Qubits & measurements

classical computer

units of data represented in bits

0s and 1s

Quantum computing

Qubits

- Two different Qubits states

① Pure state :- In pure state, behaivour is the exact same as the classical state

② Quantum state / superposition :-

0 & 1

classical computer

0 or 1

Quantum State / superposition

0 & 1

prob p

prob $1 - p$

Math reepresentation

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

- where α & β are amplitudes of ψ notation
- $\alpha, \beta \in \mathbb{C}$ [they can also be negative values]

probability of $0 = |\alpha|^2$
probability of $1 = |\beta|^2$

$|\alpha|^2 = \alpha^* \alpha$ $\quad |\beta|^2 = \beta^* \beta$

$$\underbrace{|\alpha|^2 + |\beta|^2 = 1}_{\text{normalization}}$$

Constraint
normalization ~~constant~~ constraint

- $|\psi\rangle = \alpha\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ ← unit vector

$\underbrace{\| |\psi\rangle \|}_{|\alpha|^2 + |\beta|^2 = 1} = 1$ or $\langle \psi | \psi \rangle = 1$

Ex

$|\psi\rangle = -\frac{4}{5}i|0\rangle + \frac{3}{5}|1\rangle$

Probability of 0

$\frac{16}{25}$

$|\alpha|^2 + |\beta|^2 = 1$

$(-\frac{4}{5}i)^2 + (\frac{3}{5})^2 = 1$

Probability of 1

$9/25$

$\rightarrow (\frac{16}{25}i^2) + (\frac{9}{25}) = 1$

$\frac{16}{25} + \frac{9}{25} = 1$

$\frac{16}{25} + \frac{9}{25} = 1$

$\frac{25}{25} = 1$

—x—x—x—x—x—x—x—

▷ How to derive a single notation [ket / ~~psi~~ bra] and find the probability of 1 & 0

1) Derive the norm equation $|\alpha|^2 + |\beta|^2 = 1$

2) gather up the value and their notations in α & β values

3) Use the normalization constraint and proof that the value exists as 1

4) probability of ~~1st vector~~ 0 is usually square of α and the probability of 1 is. 1 – [square α value]

how to derive in multiple notations

$$|\psi\rangle = -\frac{4}{5}i|0\rangle + \frac{3}{5}|1\rangle$$

$$|\phi\rangle = \frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}$$

$$|\psi\phi\rangle = \left(-\frac{4}{5}i|0\rangle + \frac{3}{5}|1\rangle\right)\left(\frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}\right)$$

$$= -\frac{4}{5\sqrt{2}}i|00\rangle - \frac{4}{5\sqrt{2}}i|01\rangle$$

$$+ \frac{3}{5\sqrt{2}}|10\rangle + \frac{3}{5\sqrt{2}}|11\rangle$$

$$\big\||\psi\phi\rangle\big\|^2$$ normalisation constraint

$$\langle\psi\phi|\psi\phi\rangle$$

$$\cancel{\left(\frac{4}{5}i\langle 0| + \frac{3}{5}\langle 1|\right)\left(\frac{\langle 0|}{\sqrt{2}} + \langle 01|\right)}$$

$$= -\frac{4}{5\sqrt{2}}(-i)\langle 00| - \left[\frac{4}{5\sqrt{2}} -(i) \langle 01|\right]$$

$$+ \frac{3}{5\sqrt{2}}\langle 10| + \frac{3}{5\sqrt{2}}\langle 11|$$

$$= \frac{4}{5\sqrt{2}}i\langle 00| + \frac{4}{5\sqrt{2}}i\langle 01| + \frac{3}{5\sqrt{2}}\langle 10| + \frac{3}{5\sqrt{2}}\langle 11|$$