∇^2 & $\Psi = (R, Y)$ Since wave function is solely dependent on r

d instead of psi

Y is not only a function of θ but both θ & ϕ

$$① \left\{ -\frac{\hbar^2}{2m} \left\{ \frac{Y}{r^2} \underbrace{\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right)} + \frac{R}{r^2 \sin\theta} \frac{\partial}{\partial \theta}(\sin\theta)\frac{\partial Y}{\partial \theta} + \frac{R}{r^2 \sin^2\theta} \frac{\partial^2 Y}{\partial \phi^2} \right\} \right\}$$

Y will come out because $\Psi(r)$ ~~not~~ but doesn't have any correlation with θ ~~&~~ ϕ

So for θ or ϕ individually we write $[\partial]$ partial derivative

① $\nabla^2 \Psi$

② $V\Psi = VRY = ERY$ ③

$\therefore -\frac{2mr^2}{\hbar^2 RY}$; multiplying both sides

$$\therefore \left\{ -\frac{2mr}{\hbar^2 RY} \left\{ \frac{1}{R} \frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) - \frac{2mr^2}{\hbar^2}(V-E) \right\} + \left\{ \frac{1}{Y} \left(\frac{1}{\sin^2\theta} \frac{\partial}{\partial \theta}(\sin\theta) \left(\frac{\partial Y}{\partial \theta}\right) + \frac{1}{\sin^2\theta} \frac{\partial^2 Y}{\partial \phi^2} \right) \right\} \right.$$

$$+ \left(\sin\theta \frac{\partial Y}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial \phi^2}$$

[separated radial and angular portion

Radial equation

$$\boxed{\left\{\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)-\frac{2mr^2}{\hbar^2}(V-E)\right\}} = C = \ell(\ell+1)$$

wave function in regards to the radius

$$-C = \boxed{\left\{\frac{1}{Y}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right\}\right\}}$$

Angular equation

wave function in regards to angle

→ $-[\ell(\ell+1)]$

- Radial equation: potential must be known to solve the radial equation

- Anguler equation: separate Y because Y is not only a function of θ but also a function of ϕ

→ multiply with $Y\sin^2\theta$

→ $\sin(\theta)\left[\frac{\partial}{\partial\theta}\right]\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)$

$$+\frac{\partial^2 Y}{\partial\phi^2} = -\ell(\ell+1)\sin^2\theta\, Y$$

$$Y(\theta, \phi) = \Theta(\theta)\,\Phi(\phi)$$

$$\frac{\Phi \sin\theta \frac{\partial}{\partial\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) + l(l+1)\sin^2\theta\,\Theta\Phi + \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = 0}{\Theta\Phi}$$

→ To separate Θ and Φ we divide with $(\Theta\Phi)$

$$= \left[\frac{1}{\Theta}\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + l(l+1)\sin^2\theta\right]$$

depends once only $= m^2$

$$+ \left[\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}\right] = 0$$

depends on $\Phi = -m^2$