$\frac{3\hbar^2}{4} + \frac{3\hbar^2}{4} + \frac{2\hbar^2}{4} |1\ 0\rangle$

$2\hbar^2 |1\ 0\rangle$

→ proving that $S^2 = 2\hbar^2$ for any ket notation

—x—x—x—x—x—x—x—

39-3 singlet state

triplet config
$$S^2 |1\ 0\rangle = \left(\frac{3}{4}\hbar^2 + \frac{3}{4}\hbar^2 + \frac{2}{4}\hbar^2\right) |1\ 0\rangle$$
$$= 2\hbar^2 |1\ 0\rangle$$

singlet configuration
$$S^2 |0\ 0\rangle = \vec{S}^{(1)} \cdot \vec{S}^{(2)} |0\ 0\rangle$$

Key note:
The only difference between a triplet configuration and singlet configuration is the [+] or [-] sig

$$S^2 |0\ 0\rangle = \vec{S}^{(1)} \cdot \vec{S}^{(2)} |0\ 0\rangle$$
$$S^2 |0\ 0\rangle = \frac{\hbar^2}{4} \frac{1}{\sqrt{2}} \{(2|\downarrow\uparrow\rangle - |\uparrow\rangle) - (2|\uparrow\downarrow\rangle$$
$$= \frac{\hbar^2}{4} \frac{1}{\sqrt{2}} \{(2|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle) + 2|\uparrow\downarrow\rangle$$

$$S^2 |0\,0\rangle \Rightarrow \vec{S}^{(1)} \cdot \vec{S}^{(2)} |0\,0\rangle$$

$$\Rightarrow \frac{\hbar^2}{4} \frac{1}{\sqrt{2}} \left\{ \left(3 |\downarrow\uparrow\rangle - 3 |\uparrow\downarrow\rangle \right) \right\} = \frac{3\hbar^2}{4}$$

$$\Rightarrow \frac{\hbar^2}{4} \cdot \frac{1}{\sqrt{2}} \left\{ 3 \left(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle \right) \right\}$$

$$\Rightarrow -3 \frac{\hbar^2}{4} |0\,0\rangle$$

$$S^2 |0\,0\rangle = \Bigg(\overset{\vec{S}^{2(1)}}{\frac{3\hbar^2}{4}} + \overset{\vec{S}^{2(2)}}{\frac{3\hbar^2}{4}} + 2 \left(S_1^{(1)} S_2^{(2)} \right) + 2 \left(-\frac{3\hbar^2}{4} \right) \Bigg) = 0$$

$$0 |0\,0\rangle$$

$$= \frac{6-6}{4} = \frac{0}{4}$$

$$S^2 |1\,1\rangle = \left(\vec{S}_1^{(1)} \right) + \left(S_2^{(2)} \right) + 2 \left(S^{(1)} \cdot S^{(2)} \right)$$

$$= \left(S_{(x)}^{(1)} + S_{(y)}^{(1)} + S_{(z)}^{(1)} \right) + \left(S_{(x)}^{(2)} + S_{(y)}^{(2)} + S_{(z)}^{(2)} \right)$$

$$+ 2 \left(S_x^{(1)} / S_y^{(1)} / S_{(z)}^{(1)} \cdot S_x^{(2)} / S_y^{(2)} / S_z^{(2)} \right)$$

Summary

e^- p^+ $S=1$

$\frac{1}{2}$ $\frac{1}{2}$ $S=0$

singlet

triplet configuration

$S=1$ $\begin{bmatrix} m=1 \\ 0 \\ -1 \end{bmatrix}$ $|1\ 1\rangle$ $|1\ 0\rangle$ $|1\ -1\rangle$

$S=0$ $[m=0]$

$S=0$

$l=1$ $S=1$

$-l \leq m_l \leq l$ $-S \leq m_s \leq S$

$-S \leq m_s \leq S$

$\boxed{0 \leq m_s \leq 0}$

only option is 0

$S=1\ [\frac{1}{2}+\frac{1}{2}]$ $S=0\ [\frac{1}{2}-\frac{1}{2}]$

$S_1+S_2=1$ $S_1-S_2=0$

$[S_1+S_2]$ $|S_1-S_2|$

By that ladder

$\begin{pmatrix} \mu+2B & -\mu+B \\ \mu+B & -2\mu+B \text{ so on} \\ 0 & \end{pmatrix}$

go further down, there are no values lower than minimum value so it will output 0

Because when subtracting two spin values here we must get a positive value regardless of whether value negative or positive - bcz my minimum value is 0 so even if we to move from Spin 1 to spin 0

$(S_1+S_2)-0$, $(S_1+S_2)-1$ $(S_2+S_2-)2$

-------------- , $|S_1-S_2|$

—X—X—X—X—X—

example of a composite system

$S_1 = 3/2$ $S_2 = 2$

maximum spin

S_1+S_2

$(\frac{3}{2})+(\frac{4}{2})$

$= \frac{7}{2}$

minimum spin

S_1-S_2

$|(\frac{3}{2})-(\frac{4}{2})|$

$= (\frac{1}{2})$

To move from $\frac{7}{2}$ to $\frac{1}{2}$

$\overset{7/2}{(S_1+S_2)-0}$, $\overset{5/2}{(S_T+S_2)-1}$, $\overset{3/2}{(S_1+S_2)-2}$

$\overset{1/2}{(S_1+S_2)-3}$

$7/2$, $5/2$, $3/2$, $1/2$