

Quantum computing

Bra-ket notation and Tensor notation

→ There are 6 notations

- $| \rightarrow$ 1) Ket notation $\langle - |$ 2) Bra-notation $| \rightarrow | \rightarrow$ 3) Ket-Ket notation
 $\langle - | \langle - |$ 4) Bra-Bra notation $| \rightarrow \langle - |$ 5) Ket-Bra notation $\langle - | \rightarrow$ 6) Bra-Ket notation

1) $| \rightarrow$

Ket-notation

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} 0^{\text{th}} \text{ location} \\ 1^{\text{st}} \text{ location} \end{matrix}$$

this indicates that 1 exists in 2D vector at location 0.

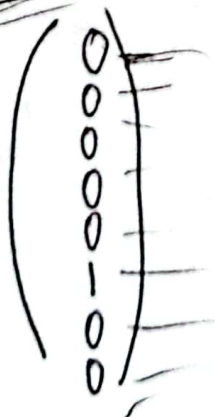
$|1\rangle \nearrow$

1 exists in the 1st location

0 → 7 for (2³) value

example

$$\begin{matrix} 4 & 2 & 1 \\ 1 & 0 & 1 \end{matrix} = 2^3 = 8$$



101 → Binary value

↳ convert it to decimal
 ↳ identify the position starting from 0 where the

example question

$$|11\rangle$$

$2^2 = 4$

values = 2
denary = 3

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Basis vectors

$$\{ |0\rangle, |1\rangle \}$$

Standard basis vector
computational basis

$$\begin{pmatrix} 7 \\ 3+5i \end{pmatrix}$$

$$7|0\rangle + (3+5i)|1\rangle$$

4D vector example

$$\begin{pmatrix} 7 \\ 6 \\ i+3 \\ 0 \end{pmatrix}$$

Basis

$$\{ |100\rangle, |01\rangle, |10\rangle, |11\rangle \}$$

$$7|100\rangle$$

position
0

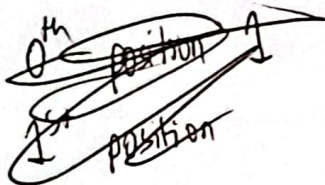
$$+ (1+3i)|10\rangle$$

position
2

[we represented a single matrix in 4D vector]

Bra - notation

~~is opposite to Ket notation~~



$$\langle 0 |$$

$$\langle 1 |$$

$$[1, 0]$$

$$[0, 1]$$

1st Column

0th column

$$|\psi\rangle^* = \langle \psi |$$

conjugate transpose of any value represented in Ket or bra notation will output bra or Ket notation respectively

+ conjugate * [change the sign from (+) to (-) or (-) to (+)]
 T transpose [change from row to column]

example

$$|\psi\rangle = (3+5i)|0\rangle + 7|1\rangle$$

$$\langle\psi| = (3-5i)\langle 0| + 7\langle 1|$$

Ket - notation

$$\begin{pmatrix} 3+5i \\ 7 \end{pmatrix}$$

2D-matrix $\begin{pmatrix} 1 & 2 \end{pmatrix}$
1 value

Bra - notation

$$[3-5i, 7]$$

Ket - notation

$$\begin{pmatrix} 3 \\ 0 \\ i \\ 7 \end{pmatrix}$$

$00 \rightarrow$ 0th location in denary
 $01 \rightarrow$ 1st location in denary
 $10 \rightarrow$ 2nd location in denary
 $11 \rightarrow$ 3rd location in denary

Bra - notation

$$[3, 0, i, 7]$$

$$3|00\rangle + 0|01\rangle + i|10\rangle + 7|11\rangle$$

$$3\langle 00| + 0\langle 01| + i\langle 10| + 7\langle 11|$$

Ket-Ket notation

tensor product

$$(\text{Ket - notation}) \times (\text{Ket - notation})$$

Bra-Bra notation

$$(\text{Bra - notation}) \times (\text{Bra - notation})$$

example of tensor product

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \end{pmatrix}$$

$A \otimes B$

$$\begin{pmatrix} 1 \cdot 2 & 3 & 0 & 0 & 0 \\ 0 & 4 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & 8 & 14 \end{pmatrix}$$

Ket-Ket notation

$$|\psi\rangle \otimes |\phi\rangle =$$

$$|\psi\rangle |\phi\rangle$$

simple

$$|\psi\phi\rangle$$

\therefore en

$$|\psi\rangle = i|0\rangle + 7|1\rangle$$

$$|\phi\rangle = \underline{1}|00\rangle + 3|10\rangle + 7|11\rangle$$

Compute

$$|\psi\rangle |\phi\rangle$$

$$\begin{pmatrix} i \\ 7 \end{pmatrix}$$

$$= i|000\rangle + 3i|010\rangle + 7i|011\rangle$$

$$+ 7|000\rangle + 21|110\rangle + 49|111\rangle$$

0 \rightarrow 6th position

49 \rightarrow 7th position

$$\begin{pmatrix} i & -0 \\ 0 & -1 \\ 3i & -2 \\ 7i & -3 \\ 7 & . \\ 0 & . \\ 21 & . \end{pmatrix}$$

Ket - Ket notation

$$(\text{Ket - notation}) \otimes (\text{Ket - notation})$$

Bra - Bra notation

$$(\text{Bra - notation}) \otimes (\text{Bra - notation})$$

example Bra - Bra notation

$$\langle \psi | = (3 \langle 0 | + 7 \langle 1 |)$$

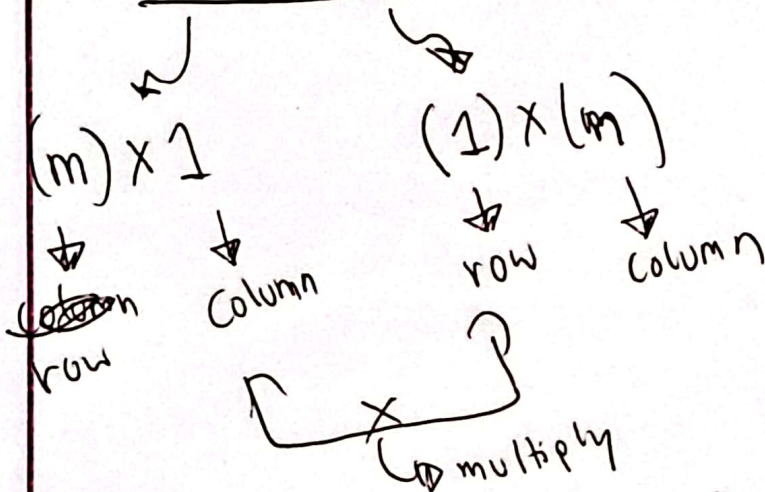
$$\langle \phi | = (\langle 0 | + i \langle 1 |)$$

$$\langle \psi \phi |$$

$$(3 \langle 00 | + 3i \langle 01 | + 7 \langle 10 | + 7i \langle 11 |)$$

Ket - Bra notation

$$|\alpha\rangle\langle\beta|$$



$m \times n$

$$|00\rangle\langle 10|$$

Example 1

0th row
2nd column $\rightarrow 1$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

number of rows
depend on the
first column

$$|\alpha\rangle = 3|0\rangle + i|2\rangle$$

$$|\beta\rangle = |00\rangle + 2|10\rangle + 7|11\rangle$$

$$|\alpha \times \beta\rangle$$

1) compute $\langle \beta | \alpha \rangle$

$$\langle \beta | = \langle 00 | + 2\langle 10 | + 7\langle 11 |$$

$$|\alpha \times \beta\rangle = \left(3|0\rangle + i|2\rangle \right)$$

$$\left(\langle 00 | + 2\langle 10 | + 7\langle 11 | \right)$$

$$= 3|0 \times 00| + 6|0 \times 10| + 21|0 \times 11|$$

$$+ i|1 \times 00| + 2i|1 \times 10| + 7i|1 \times 11|$$