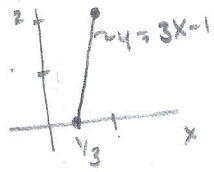
Example 6.6 (Δ of variable / Method of transformations)

$$f_x(x)=\begin{cases}2x & ; 0<x<1\\ 0 & ; \text{otherwise}\end{cases}$$

$y = 3x - 1 = g_y(x)$ (increasing function)

$\Rightarrow g_x^{-1}(y) = x = \frac{y+1}{3}$



Step 1) Support for y?

$$3\overset{\uparrow}{x}\overset{0}{\downarrow} - 1 = \overset{2}{\uparrow}\,y\,\overset{-1}{\downarrow} \rightarrow \quad -1 < y < 2$$

Step 2)

$$f_y(y) = f_x(g_x^{-1}(y))\left|\frac{d}{dy}g_x^{-1}(y)\right| = \overbrace{2\left[\overbrace{\tfrac{y+1}{3}}^{x}\right]}^{f_x}\left|\frac{d}{dy}\left(\tfrac{y+1}{3}\right)\right|$$

$$= \frac{2y+2}{3}\left|\tfrac{1}{3}+0\right| = \frac{2y+2}{3}\left(\tfrac{1}{3}\right) = \begin{cases}\frac{2}{9}(y+1) & ; -1<y<2\\ 0 & ; \text{otherwise}\end{cases}$$

Example 6.7 (Δ of variable / Method of transformations)

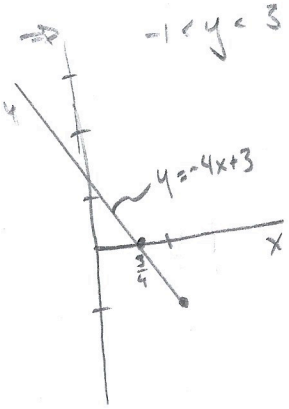
$$f_x(x)=\begin{cases}2x & ; 0<x<1\\ 0 & ; \text{otherwise}\end{cases}$$

$y = -4x + 3 = g_y(x)$

$\Rightarrow g_x^{-1}(y) = \frac{y-3}{-4} = x = \frac{3-y}{4}$

Step 1) Support of $y = -4\overset{1}{\underset{0}{\downarrow}}x + 3$ (3 ↑ y ↓ −1)

$\Rightarrow -1 < y < 3$



(decreasing function)

Step 2)

$$f_y(y) = f_x(g_x^{-1}(y))\left|\frac{d}{dy}g_y^{-1}(x)\right|$$

$$= 2\left[\frac{3-y}{4}\right]\left|\frac{d}{dy}\left(\tfrac{3}{4}-\tfrac{y}{4}\right)\right| = \frac{6-2y}{4}\left|0-\tfrac{1}{4}\right| = \frac{6}{4}\cdot\frac{y}{2}\left(\tfrac{1}{4}\right) = \frac{6}{16}-\frac{y}{8}$$

$$f_y(y) = \begin{cases}\frac{3}{8}-\frac{y}{8} & ; -1<y<3\\ 0 & ; \text{otherwise}\end{cases}$$