Full and partial Q-bit measurements

Full measurement

→ given n number of Qubits

you measure all of it

Partial measurement

→

you measure partially

Rules

(1) When Qubits measured which is in superposition state, after measurements, it converts to a pure state. [classical bit]

(2) You must ensure that the converted state fullfil the normalisation constraint

example of full measurement

$$|\psi\rangle = \underbrace{\tfrac{1}{2}|00\rangle}_{\cancel{\text{not }0}} + \underbrace{\tfrac{1}{2}|10\rangle}_{\cancel{\text{not }0}} + \underbrace{\tfrac{1}{\sqrt{2}}|11\rangle}_{\text{not }0}$$

1 1

full measurements

The Qubits must not have 0

- Find the full measurement of $|\psi\rangle$

measurement	prob	Resultant
00	$\left(\frac{1}{2}\right)^*\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	$\|\psi\rangle = \|00\rangle$
10	$\left(-\frac{1}{2}i\right)\left(\frac{1}{2}i\right) = -\frac{1}{4}(-i) = \frac{1}{4}$	$\|\psi\rangle = \|10\rangle$
11	$\frac{1}{2}$	$\|\psi\rangle = \|11\rangle$

$P(A \cup B) = P(A) + P(B)$ ~~$+ P(A \cap B)$~~

example of partial measurement

measurement	prob	Resultant
1st Qubit = 1	$\left\|-\frac{i}{2}\right\|^2 + \left\|\frac{1}{\sqrt{2}}\right\|^2$ $\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right) = \frac{3}{4}$	$\|\psi\rangle = -\frac{i}{2}\|10\rangle + \frac{1}{\sqrt{2}}\|11\rangle$

② $\dfrac{-\frac{1}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle}{\sqrt{3/4}}$

③ $-\dfrac{1}{\sqrt{3}}|10\rangle + \dfrac{\sqrt{2}}{\sqrt{3}}|11\rangle$

example of partial measurements

Measurement	Probability	Resultant state
1st Qubit = 1	$\left\|-\frac{1}{2}\right\|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 =$ $\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right) = \frac{3}{4}$	① $\|\psi\rangle = \dfrac{-\frac{1}{2}\|10\rangle + \frac{1}{\sqrt{2}}\|11\rangle}{\left\| -\frac{1}{2}\|10\rangle + \frac{1}{\sqrt{2}}\|11\rangle \right\|}$
2nd Qubit = 0	$\left\|\frac{1}{2}\right\|^2 + \frac{1}{4}$ $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$	$\|\psi\rangle = \dfrac{-\frac{1}{2}\|00\rangle + \frac{1}{2}\|00\rangle}{\frac{1}{\sqrt{2}}}$

X —— X —— X —— X —— X —

harder example on next page

$$\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{1} = \frac{\sqrt{5}}{5}$$

ex:

$$\frac{1}{\sqrt{5}}|0000\rangle - \sqrt{\frac{2}{5}}|0100\rangle + \sqrt{\frac{1}{3}}|1111\rangle + \sqrt{\frac{1}{5}}|0110\rangle$$

What is the probability and resultant force if 1st and 4th qubits are 0

measurement | Probability | Resultant state

$$\left|\frac{1}{\sqrt{3}}\right|^2 + \left(-\sqrt{\frac{2}{3}}\right)^2 + \left(\sqrt{\frac{1}{3}}\right)^2$$

$$\frac{1}{5} + \frac{2}{3} + \frac{1}{5} = \frac{4}{5}$$

resultant

$$|\psi\rangle = \frac{\frac{1}{\sqrt{5}}|0000\rangle - \left(\frac{\sqrt{2}}{\sqrt{5}}\right)|0100\rangle + \sqrt{\frac{1}{5}}|0110\rangle}{\sqrt{\frac{4}{5}}}$$

Whatever the value