Example 2: A parking lot attendant observes three vehicles enter the lot in a certain minute. If the number of vehicles entering this parking lot in a minute can be modeled with a Poisson distribution, what is the best guess for the value of λ? That is, of four possible values of λ: $3, 4, 5, or\ 6$ what value would you estimate λ to be?

we don't know what λ is but what if we believe it is between 3 & 6?

if $X = 3 \Rightarrow \lambda$?

$$P(X=3 \mid \lambda) = \frac{e^{-\lambda}\lambda^3}{3!}$$

let $\lambda = 3, 4, 5, 6, \dots$, Support $x = 0, 1, 2, \dots$

$\lambda =$	3	4	5	6
$P(X=3 \mid \lambda)$	0.2240	0.1954	0.1404	0.0892
	dpois(3,3)	dpois(3,4)	dpois(3,5)	dpois(3,6)

max prob!

dpois(3, 0:10)

Based on sample of one, 1-minute interval...

What would you guess if three, 1-minute intervals revealed 3 cars, 4 cars, 5 cars...?

Example continued: $X_i \sim Poisson$, find MLE for λ

$$f(x_i \mid \lambda) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$L(\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = \frac{e^{-\lambda}\lambda^{x_1}}{x_1!} \cdots \frac{e^{-\lambda}\lambda^{x_n}}{x_n!}$$

$$= \frac{e^{-n\lambda}\lambda^{\sum x_i}}{\prod(x_i!)} \Rightarrow \ln(L(\lambda)) = -n\lambda + \left(\sum x_i\right)\ln(\lambda) - \ln\left(\prod(x_i!)\right)$$

$$\frac{\partial \ln(L(\lambda))}{\partial \lambda} = -n + \frac{\sum x_i}{\lambda} - 0 = \frac{\sum x_i}{\lambda} - n \overset{\text{set}}{=} 0 \Rightarrow \frac{\sum x_i}{\lambda} = n \Rightarrow \lambda = \frac{\sum x_i}{n} = \bar{X}$$

$$\frac{\partial^2}{\partial \lambda^2} \ln(L(\lambda)) = -\frac{\sum x_i}{\lambda^2} \Rightarrow f''(x_0) = \frac{\partial^2}{\partial \lambda^2}\left[\ln(L(\bar{x}))\right] = -\frac{\sum x_i}{\left(\frac{\sum x_i}{n}\right)^2} = -\frac{n^2}{\sum x_i} < 0$$

must be

So $\bar{X}$ is maximum

$$\text{MLE for } \lambda = \frac{\sum x_i}{n} = \bar{x} = MLE(\lambda).$$