Example 2: Suppose we obtain a single observation, $X_1 = 100.1682$, from a normal distributed random variable, with mean $\mu, \sigma = 10$. Use $\frac{X_1-\mu}{\sigma}$ to form a 95% confidence interval for μ.

a) Is $\frac{X_1-\mu}{\sigma}$ a pivotal quantity?

1. function of sample? Yes! μ is included but is only unknown? Yes!

2. Dist does not depend on μ?

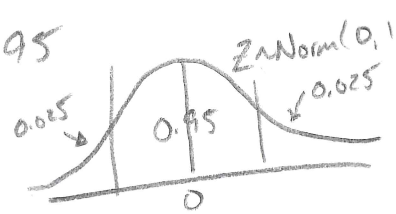
$\frac{X_1-\mu}{\sigma} \sim Norm(0,1)$ Since $X_1 \sim Norm(\mu, \sigma)$ we proved this Univariate Booklete Example 11

So Does Not depend on μ

$\therefore \frac{X_1-\mu}{\sigma}$ is a pivotal quantity!

b) $P\left(a \le \frac{X_1-\mu}{\sigma} \le b\right) = 0.95$, find a and b. Form a 95% confidence interval for μ.

$P\left(a \le \frac{X_1-\mu}{\sigma} \le b\right) = P(a \le Z \le b) = 0.95$



$a = qnorm(0.025)$
$\cong -1.96$
$b = qnorm(0.975)$
$\cong 1.96$.

$P\left(a \le \frac{X_1-\mu}{\sigma} \le b\right) = P(\sigma a \le X_1 - \mu \le \sigma b) = P(\sigma a - X_1 \le -\mu \le \sigma b - X_1)$

$= P(X_1 - \sigma a \ge \mu \ge X_1 - \sigma b) = P(X_1 - \sigma b \le \mu \le X_1 - \sigma a) = 0.95$

$\Rightarrow$ here $a \cong -1.96$ $\Rightarrow P(X_1 - 10(1.96) \le \mu \le X_1 - 10(-1.96)) = 0.95$
$b \cong 1.96$
$\sigma = 10$
$X_1 = 100.1682$

$P(80.56856 \le \mu \le 119.7684) = 0.95$

We are 95% Sure μ is between 80.56856 & 119.7684