Quantum bit - Qbits

Classical computer pov

- unit of data is called bits
- Bits store 0 or 1

Quantum computer pov

- unit :- Quantum bits or Q bits (qubit)
- qubit in two different state
  - ① pure state or classical state
    - bits store 0 or 1
  - ② superposition :- quantum bits
    - 0 and 1
    - 0 → prob p
    - 1 → prob 1-p

Math representation

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

α & β amplitudes of Qubits

α & β complex numbers that takes negative values

Prob of 0 $= |\alpha|^2$

Prob of 1 $= |\beta|^2$

$$\begin{bmatrix} |\alpha|^2 = \alpha^* \alpha \\ |\beta|^2 = \beta^* \beta \end{bmatrix}$$

$$\underbrace{|\alpha|^2 + |\beta|^2 = 1}$$

normalization ~~constant~~ constraint

— x — x — x — x — x — x —

Math representation in vector format

$$|\psi\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

unit vector by normalization constraint $= \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$$\| \, |\psi\rangle \| = 1 \quad \text{or,} \quad \langle \psi | \psi \rangle = 1$$

$\frac{16}{25} + \frac{9}{25}$

example

$|\psi\rangle = -\frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle$

if this value represents a valid Qubit

- $|\alpha|^2 + |\beta|^2 = 1$ ☑
- $\| |\psi\rangle \| = 1$
- $\langle\psi|\psi\rangle = 1$

$\left|-\frac{4}{5}i\right|^2 + \left|\frac{3}{5}\right|^2 = 1$

$\left[\left(\frac{4}{5}i\right) \times \left(-\frac{4}{5}i\right)\right] + \left[\frac{3}{5} \times \frac{3}{5}\right] = 1$

$\left[\left(-\frac{16}{25}\right)(-1)\right] + \left[\frac{9}{25}\right] = 1$

$\left(\frac{16}{25}\right) + \left[\frac{9}{25}\right] = 1$

$\frac{25}{25} = 1$

$1 = 1$

Prob of 0

$|-\frac{4}{5}i|^2$

$(\frac{4}{5}i) * (-\frac{4}{5}i)$

$|\frac{16}{25}|$

Prob of 1

$|1 - ?)$

or, $|\beta|^2$

$(\frac{3}{5})^2 = \frac{9}{25}$

example with multiple notations

$|\psi\rangle = -\frac{4}{5}i\,|0\rangle + \frac{3}{5}\,|1\rangle$

$|\phi\rangle = \frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}$

Tensor product

$|\psi\rangle \otimes |\phi\rangle$

or, $|\psi\phi\rangle$

$$|\psi\,\phi\rangle = \left(-\frac{4}{5}\,i|0\rangle + \frac{3}{5}|1\rangle\right) \otimes \left(\frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}\right)$$

$$= \left(\frac{-\frac{4}{3}\,i|00\rangle}{\sqrt{2}} + \frac{-\frac{4}{5}\,i|01\rangle}{\sqrt{2}} + \frac{3|10\rangle}{5\sqrt{2}} + \frac{3|11\rangle}{5\sqrt{2}}\right)$$

$$= \overset{\alpha}{\frac{-4}{5\sqrt{2}}\,i}\,|00\rangle + \overset{\beta}{-\frac{4}{5\sqrt{2}}}\,i|01\rangle + \overset{\gamma}{\frac{3}{5\sqrt{2}}}|10\rangle + \overset{\delta}{\frac{3}{5\sqrt{2}}}|11\rangle$$

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

$\sqrt{50}$

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

$$|\alpha|^2 = |\alpha^* \alpha| \qquad |\gamma|^2 = |\gamma^* \gamma|$$

$$|\beta|^2 = |\beta^* \beta| \qquad |\delta|^2 = |\delta^* \delta|$$

$$\Rightarrow \begin{bmatrix} -\frac{4}{5\sqrt{2}} i & \frac{3}{5\sqrt{2}} \\ -\frac{4}{5\sqrt{2}} i & \frac{3}{5\sqrt{2}} \end{bmatrix}$$

$$\Rightarrow \left(-\frac{4}{5\sqrt{2}}\right)\left(\frac{4}{5\sqrt{2}}\right) + \left(\frac{3}{5\sqrt{2}}\right)\left(\frac{3}{5\sqrt{2}}\right) + \left(-\frac{4}{5\sqrt{2}} i\right)\left(\frac{4}{5\sqrt{2}}\right)$$

$$+ \left(\frac{3}{5\sqrt{2}}\right)\left(\frac{3}{5\sqrt{2}}\right)$$

$$\Rightarrow \left(-\frac{16}{50}\right)(-1) + \left(\frac{9}{50}\right) + \left(-\frac{16}{50}\right)(-1)$$

$$+ \left(\frac{9}{50}\right)$$

$$= \left(\frac{16}{30}\right) + \left(\frac{9}{30}\right) + \left(\frac{16}{50}\right) + \left(\frac{9}{50}\right)$$

$$= \left(\frac{32}{50}\right) + \left(\frac{18}{30}\right)$$

$$= \left(\frac{50}{50}\right) = 1$$

operetions on Qubits

full measure

partielly measurement