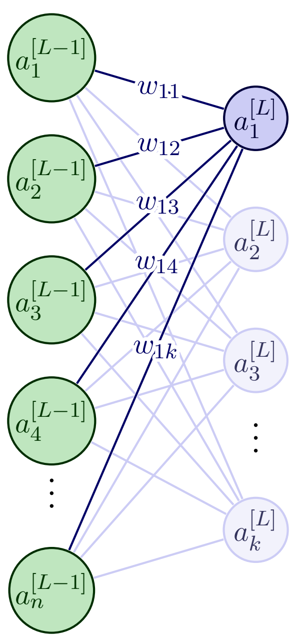
Forward Propagation

Assume that we have **m** examples. Let's review the general formulas we know.

$n^{[L]}$: number of units in layer L
$A^{[L]} = \sigma(Z^{[L]}) = \sigma(W^{[L]} \cdot A^{[L-1]} + b^{[L]})$
$\mathrm{W}^{[L]}.shape = (n^{[L]}, n^{[L-1]})$
$\mathrm{b}^{[L]}.shape = (n^{[L]}, 1)$
$\mathrm{A}^{[L]}.shape = Z^{[L]}.shape = (n^{[L]}, n^{[L-1]})$



$$
\begin{aligned}
a_1^{[L]} &= \sigma\left(z_1^{[L]}\right) \\
&= \sigma\left(w_{11} \cdot a_1^{[L-1]} + w_{12} \cdot a_2^{[L-1]} + \ldots + w_{1n} \cdot a_n^{[L-1]} + b_1^{[L]}\right) \\
&= \sigma\left(\sum_{i=1}^{n} w_{1i} \cdot a_i^{[L-1]} + b_1^{[L-1]}\right)
\end{aligned}
$$

$$
\begin{aligned}
a_2^{[L]} &= \sigma\left(z_2^{[L]}\right) \\
&= \sigma\left(w_{21} \cdot a_1^{[L-1]} + w_{22} \cdot a_2^{[L-1]} + \ldots + w_{2n} \cdot a_n^{[L-1]} + b_2^{[L]}\right) \\
&= \sigma\left(\sum_{i=1}^{n} w_{2i} \cdot a_i^{[L-1]} + b_2^{[L-1]}\right)
\end{aligned}
$$

$$
\begin{aligned}
a_3^{[L]} &= \sigma\left(z_2^{[L]}\right) \\
&= \sigma\left(w_{31} \cdot a_1^{[L-1]} + w_{32} \cdot a_2^{[L-1]} + \ldots + w_{3n} \cdot a_n^{[L-1]} + b_3^{[L]}\right) \\
&= \sigma\left(\sum_{i=1}^{n} w_{3i} \cdot a_i^{[L-1]} + b_3^{[L-1]}\right)
\end{aligned}
$$

$$\vdots$$

$$
\begin{aligned}
a_m^{[L]} &= \sigma\left(z_m^{[L]}\right) \\
&= \sigma\left(w_{m1} \cdot a_1^{[L-1]} + w_{m2} \cdot a_2^{[L-1]} + \ldots + w_{mn} \cdot a_n^{[L-1]} + b_m^{[L]}\right) \\
&= \sigma\left(\sum_{i=1}^{n} w_{3i} \cdot a_i^{[L-1]} + b_k^{[L-1]}\right)
\end{aligned}
$$

Getting Matrix Dimensions

From above operations we know:

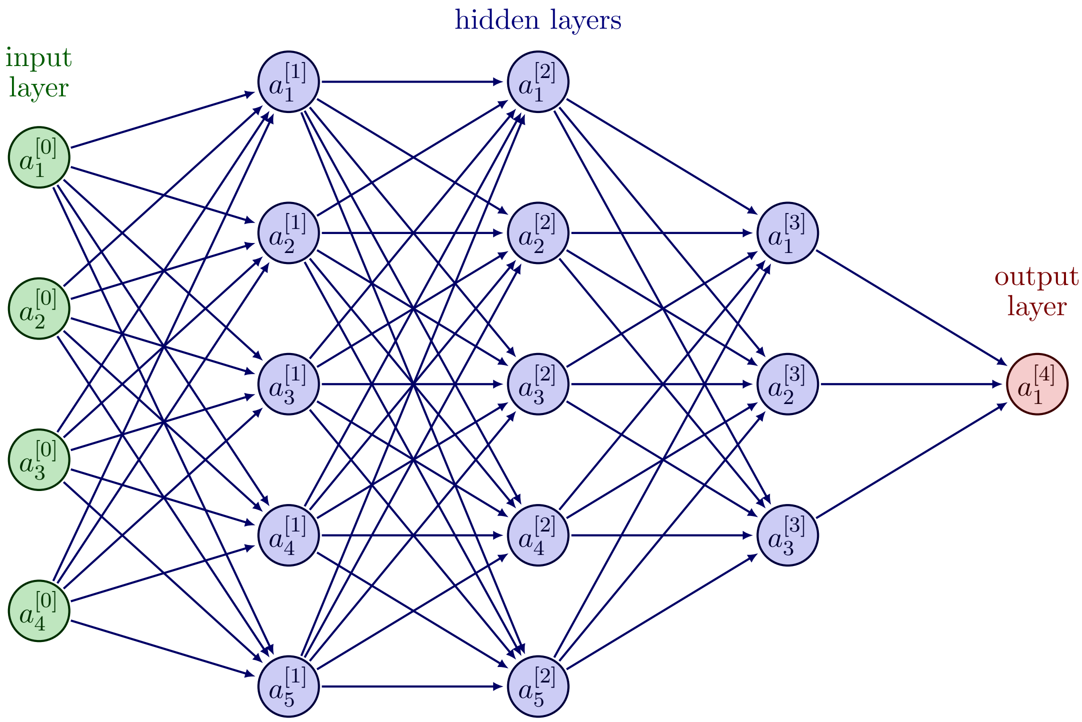
$$
\begin{gathered}
w_{11} \cdot a_1^{[L-1]} + w_{12} \cdot a_2^{[L-1]} + \ldots + w_{1n} \cdot a_n^{[L-1]} + b_1^{[L-1]} = z_1^{[L]} \\
w_{21} \cdot a_1^{[L-1]} + w_{22} \cdot a_2^{[L-1]} + \ldots + w_{2n} \cdot a_n^{[L-1]} + b_1^{[L-1]} = z_2^{[L]} \\
w_{31} \cdot a_1^{[L-1]} + w_{32} \cdot a_2^{[L-1]} + \ldots + w_{3n} \cdot a_n^{[L-1]} + b_1^{[L-1]} = z_3^{[L]} \\
\vdots \\
w_{m1} \cdot a_1^{[L-1]} + w_{m2} \cdot a_2^{[L-1]} + \ldots + w_{mn} \cdot a_n^{[L-1]} + b_1^{[L-1]} = z_k^{[L]}
\end{gathered}
$$

The equations look like system of linear equations from linear algebra. In order to get the dimensions we're gonna treat them like it. Let's rewrite the system as matrices.

$$
\begin{bmatrix} w_{1,1} & w_{1,2} & \cdots & w_{1,n} \\ w_{2,1} & w_{2,2} & \cdots & w_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{k,1} & w_{k,2} & \cdots & w_{k,n} \end{bmatrix}
\begin{bmatrix} a_1^{[L-1]} \\ a_2^{[L-1]} \\ \vdots \\ a_n^{[L-1]} \end{bmatrix}
+
\begin{bmatrix} b_1^{[L]} \\ b_2^{[L]} \\ \vdots \\ b_k^{[L]} \end{bmatrix}
\begin{bmatrix} z_1^{[L]} \\ z_2^{[L]} \\ \vdots \\ z_k^{[L]} \end{bmatrix}
$$

$$
\underset{k \times n}{W^{[L]}} \times \underset{n \times m}{A^{[L-1]}} + \underset{k \times 1}{b^{[L]}} = \underset{k \times m}{\mathrm{Z}^{[\mathrm{L}]}}
$$

Back Propagation

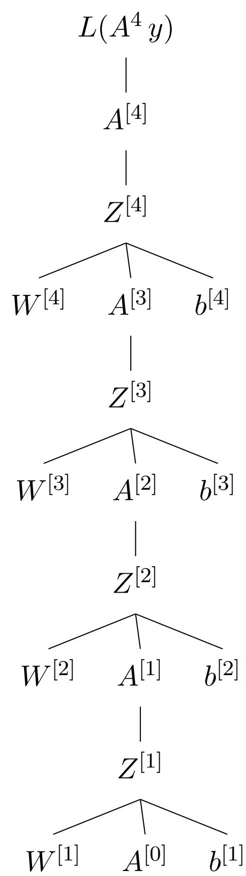


We know that in the first step of backward prop, we're given $dA^{[L]}$. Using $dA^{[L]}$ and the given formulas below, we calculate $dA^{[L-1]}, dW^{[L]}$ and $db^{[L]}$. However, now we gonna just do two of them like we did in the meeting. I'm planning to update this document and add other calculations as well.

Formulas

$$dZ^{[L]} = dA^{[L]} \cdot \sigma(Z^{[L]})$$
$$dW^{[L]} = \frac{1}{m} dZ^{[L]} \cdot A^{[L-1]T}$$

Remember the tree structure. We'll draw it with respect to the neural network top of the page. Then we'll find the derivatives and compare them with the formulas above.



Instead of Andrew's notation, we're gonna use some "real mathematics" notation :) $\frac{\partial L}{\partial Z^{[L]}} = dZ^{[L]}$, $\frac{\partial L}{\partial W^{[L]}} = dW^{[L]}$. Now let's take some derivative

$$\underbrace{\frac{\partial L}{\partial Z^{[4]}}}_{} = \underbrace{\frac{\partial L}{\partial A^{[4]}}}_{} \cdot \underbrace{\frac{\partial A^{[4]}}{\partial Z^{[4]}}}_{}$$

$$\underbrace{\frac{\partial L}{\partial Z^{[3]}}}_{} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}}}_{} \cdot \underbrace{\frac{\partial A^{[3]}}{\partial Z^{[3]}}}_{}$$

$$\underbrace{\frac{\partial L}{\partial Z^{[2]}}}_{} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}} \cdot \frac{\partial A^{[3]}}{\partial Z^{[3]}} \cdot \frac{\partial Z^{[3]}}{\partial A^{[2]}}}_{} \underbrace{\frac{\partial A^{[2]}}{\partial Z^{[2]}}}_{}$$

$$\underbrace{\frac{\partial L}{\partial W^{[4]}}}_{} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}}}_{} \cdot \underbrace{\frac{\partial Z^{[4]}}{\partial W^{[4]}}}_{}$$

$$\underbrace{\frac{\partial L}{\partial W^{[3]}}}_{} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}} \cdot \frac{\partial A^{[3]}}{\partial Z^{[3]}}}_{} \cdot \underbrace{\frac{\partial Z^{[3]}}{\partial W^{[3]}}}_{}$$

$$\underbrace{\frac{\partial L}{\partial W^{[2]}}}_{} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}} \cdot \frac{\partial A^{[3]}}{\partial Z^{[3]}} \cdot \frac{\partial Z^{[3]}}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}}}_{} \cdot \underbrace{\frac{\partial Z^{[2]}}{\partial W^{[2]}}}_{}$$