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In [1]: import pandas as pd
import numpy as np
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.preprocessing import PolynomialFeatures

# Generate random data
def generate_data(n_samples=100, n_features=1):
    X = np.random.rand(n_samples, n_features)
    y = 3 * X**2 + 4 * X + 7 + np.random.randn(n_samples)
    return X, y

# Load training and test data sets from a csv file
def load_data(csv_file):
    data = pd.read_csv(csv_file)
    X_train = data[['x']]
    y_train = data['y']
    X_test = data[['x']]
    y_test = data['y']
    return X_train, y_train, X_test, y_test

# Create polynomial features
def create_polynomial_features(X, degree):
    poly = PolynomialFeatures(degree=degree)
    X_poly = poly.fit_transform(X)
    return X_poly

# Fit a polynomial regression model
def fit_polynomial_model(X_poly, y):
    model = LinearRegression()
    model.fit(X_poly, y)
    return model

# Evaluate the model
def evaluate_model(model, X_test, y_test):
    y_pred = model.predict(X_test)
    mse = mean_squared_error(y_test, y_pred)
    r2 = r2_score(y_test, y_pred)
    return mse, r2

# 1. Build a degree 4 polynomial model
In [2]: # Create polynomial features of degree 4
X_train, y_train, X_test, y_test = generate_data(n_samples=100, n_features=1)
X_train_poly4 = create_polynomial_features(X_train, degree=4)
X_test_poly4 = create_polynomial_features(X_test, degree=4)

# Fit a degree 4 polynomial regression model
model4 = fit_polynomial_model(X_train_poly4, y_train)

# Evaluate the model
mse4, r24 = evaluate_model(model4, X_test_poly4, y_test)

print("MSE: %.4f, R^2: %.4f" % (mse4, r24))

# 2. 5-fold cross validation
In [3]: # Perform 5-fold cross validation
mse_cv, r2_cv = cross_val_score(model4, X_train, y_train, cv=5, scoring='neg_mean_squared_error')

print("5-fold cross validation MSE: %.4f, R^2: %.4f" % (mse_cv, r2_cv))

# 3. Degree 5 polynomial model using statsmodel
In [4]: # Fit a degree 5 polynomial regression model using statsmodels
import statsmodels.api as sm
X_train, y_train, X_test, y_test = generate_data(n_samples=100, n_features=1)
X_train_poly5 = create_polynomial_features(X_train, degree=5)
X_test_poly5 = create_polynomial_features(X_test, degree=5)

# Add a constant term
X_train_poly5 = sm.add_constant(X_train_poly5)
X_test_poly5 = sm.add_constant(X_test_poly5)

# Fit the model
model5 = sm.OLS(y_train, X_train_poly5).fit()

# Predict
y_pred5 = model5.predict(X_test_poly5)

# Evaluate
mse5, r25 = evaluate_model(model5, X_test_poly5, y_test)

print("MSE: %.4f, R^2: %.4f" % (mse5, r25))

# 4. Hypothesis testing
In [5]: # Perform hypothesis testing
import statsmodels.api as sm
X_train, y_train, X_test, y_test = generate_data(n_samples=100, n_features=1)
X_train_poly5 = create_polynomial_features(X_train, degree=5)
X_test_poly5 = create_polynomial_features(X_test, degree=5)

# Add a constant term
X_train_poly5 = sm.add_constant(X_train_poly5)
X_test_poly5 = sm.add_constant(X_test_poly5)

# Fit the model
model5 = sm.OLS(y_train, X_train_poly5).fit()

# Predict
y_pred5 = model5.predict(X_test_poly5)

# Evaluate
mse5, r25 = evaluate_model(model5, X_test_poly5, y_test)

print("MSE: %.4f, R^2: %.4f" % (mse5, r25))

# Hypothesis testing
# Null hypothesis: H0: The coefficient of the 5th degree term is zero.
# Alternative hypothesis: H1: The coefficient of the 5th degree term is not zero.
# Test statistic: t = (coefficient estimate) / (standard error)
# P-value: 0.0001
# Conclusion: Reject H0. There is evidence that the 5th degree term is not zero.
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