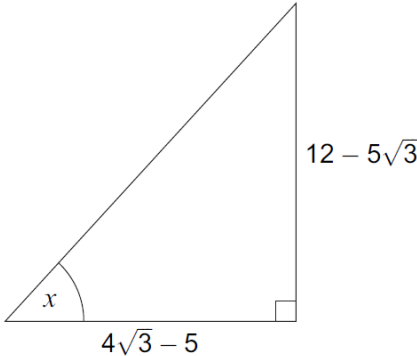
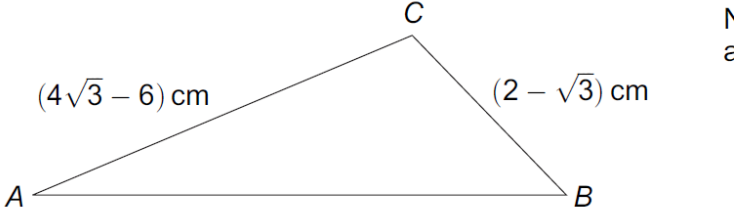
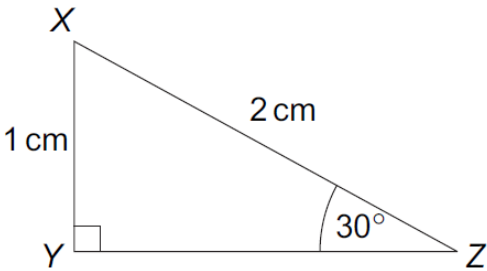


Further Maths GCSE Topics Tests

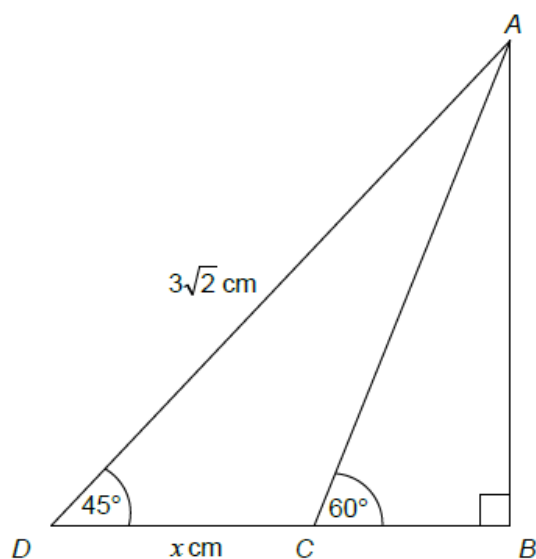
x means non Calculator Paper and ✓ means Calculator Paper.

1. Trigonometry

1	<p>Show that angle $x = 60^\circ$</p>  <p>You must show your working.</p>
2	<p>Triangle ABC has an obtuse angle at C.</p>  <p>Given that $\sin A = \frac{1}{4}$, use triangle ABC to show that angle $B = 60^\circ$</p>
3	<p>XYZ is a right-angled triangle.</p>  <p>Use triangle XYZ to show that $\sin 60^\circ = \frac{\sqrt{3}}{2}$</p>

4

In the diagram, DCB is a straight line.



Not drawn accurately

Work out the length of DC , marked x on the diagram.

Write your answer in the form $a - \sqrt{b}$

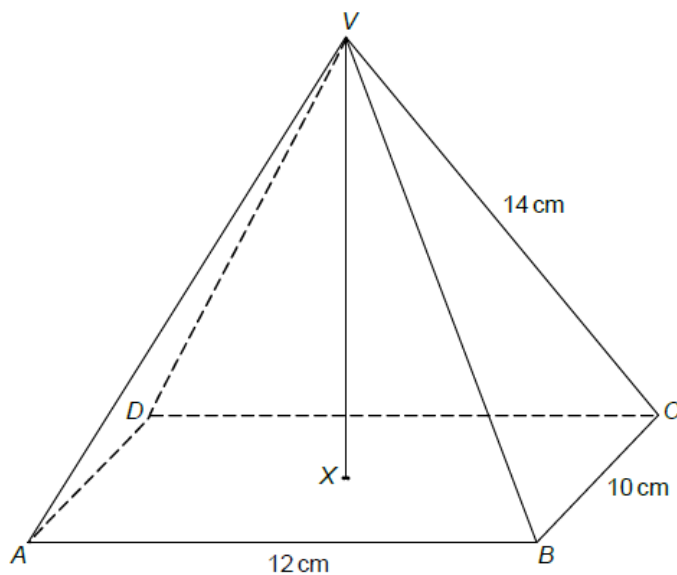
✗

5

$VABCD$ is a rectangular based pyramid.

$AB = 12$ cm, $BC = 10$ cm and $VC = 14$ cm

The base $ABCD$ is horizontal and the vertex V is directly above X , the centre of the base.



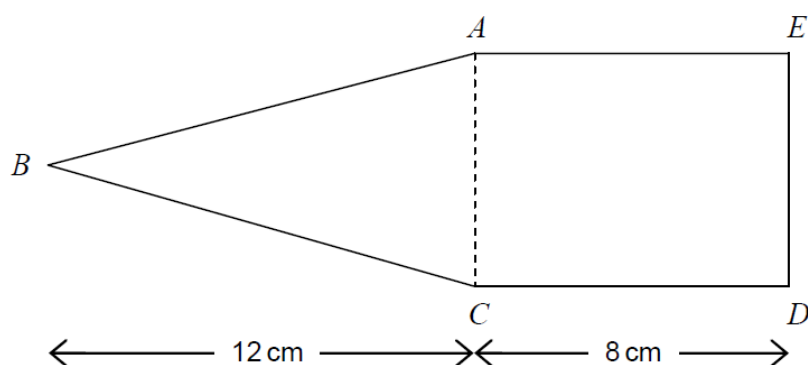
Work out the height of the pyramid, VX .

Work out the angle between VC and the plane $ABCD$

✓

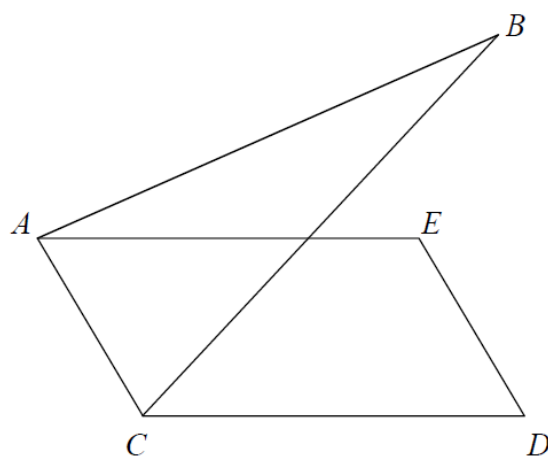
6

$ABCDE$ is a piece of card in the shape of a rectangle and an isosceles triangle.



Not drawn accurately

$ABCDE$ is folded along AC so that B is vertically above the midpoint of DE .

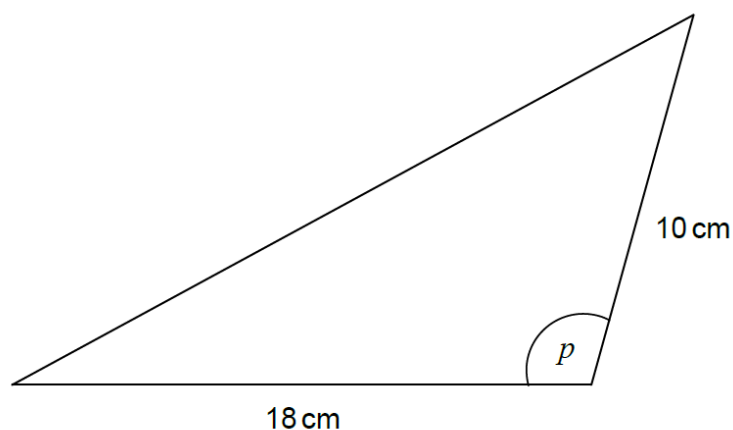


Work out the angle between the planes ABC and $ACDE$.



7

The area of this triangle is 27 cm^2 .



Work out the size of obtuse angle p .



8

The diagram shows a vertical mast, AB , 12 metres high.

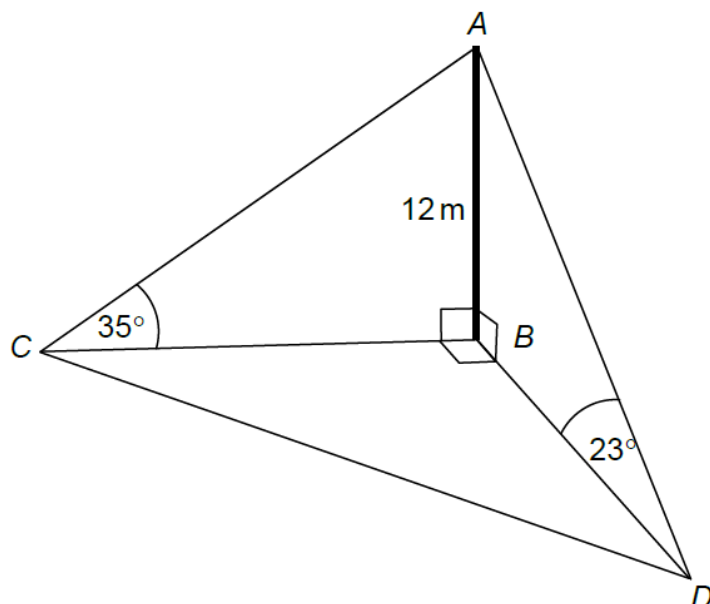
Points B , C and D are on a horizontal plane.

Point C is due West of B .

The angle of elevation of A from C is 35° .

Point D is due South of B .

The angle of elevation of A from D is 23° .



Not drawn
accurately

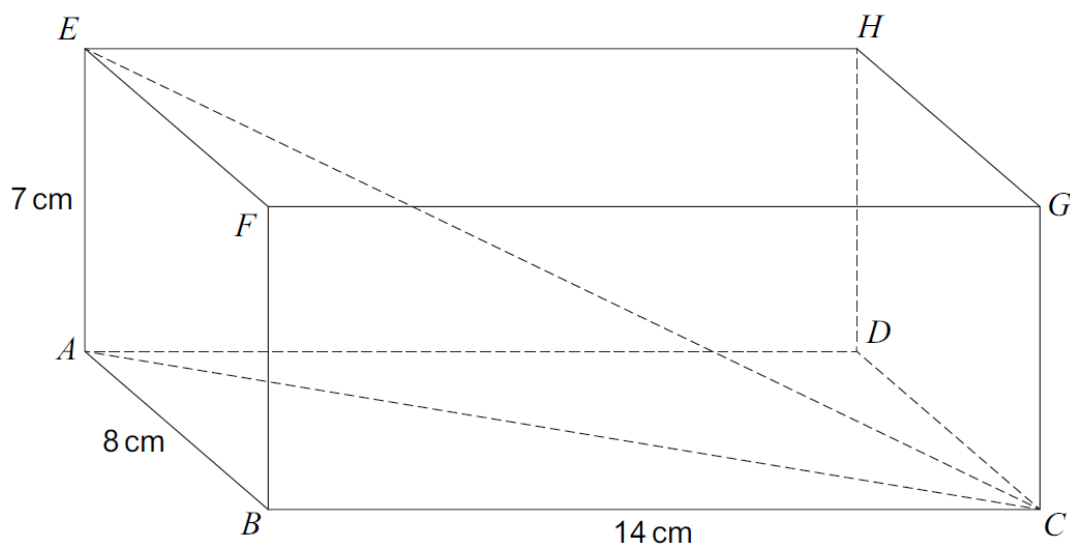
Calculate the distance CD .

Calculate the Bearing of D from C to the nearest degree



9

$ABCDEFGH$ is a cuboid.

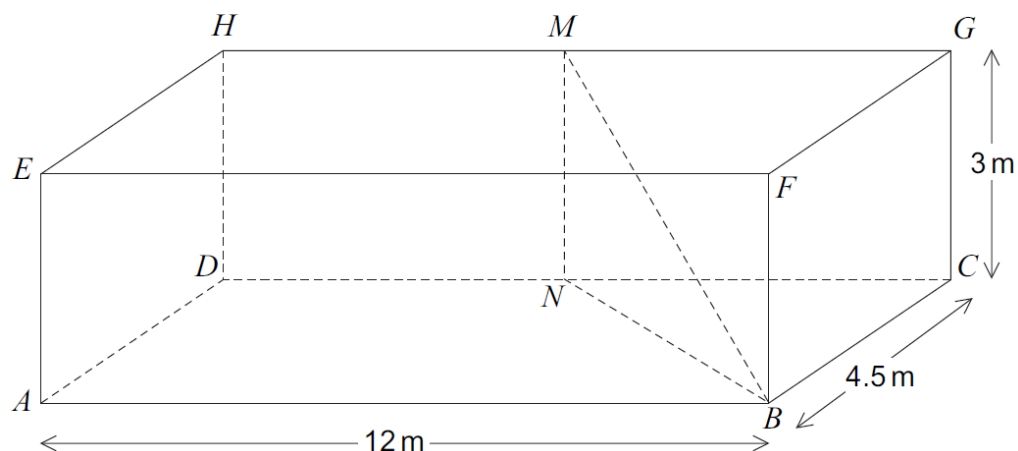


Work out the angle between EC and $ABCD$.



10

$ABCDEFGH$ is a cuboid.
 M is the midpoint of HG .
 N is the midpoint of DC .



Show that $BN = 7.5$ m

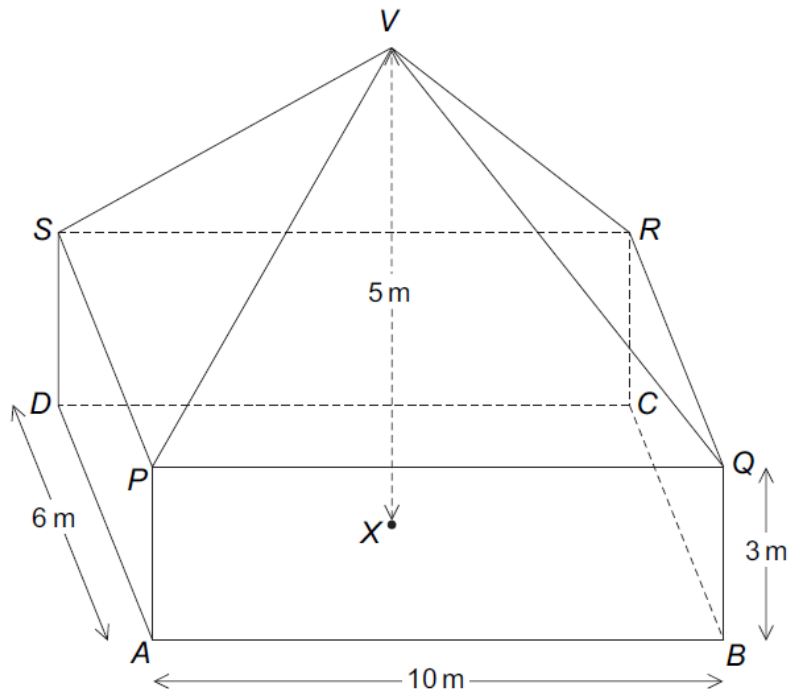
Work out the angle between MB and the plane ABCD

Work out the obtuse angle between the planes MNB and CDHG



11

The diagram shows a cuboid $ABCDPQRS$ and a pyramid $PQRSV$.
 V is directly above the centre, X , of $ABCD$.



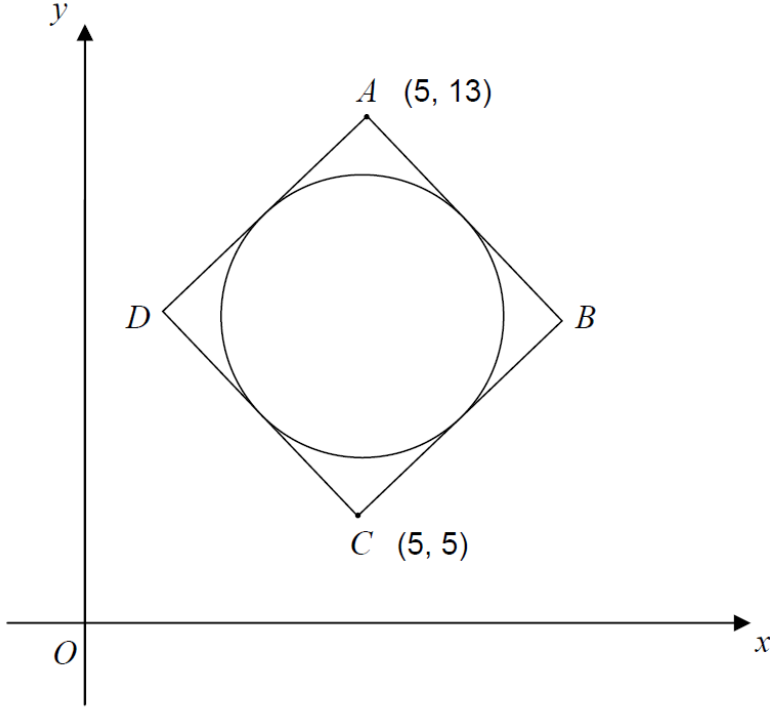
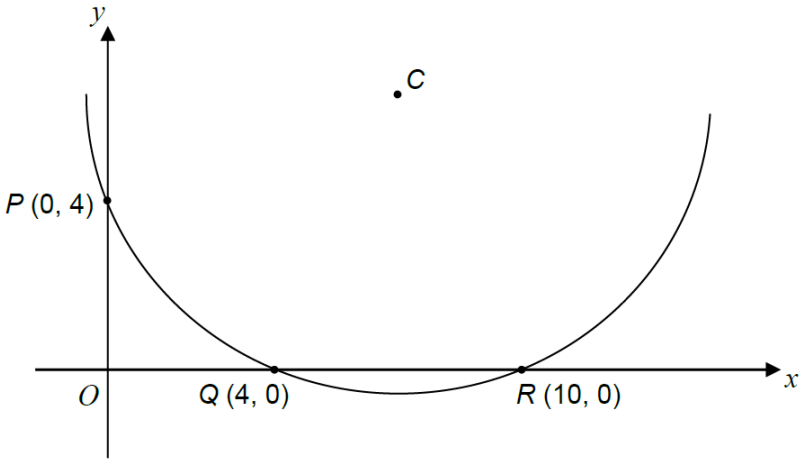
The total height, VX , is 5 metres.

Work out the angle between VA and the plane ABCD

Work out the angles between the planes VQR and PQRS

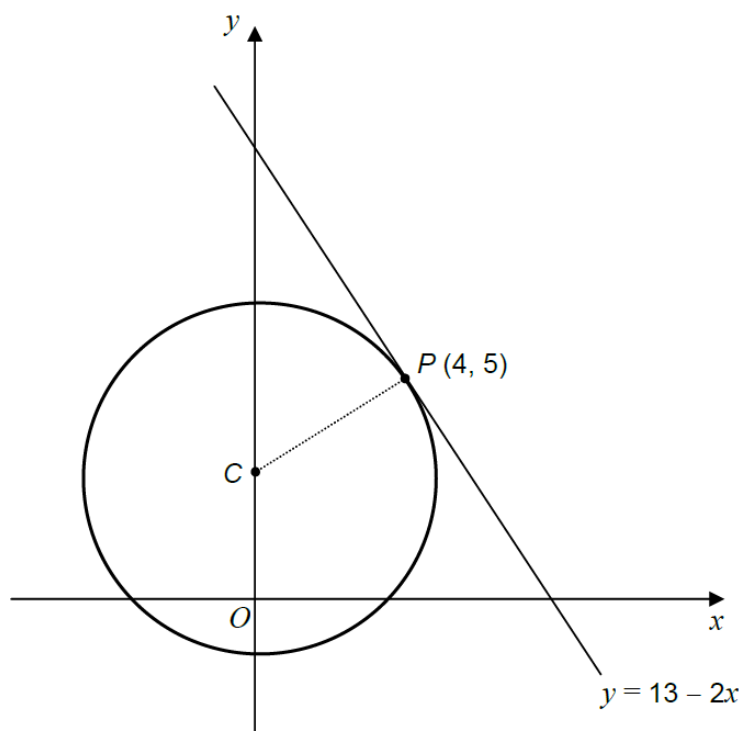


2. Circles

1	<p>Circle A has equation $x^2 + y^2 = 16$</p> <p>Circle B has equation $(x + 6)^2 + (y - 8)^2 = 25$</p> <p>Work out the distance between the centres of the circles.</p> <div style="text-align: right;">✓✗</div>
2	 <p>Work out the equation of the circle.</p> <div style="text-align: right;">✓✗</div>
3	 <p>Explain why the centre of the circle lies on the line $x = 7$.</p> <p>Work out the equation of the circle.</p> <div style="text-align: right;">✓✗</div>

4

The sketch shows point P on a circle, centre C .
The equation of the tangent at P is $y = 13 - 2x$



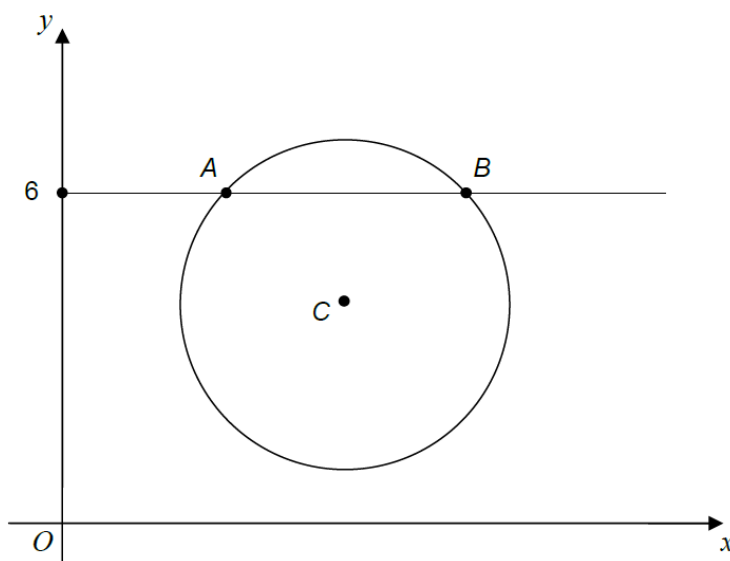
Work out the gradient of PC .
Work out the equation of the circle.

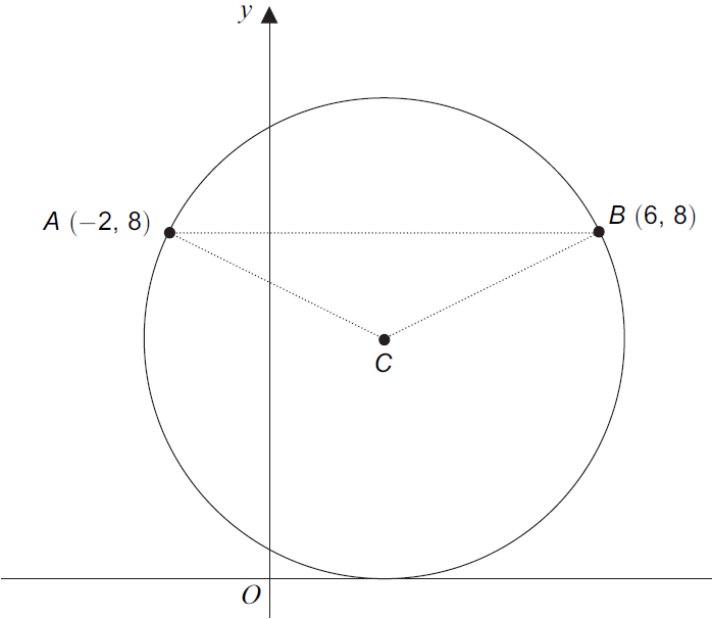
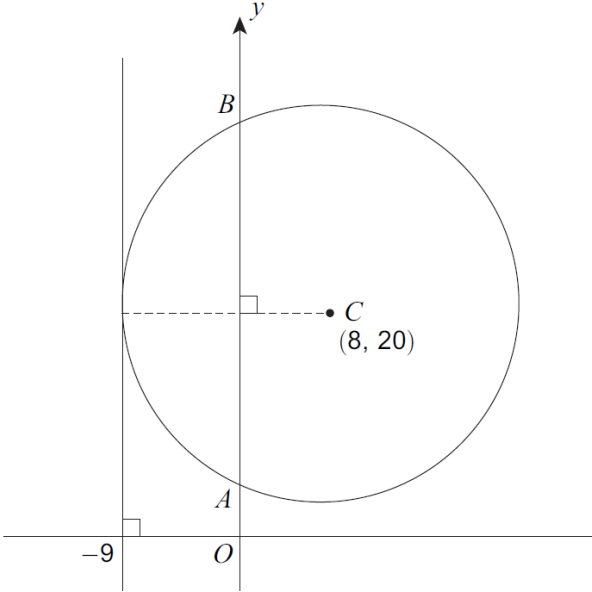


5

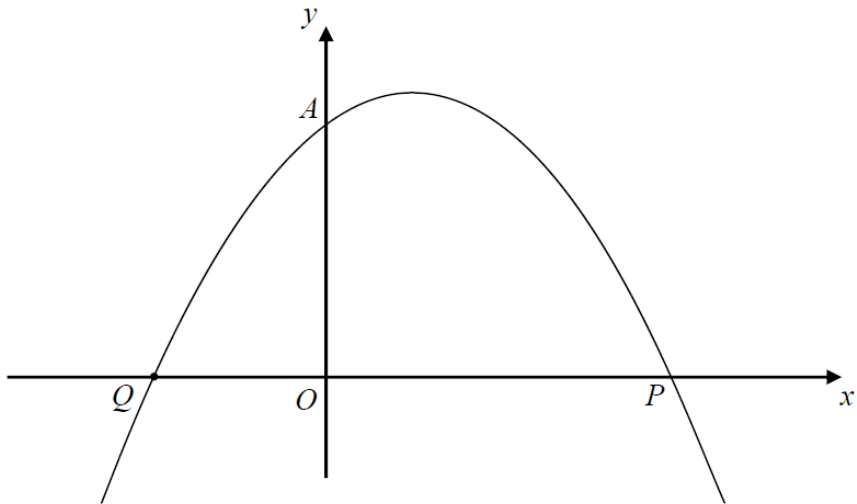
The diagram shows a sketch of the circle $(x - 7)^2 + (y - 4)^2 = 9$ with centre C .
The line $y = 6$ intersects the circle at A and B .

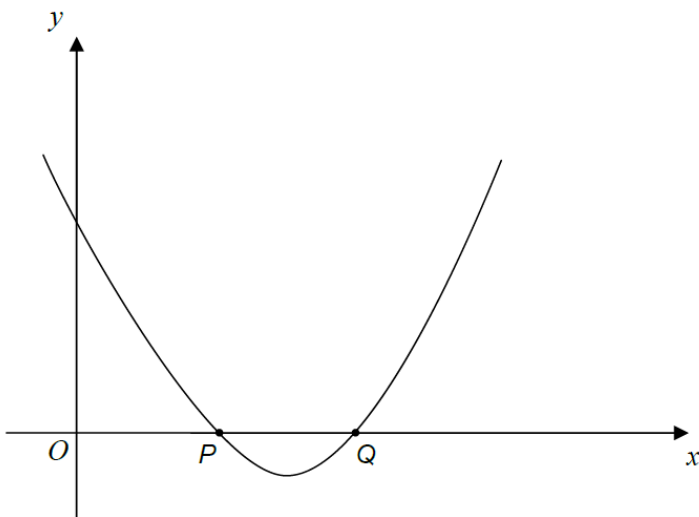
Show that $AB = 2\sqrt{5}$



6	 <p>Work out the equation of the circle.</p> <p style="text-align: right;">✓ x</p>
7	<p>The line $x = -9$ is a tangent to the circle, centre $C(8, 20)$</p>  <p>Show that the radius of the circle is 17.</p> <p>The circle intersects the y-axis at A and B.</p> <p>Show that the length AB is 30.</p> <p style="text-align: right;">✓</p>
8	<p>$x^2 - 2x + y^2 - 6y = 0$ is the equation of a circle.</p> <p>By writing the equation in the form $(x - a)^2 + (y - b)^2 = r^2$ work out the centre and radius of the circle.</p> <p style="text-align: right;">✓ x</p>

3. Differentiation Questions

1	Work out the coordinates of the stationary point for the curve $y = x^2 + 3x + 4$	✓✗
2	<p>A sketch of the curve $y = (x + 1)(2 - x)$ is shown.</p> <p>$A(0, 2)$, $P(2, 0)$ and Q are points on the curve.</p>  <p>Show that the normal to the curve at A intersects the curve again at P.</p>	✓✗
3	<p>The equation of the curve is $y = px^3 - 3x^2 + 8x + r$ where p and r are constants.</p> <p>Use the fact that there is a maximum point at $(2, 10)$ to work out the values p and r.</p>	✓✗
4	$y = \frac{3x(2x^4 - 5x)}{x^2}$ Work out $\frac{dy}{dx}$	✓✗
5	Work out the gradient of the curve $y = (3x - 4)(x + 2)$ at the point $(2, 8)$	✓✗
6	<p>$y = 10 - 8x - x^3$ for all values of x.</p> <p>Show that y is a decreasing function for all values x.</p>	✓✗
7	<p>A curve has gradient function $\frac{dy}{dx} = 2x^2 - 7$</p> <p>Work out the gradient of the curve when $x = -3$</p> <p>Work out the values of x for which the rate of change of y with respect to x is 1.</p>	✓✗
8	$y = x^{\frac{1}{2}}(x^{\frac{7}{2}} - x^{\frac{1}{2}})$ Work out $\frac{dy}{dx}$.	✓✗

9	<p>A curve has equation $y = 4x^3 + 6x^2 + 3x + 5$</p> <p>Work out the coordinates of any stationary points on this curve and determine their nature.</p>	✓ ✗
10	<p>The graph shows a sketch of $y = (x - 2)(x - 3)$</p> <p>The curve intersects the x-axis at P and Q.</p>  <p>Show that the tangents at P and Q are perpendicular.</p>	✓ ✗
11	<p>A curve has equation $y = x^2(x - 2)$</p> <p>Work out the gradient of the curve at the point $(3, 9)$.</p>	✓ ✗
12	<p>$y = (5x - 3)^2$</p> <p>Work out $\frac{dy}{dx}$</p> <p>Give your answer in the form $a(bx - c)$ where a, b and c are integers > 1</p>	✓ ✗
13	<p>The curve $y = x^3 + bx + c$ has a stationary point at $(-2, 20)$.</p> <p>Work out the values of b and c.</p>	✓ ✗
14	<p>For what values of x is $y = 150x - 2x^3$ an increasing function?</p>	✓ ✗

15	<p>The curve $y = f(x)$ is such that $\frac{dy}{dx} = -x(x - 2)^2$</p> <p>The stationary points of the curve are at $\left(0, \frac{4}{3}\right)$ and $(2, 0)$.</p> <p>Determine the nature of each stationary point. You must show your working.</p>	✓ ✗
16	<p>$y = (x^3 - 1)^2 + (\sqrt{x})^8$ Work out $\frac{dy}{dx}$.</p>	✓ ✗
17	<p>$y = 2x^3 + ax$, where a is a constant.</p> <p>The value of $\frac{dy}{dx}$ when $x = 2$ is twice the value of $\frac{dy}{dx}$ when $x = -1$</p> <p>Work out the value of a.</p>	✓ ✗

4. Factor Theorem Questions

1	<p>Show that $(x - 2)$ is a factor of $x^3 + 8x^2 + x - 42$</p>	✓ ✗
2	<p>Prove that, for all values of x, $2x^2 - 8x + 9 > 0$</p>	✓ ✗
3	<p>$x^3 + ax^2 + bx + 150$ factorises to $(x + c)^2(x + d)$ a, b, c and d are positive integers and $c \neq 1$</p> <p>Work out the values of a, b, c and d.</p>	✓ ✗
4	<p>$(x + 3)$ is a factor of $x^3 + 6x^2 + ax - 12$</p> <p>Show that the value of a is 5</p> <p>Hence, factorise fully $x^3 + 6x^2 + 5x - 12$</p>	✓ ✗
5	<p>$(x - 5)$ is a factor of $x^3 - 6x^2 + ax - 20$</p> <p>Work out the value of a.</p>	✓ ✗
6	<p>$(x - a)$ is a factor of $2x^3 - 7ax + 3a$</p> <p>Work out the largest possible value of a.</p>	✓ ✗

7	Use the factor theorem to show that $(x - 1)$ and $(x - 4)$ are factors of $x^3 - 21x + 20$	✓ ✗
8	$f(x) = x^3 + ax^2 + bx + 24$ for all values of x . Two of the factors of $f(x)$ are $(x - 2)$ and $(x + 3)$. Work out the values of a and b .	✓ ✗
9	Show that $(x - 1)$ and $(x - 4)$ are also factors of $x^3 - 10x^2 + 29x - 20$	✓ ✗

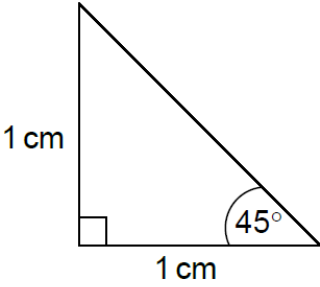
5. Sequences Questions

1	The n th term of a different sequence is $n^2 - 6n + 14$ By completing the square, or otherwise, show that every term is positive.	✓ ✗
2	The n th term of a sequence is $n^2 + n$ Two consecutive terms in the sequence have a difference of 32. Work out the two terms.	✓ ✗
3	The first five terms of a quadratic sequence are 4 10 18 28 40 Work out an expression for the n th term.	✓ ✗
4	The first four terms of a quadratic sequence are $a + 5b$ $2a + 8b$ $3a + 12b$ $4a + 17b$ The n th term of the sequence is $n^2 - 2n + 6$ Work out the values of a and b .	✓ ✗
5	The n th term of sequence X is $an + b$ The n th term of sequence Y is $bn + a$ Show that the sequences have the same first term. The 2nd term of sequence X is equal to the 3rd term of sequence Y. Show that $a = 2b$	

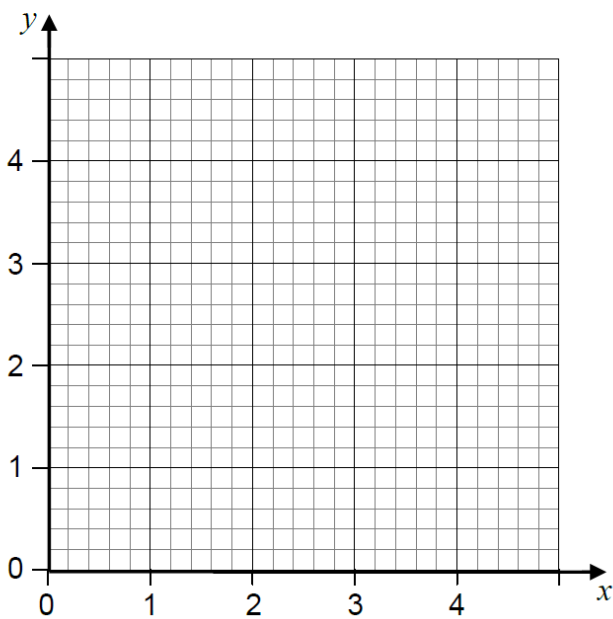
	<p>Prove that</p> $\frac{n \text{th term of sequence } X}{n \text{th term of sequence } Y} = \frac{2n + 1}{n + 2}$ <p style="text-align: right;">✓✗</p>
6	<p>Here are the first four terms of a sequence.</p> <p style="text-align: center;">$4a \qquad 9a \qquad 14a \qquad 19a$</p> <p>The nth term of the sequence is $\frac{10n - 2}{3}$</p> <p>Work out the value of a.</p> <p style="text-align: right;">✓✗</p>
7	<p>The nth term of a sequence is $\frac{2n^2 + 7}{3n^2 - 2}$</p> <p>Work out the 7th term.</p> <p>Give your answer as a fraction in its simplest form.</p> <p style="text-align: right;">✓</p>
8	<p>Show that the limiting value of $\frac{2n^2 + 7}{3n^2 - 2}$ as $n \rightarrow \infty$ is $\frac{2}{3}$</p> <p style="text-align: right;">✓✗</p>
9	<p>The n^{th} term of the linear sequence $2 \quad 7 \quad 12 \quad 17 \quad \dots$ is $5n - 3$</p> <p>A new sequence is formed by squaring each term of the linear sequence and adding 1.</p> <p>Prove algebraically that all the terms in the new sequence are multiples of 5.</p> <p style="text-align: right;">✓✗</p>

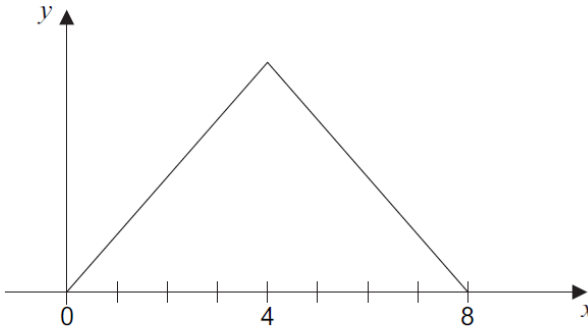
6. Trigonometric Equations and Identities

1	<p>Prove that $\tan^2 x - 1 \equiv \frac{1 - 2\cos^2 x}{\cos^2 x}$</p> <p style="text-align: right;">✓✗</p>
2	<p>Angle θ is obtuse and $\sin \theta = \frac{\sqrt{5}}{3}$</p> <p>Work out the values of $\cos \theta$.</p> <p style="text-align: right;">✗</p>
3	<p>Expand and simplify $(2s + 1)(s - 1)$</p> <p>Hence, or otherwise, solve $2\sin^2 \theta - \sin \theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$</p> <p style="text-align: right;">✓</p>

4	<p>Here is a right-angled triangle.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center;">  <p>1 cm</p> <p>1 cm</p> <p>45°</p> </div> <div style="margin-left: 20px;"> <p>Not drawn accurately</p> </div> </div> <p>Show clearly that $\sin 45^\circ = \frac{1}{\sqrt{2}}$</p>	✗
5	Solve $5\sin^2 x - 2\sin x = 0$ for $0^\circ \leq x \leq 360^\circ$	✓
6	<p>$f(x) = \sin x \quad 180^\circ \leq x \leq 360^\circ$</p> <p>$g(x) = \cos x \quad 0^\circ \leq x \leq \theta$</p> <p>Calculate the value of $f(210^\circ)$.</p> <p>Complete this inequality for the range of $f(x)$.</p> <p style="text-align: center;">Answer $\leq f(x) \leq$</p> <p>You are given that $0 \leq g(x) \leq 1$</p> <p>Work out the value of θ.</p>	✓
7	Express $1 - \tan \theta \sin \theta \cos \theta$ in terms of $\cos \theta$.	✓
8	Solve $3\cos^2 \theta - 1 = 0$ for $0^\circ \leq \theta \leq 180^\circ$	✓
9	Prove that $\tan \theta + \frac{1}{\tan \theta} \equiv \frac{1}{\sin \theta \cos \theta}$	✓
10	Solve $\tan^2 \theta + 3 \tan \theta = 0$ for $0^\circ < \theta < 360^\circ$	✓

7. Functions Questions

1	<p>A function $f(x)$ is defined as</p> $\begin{aligned} f(x) &= 3x & 0 \leq x < 1 \\ &= 3 & 1 \leq x < 3 \\ &= 12 - 3x & 3 \leq x \leq 4 \end{aligned}$ <p>Calculate the area enclosed by the graph of $y = f(x)$ and the x-axis.</p>  <div style="text-align: right;">✓✗</div>
2	<p>$f(n) = n^2 + n$</p> <p>Show that $f(n + 1) - f(n) = 2n + 2$</p> <div style="text-align: right;">✓✗</div>
3	<p>A cubic function $f(x)$ has domain all values of x.</p> <p>The curve $y = f(x)$ has two stationary points.</p> <p>There is a minimum point at $(-2, 1)$.</p> <p>There is a maximum point at $(2, 5)$.</p> <p>Sketch the graph of $y = f(x)$ on these axes.</p> <div style="text-align: right;">✓✗</div>
4	<p>$g(x) = x^3 + 3x^2$ for all values of x.</p> <p>Show that $g(3x) = kx^2(x + 1)$ where k is an integer.</p> <div style="text-align: right;">✓✗</div>

5	<p>$y = f(x)$ is a continuous graph.</p> <p>When $0 \leq x < 3$ $\frac{dy}{dx} = 2$ and this part of the graph passes through (2, 1).</p> <p>When $3 \leq x < 5$ $\frac{dy}{dx} = 0$</p> <p>When $5 \leq x \leq 8$ $\frac{dy}{dx} = -1$</p> <p>Draw the graph of $y = f(x)$ for $0 \leq x \leq 8$</p>
6	<p>A function $f(x)$ is defined as</p> $\begin{aligned} f(x) &= x + 3 & -3 \leq x < 0 \\ &= 3 & 0 \leq x < 1 \\ &= 5 - 2x & 1 \leq x \leq 2 \end{aligned}$ <p>Draw the graph of $y = f(x)$ for $-3 \leq x \leq 2$</p>
7	<p>The graph of $y = f(x)$ is a straight line.</p> <p>The domain of $f(x)$ is $1 \leq x \leq 5$</p> <p>The range of $f(x)$ is $3 \leq f(x) \leq 11$</p> <p>Work out one possible expression for $f(x)$.</p>
8	<p>$f(x) = 3x - 5$ for all values of x.</p> <p>Solve $f(x^2) = 43$</p>
9	<p>A sketch of $y = g(x)$ for domain $0 \leq x \leq 8$ is shown.</p>  <p>The graph is symmetrical about $x = 4$</p> <p>The range of $g(x)$ is $0 \leq g(x) \leq 12$</p> <p>Work out the function $g(x)$.</p>

10	<p>A function $f(x)$ is defined as</p> $\begin{aligned} f(x) &= 4 & x < -2 \\ &= x^2 & -2 \leq x \leq 2 \\ &= 12 - 4x & x > 2 \end{aligned}$ <p>Draw the graph of $y = f(x)$ for $-4 \leq x \leq 4$</p> <p style="text-align: right;">✓✗</p>
11	<p>A cubic function $f(x)$ has domain $-4 \leq x \leq 4$</p> <p>The curve $y = f(x)$</p> <ul style="list-style-type: none"> • has a minimum point at $(-2, 0)$ • has a maximum point at $(1, 4)$ • meets the x-axis at $(4, 0)$. <p>Sketch the graph of $y = f(x)$ on these axes. Label any points where the graph meets the x-axis.</p> <p style="text-align: right;">✓✗</p>
12	<p>$f(x) = 10 - x^2$ for all values of x.</p> <p>$g(x) = (x + 2a)(x + 3)$ for all values of x.</p> <p>Circle the correct value of $f(-4)$</p> <p style="text-align: center;">26 -6 36 16 196</p> <p>Write down the range of $f(x)$.</p> <p>$g(0) = 24$</p> <p>Show that $a = 4$</p> <p>Hence solve $f(x) = g(x)$</p> <p style="text-align: right;">✓✗</p>

8. Matrices Questions

1	<p>$M = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$</p> <p>$M$ represents a single transformation.</p> <p>Describe the transformation fully.</p> <p style="text-align: right;">✓✗</p>
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2	$\mathbf{P} = \begin{pmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{pmatrix}$ <p>Work out PQ</p> <p>Give your answer in its simplest form.</p>	✓✗
3	$\begin{pmatrix} 2 & a \\ 1 & -3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ <p>Work out all possible pairs of values of a and b.</p>	✓✗
4	$\begin{pmatrix} -7 & 4 \\ 5 & -3 \end{pmatrix} \begin{pmatrix} -3 & -4 \\ -5 & t \end{pmatrix} = \mathbf{I}$ <p>Work out the value of t.</p>	✓✗
5	<p>On the grid, draw the image of the unit square after it is transformed using the matrix</p> $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}.$	✓✗
6	<p>Work out the matrix that transforms the unit square by a 270° rotation about O.</p>	✓✗
7	<p>The matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point $(5, 4)$ onto the point $(1, 17)$.</p> <p>Work out the values of a and b.</p>	✓✗
8	<p>Matrix $\mathbf{P} = \begin{pmatrix} 2 & 3 \\ a & b \end{pmatrix}$ Matrix $\mathbf{Q} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$</p> <p>You are given that $\mathbf{PQ} = \mathbf{QP}$</p> <p>Work out the values of a and b.</p>	✓✗
9	<p>The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point $(3, 4)$ onto the point $(2, b)$.</p> <p>Work out the values of a and b.</p>	✓✗

10	$\mathbf{M} = \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$ <p>Show that $\mathbf{M}^3 = \mathbf{I}$</p>	✓✗
11	<p>Work out $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 0 & b \\ a & c \end{pmatrix}$</p> <p>Give your answer in terms of a, b and c.</p>	✓✗
12	<p>The transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps point P to point Q.</p> <p>The transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ maps point Q to point R.</p> <p>Point R is $(-4, 3)$.</p> <p>Work out the coordinates of point P.</p>	✓✗
13	Describe fully the single transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	✓✗
14	<p>$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a reflection in the y-axis.</p> <p>$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$</p> <p>Work out the matrix that represents a reflection in the y-axis followed by a reflection in the line $y = x$</p>	✓✗
15	<p>A transformation is given by the matrix \mathbf{M}, where $\mathbf{M} = \begin{pmatrix} 1 & a \\ 0 & 2 \end{pmatrix}$</p> <p>The image of the point $(b, 5)$ under \mathbf{M} is $(5, b)$.</p> <p>Work out the values of a and b.</p>	✓✗

9. Other Topics Questions

1	<p>Work out $2\frac{2}{3} - 1\frac{3}{4} \div 1\frac{1}{8}$</p> <p>Give your answer as a fraction in its simplest form.</p>	✗
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2	<p>The points $A(-1, -7)$ and $B(24, 23)$ are on a straight line ACB. $AC : CB = 2 : 3$</p> <p>Work out the coordinates of C.</p>	x
3	<p>Explain why the equation $x^2 + 3x + 4 = 0$ has no real solutions.</p>	x
4	<p>Solve the simultaneous equations</p> $\begin{aligned}x + y &= 4 \\ y^2 &= 4x + 5\end{aligned}$ <p>Do not use trial and improvement.</p>	✓
5	<p>Factorise fully $(x + y)^2 + (x + y)(2x + 5y)$</p>	✓ x
6	<p>Factorise fully $(w + 4)^3 - (w + 4)^2(w + 1)$</p>	✓ x
7	<p>Simplify $\frac{x^2 + 4x - 12}{x^2 - 25} \div \frac{x + 6}{x^2 - 5x}$</p>	✓ x
8	<p>Simplify fully $\frac{4x^2 + 19x - 5}{9x^2 - 16} \div \frac{x + 5}{3x - 4}$</p>	✓ x
9	<p>Solve the simultaneous equations</p> $\frac{x - 1}{y - 2} = 3 \quad \frac{x + 6}{y - 1} = 4$ <p>Do not use trial and improvement. You must show your working.</p>	✓ x
10	<p>Write this ratio in its simplest form</p> $\sqrt{12} : \sqrt{48} : \sqrt{300}$	x

11	$x^{\frac{3}{2}} = 8$ where $x > 0$ and $y^{-2} = \frac{25}{4}$ where $y > 0$ Work out the value of $\frac{x}{y}$.	x
12	Solve $\sqrt{(33 + \sqrt{x})} = 6$	✓ x
13	Solve $y(\sqrt{3} - 1) = 8$ Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.	x
14	Solve $x^{-\frac{2}{3}} = 7\frac{1}{9}$ Write your answer as a proper fraction.	x
15	The value of x is 50% more than the value of t . The value of y is 10% less than the value of w . $x = y$ Work out $\frac{t}{w}$ Give your answer as a decimal.	✓ x
16	A sphere has radius x centimetres. A hemisphere has radius y centimetres. The shapes have equal volumes. Work out the value of $\frac{y}{x}$. Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.	✓ x