Example 2: A political candidate is pondering running for public office. This candidate will only do so if there is some indication that he will not receive less than 50% of the vote. If at least 50% of votes ⇒ will run

$p_0 = 0.5$

A random sample of 15 voters is taken. **If three or less of the voters say they will vote for this candidate, then he will not run for public office.** [Rejection region ; level of tolerance ; "margin of error"]

(a) State the most appropriate null and alternative hypothesis.

let X be # out of 15 votes for Him $X \sim Bin(n=15, p_0=0.5)$

$H_0: X \geq 7.5$
$H_a: X < 7.5$

OR

let $p = \frac{X}{15}$, be prop. of votes for Him.

$H_0: p \geq 0.5$
$H_a: p < 0.5$

(b) Defining X as the number of voters, out of 15, that would vote for this particular candidate, find α, the probability of committing a Type I error.
[Define X as the number of voters, out of 15 randomly chosen that would vote for this particular candidate, the rejection region is: $X : X \leq 3$.]

$\alpha = P[\text{Type I Error}] = P[R\,H_0 \mid H_0 \text{ true}]$

$= P[X \leq 3 \mid p_0 = 0.5]$

X ⇒ # of votes for him out of 15 ...
$H_a: p < 0.5$ But level of tolerance because we recognise we are dealing with sample data.
half of 15 is 7.5 So technically 7 or fewer votes out of 15 would be a "fail!" But when considered sample data we are going to say we aren't sure it is significantly less than half unless $X \leq 3$!

$X \sim Bin(n=15, p_0=0.5)$ * if H_0 true ⇒ $p = p_0 = 0.5$

$\alpha = P[X \leq 3 \mid p = 0.5]$
$= \binom{15}{0} 0.5^0 (1-0.5)^{15-0} \cdots \binom{15}{3} 0.5^3 (1-0.5)^{15-3}$
$=$ pbinom(3, size=15, prob=0.5) $\approx$ $\boxed{0.01757813 = \alpha}$

we will have about 1.75% chance of saying p is less than 0.5 when it is really $= 0.5$ [if using a sample size of $n=15$]

* Notice: α has Nothing to do with actual sample data points it depends on level of tolerance [rejection region ; risk] and samples size! But Not sample points themselves!

*Note must know what truth is to calc. Power!

$p = 0.4$ Meaning $p_0 = 0.5$ is false!

(c) Unbeknownst to the interested candidate, he is to receive 40% of the vote. What is the probability of concluding from the sample of 15 that he will receive at least 50% of the vote? Is this a good thing? Re-write the meaning of this in context but in your own words?

Concluding $p \geq 0.5$ implies we FRHo. $\rightarrow X > 3$ or $X \geq 4$

Could be written this way → $H_0: p \geq 0.5$

$H_a: p < 0.5$

$\beta = P[\underbrace{\text{FRHo} \mid \text{Ho false}}_{\text{failing to reject things that are false is a "bad" thing Error}}] = P[X \geq 4 \mid p = 0.4] = P[\text{Type II Error}]$

$= 1 - P[X \leq 3 \mid p = 0.4] = 1 - \text{pbinom}(3, \text{size} = 15, \text{prob} = 0.4)$

$\cong 0.9094981 = P[\text{Type II error}]$

We have a ~91% chance of making a type II error if true $p = 0.4$.
Meaning if The true prop. of votes (in total) That would vote for Him is actually 0.4
we have ~91% chance of thinking The proportion is Not sig. less than 0.5!

This may be that high 2 Reasons 1 → 0.4 isn't sig less 0.5 2 → sample size low.

(d) Returning to parts (b) and (c): consider a new suggested decision rule which states the null hypothesis in (a) can be rejected if $X \leq 5$. Find the probability of committing a Type II error with the rejection region $\{X : X \leq 5\}$. How does this impact the possible errors? Which error would you prefer?

$\alpha = P[\text{RHo} \mid \text{Ho true}] = P[X \leq 5 \mid p = 0.5] \cong \text{pbinom}(5, \text{size} = 15, \text{prob} = 0.5)$

$\cong 0.1508789$

$\beta = P[\text{FRHo} \mid \text{Ho false}] = P[X \geq 6 \mid p = 0.4] = 1 - \text{pbinom}(5, 15, 0.4)$

$\cong 0.7827223$

when $\alpha \uparrow$, $\beta \downarrow$

* Don't try to Assess which Error is worse by looking at The Probabilities!
That makes No Sense! let's interpret what The Errors mean!

$H_0: p \geq 0.5$
$H_a: p < 0.5$

Type I: RHo when Ho true!
Believes this support is sig. less than 50% when it is actually Not! ⇒ Doesn't run but would have won ...

Type II: FRHo when Ho false
Believes support is Not Sig. less Than 50% when it actually is!
⇒ Run's but doesn't win ...

Assuming this person has a current job that is satisfactory
I would say type II is more serious since they may Spend money or quit current job to chase dream that doesn't Happen ...

Suggestion: increase α so that β becomes lower (more Acceptable) also increase sample size from 15 to something more telling!

test statistic

(e) A random sample of 15 was taken, of which 8 voters said they would voter for this particular candidate. What decision can be made about the null hypothesis in (a)? Identify the Test Statistic, give the p-value and interpret it.

$H_0: p \geq 0.5$

$H_a: p < 0.5$

recall rejection region in (a) if $X \leq 3 \Rightarrow$ RHo

we are clearly on FRHo "side"

RHo ← | → FRHo

X: 0 ... 3 | 4 ... 8 ... 15

p̂: 0 ... 0.2 | 0.26̄ ... 0.53̄ ... 1

we would FRHo ⇒ Based on this sample level of sig of $\alpha \cong 0.0175$..

Believe our support is Not sig. less than 0.5!

$X \sim Bin(n=15, p=0.5)$

test stat; Ho true

p-value $= P[X < 8 \mid p_0 = p = 0.5] = \text{pbinom}(7, 15, 0.5) = 0.5$

p-value $> \alpha \iff 0.5 > 0.0175 \Rightarrow$ FRHo.

if prop of voters for can. is 0.5 The prob. of collecting another sample, n=15, that would produce more evidence against claim ($p=0.5$) is 0.5 or 50%

Since this is high, it implies our sample is close to what we expected since large prob of seeing worse!

(f) Does your p-value change when considering rejection region proposed in part (b) vs. part (d)?

Calculating. $P[X < 8 \mid p_0 = p = 0.5] = \text{pbinom}(7, 15, 0.5)$ has Nothing to do with $\alpha \iff$ or Rejection region. P-value will be the same No matter what α is!

So for part b & d p-value = 0.5

a. What if $H_A: \theta \neq 0.5$ instead of $H_A: \theta < 0.5$

if $H_a: p \neq 0.5 \Rightarrow$ two tailed! means we must have two rejection regions

lets make them symmetric!

0 1 2 3 | 4 ... 11 | 12 13 14 15

Rejection Region; Rejection Region

$\frac{\alpha}{2} \cong 0.0175 \Rightarrow$ critical values $\Rightarrow X \leq 3$ & $X \geq 12$

$\alpha \cong (0.0175)2 \cong 0.03515625$

if test stat $X = 12 \Rightarrow \hat{p} = \frac{12}{15} = 0.8$ ← over $0.5 = p_0$

$2 * P[X \geq 12 \mid p = 0.5] = 2 * (1 - \text{pbinom}(11, 15, 0.5)) \cong 0.03515625$

what to do when α = p-value? best to suggest increased sample size!, but here RHo

(g) What type of error could have been made?
(i) use rejection region proposed in part (b)
(ii) part (d)

if Test stat = 4

b) RHo when $X \leq 3 \Rightarrow$ FRHo

d) RHo when $X \leq 5 \Rightarrow$ RHo

Nothing from sample changed But two different conclusions why? default to believe Ho and different level of α...

we choose alpha... so we can always determin which conclusion

p-value is the level of sig (α) when the Ho would be rejected

less subjective to report p-value rather then FRHo or RHo...