Power series

Example

$$g(x) = \sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

ratio test

$a_n = \frac{x^n}{n \times 3^n}$ $b_n = \frac{x^{n+1}}{(n+1) \times 3^{n+1}}$

$$\frac{b_n}{a_n} = \frac{\frac{x^{n+1}}{(n+1) \cdot 3^{n+1}}}{\frac{x^n}{n \cdot 3^n}}$$

$$\frac{b_n}{a_n} = \frac{x^{n+1}}{(n+1) \cdot 3^{n+1}} \times \frac{n \cdot 3^n}{x^n}$$

$$= \frac{\cancel{x^n} \cdot x}{(n+1) \cdot (\cancel{3^n} \cdot 3)} \times \frac{n \cdot \cancel{3^n}}{\cancel{x^n}}$$

$$= \frac{x}{3(n+1)} \cdot n$$

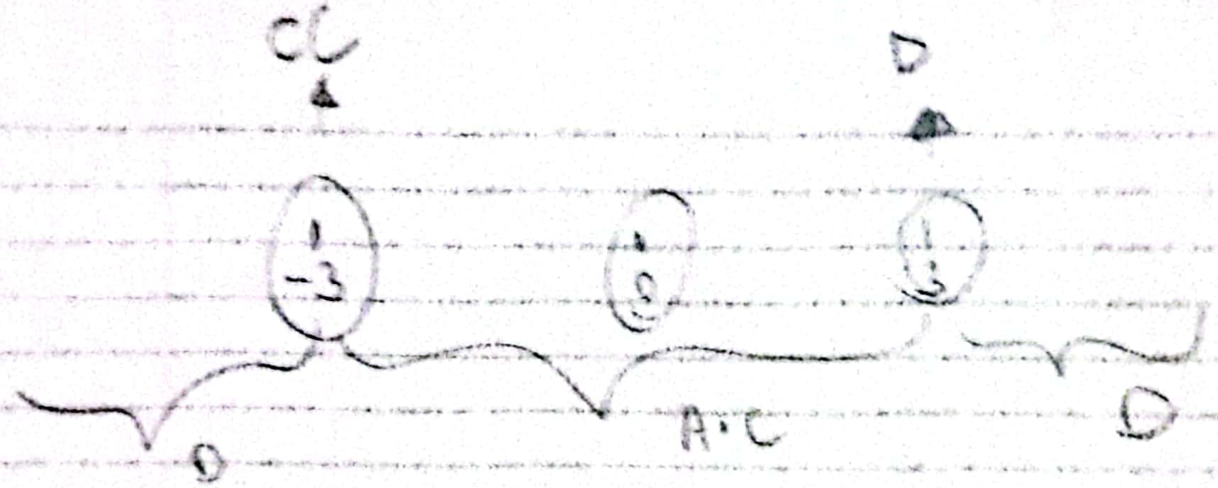
$$= \frac{xn}{3(n+1)} \quad \lim_{n \to \infty} \frac{x}{3} \frac{n}{n+1}$$

$$\lim_{n \to \infty} \frac{n}{n+1} \frac{\infty}{\infty+1} = \frac{\infty}{\infty} = 1$$

$$\therefore \lim_{n \to \infty} \frac{x}{3}(1) = \frac{x}{3}$$

$\boxed{g(x) \text{ divergent when } x > 3}$

$\boxed{g(x) \text{ convergent } x < 3}$



$g(3) = \frac{3^n}{n \cdot 3^n} = \frac{1}{n \cdot 1} = \frac{1}{n} = D$ [harmonic series]

$g(-3) = \frac{(-3)^n}{n \cdot (-3)^n} = \frac{(-1)^n}{n}$ = convergent by alternate series test

divergent by absolute value theorem

∴ conditionally convergent