ETH zürich D BSSE Computational Biology Group

Evolutionary Dynamics

Exercises 1

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Exercises marked with a "💻" are programming exercises. These can be solved in a programming language of your choice. Please make sure to hand in your code along with your answers to these exercises.

Problem 1: Discrete time (tutorial question)
Suppose you have a difference equation $x_{t+1} = f(x_t)$ of a discrete time model with

$$f(x) = 5x^2(1-x).$$

(a) Determine the equilibrium points x^* of the system.

(b) Which of the equilibrium points x^* are stable?

Problem 2: Logistic difference equation
In a discrete time model for population growth, the value x (number of cells divided by the maximum number supported by the habitat) at time $t+1$ is calculated from the value at time t according to the difference equation

$$x_{t+1} = r x_t (1 - x_t).$$

(a) Determine the equilibrium points x^* of the system. **(1 point)**

(b) Are the points stable for $r = 0.5$, $r = 1.5$, $r = 2.5$? **(1 point)**

(c) Confirm this by numerically iterating the difference equation. 💻 **(1 point)**
Hint: Plot the value x for a series of time steps.

(d) Examine the stability and behaviour for $r = 3.5$. 💻 **(1 point)**
Hint: Plot the *Poincaré section* of x_t against x_{t-1}.

(e) What happens for $r = 3.9$? 💻 **(1 point)**

Problem 3: Logistic growth in continuous time
The logistic model for population growth is:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \lambda x(t)\left(1 - \frac{x(t)}{K}\right) \tag{1}$$

(a) Show, by direct integration of (1), that the solution is given by: **(2 points)**

$$x(t) = \frac{K x_0 e^{\lambda t}}{K + x_0(e^{\lambda t} - 1)}.$$

Hint: Use *separation of variables* and *partial fractions*.

(b) Find the equilibrium points of the system and discuss their stability. **(1 point)**
Hint: Consider the cases $\lambda > 0$ and $\lambda < 0$.

(c) Numerically integrate to demonstrate the results above for $K = 1$. **(2 points)**