Restrictions

$\langle 0\,0 | 0\,0 \rangle = 1$

$S_{00} = 1$

$m\,n$

$m = n \Rightarrow 1$

$m \neq n \Rightarrow 0$

$\alpha_2^* \alpha_2 + \alpha_3^* \alpha_3 = 1$

$|\alpha_2^2| + |\alpha_3|^2 = 1$ ✗

$\langle 1\,-1 | 1\,-1 \rangle = |\beta_4|^2 = 1$

$|\beta_4^* \beta_4| = 1$ ✓

$\langle 1\,0 | 1\,0 \rangle = r_2^2 + r_3^2 = 1$ ✗

$\langle 1\,1 | 1\,1 \rangle = |\delta_1^2| = 1$ ✓

$\langle 1\,0 | 0\,0 \rangle = 0$ [not same]

$\gamma_2^* \alpha_2 + r_3 \alpha_3 = 0$

$\Rightarrow S^2|\psi\rangle = s(s+1)\hbar^2|\psi\rangle$

$S^2|1\ 1\rangle = 1(1+1)|1\ 1\rangle$

$= 1(2)|1\ 1\rangle$

$= 2|1\ 1\rangle$

$\Rightarrow J^2|1\ 0\rangle = 1(1+1)|1\ 0\rangle$

$= 1(2)|1\ 0\rangle$

$= 2|1\ 0\rangle$

$\Rightarrow J^2|0\ 0\rangle = 0(0+1)|0\ 0\rangle$

$= 0(1)|0\ 0\rangle$

$= 0$

$J^2 = J_1^{(2)} + J_2^{(2)} + 2\,J_1 \cdot J_2$

[J chi 1] [J chi 2]

$J_1 + J_2 + [J_{1-}J_{2+}] + 2J_{1z}J_{2z}$

Break down

$J^2|1\ 0\rangle = \{J_1^2 + J_2^2 + J_{1+}J_{2-} + J_{1-}J_{2+} + 2J_{1z}J_{2z}\}$

$\left[r_2\left|+\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle + r_3\left|-\tfrac{1}{2}\ +\tfrac{1}{2}\right\rangle\right]$

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