

Lab W1D5

Question 1. Goal of this question is to understand inversions.

In Slide 13 (Lesson 4), you have been introduced to inversions. In fact it has the following example.

Example. The array $arr = \{34, 8, 64, 51, 32, 21\}$ has nine inversions:

$(34, 8), (34, 32), (34, 21), (64, 51), (64, 32),$

$(64, 21), (51, 32), (51, 21), (32, 21).$

Apply Bubble Sort on array arr . List all inversions and the number of inversions **after each iteration of the outer loop**. (Please complete the table. Add/delete rows as required.)

Iteration	inversions	# inversions
0	$(34, 8), (34, 32), (34, 21), (64, 51), (64, 32), (64, 21), (51, 32), (51, 21), (32, 21).$	9

Total number of inversions for Bubble Sort =

Apply Selection Sort on array arr . List all inversions and the number of inversions **after each iteration of the outer loop**. (Please complete the table. Add/delete rows as required.)

Iteration	inversions	# inversions
0	$(34, 8), (34, 32), (34, 21), (64, 51), (64, 32), (64, 21), (51, 32), (51, 21), (32, 21).$	9

Total number of inversions for Selection Sort =

Apply Insertion Sort on array arr . List all inversions and the number of inversions **after each iteration of the outer loop**. (Please complete the table. Add/delete rows as required.)

Iteration	inversions	# inversions
0	$(34, 8), (34, 32), (34, 21), (64, 51), (64, 32), (64, 21), (51, 32), (51, 21), (32, 21).$	9

Total number of inversions for Insertion Sort =

Question 2. Aim of this question is to understand amortized cost analysis.

Consider Slide 30 of Lecture 4. What you see there we are going to present in a table form IN A SLIGHTLY DIFFERENT INTERPRETATION so that we can better understand the concepts involved.

The function c is the “real” cost. Function \hat{c} is what we are charging our customer. Balance shows how much cash we have. We never want the balance to be negative.

c function: 1 to add.

3k to resize (if $k > 0$. Note: k is the size of the “completely filled array”)

\hat{c} function: 7 to add. (customer is willing to pay for add)

0 to resize (customer do not want to pay for resizing. It is not his/her concern)

Item #	Operation	Cost for us	Customer paid	Profit	Balance
1	Add	We assume we start with 1 slot. We add 1 item at the cost of 1.	7	6	6
2	Add	3 to resize (We have two slots) 1 to add	7	6	9
3	Add	6 to resize (We have 4 slots) 1 to add	7	6	9
4	Add	1 to add	7	6	15
5	Add	12 to resize (We have 8 slots) 1 to add	7	6	9
6	Add	1 to add	7	6	15

I have filled in first 6 rows for you. Your task is to fill in 12 more rows!

After inspecting 18 rows, can you predict the minimum value of “Balance”?