part 0 :- Quantum computing introduction

① Bra - ket notation

- allows large vectors and matrices to be calculated in a precise and conscience manner

$\langle \alpha | \, | \infty \rangle$ [normal basic bra - ket notation]

- makes critical use of linear algebra prob + number theory
- Quantum algorithms
- extensive research

—x—x—x—x—x—x—x—x—

Dirac's Bra - ket notation

- Six types of notations

1) $|-\rangle$ ket notation
2) $\langle -|$ Bra notation
3) $|-\rangle|-\rangle$ ket - ket notation
4) $\langle -| \langle -|$ Bra - Bra notation
5) $|-\rangle\langle -|$ ket Bra notation
6) $\langle -|-\rangle$ Bra - ket notation.

2) Ket notation

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{array}{l} \to 0^{th} \text{ location} \\ \to 1^{st} \text{ location} \end{array}$ $\quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{array}{l} \to 0^{th} \\ \to 1^{st} \text{ location} \end{array}$

$|101\rangle$

3 binary digits $= 2^3$ dimensions

1) convert the binary value $\overset{4\,2\,1}{101}$ to ~~d~~denary

2) $|5\rangle$ =

0
0
0
0
0
1
0
0

if 8 possible locations meaning 7 possible values

3) $|11\rangle$

$|3\rangle$ =

0
0
0
1

How to derive ket-notation

1) look at the diagram used and observe the binary value represented in ket notation

2) convert the binary value into denary

3) when deriving ket notation, 1st position is the 0^{th} position. and place the value 1 at $(n-1)^{th}$ location

2D $\{|0\rangle, |1\rangle\}$ standard computational basis

example

$$\begin{pmatrix} 7 \\ 3+5i \end{pmatrix} = 7|0\rangle + (3+5i)|1\rangle$$

$$7\begin{pmatrix} 1 \\ 0 \end{pmatrix} + (3+5i)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3+5i \end{pmatrix}$$

4D example

4D basis

Ket notation

$$\begin{pmatrix} 7 \\ 0 \\ i+3 \\ 0 \end{pmatrix} \quad \therefore \left\{ \underset{7}{\underline{|00\rangle}}, \underset{0}{\underline{|01\rangle}}, \underset{i+3}{\underline{|10\rangle}}, \underset{0}{\underline{|11\rangle}} \right\}$$

$$= 7|00\rangle + (i+3)|10\rangle$$

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how to derive any ket → notation in any dimension

1) Identify the number of binary values inside the ket-notation and ~~multiply~~ use $2^{[n]}$ n being the number of ket-notation values

2) convert the binary value into denorary and plot the given value at their respected locations

quick note

- If one value inside the ket notation
  then 2^1 possible values
- If two values
  then $2^2 = 4$ possible values
- If three values
  then $2^3 = 8$ possible value
- If n values
  then $2^n = z$ possible values

—x—x—x—x—x—x—x—x—x—

Bra-notation

$\langle 0| = [1, 0] \qquad \langle 1| = [0, 1]$

1 will be placed in 0^{th} location row vector

- Bra-notation denoted row wise while ket-notation denoted column wise

transposed Bra-ket notation

$|\psi\rangle^{\dagger} = \langle\psi| \quad$ and $\quad \langle\psi|^{\dagger} = |\psi\rangle$

$*$: conjugate $\qquad$ $\dagger$: transpose

$\circledast$: changes any i sign from [+] to [-] or [-] to [+]

example

$|\psi\rangle = (3+5i)|0\rangle + 7|1\rangle$

$\langle\psi| = |\psi\rangle^*$

$|\psi\rangle^* = (3-5i)\langle 0| + 7\langle 1|$

$|\psi\rangle = \begin{bmatrix} 3+5i \\ 7 \end{bmatrix}$ $\langle\psi| = \begin{bmatrix} 3-5i & 7 \end{bmatrix}$

example $4 = 2^n$, $n = 2$ 4D vector so 2 values inside the notation

$\overbrace{\begin{bmatrix} 3 & 0 & i & 7 \end{bmatrix}}$

$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$

$3\langle 00| + i\langle 10| + 7\langle 11|$

—x—x—x—x—x—x—x—x—

ket - ket notation and bra - bra notation

Tensor product

$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \end{pmatrix}$

$$A \otimes B = \begin{bmatrix} 1 \cdot B & 0 \cdot B \\ 0 \cdot B & 2 \cdot B \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 4 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & 8 & 14 \end{bmatrix}$$

Ket-Ket notation

$$|\psi\rangle \otimes |\phi\rangle$$

$$= |\psi\phi\rangle$$

example

$$|\psi\rangle = i|0\rangle + 7|1\rangle$$

$$|\phi\rangle = |00\rangle + 3|10\rangle + 7|11\rangle$$

$$|\psi\phi\rangle = \begin{pmatrix} i \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 7 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 7 \end{bmatrix} \quad \begin{pmatrix} i \\ 7 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 7 \end{bmatrix}$$

$$= i|000\rangle + 3i|010\rangle + 7i|011\rangle$$

$$+ 7|00\rangle + 21|110\rangle + 49|111\rangle$$

$$i|000\rangle + 3i|010\rangle + 7i|011\rangle + 7|100\rangle + 2i|110\rangle + 49|111\rangle$$

$$\begin{bmatrix} i \\ 0 \\ 3i \\ 7i \\ 7 \\ 0 \\ 2i \\ 49 \end{bmatrix}$$

How to derive ket-ket notation

1) Identify the two given ket notations
2) Combine the signs or equation derived in the ket notation and combine them as one ket notation

3) multiply 1st ket notation value with the second ket notation values all of it and repeat the step again for the next one

4) by the given ket notation position values, which are represented in binary, convert them into denary

Bra- Bra notation

ex

$$\langle \psi | = 3\langle 0| + 7i \langle 1|$$

$$\langle \phi | = \langle 0| + i \langle 1|$$

$$\langle \psi \phi |$$

$$= 3\langle 00| + 3i \langle 01| + 7\langle 10| + 7i \langle 11|$$

$$\begin{bmatrix} 3 & 3i & 7 & 7i \end{bmatrix}$$

—x—x—x—x—x—x—x—

Ket -Bra notation

$$|\alpha\rangle\langle\beta| = |\alpha\rangle\langle\beta|$$

1 x n

n number of columns but one row

$$\begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{pmatrix}$$

n number of rows but one column

$\therefore |\alpha\rangle\langle\beta|$

$= m \times \cancel{r} = \cancel{r} \times n$

$m \times n$

Ex

$|00\rangle\langle 10|$

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \begin{pmatrix}0 & 0 & 1 & 0\end{pmatrix}$$

0^{th} row

2nd column

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$|\alpha\rangle = 3|0\rangle + i|1\rangle$

$|\beta\rangle = |00\rangle + 2|10\rangle + 7|11\rangle$

$|\alpha\rangle\langle\beta|$

$|\alpha\rangle = 3|0\rangle + i|1\rangle$

$\langle\beta| = \langle 00| + 2\langle 10| + 7\langle 11|$

$|\alpha\rangle = 3|0\rangle + i|1\rangle$

$\langle\beta| = \langle 00| + 2\langle 10| + 7\langle 11|$

$|\alpha\rangle\langle\beta|$

$= (3|0\rangle + i|1\rangle)(\langle 00| + 2\langle 10| + 7\langle 11|)$

$$\begin{pmatrix} 3 \\ i \end{pmatrix} \begin{bmatrix} 1 & 0 & 2 & 7 \end{bmatrix}$$

$= 3|0\rangle\langle 00| + 6|0\rangle\langle 01|| + 2i|0\rangle\langle 11|$

$+ i|\,|1\rangle\langle 00|| + 2i|1\rangle\langle 10|$

$+ 7i|1\rangle\langle 11|$

$$\begin{bmatrix} 3 & 0 & 6 & 2i \\ i & 0 & 2i & 7i \\ & & & \\ & & & \end{bmatrix}$$

m rows
1 colum ket

1 row
n colums bra 4 rows
2 colums

~~$|0\rangle|1\rangle$~~

Matrix given

$$
A = \begin{pmatrix} 0 & 1 \\ 3 & i \\ 7 & 0 \\ 0 & 13 \end{pmatrix}
\begin{array}{l} \leftarrow 00 \\ \leftarrow 01 \\ \leftarrow 10 \\ \leftarrow 11 \end{array}
$$

$|00\rangle$ $\langle 0|$
$|01\rangle$ $\langle 1|$
$|10\rangle$
$|11\rangle$

$\uparrow$ 0 $\uparrow$ 1

4 rows 4 rows
1 column 1 colum

~~$1\langle 1|01\rangle + 3\langle 0|$~~ $|01$

$$1|00 \times 1| + 3|01 \times 0| + i|01 \times 1|$$
$$+ 7|10 \times 0| + 13|11 \times 1|$$

—X—X—X—X—X—X—X—

how to derive ket-bra notation

- Identify the given ket notation and bra notation equations
- Draw a matrix and identify their ket notation values

Bra-Ket notation

$\langle \alpha | \beta \rangle$

1 rows — n row

m column — 1 colums

$n \times m$ if $N = M$

example

$\langle 0 | 0 \rangle = [1 \quad 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= (1 \times 1) + (0 \times 0) = 1$

$= (1 \times 1) + (1 \times 0) + (0 \times 1) + (0 \times 0)$

$= 1$

norm

$|\alpha\rangle$ is $\sqrt{\langle \alpha | \alpha \rangle}$

if $\sqrt{\langle \alpha | \alpha \rangle} = 1$ [unit vector

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orthogonel

$\langle 1 | 0 \rangle = 0$

when $\langle \alpha | \beta \rangle = 0$

$$\langle 0|1\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$(1\times 0)+(0\times 1)$$

$$=0$$

Inner product

example

$$|\alpha\rangle = i|0\rangle + 7|1\rangle$$

$$|\beta\rangle = 3|0\rangle + |1\rangle$$

inner product

$$\langle\alpha|\beta\rangle = \left[-i\langle 0| + 7\langle 1|\right]\left[3|0\rangle + |1\rangle\right]$$

NOTE:

Before performing the inner product, verify whether the metrices for each of the notation are the same

1 row 1 column $|\alpha\rangle$

1 row 1 colum $\langle\alpha|$

$$\langle\alpha|\beta\rangle = \left(-i\langle 0| + 7\langle 1|\right)\left(3|0\rangle + 1|1\rangle\right)$$

$$\left[-3i\underbrace{\langle 0|0\rangle}_{1} - i\underbrace{\langle 0|1\rangle}_{0} + 21\underbrace{\langle 1|0\rangle}_{0} + 7\underbrace{\langle 1|1\rangle}_{1}\right]$$

$$\left[-3i + 7\right]$$