Differential equation

$$\frac{d^2u}{d\delta^2} = \left(1 - \cancel{\frac{\delta_0}{\delta}}^{\,0} + \cancel{\frac{\ell(\ell+1)}{\delta^2}}^{\,0}\right)u$$

$$\Rightarrow \lim_{\delta \to \infty} : \frac{d^2u}{d\delta^2} = (1)u = u \qquad \Rightarrow u(\delta) = Ae^{-\delta} + Be^{-\delta}$$

⇒ $u = rR$ → wave function

$\rho = kr$

$\rho \to \infty$ $\quad r \to \infty$ $\quad R \to 0$ [wave function]

⇒ $R = u(\rho)$

a) $\rho \to \infty$

$u(\infty) = Ae^{-\infty} + \cancel{Be^{\infty}}$ 0

$u(\rho) = Ae^{-\rho}$ $\quad B = 0$

When ρ goes to infinite

b) $\cancel{\rho \to -\infty}$

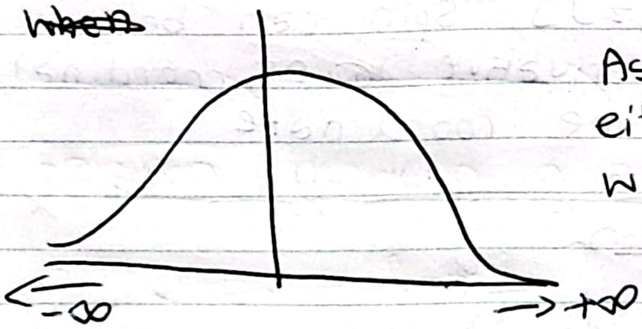
$\cancel{u(-\infty) = Ae^{\infty} + Be^{-\infty}}$

$\cancel{u(-\infty) = Be^{-\infty}}$

$$\frac{d^2u}{d\rho^2} = \frac{l(l+1)}{\rho^2}u$$

$$u(\rho) = C\rho^{l+1} + D\rho^{-l}$$

~~When~~



As the ρ goes to either $+\infty$, or $-\infty$, our wave function will deacrease

$|\psi| = 0$ $\quad |\psi|^2 = 0$

→ Since our wave function will be 0 as the ρ goes to $[+\infty]$ or $[-\infty]$ due to the gaussian distribution, the probability density becomes 0.

$u(\rho) = C\rho^{l+1} + \cancel{D\rho^{-l}}$ 0 $\quad$ when $\rho = 0$

⇒ $u(\rho) = C\rho^{l+1}$ $\quad D = 0$

Two solutions for wave function

$$u(\rho) = Ae^{-\rho}$$
$$u(\rho) = C\rho^{l+1}$$

© $u(\rho) = Ae^{-\rho} C\rho^{l+1}$ A & C are constants

$AC = \nu(\rho)$

product rule differentiation

$$u(\rho) = \underbrace{e^{-\rho}}\underbrace{\rho^{l+1}}\ \underbrace{\nu(\rho)}_{\text{constants}}$$

[~~co~~ changing position of value doesn't change the solution in multiplication]

$$\Rightarrow \rho\frac{d^2\nu}{d\rho^2} + 2(l+1-\rho)\frac{d\nu}{d\rho} + [\rho_0 - 2(l+1)]\nu = 0$$

ρ ↓ rho

power series solution of this equation

$$C_{j+1} = \left[\frac{2(j+l+1) - \rho_0}{(j+1)(j+2l+2)}\right] C_j \qquad \nu(\rho) = \sum_{j=0}^{\infty} C_j \rho^j$$

recursion relation

C_0 :- normalisation constant

$$C_j = \frac{2^j}{j!} C_0 \qquad \nu(\rho) = C_0 \sum_{j=0}^{\infty} \frac{2^j}{j!} \rho^j$$
$$= C_0 e^{2\rho}$$

$$u(\delta) = e^{-\delta} \delta^{l+1} C_0 e^{2\delta}$$

$$u(\delta) = C_0 \delta^{l+1} e^{\delta}$$

$$\lim_{\delta \to \infty} U(\delta) \to \infty$$

$\Rightarrow C_{j_{max}+1} = 0$

when

$$\left[\text{numeretor} \left(2(j+l+1) \right) - \delta) = 0 \right]$$

$\therefore n \equiv j_{max} + l + 1 \equiv$ principle quantum number

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Relation between n and l

$\rightarrow$
$$\left. \begin{array}{l} n = J_{max} + l + 1 \\ l = n - J_{max} - 1 \end{array} \right]$$

1) $\Rightarrow l = n$ $\qquad 1 = 1 - 1 - J_{max}$

derivetion $\Rightarrow$ $\qquad 1 = -J_{max}$ $\qquad J_{max} = -1 < 0$

$\Rightarrow$ Because J_{max} is some[∞] value $l = n$ X

$l = n - 1 - J_{max}$

larger J_{max} value $\times(-1)$ outputs a smaller J_{max} value

l becomes smaller

- $l \neq n : l < n$

$\therefore l + J_{max} = n - 1$

$l = n - 1$ when $J = 0$

- $l = n - 1 - j_{max}$

- when J reaches maximum, l deacreases

$l = 0, 1, 2, 3, 4, 5, \ldots$

$K^2 = \frac{-2mE}{\hbar^2}$ $\quad K^2\hbar^2 = -2mE$ $\quad \boxed{\frac{K^2\hbar^2}{-2m} = E}$

when n is some maximum integer

l can have ~~a~~ some maximum integer $(n-1)$

previous slide

$\Rightarrow 2n = \delta_0$ $\quad \boxed{\delta_0 = \frac{me^2}{2\pi\varepsilon_0\hbar^2}}$ $\quad \frac{me^2}{2\pi\varepsilon_0\hbar^2} = 2n$ $\quad \boxed{K = \sqrt{\frac{-2mE}{\hbar^2}}}$

$\Rightarrow E = -\frac{\hbar^2 K^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{me^2}{2\pi\varepsilon_0\hbar^2 \delta}\right)^2$

~~$E = -\frac{\hbar^2}{2m} \times K^2$~~

~~$E = -\frac{\hbar^2}{2m}\left(\frac{-2me}{\hbar^2}\right)$~~

$E = -\frac{K^2\hbar^2}{2m}$

$$\boxed{E = \underbrace{\left\{\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)\right\}^2}_{E_1}\frac{1}{n^2}}$$

$E_n = \frac{E_1}{n^2}$

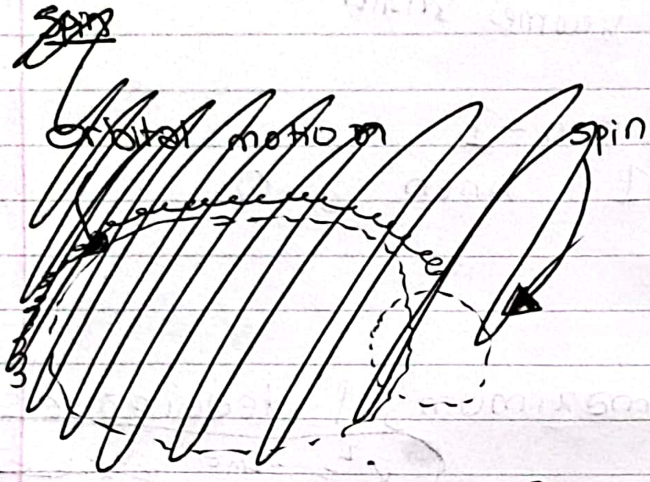
$E_1 = \frac{me^4}{2\hbar^2}\left(\frac{me^4}{2\hbar^2 16\pi^2\varepsilon_0^2}\right)$

$= -13.6\,eV$

= ground state energy

- $E = -\frac{\hbar^2 \cancel{m} e^2}{(2\cancel{m})(2\pi\varepsilon_0\hbar^2\delta_0)}$

~~$E = -\frac{\hbar^2 e^2}{(2)(2\pi\varepsilon_0\hbar^2 S_0)}$~~



$$E = -\frac{\hbar^2}{2m}\left(\frac{me^2}{2\pi\varepsilon_0\hbar^2 S_0}\right)^2$$

$$E = -\frac{\hbar^2}{2m}\left(\frac{\cancel{e^2}\,m^2e^4}{4\pi^2\varepsilon_0^2\hbar^4 S_0^2}\right)$$

$$E = -\frac{\hbar^2}{2m}\left(\frac{m^2e^4}{4\pi^2\varepsilon_0^2\hbar^4 S_0^2}\right)$$

$$E = \left(-\frac{\hbar^2 m^2 e^4}{2m\,4\pi^2\varepsilon_0^2\hbar^4 S_0^2}\right)$$

$$E = \left(-\frac{\cancel{\hbar^2} m^2 e^4}{8m\pi^2\varepsilon_0^2\hbar^{\cancel{4}\,2} S_0^2}\right)$$

$$E = \left(-\frac{m^{\cancel{2}}e^4}{8\cancel{m}\pi^2\varepsilon_0^2\hbar^2 S_0^2}\right)$$

$$E = \left(-\frac{me^4}{8\pi^2\varepsilon_0^2\hbar^2 S_0^2}\right) = -\frac{13.6\,eV}{n^2} \qquad \boxed{-13.6\,eV}$$ ground state

$$E_n = -\frac{13.6\,eV}{n^2}$$

$n = J_{max} + l + 1$

J_{max} minimum value $= 0$ | $l = 0, 1, 2, 3, 4, \ldots$

$l = 0$

$n = 0 + 0 + 1$ [when J_{max} and l are minimum]

$n = 1$

— X — X — X — X — X —

Total solution for wave function

$$\Psi(r, \theta, \phi) = \Psi_{nlm} = \underbrace{R(r)}_{nl} \; Y_l^m(\theta, \phi)$$

quantum number

Radial wave function

$$R_{nl} = \frac{1}{r} e^{-\delta} \delta^{l+1} \nu(\delta)$$

$u = rR$

$\frac{u}{r} = R$

R

$$u = e^{-\delta} \delta^{l+1} \nu(\delta)$$

$$R_{nl} = \frac{1}{r}(u)$$

substituted