heisenberg uncertainty derivation

Derivation

$$\langle \Phi_m | \Phi_n \rangle = \delta_{mn} \rightarrow \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$$

$\langle \Phi_m | \Phi_n \rangle \geq 0$ [maximum is one>

or, $\langle \phi | \phi \rangle \geq 0$

proof $\sigma_x \sigma_p \geq \frac{\hbar}{2}$ | $\Delta x \Delta p \geq \hbar/2$ uncertainity

example

$\hat{A}$ & $\hat{B}$: Hermitian operators

$\hat{A}\psi = a\psi$; $\hat{B}\psi = b\psi$

$(\hat{A} - a)\psi = 0$; $(\hat{B} - b)\psi = 0$

$\sigma_A^2 = \langle (\hat{A} - a)^2 \rangle$, $\sigma_B^2 = \langle (\hat{B} - b)^2 \rangle$ Bra-Ket

$\sigma_A^2 = \langle \psi | (\hat{A} - a)^2 | \psi \rangle$, $\sigma_B^2 = \langle \psi | (\hat{B} - b)^2 | \psi \rangle$ notel

$(\hat{A} - a) + i\lambda(\hat{B} - b)$; ψ ; $\tilde{\psi}$

$((\hat{A} - a) + i\lambda)(\hat{B} - b) | \psi \rangle = | \tilde{\psi} |$

$\langle \tilde{\psi} | \tilde{\psi} \rangle \geq 0$

I call it big ~~notation~~ conjugate

$$\langle \tilde{\psi}| = \langle \psi| \left((\hat{A} - a) + i\lambda(\hat{B} - b) \right)^{\dagger}$$

$$= \langle \psi| \left((\hat{A} - a) - i\lambda(\hat{B} - b) \right)$$

Changing from ket-notation to bra-notation

$$\langle \tilde{\psi}|\tilde{\psi}\rangle$$

$$\underbrace{\left[\langle \tilde{\psi}| \left((\hat{A} - a) - i\lambda(\hat{B} - b) \right) \right]}_{\text{Bra}} \underbrace{\left[\left(\hat{A} - a \right) + i\lambda(\hat{B} - b) \right]}_{\text{Ket}} \geq 0$$

— notation

$$\left[(\hat{A} - a)(\hat{A} - a) \right] + \cancel{\left[(\hat{A} - a)(i\lambda(\hat{B} - b)) \right]}$$

$$\hat{B}\left[i\lambda(\hat{B} - B) \right]\left[-i\lambda(\hat{B} - B) \right]$$

$$= \left[(\hat{A} - a), (\hat{B} - b) \right]$$