

BISECTION METHOD:

This method is a root-finding method that applies to any continuous functions with two known values of opposite signs.

FORMULA:

$$x_m = \frac{x_l + x_u}{2}$$

ADVANTAGES:

1. Always convergent.
2. The roots get halved with each iteration.

DISADVANTAGES:

1. Slow convergence.
2. If one of the initial guesses is close to the root, the convergence is slower.
3. If a function $f(x)$ is such that it just touches the x-axis it will be unable to find the lower and upper guesses.
4. Function changes sign but roots do not exist.

EXAMPLE:

$$\cos x - xe^x = 0$$

$$F(0) = \cos(0) - 0e^0 = 0$$

$$F(2) = \cos(2) - 2e^2 \neq 0$$

So, the roots are (0,2).

$$x_l = 0$$

$$x_u = 2$$

Now,

$$x_m = \frac{0+2}{2} = 1$$

Now,

New roots are

$$x_m = \frac{1+2}{2} = 1.5$$

New roots are (1.5,2)

$$x_m = \frac{1.5+2}{2} = 1.7$$

Error:

$$\begin{aligned} |E_a| &= \left| \frac{x_{new} - x_{old}}{x_{new}} \right| \\ &= \left| \frac{1.7 - 1.5}{1.7} \right| \times 100 \\ &= 11\% \end{aligned}$$