$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = -m^2$$

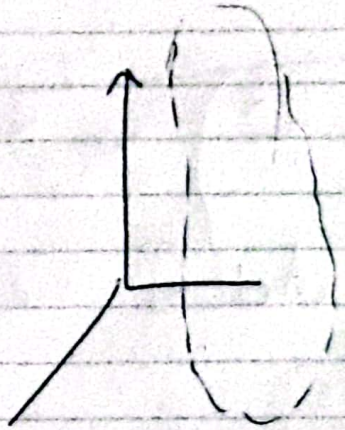
$$\frac{d^2\Phi}{d\phi^2} = -m^2\Phi \Rightarrow \Phi(\phi) = e^{\pm im\phi}$$

$\therefore \pm m$

exponential format

$\cos(im\phi) + i\sin(im\phi)$

$$= \Phi = e^{im\phi}$$



$$\Rightarrow \Phi(\phi) = \Phi(\phi + 2\pi)$$

$$\therefore e^{im(\phi)} = e^{im(\phi+2\pi)}$$

$$\left[e^{im(\phi)}\, e^{im(2\pi)}\right]$$

$$\therefore e^{im(2\pi)} = 1 \quad \because e^{in} = \cos(n) + i\sin(n)$$

$$\left\{\underbrace{\cos(2\pi m) + i\sin(2\pi)m}_{[=1]}\right\}$$

when $m = 0$

m is an integer : $m = 0, \pm 1, \pm 2, \pm 3$

↳ we don't know what the value is but we know its an integer

Associated legendre equation

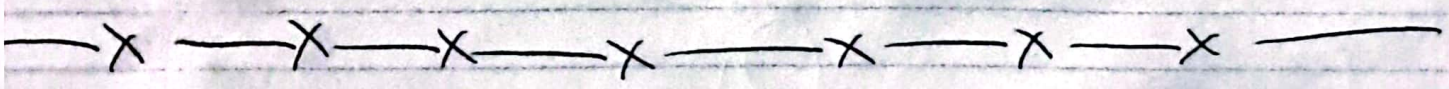
$$\Rightarrow \sin\theta \frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) + \left[l(l+1)\sin^2\theta - m^2\right]\Theta = 0$$

$$\Rightarrow \Theta(\theta) = A P_l^m(\cos\theta)$$

$\therefore x = r\cos(\theta)$

$r = 1$

$\therefore x = \cos(\theta)$



l :- positive integer

$$P_l^m(x) = (1-x^2)^{|m|/2}\left(\frac{d}{dx}\right)^{|m|} P_l(x)$$

↳ legendre polynomial