Progress Report

Lifting Linearization of a UAV

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New Model

$$
\begin{aligned}
\dot{\boldsymbol{p}} &= \boldsymbol{v} \\
\dot{\boldsymbol{v}} &= \boldsymbol{r}_z(\boldsymbol{e})T + \boldsymbol{g} \\
\begin{bmatrix} \dot{\boldsymbol{\epsilon}} \\ \dot{\eta} \end{bmatrix} &= \frac{1}{2}\boldsymbol{J}_E\omega = \frac{1}{2}\begin{bmatrix} \eta\boldsymbol{I} - \boldsymbol{\epsilon}\times \\ -\boldsymbol{\epsilon}^T \end{bmatrix}\boldsymbol{\omega}, \\
\dot{\boldsymbol{\omega}} &= \boldsymbol{w}_1 \\
\dot{T} &= w_2
\end{aligned}
\tag{1}
$$

$$
\boldsymbol{r}_z(\boldsymbol{e}) = \boldsymbol{R}(e)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2\left(\epsilon_1\epsilon_3 + \epsilon_2\eta\right) \\ 2\left(\epsilon_2\epsilon_3 - \epsilon_1\eta\right) \\ 1 + 2\left(-\epsilon_1^2 - \epsilon_2^2\right) \end{bmatrix}, \boldsymbol{\epsilon}\times = \begin{bmatrix} 0 & -\epsilon_3 & \epsilon_2 \\ \epsilon_3 & 0 & -\epsilon_1 \\ -\epsilon_2 & \epsilon_1 & 0 \end{bmatrix}, \boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}
$$

- States: $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}^T & \boldsymbol{v}^T & \boldsymbol{\epsilon}^T & \eta & \boldsymbol{\omega}^T & T \end{bmatrix}^T_{14\times1}$
- Inputs: $\boldsymbol{w} = \begin{bmatrix} \boldsymbol{w}_1^T & w_2 \end{bmatrix}^T_{4\times1}$
- $\dot{p}, \dot{v}, \boldsymbol{\omega}, \boldsymbol{g}$ are measured in Global Frame.
- T is measured in Body Fixed Frame.

Expanded Model

$$\dot{x} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ v_1 \\ v_2 \\ v_3 \\ \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \eta \\ \omega_2 \\ \omega_3 \\ \omega_1 \\ T \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ T\left(2\epsilon_1\epsilon_3 + 2\epsilon_2\eta\right) \\ T\left(2\epsilon_2\epsilon_3 - 2\epsilon_1\eta\right) \\ 9.81 - T\left(2\epsilon_1^2 + 2\epsilon_2^2 - 1\right) \\ \frac{\epsilon_3\omega_2}{2} - \frac{\epsilon_2\omega_3}{2} + \frac{\eta\omega_1}{2} \\ \frac{\epsilon_1\omega_3}{2} - \frac{\epsilon_3\omega_1}{2} + \frac{\eta\omega_2}{2} \\ \frac{\epsilon_2\omega_1}{2} - \frac{\epsilon_1\omega_2}{2} + \frac{\eta\omega_3}{2} \\ -\frac{\epsilon_1\omega_1}{2} - \frac{\epsilon_2\omega_2}{2} - \frac{\epsilon_3\omega_3}{2} \\ w_{1,1} \\ w_{1,2} \\ w_{1,3} \\ w_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 2\boldsymbol{\eta}_1 + 2\boldsymbol{\eta}_2 \\ 2\boldsymbol{\eta}_3 - 2\boldsymbol{\eta}_4 \\ T - \boldsymbol{\eta}_6 - 2\boldsymbol{\eta}_5 \\ \boldsymbol{\eta}_7 - \boldsymbol{\eta}_8 + \boldsymbol{\eta}_9 \\ \boldsymbol{\eta}_{10} - \boldsymbol{\eta}_{11} + \boldsymbol{\eta}_{12} \\ \boldsymbol{\eta}_{13} - \boldsymbol{\eta}_{14} + \boldsymbol{\eta}_{15} \\ -\boldsymbol{\eta}_{16} - \boldsymbol{\eta}_{17} - \boldsymbol{\eta}_{18} \\ w_{1,1} \\ w_{1,2} \\ w_{1,3} \\ w_2 \end{pmatrix} \tag{3}$$

Auxiliary Parameters

$$\begin{pmatrix} T\epsilon_1\epsilon_3 \\ T\epsilon_2\eta \\ T\epsilon_2\epsilon_3 \\ T\epsilon_1\eta \\ T\epsilon_1 2 \\ 2T\epsilon_2^2 + g \\ \frac{\epsilon_3\omega_2}{2} \\ \frac{\epsilon_2\omega_3}{2} \\ \frac{\eta\omega_1}{2} \\ \frac{\epsilon_1\omega_3}{2} \\ \frac{\epsilon_3\omega_1}{2} \\ \frac{\eta\omega_2}{2} \\ \frac{\epsilon_2\omega_1}{2} \\ \frac{\epsilon_1\omega_2}{2} \\ \frac{\eta\omega_3}{2} \\ \frac{\epsilon_1\omega_1}{2} \\ \frac{\epsilon_2\omega_2}{2} \\ \frac{\epsilon_3\omega_3}{2} \end{pmatrix} = \begin{pmatrix} \eta1 \\ \eta2 \\ \eta3 \\ \eta4 \\ \eta5 \\ \eta6 \\ \eta7 \\ \eta8 \\ \eta9 \\ \eta10 \\ \eta11 \\ \eta12 \\ \eta13 \\ \eta14 \\ \eta15 \\ \eta16 \\ \eta17 \\ \eta18 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} {\epsilon_1}^2 \\ {\epsilon_2}^2 \\ {\epsilon_3}^2 \\ {\omega_1}^2 \\ {\omega_2}^2 \\ {\omega_3}^2 \end{pmatrix} = \begin{pmatrix} \gamma1 \\ \gamma2 \\ \gamma3 \\ \gamma4 \\ \gamma5 \\ \gamma6 \end{pmatrix} \tag{5}$$

T4

$$\epsilon = [0, 0, \sin(5)]$$
$$\eta = \cos(5)$$

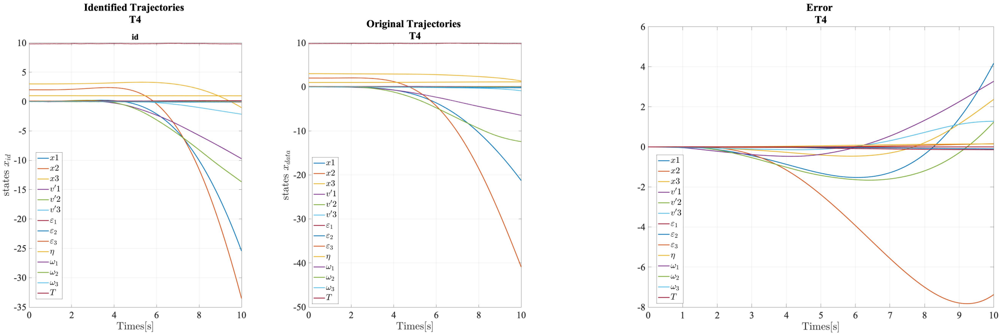

Figure: Linearized vs Real System

T5

$$\epsilon = [0, 0, \sin(10)]$$
$$\eta = \cos(10)$$

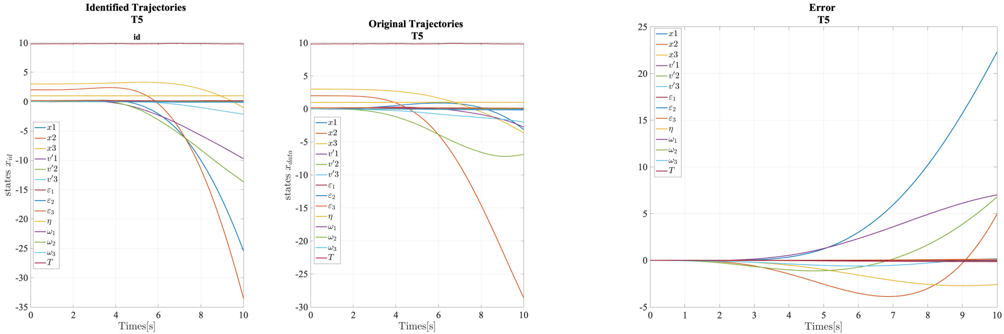

Figure: Linearized vs Real System

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LQR in Matlab

```
%% Controller Settings
Q_x = 2.3e3*[1;1;1];
Q_v = 1e3*[5;5;5];
Q_e = .5e3*[1;1;1;2];
Q_omega =200 * [10;10;10];
Q_dfl = diag([Q_x;Q_v;Q_e;Q_omega]);

R = 1000 * diag([1;1;1;1]);
C_z = [eye(13,13),zeros(13,25)];
K = lqr(A,B,C_z'*Q_dfl*C_z,R);
```

LQR Parameters

Initial State

- Position: $p_0 = \begin{bmatrix} 0 & 2 & 3 \end{bmatrix}^T$
- Velocity: $v_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
- Orientation: $\epsilon_0 = \begin{bmatrix} 0 & 0 & \sin 10 \end{bmatrix}^T, \eta_0 = \cos 10$
- Angular Velocity: $\omega_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
- Thrust: $T_0 = 9.81$

Desired State

- Position: $p = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
- Velocity: $v = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
- Orientation: $\epsilon = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, \eta = 1$
- Angular Velocity: $\omega = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
- Thrust: $T = 9.81$

Postion and Velocity

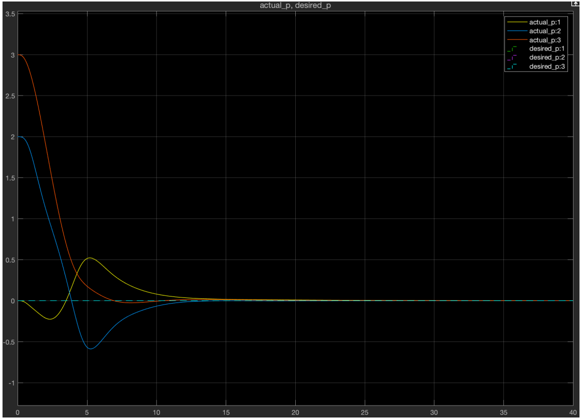

Figure: Position plot

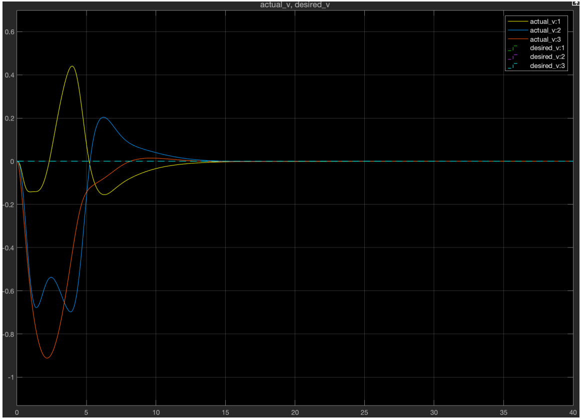

Figure: Velocity plot

Euler Parameters

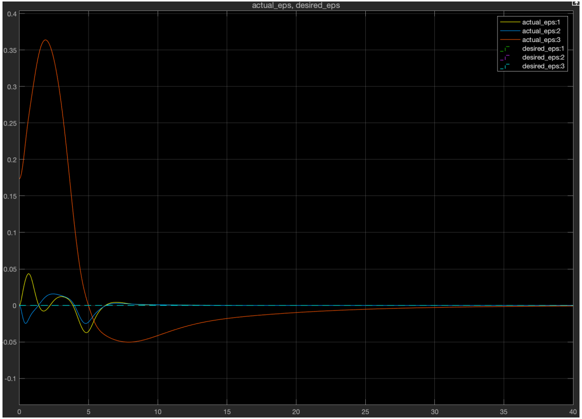

Figure: Epsilon plot

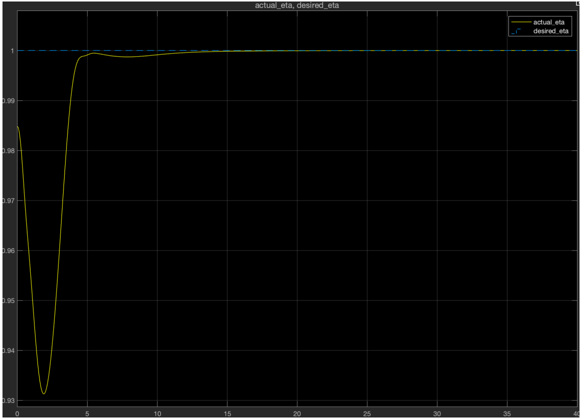

Figure: eta plot

Angular Velocity and Inputs

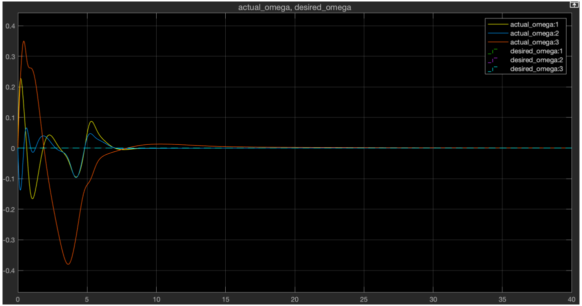

Figure: Angular Velocity plot

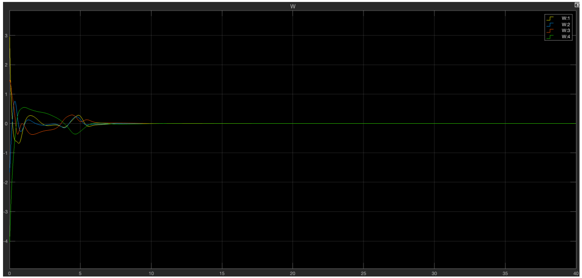

Figure: Inputs plot

Auxiliary Parameters

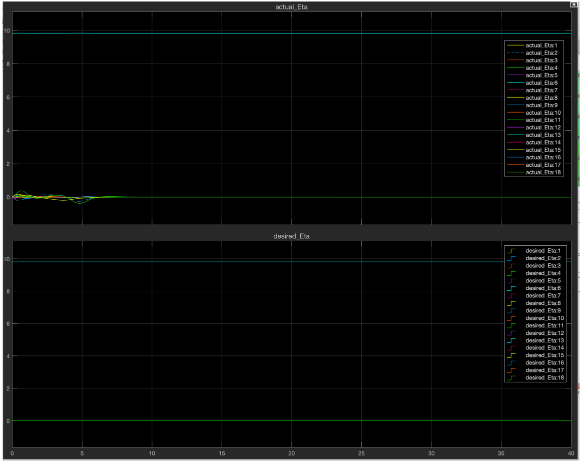

Figure: Etas plot

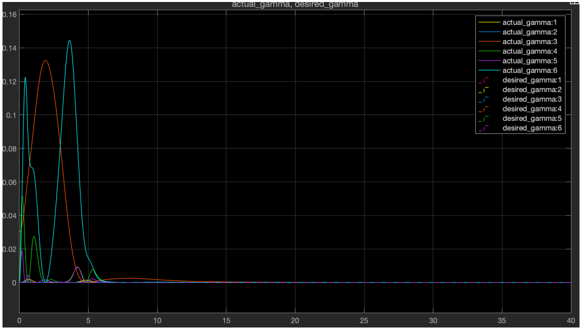

Figure: Gamma plot