Moscow Institute of Physics and Technology

Chair of Discrete Mathematics

Variations of the Theorem of Barany and Grinberg about Vector Sums

Bachelor's Thesis

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Abstract

Call a set $V \subset \mathbb{R}^d$ *balanced* if $0 \in \operatorname{conv} V$. Define a *transversal* of the system of sets $V_1, \dots, V_n$ as any set $\{v_1, \dots, v_n\}$ where $v_i \in V_i$ for all i. In 1981 Barany and Grinberg proved that for any system $\{V_i\}, i \in \{1, \dots, n\}$ of balanced sets in the unit ball of d-dimensional real normed space exists a transversal the norm of whose element sum does not exceed d. In this work we improve this bound to $\sqrt{d}$ for the Euclidean norm. Furthermore, if $|V_i| = m$ for all i for some m then we prove that the system $\{V_i\}$ can be partitioned to m transversals the norm of element sum of each of which does not exceed $m\sqrt{d}$. Additionally, we prove that in the same setting a transversal can be extracted the norms of whose partial sums do not exceed d.

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