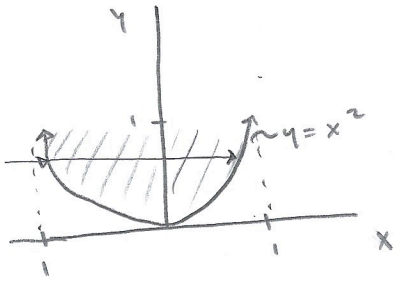
Example 6.4 (CDF / Method of Distributions)

$$f_X(x) = \begin{cases} \frac{x+1}{2}, & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$g_Y(x): X^2 = Y \Rightarrow$ inverse function $g_X^{-1}(y) = \pm\sqrt{y} = X$

Step 1) Support of Y?


Step 2) graph transformation function




$\Rightarrow -\sqrt{y} \le x \le \sqrt{y} \wedge 0 \le x \le 1 \wedge 0 \le y \le 1$

Step 3)

$$F_Y(y) = P(Y \le y) = P(g_Y(x) \le y) = P(x^2 \le y) = P(x \le \pm\sqrt{y}) = P(-\sqrt{y} \le X \le \sqrt{y})$$

Step 4)

$$F_Y(y) = \int_x f_X(x)\,dx = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{x+1}{2}\,dx = \sqrt{y}$$

So $$F_Y(y) = \begin{cases} 0; & -\infty < y < 0 \\ \sqrt{y}; & 0 < y < 1 \\ 1; & y \ge 1 \end{cases}$$

Step 5)

$$f_Y(y) = \frac{d}{dy}[F_Y(y)] = \begin{cases} \frac{1}{2\sqrt{y}}; & 0 \le y < 1 \\ 0; & \text{otherwise} \end{cases} = \frac{1}{2} y^{-1/2}(1-y)^0; \quad 0 < y < 1$$

$Y \sim \text{Beta}(\alpha = 1.5, \beta = 1)$