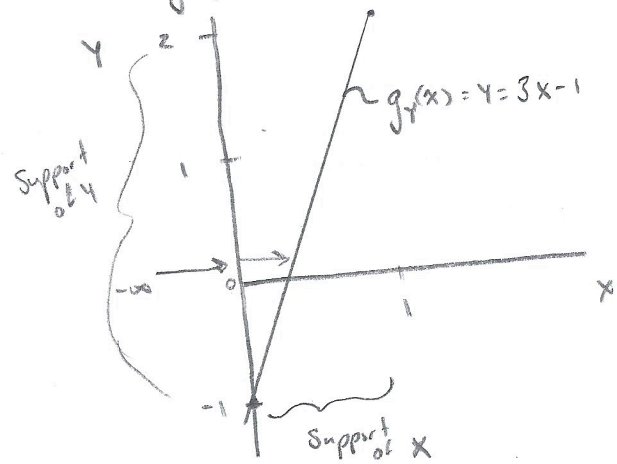
Example 6.1 (CDF / Method of Distributions)

$g_y(x) = Y = 3X - 1$ → inverse function $g_x^{-1}(y) = X = \frac{y+1}{3}$

$$f_x(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Step 1) Support for Y? $\underset{-1}{\overset{2}{Y}} = 3\underset{0}{\overset{1}{X}} - 1$

Step 2)



Step 3)

$$F_Y(y) = P(Y \le y)$$
$$= P(g_y(x) \le y) = P(3X - 1 \le y)$$
$$= P(3X \le y + 1)$$
$$= P\left(X \le \frac{y+1}{3}\right) \quad g_x^{-1}(y)$$

Step 4)

So $F_x(x) = P(X \le x) = \int_{-\infty}^{x} f_x(x)\,dx$

So $F_Y(y) = P\left(X \le \frac{y+1}{3}\right) = \int_{-\infty}^{x = \frac{y+1}{3}} f_x(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{\frac{y+1}{3}} 2x\,dx$

$= 0 + \frac{(y+1)^2}{9}$ So $F_Y(y) = \begin{cases} 0, & -\infty < y < -1 \\ \frac{(y+1)^2}{9}, & -1 \le y < 2 \\ 1, & 2 \le y < \infty \end{cases}$

Step 5)

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$
$$= \frac{d}{dy}\left[\frac{(y+1)^2}{9}\right] = \frac{2(1+y)}{9}$$

So $f_Y(y) = \begin{cases} \frac{2(1+y)}{9}, & -1 \le y < 2 \\ 0, & \text{otherwise} \end{cases}$