Problem Set 1

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Problem 1

The link to the repository is here.

Problem 2

The code is in the "ps1_integration.R" file. The following table shows computation time and result of the integration:

Method	Result	Time (sec.)
Midpoint	-18.20953	0.025
Trapezoid	-18.20953	0.015
Simpson	-18.20953	0.022
Monte Carlo	-17.98814	0.003

We can see that quadrature methods are more exact but also slower than Monte Carlo. Monte Carlo result also depends on the seed.

Problem 3

The code is in the "ps1_optimization.R" file. The following table shows computation time, optimal x and y, and optimal value:

Method	x	y	Time (sec.)
Newton-Raphson	1	1	0.131
BFGS	1	1	0.042
Steepest descent	0.956	0.913	0.032
Conjugate gradient	0.956	0.912	0.034

Steepest descent and conjugate gradient methods are faster but less precise than Newton-Raphson and BFGS.

Problem 4

The code is in the "ps1_pareto.R" file. I first compute Pareto-efficient distribution for homogenous case. Initial values are equal distribution. Parameter values (identical for all agents in this case) can be found in the code. I use NLOPT package. Specifically, I use ISRES method for optimization. I then introduce heterogeneity and do the same thing for 3 agents and 3 goods. Finally, I solve for 10 homogeneous agents and 10 goods. The optimization is subject to the resource constraint.

Results for the homogeneous case are the following:

x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_1^3	x_2^3	x_3^3	Time (sec.)
1.4035	1.3343	1.2993	1.4283	1.3426	1.2652	1.4086	1.327	1.2908	5.42

As expected, all agents get the same quantities of each good. The sum of goods is equal to the sum of endowments.

Results for the heteregeneous case are the following:

x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_1^3	x_2^3	x_3^3	Time (sec.)
0.96702	0.2981	0.6235	0.2492	0.2723	0.0645	3.0914	4.6004	1.9337	5.37

Computation for 10 agents and 10 goods takes 49 seconds. I don't display results here.

Problem 5

The code is in the "ps1_price.R" file. I compute prices by solving the system of 6 equations and 6 unknowns. Unknowns are prices and Lagrande multipliers. Equations are budget constraints and excess demands. I solve equations by minimizing the sum of squares. The prices are 0.010, 0.012, and 0.018. The time is 37 seconds.