Quantum computing

Bra-ket notation and Tensor notation

→ There are 6 notations

$\vert - \rangle$ 1) Ket notation	$\langle - \vert$ 2) Bra-notation	$\vert - \rangle \vert - \rangle$ 3) Ket-ket notation
4) Bra-Bra notation $\langle - \vert \langle - \vert$	5) Ket-Bra notation $\vert - \rangle \langle - \vert$	6) Bra-ket notation $\langle - \vert - \rangle$

1) $\vert - \rangle$

Ket-notation

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{array}{l} \to 0^{th} \text{ location} \\ \to 1^{st} \text{ location} \end{array}$$

↳ this indicates that 1 exists in a 2D vec at location 0.

$|1\rangle$ ⤴ 1 exists in the 1^{st} location

example

$\overset{4\,2\,1}{101}$ ↳ $2^3 = 8$:

0 → 7 for (2^3) values

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

— 0^{th} location

101 → Binary value
↳ convert it to decimal
↳ Identify the position starting from 0 where the

example question

$|11\rangle$ values $= 2$

$\underbrace{11}_{2^2=4}$ denary $= 3$

$\therefore \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Basis vectors

$\{|0\rangle, |1\rangle\}$ standard basis vector → computational basis

$\begin{pmatrix} 7 \\ 3+5i \end{pmatrix}$ $7|0\rangle + 3+5i\,|1\rangle$

4D vector example

[we represented a simple matrix in 4D vector]

$\begin{pmatrix} 7 \\ 0 \\ i+3 \\ 0 \end{pmatrix}$

Basis

$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$\underbrace{7|00\rangle}_{\text{position } 0} + \underbrace{(i+3)|10\rangle}_{\text{position } 2}$

1 —

Bra -notation

~~$\langle 0 |$ opposite to Ket notation~~

~~0^{th} position~~

~~1^{st} position~~

$\langle 0 |$

$[1, 0]$

0^{th} column

$\langle 1 |$

$[0, 1]$

1st Column

$$|\psi\rangle^{*} = \langle \psi |$$

conjugate transpose of any value represented in Ket or bra notation will output bra or Ket notation respectively

- conjugate $*$ [Change the iota sign from $(+)$ to $(-)$ or $(-)$ to $(+)$]
- transpose T [Change from row to column]

example

$|\psi\rangle = (3+5i)^* |0\rangle + 7|1\rangle$

$\langle\psi| = (3-5i)\langle 0| + 7\langle 1|$

2D-matrix 2^1
1 value

Ket-notation

$$\begin{pmatrix} 3+5i \\ 7 \end{pmatrix}$$

Bra-notation

$[3-5i, 7]$

Ket-notation

$$\begin{pmatrix} 3 \\ 0 \\ i \\ 7 \end{pmatrix}$$

00 → 0^{th} location in denary
01 → 1^{st} location in denary
10 → 2nd location in denary
11 → 3^{rd} location in denary

Bra-notation

$[3, 0, i, 7]$

$$\boxed{3|00\rangle + 0|01\rangle + i|10\rangle + 7|11\rangle}$$

$3\langle 00| + 0\langle 01| + i\langle 10| + 7\langle 11|$

Ket-Ket notation

tensor product

(Ket-notation) X (Ket-notation)

Bra-Bra notation

(Bra-notation) X (Bra-notation)

example of tensor product

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \end{pmatrix}$$

$A \otimes B$

$$\begin{pmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 4 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & 8 & 14 \end{pmatrix}$$

Ket-Ket notation

$|\psi\rangle \otimes |\phi\rangle =$

$|\psi\rangle|\phi\rangle$

simple

$\boxed{|\psi\phi\rangle}$

∴ en

$|\psi\rangle = i|0\rangle + 7|1\rangle$

$|\phi\rangle = |00\rangle + 3|10\rangle + 7|11\rangle$

Compute

$|\psi\rangle|\phi\rangle$

$\begin{pmatrix} i \\ 7 \end{pmatrix}$ $= i|000\rangle + 3i|010\rangle + 7i|011\rangle$

$+ 7|\phi 00\rangle + 21|110\rangle + 49|111\rangle$

$\begin{pmatrix} i & -0 \\ 0 & -1 \\ 3i & -2 \\ 7i & -3 \\ 7 & \\ 0 & \\ 21 & \\ 0 & \\ 49 & \end{pmatrix}$

$0 \to 6^{th}$ location

$49 \to 7^{th}$ location

Ket - Ket notation

(Ket - notation) $\otimes$ (Ket - notation)

Bra - Bra notation

(Bra - notation) $\otimes$ (Bra - notation)

example Bra - Bra notation

$$\langle \psi | = \left(3 \langle 0 | + 7 \langle 1 | \right)$$

$$\langle \phi | = \left(\langle 0 | + i \langle 1 | \right)$$

$$\langle \psi \phi | \quad \left(3 \langle 00 | + 3i \langle 01 | + 7 \langle 10 | + 7i \right.$$

Ket - Bra notation

$$|\alpha\rangle\langle\beta|$$

$(m) \times 1$ — ~~column~~ row, column

$(1) \times (m)$ — row, column

multiply

$m \times n$

example 1

$|\tilde{00}\rangle\langle 10|$

0th row
2nd column] $\to$ 1

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \begin{pmatrix}[0 & 0 & 1 & 0]\end{pmatrix}$$

$$\begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

number of rows depend on the first column

$$|\alpha\rangle = 3|0\rangle + i|1\rangle$$

$$|\beta\rangle = |00\rangle + 2|10\rangle + 7|11\rangle$$

3

$$|\alpha\rangle\langle\beta|$$

1) compute Bra beta

$$\langle\beta| = \langle 00| + 2\langle 10| + 7\langle 11|$$

$$|\alpha\rangle\langle\beta| = \left(3|0\rangle + i|1\rangle\right)\left(\langle 00| + 2\langle 10| + 7\langle 11|\right)$$

$$= 3|0 \times 00| + 6|0 \times 10| + 21|0 \times 11|$$

$$+ i|1 \times 00| + 2i|1 \times 10| + 7i|1 \times 11|$$