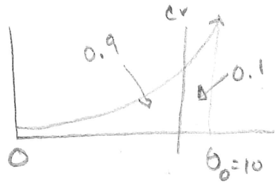
Example 4: recall previous example. A random sample $X_1, X_2, \cdots, X_8$ is taken from a population of values that is modeled by the uniform distribution from 0 to θ. From this sample, one wishes to test

$$H0: \theta = 10 \qquad HA: \theta > 10$$

Use $X_{(n)}$ as a test statistic, recall $f_{X_{(n)}}(x) = \frac{n}{\theta^n} x^{n-1}\ for\ 0 < x < \theta$

Derive a decision rule, based on the previously derived probability density function of $X_{(n)}$, setting the probability of committing a Type I error to be 0.10 then find the power of this test if $\theta = 11$. Can you find the power curve?



$\frac{n}{\theta^n} x^{n-1}$

$1-\alpha = P[FRH_0 \mid H_0 \text{ true}] = P[X_{(n)} < cv \mid \theta = 10]$

$0.9 = \int_0^{cv} \frac{8x^7}{10^8}\,dx \Rightarrow 0.9 = \frac{cv^8}{10^8} \Rightarrow cv = (10^8(0.9))^{1/8}$

$\cong 9.8691$

*No computer code pre written for this . . .

So if we get $X_{(8)}$ out of 8 to be $\geq 9.8691 \Rightarrow RH_0$ otherwise FRH_0.

$1-\beta = P[RH_0 \mid H_0 \text{ false}] = P[X_{(n)} > cv \mid \theta = 11]$

$= \int_{cv}^{11} \frac{8x^7}{\theta^8}\,dx \Big|_{\theta=11,\ cv \cong 9.8691} = \frac{11^8}{11^8} - \frac{\left[[10^8(0.9)]^{1/8}\right]^8}{11^8} = 1 - \frac{10^8(0.9)}{11^8}$

$\cong 0.5801433578$

58% chance of detecting $\theta > 10$ when we think it is 10 but it is actually 11!