Example 5: Consider a random sample of values $n \ldots$ taken from a population that follows a normal distribution with a mean of μ and a known variance 1. Consider the first observation X_1 as an estimator for μ.

a) Show that X_1 is an unbiased estimator for μ.

$E[X_i] = \mu$ So $E[X_1] = \mu \leftarrow$ unbiased. $\quad X_i \sim Norm(\mu, 1)$

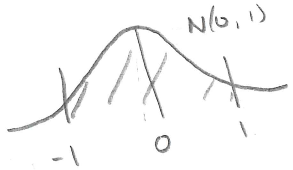
$V[X_1] = 1$

$\Rightarrow \lim_{n \to \infty} V[X_1] = 1$

$\Rightarrow$ Not Consistent ... Since $V[\hat{\theta}_n] = 0$ when $\hat{\theta}_n$ is unbiased!

b) Find $P(|X_1 - \mu| \leq 1)$.

$X_1 \sim Norm(\mu, 1)$. $\Rightarrow X_1 - \mu \sim Norm(0, 1)$



$\Rightarrow P(|X_1 - \mu| \leq 1) = P(|Z| \leq 1)$

$= P[-1 \leq Z \leq 1] = pnorm(1) - pnorm(-1)$

$\cong 0.6826895$ which is Not 1 like it should be.

X_1 is Not a Consistent Estimator for μ!

c) Look at the basic definition of consistency given, based on the result of part (b), is X_1 a consistent estimator for μ?

$\lim_{n \to \infty} P(|X_1 - \mu| \leq \varepsilon) \cong 0.6826895 \neq 1$

$\Rightarrow$ Not Consistent