{1}{2} \right\rangle + \epsilon_2 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle + \epsilon_3 \left| -\frac{1}{2}, \frac{1}{2} \right\rangle + \epsilon_4 \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle \end{aligned}$$

$\otimes_1, \otimes_2, \otimes_3, \otimes_4$  stay the same for all

possible spin values followed by  $\alpha, \beta, \delta, \epsilon$

[Any ket notation values which doesn't output the m value