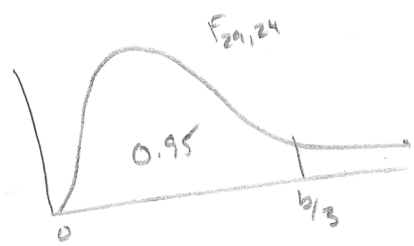
Example 4: S_1^2 and S_2^2 are sample variances observed from two independent samples of sizes $n_1 = 30$ and $n_2 = 25$, taken from two different populations, each being normally distributed. That is, $X_1 \sim Norm(\mu_1, \sigma_1^2)$, and $X_2 \sim Norm(\mu_2, \sigma_2^2)$. If $\sigma_1^2 = 3\sigma_2^2$, find the value b such that

$$P\left(\frac{S_1^2}{S_2^2} < b\right) = 0.95$$

$$P\left(\frac{S_1^2}{S_2^2} < b\right) = 0.95 \iff P\left(\frac{S_1^2\sigma_2^2}{S_2^2\sigma_1^2} < \frac{b\,\sigma_2^2}{\sigma_1^2}\right) = 0.95$$

$$\iff P\left(F_{n_1-1,n_2-1} < \frac{b\,\sigma_2^2}{3\sigma_2^2}\right) = 0.95$$

$$\iff P\left(F_{29,24} < \frac{b}{3}\right) = 0.95$$



qf(0.95, df1 = 29, df2 = 24)
$\approx 1.945259 \approx b/3$

So $b = 3\,(qf(0.95, df1=29, df2=24)) \approx 5.835778$

Why would we be interested in Ratio of SD?

well we know if $\frac{S_1^2}{S_2^2} = 1 \Rightarrow S_1^2 = S_2^2$ we Assume

S.D's of pop1 & pop2 are the same!

Here the chance that the ratio of sample SD's is $\leq 5.835\ldots$ is "High"... This leads one to believe SDs are Not that similar.

And They aren't remember $3\sigma_2^2 = \sigma_1^2$!