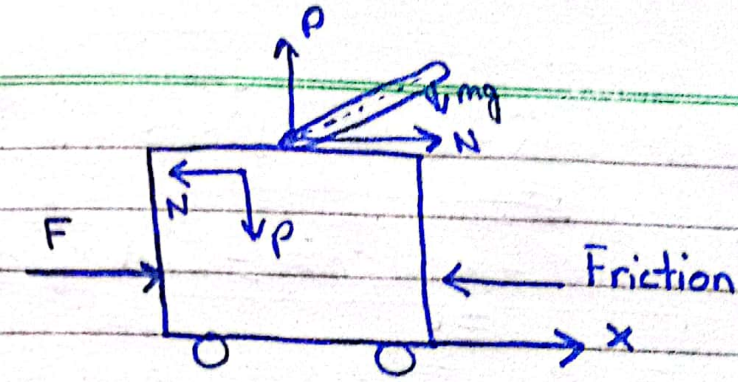

fx cart:

$M\ddot{x} + b\dot{x} + N = F$

fx pendulum:

$F_x = N$

$F_x = m\ddot{x}_{pendulum}$

$N = m\ddot{x}_p$

fy pendulum:

$F_y = P - mg$

$m\ddot{y}_p = P - mg$

$P = m(\ddot{y} + g)$

We define xp & yp interms of x & θ:

$x_p = x + L\cos(\theta - 90)$

$\because \cos(\theta - 90) = \sin\theta$

$x_p = x + L\sin\theta$

$\dot{x}_p = \dot{x} + L\dot{\theta}\sin\theta$

$\ddot{x}_p = \ddot{x} - L\dot{\theta}^2\sin\theta + L\ddot{\theta}\cos\theta$

$y_p = L\sin(\theta - 90)$

$y_p = -L\cos\theta$

$\dot{y}_p = L\dot{\theta}\sin\theta$

$\ddot{y}_p = L\dot{\theta}^2\cos\theta + L\ddot{\theta}\sin\theta$

we put $\ddot{x}_p$ & $\ddot{y}_p$ back in pendulum equations:

$N = m\ddot{x}_p$ & $P = m(\ddot{y}_p + g)$

$N = m(\ddot{x} - L\dot{\theta}^2\sin\theta + L\ddot{\theta}\cos\theta)$

$P = m(L\dot{\theta}^2\cos\theta + L\ddot{\theta}\sin\theta + g)$

we use N & P in our cont & pend eq.

$$\boxed{F = (M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta}$$

$$\boxed{(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta}$$

For Transfer Function we must linearise:

$\theta = \pi$ (upward)
$\theta = \pi + \phi$ (For slight changes)

$\cos\theta = \cos(\pi + \phi) = -1$
$\sin\theta = \sin(\pi + \phi) = -\phi$

$\dot{\theta}^2 = \dot{\phi}^2 \approx 0$

$(I + ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x}$ $(F = u)$
$(M+m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u$

Transfer Function:

$(I + ml^2)\,\phi(s)\,s^2 - mgl\,\phi(s) = ml\,X(s)\,s^2$

$(M+m)X(s)s^2 + bX(s)s - ml\,\phi(s)s^2 = U(s)$ —— (i)

To Find $X(s)$:

$$X(s) = \frac{(I + ml^2)\cancel{s^2}\phi(s)}{ml\cancel{s^2}} - \frac{mg\cancel{l}\phi(s)}{\cancel{m}\cancel{l}s^2}$$

$$X(s) = \left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)$$

putting $X(s)$ in (i) :

$$\Rightarrow (M+m)\left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)s^2 + b\left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)s$$

$$- ml\,\phi(s)s^2 = U(s)$$

$$\Rightarrow \left[\frac{(M+m)(I+ml^2)s^2 - (M+m)mlg}{mls^2}\right]\phi(s)s^2$$

$$+ \left[\frac{b(I+ml^2)s^2 - bgml}{mls^2}\right]\phi(s)s - ml\phi(s)s^2 = U(s)$$

$$\Rightarrow \phi(s)\left[\frac{(M+m)(I+ml^2)s^4 - (M+m)mlgs^2}{mls^2} + \frac{b(I+ml^2)s^3 - mlgbs}{mls^2}\right.$$

$$\left. - mls^2\right] = U(s)$$

$$\Rightarrow \phi(s)\left[\frac{[(M+m)(I+ml^2) - (ml)^2]s^4 + b(I+ml^2)s^3 - (M+m)mgls^2 - mgbls}{mls^2}\right]$$

$$= U(s)$$

$$\Rightarrow \frac{\phi(s)}{U(s)} = \frac{mls^2}{[(M+m)(I+ml^2) - (ml)^2]s^4 + b(I+ml^2)s^3 - (M+m)mgls^2 - mgbls}$$

Let $(M+m)(I+ml^2) - (ml)^2 = z = \boxed{0.0132}$

$$= \frac{\frac{ml s^2}{Z}}{s^4 + \frac{b(I+ml^2)}{Z}s^3 - \frac{(M+m)mgl}{Z}s^2 - \frac{mgbl}{Z}s}$$

$$= \frac{\frac{ml s}{Z}}{s^3 + \frac{b(I+ml^2)}{Z}s^2 - \frac{(M+m)mgl}{Z}s - \frac{mgbl}{Z}}$$

$$= \frac{4.5454}{s^3 + 0.1818 s^2 - 31.1818 s - 4.4545}$$