Addition of angular momenta

Triplet configurations for $s = 1$

$|1\ \ 1\rangle = |\uparrow\uparrow\rangle$

$|1\ \ 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow + \downarrow\uparrow\rangle)$

$|1\ -1\rangle = |\downarrow\downarrow\rangle$

: triplet $S=1$ configuration

singlet configuration

$|0\ \ 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow - \downarrow\uparrow\rangle)$ $s=0$: singlet configuration

Two particle system

e^- → spin $1/2$

three possible combinations

p^+ → spin $1/2$

$|\uparrow\uparrow\rangle$

$|\downarrow\downarrow\rangle$

$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

Eigen functions of total spin

[z component of spin angular momentum] S^2 :- Total spin value

S

S_z

[magnetic spin]

m_s :- orientation for spin angular momentum

L^2

l

L_z [z component of orbital angular momentum]

[magnetic orbital]

m_l :- orientation for the orbital angular momentum

eigen functions of S^2

$S^2 := 2\hbar^2$

$S_x^2 \quad S_y^2 \quad S_z^2$

$S_x \cdot S_x + S_y \cdot S_y + S_z \cdot S_z$

two different spin state values

Keep in mind

$$|1\ 1\rangle = |\overbrace{\uparrow\ \uparrow}^{e\ \ p}\rangle$$

one S [spin] value — another s[spin] value

$$S^2 = (\underset{electron}{S^{(1)}} + \underset{proton}{S^{(2)}})^2$$

$$= (\vec{S}^{(1)} + \vec{S}^{(2)})\cdot(S^{(1)} + S^{(2)})$$

$$(S^{(1)})^2 + 2S^{(1)}S^{(2)} + (S^{(2)})^2$$

$\Rightarrow$ $S^{(1)} = S_x^{(1)} + S_y^{(1)} + S_z^{(1)}$

Total spin value

same goes for $S^{(2)} = S_x^{(2)} + S_y^{(2)} + S_z^{(2)}$

back to the S^2 equation

$$S^2 = (S^{(1)})^2 + \underline{2S^{(1)}S^{(2)}} + (S^{(2)})^2$$

⊛ $S_-^{(1)}S_-^{(2)}$ derived from the $|1\ 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Because $|1\ 1\rangle$ and $|1\ -1\rangle$ ~~are~~
$\uparrow\uparrow$ $\downarrow\ \downarrow$

$$\Rightarrow S_-^{(1)}S_-^{(2)}|\uparrow\downarrow\rangle = \underbrace{(S_x^{(1)}|\uparrow\rangle)\,(S_x^{(2)}|\downarrow\rangle)}$$

⊛ Because χ_1 and χ_2 are independent, the state spin χ_1 has nothing to do with spin state χ_2

$$\vec{S}^{(1)}\vec{S}^{(2)}|\uparrow\downarrow\rangle = \left(S_x^{(1)}|\uparrow\rangle\right)\left(S_x^{(2)}|\downarrow\rangle\right) + \left(S_y^{(1)}|\uparrow\rangle\right)\left(S_y^{(2)}|\downarrow\rangle\right)$$

⊛ axis x, y, z and the mein objective is that the core reeson for $S_x + S_y + S_z = S$

⊛ $S^{(1)}S^{(2)}$ $\quad S = S_x + S_y + S_z$

$+ S_x^{(1)} S_x^{(2)}$
$+ S_y^{(1)} S_y^{(2)}$
$+ S_z^{(1)} S_z^{(2)}$

$$S^2 = \left(S^{(1)}\right)^2 + \left(S^{(2)}\right)^2 + \underbrace{2 S^{(1)} S^{(2)}}$$

$$= 2\left[\left(S_x^{(1)}|\uparrow\rangle\right)\left(S_x^{(2)}|\downarrow\rangle\right) + \left(S_y^{(1)}|\uparrow\rangle\right)\left(S_y^{(2)}|\downarrow\rangle\right) + \left(S_z^{(1)}|\uparrow\rangle\right)\left(S_z^{(2)}|\downarrow\rangle\right)\right]$$

$$S_x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\frac{\hbar}{2}$$

$$S_y = \frac{i\hbar}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$