Data Science Notes by Sarowar Ahmed

📊 Chapter: Probability Theory

📊 Topic: Bayes' Theorem

🔎 Hello, GitHub family! Today, let's embark on a journey into the profound world of Bayes' Theorem, a cornerstone of probability theory that illuminates the process of updating beliefs in the face of new evidence. I'll guide you through this topic with clarity and insight, ensuring that it's accessible to everyone, regardless of age or background.

🌟 What is Bayes' Theorem?

▪ Imagine you're a detective investigating a crime. You have initial beliefs about who the culprit might be based on past experience and evidence. As new clues emerge, Bayes' Theorem allows you to update your beliefs by incorporating this new evidence, enabling you to make more informed decisions and predictions.

📐 Formula for Bayes' Theorem:

Bayes' Theorem relates the conditional probability of an event A given another event B to the conditional probability of event B given event A. The formula is as follows:

$P(A|B) = P(B|A) \times P(A)/P(B)$

Where:
▪ $P(A|B)$ is the probability of event A occurring given that event B has occurred.
▪ $P(B|A)$ is the probability of event B occurring given that event A has occurred.
▪ $P(A)$ and $P(B)$ are the probabilities of events A and B occurring independently.

📊 Understanding Bayes' Theorem with an Example:

<u>Example 1: Medical Diagnosis</u>
▪ Imagine a rare disease affects 1 in 1000 people. You have a test for this disease, which is 99% accurate for both those who have the disease and those who don't.
Let's use Bayes' Theorem to calculate the probability that a person who tests positive actually has the disease.

▪ Let A be the event that a person has the disease.
▪ Let B be the event that the person tests positive.

Given:
▪ $P(A)=0.001$ (probability of having the disease)
▪ $P(B|A)=0.99$ (probability of testing positive given that the person has the disease)
▪ $P(B|\neg A)=0.01$ (probability of testing positive given that the person does not have the disease)

▪ We want to find $P(A|B)$, the probability of having the disease given that the person tests positive.

Using Bayes' Theorem: $P(A|B)=P(B|A)\times P(A)/P(B)$

We need to calculate P(B) using the law of total probability:

$P(B) = P(B|A)\times P(A)+P(B|\neg A)\times P(\neg A)$

$P(B)=0.99\times 0.001+0.01\times 0.999=0.00199$

Now, plugging into Bayes' Theorem:
$P(A|B) = 0.99\times 0.001/0.00199\approx 0.497$

So, if a person tests positive for the disease, the probability that they actually have the disease is approximately 49.7%.

Example 2: Coin Toss
▪ Suppose you have two coins in a bag: one fair coin and one biased coin that comes up heads 80% of the time.

You draw a coin from the bag at random and toss it. It comes up heads. What is the probability that you drew the biased coin?

- Let A be the event of drawing the biased coin.
- Let B be the event of getting heads.

Given:
- $P(A)=0.5$ (probability of drawing the biased coin)
- $P(B|A)=0.8$ (probability of getting heads given that the biased coin is drawn)
- $P(B|\neg A)=0.5$ (probability of getting heads given that the fair coin is drawn)

Using Bayes' Theorem:
$P(A|B)= P(B|A)\times P(A)/P(B)$

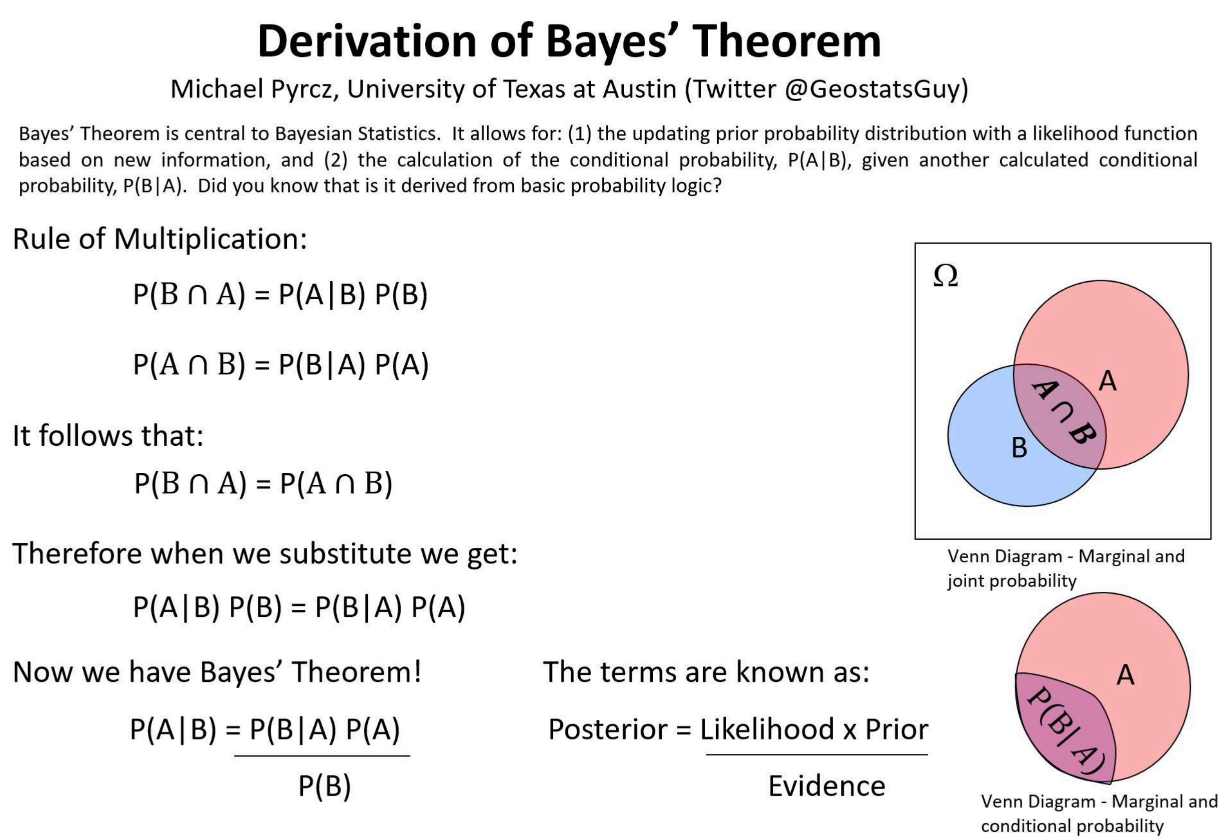
We again need to calculate P(B) using the law of total probability:
- $P(B)=P(B|A)\times P(A)+P(B|\neg A)\times P(\neg A)$

$P(B)=0.8\times 0.5+0.5\times 0.5=0.65$

Now, plugging into Bayes' Theorem:
$P(A|B) = 0.8\times 0.5/0.65\approx 0.615$

So, if you toss a coin and it comes up heads, the probability that you drew the biased coin is approximately 61.5%.

🖼️ Visualizing the Beta Distribution:



Derivation of Bayes' Theorem

Michael Pyrcz, University of Texas at Austin (Twitter @GeostatsGuy)

Bayes' Theorem is central to Bayesian Statistics. It allows for: (1) the updating prior probability distribution with a likelihood function based on new information, and (2) the calculation of the conditional probability, P(A|B), given another calculated conditional probability, P(B|A). Did you know that is it derived from basic probability logic?

Rule of Multiplication:

$$P(B \cap A) = P(A|B)\,P(B)$$

$$P(A \cap B) = P(B|A)\,P(A)$$

It follows that:

$$P(B \cap A) = P(A \cap B)$$

Therefore when we substitute we get:

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

Now we have Bayes' Theorem!

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The terms are known as:

$$\text{Posterior} = \frac{\text{Likelihood x Prior}}{\text{Evidence}}$$

Venn Diagram - Marginal and joint probability

Venn Diagram - Marginal and conditional probability

▪ This visual aid helps to illustrate the sequential process of updating beliefs based on new evidence, making it easier to grasp the concept.

🚀 Why Does This Matter?

▪ Bayes' Theorem has far-reaching implications in various fields, including medicine, finance, and artificial intelligence. It enables us to make rational decisions and predictions in the face of uncertainty, leading to more effective problem-solving and decision-making.

Got any questions about Bayes' Theorem!? Feel free to ask me via Linkedin! Let's keep learning together.

My LinkedIn Date: 18/04/2024