Want to "win" a vote. Aka $p \geq 0.5$ [worst case $p=0.5$]

You Believe you will win. So You Believe.

Meaning out of the whole population half or more would vote for You!

However, if you took a sampling say $n=15$

What could happen? would the sample Be a true representation of what you believe The population is? Does it have to Be?

If X is the number of people out of 15 who voted for you, And you Believe this sample is being Drawn from a population You Believe that Has 0.5 voting for You.

What is the Dist. of X?

Believe $p \geq 0.5$

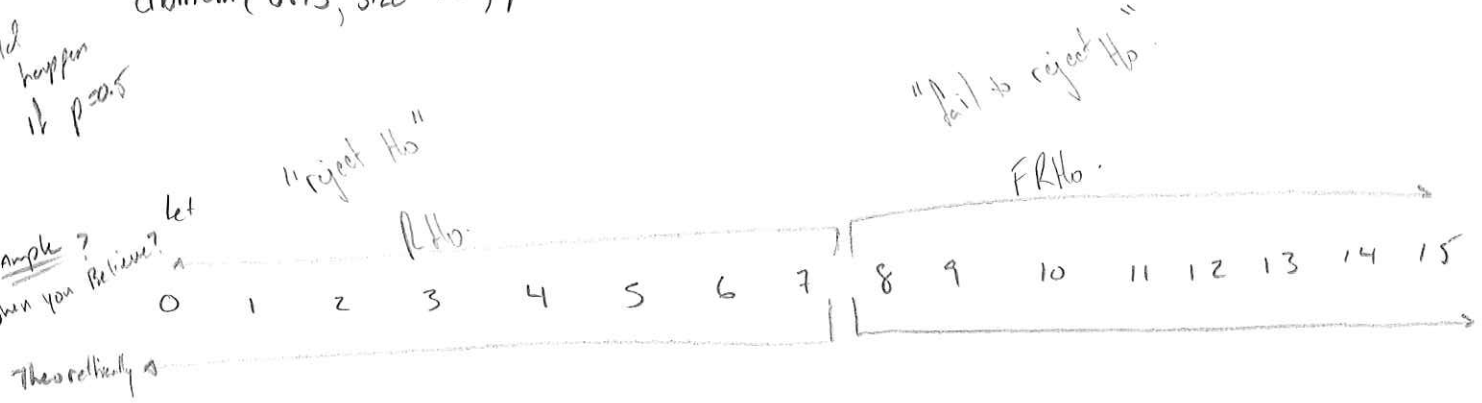
worst case.

$X \underset{H_0}{\sim} Bin(n=15, p=0.5)$

$H_0: p \geq 0.5$ worst case $p = 0.5$

dbinom(0:15, size = 15, prob = 0.5)

What could happen if $p = 0.5$



Sum (dbinom(8:15, 15, 0.5)) = 0.5

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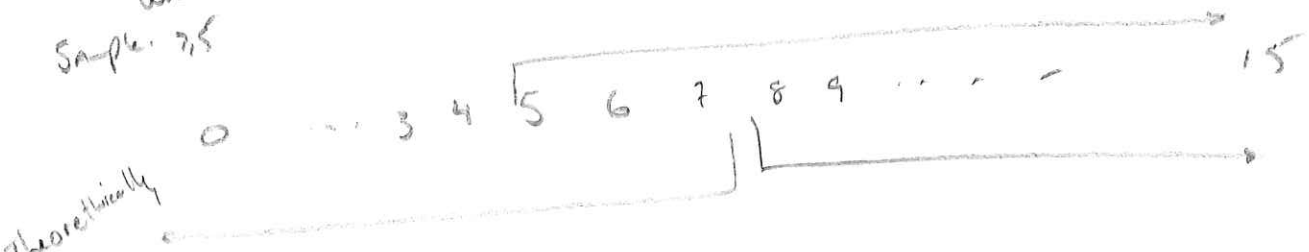
When Support is 0.5, The chance of getting 8 or more votes out of 15 is 50%

Say you will Believe when Sample. ?,5

Decision Rule: "?"



Sum (dbinom(5:15, 15, 0.5)) = 0.9407654.

When support is 0.5, The chance of getting 5 or more votes out of 15 is 94.07654%.

Say the "true" prop. of the population is only 40% (Not 50% like you thought)

You will lose the vote But you Don't know it yet...

$X \sim_{\text{true}} \text{bin}(n=15, p=0.4)$.

Remember you said you would Believe you had $\underbrace{p=0.5}_{\text{Null}}$ if out of 15 samples at least 5 votes out of 15.

That was assuming you had 50% of population support.

If the truth is only 0.4 of the population supported you...

Believe under Assumption $p=0.5$.

Sample	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Theoretical under $p=0.4$

Sum(dbinom(5:15, 15, 0.4)) $\approx$ 0.7827223

To sum up...

Under the belief that you have 50% or more (worst case = 0.5) of the populations support; The probability you receive 5 or more votes out of 15 is at least 94.07654%

Not making type I error = Confidence = $1-\alpha$ $P[R|H_0 \text{ true}]$

"think you have support and you do"

Under the belief you have 50% of the populations support.

The probability you only get 4 or less votes out of a random sample of 15 is at most 5.923463%.

type I error = significance = α $P[R|H_0 \text{ true}]$

"think you don't have 50% but you Do"

However, when you believe pop support is 50%. So you believe getting at least 5 out of 15 votes supports your belief. The probability you receive 5 (or more) votes when the true population support is 40%.

is 78.27223.

Type II error = β $P[FR H_0 \mid H_0 \text{ false}]$

"Think you have support, But you don't"

Then again you may have received 4 (or less) votes out of 15 (leading you to believe you don't have 50% under orginal Belief).

and the true population support is 0.4

Prob of this is 0.2172777

Power = Not making Type II error = $1-\beta$ $P[R H_0 \mid H_0 \text{ false}]$

"Think you don't have support and you don't"

Back to square one. You Believe $p \geq 0.5$ (you have support).

Then you take a random sample of 15.

Say X_{obs} is the number from the sample that did support you!

under our current Assumption of the population $p \geq 0.5$

worst case $p = 0.5$

$$X_{obs} \sim Bin(n=15, \underbrace{p=0.5}_{H_0})$$

And you have developed the rule that if 5 or more out of 5 will support your idea of $p \geq 0.5$

$H_0: p \geq 0.5$

$H_a: p < 0.5$

Then

$$P(\underset{\uparrow H_0}{X} < X_{obs}) = \text{p-value}$$

Under H_0, Probability that your observed sample will have more evidence against H_0

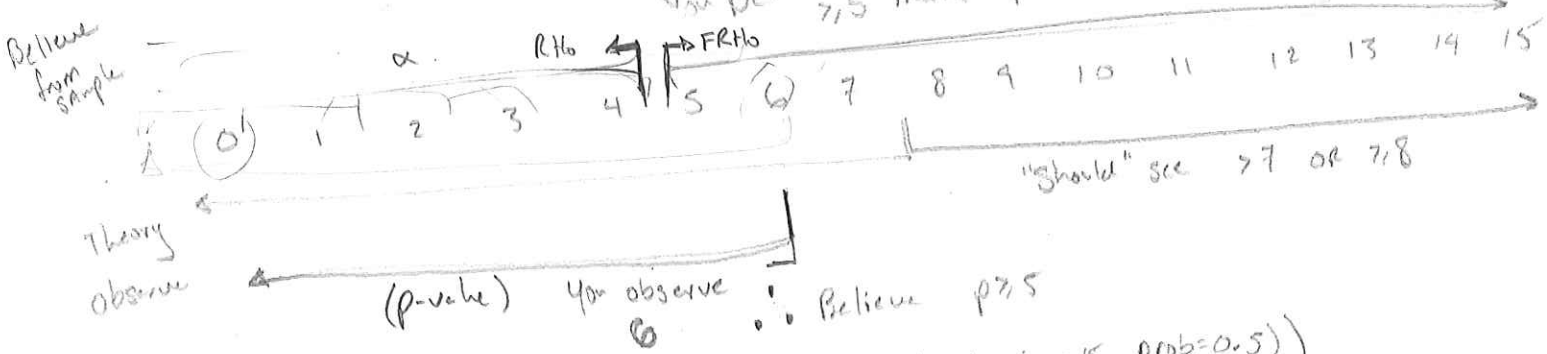
implication when p-value $\geq \alpha$ FR H_0

p-value $< \alpha$ R H_0

for example. $X_{obs} = 6$.

Decision rule:

You Believe $\geq .5$ means $\underset{\sim}{H_0}$ $p \geq 5$ $(1-\alpha)$



p-value = sum(dbinom(0:6, size=15, $\underbrace{prob=0.5}_{H_0}$)) $\cong 0.9036194$

$P(R H_0 \mid H_0 \text{ true}) \rightarrow \alpha = sum(dbinom(0{:}4, size=15, prob=0.5)) \cong 0.05923462$