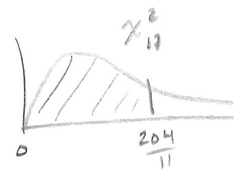
Example 6: A random sample of $n = 18$ is taken from a population which is Normally distributed with a mean of 105 and a variance of 5.5. What is the probability…

(a) the variance of the sample is less than 6?

$$P(s^2 < 6) = P((n-1)s^2 < 17(6)) = P\left(\frac{(n-1)s^2}{\sigma^2} < \frac{17(6)}{5.5}\right) = P\left(\chi^2_{df=17} < \frac{204}{11}\right)$$

$$= \text{pchisq}(204/11, df=11) \doteq 0.6447626$$

0.645 prob that s^2 will be < 6.

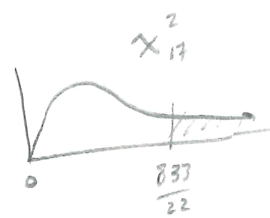


(b) the standard deviation of the sample is at least 3.5?

$$P(s > 3.5) = P(s^2 > 3.5^2) = P\left(\frac{(n-1)s^2}{\sigma^2} > \frac{3.5^2(17)}{5.5}\right)$$

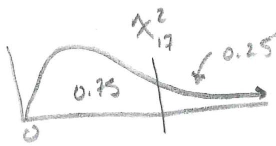
$$P\left(\chi^2_{df=17} > 833/22\right) \doteq 1 - \text{pchisq}(833/22, df=17)$$

$$\doteq 0.002566039$$



(c) 75% of the time, the variance of such a sample will be less than or equal to what value?

$P(s^2 \le a) = 0.75$



$\uparrow$ qchisq(0.75, df=17)
$\cong 20.48868$

$$P\left(\frac{(n-1)s^2}{\sigma^2} \le \frac{a(17)}{5.5} \right) = 0.75$$

So $\dfrac{a(17)}{5.5} = 20.48868$

$$\Rightarrow a \cong \frac{20.48868(5.5)}{17} \cong 6.628689$$

$P(s^2 \le 6.628689) = 0.75$

$$\int_0^{\frac{(6.6)(17)}{5.5}} \frac{z^{\frac{17}{2}-1} e^{-z/2}}{\Gamma(\frac{17}{2})\, 2^{17/2}} \, dz \cong 0.75$$