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# A Brief Introduction to Deep Neural Networks

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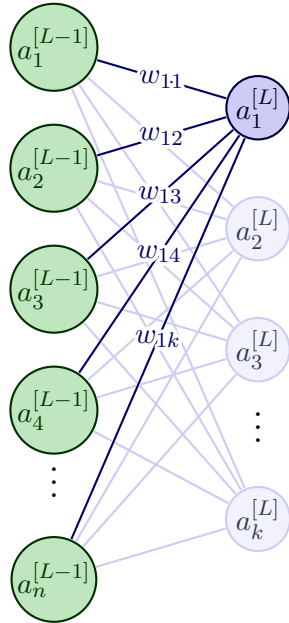
## Introduction

One should have a basic knowledge of linear algebra and calculus to understand some concepts in neural networks. If you don't have any prior knowledge, you can have a basic understanding by looking at the notes on [systems of linear equations](#) and [matrix operations](#). These notes are not complete yet. However, they are sufficient for the concepts below. (Calculus is not included.)

## Forward Propagation

Assume that we have  $\mathbf{m}$  examples. Let's review the general formulas we know.

$n^{[L]}$ : number of units in layer L  
 $A^{[L]} = \sigma(Z^{[L]}) = \sigma(W^{[L]} \cdot A^{[L-1]} + b^{[L]})$   
 $W^{[L]}.shape = (n^{[L]}, n^{[L-1]})$   
 $b^{[L]}.shape = (n^{[L]}, 1)$   
 $A^{[L]}.shape = Z^{[L]}.shape = (n^{[L]}, m)$



$$\begin{aligned} a_1^{[L]} &= \sigma(z_1^{[L]}) \\ &= \sigma(w_{11} \cdot a_1^{[L-1]} + w_{12} \cdot a_2^{[L-1]} + \dots + w_{1n} \cdot a_n^{[L-1]} + b_1^{[L]}) \\ &= \sigma\left(\sum_{i=1}^n w_{1i} \cdot a_i^{[L-1]} + b_1^{[L-1]}\right) \end{aligned}$$

$$\begin{aligned} a_2^{[L]} &= \sigma(z_2^{[L]}) \\ &= \sigma(w_{21} \cdot a_1^{[L-1]} + w_{22} \cdot a_2^{[L-1]} + \dots + w_{2n} \cdot a_n^{[L-1]} + b_2^{[L]}) \\ &= \sigma\left(\sum_{i=1}^n w_{2i} \cdot a_i^{[L-1]} + b_2^{[L-1]}\right) \end{aligned}$$

$$\begin{aligned} a_3^{[L]} &= \sigma(z_3^{[L]}) \\ &= \sigma(w_{31} \cdot a_1^{[L-1]} + w_{32} \cdot a_2^{[L-1]} + \dots + w_{3n} \cdot a_n^{[L-1]} + b_3^{[L]}) \\ &= \sigma\left(\sum_{i=1}^n w_{3i} \cdot a_i^{[L-1]} + b_3^{[L-1]}\right) \end{aligned}$$

⋮

$$\begin{aligned} a_k^{[L]} &= \sigma(z_m^{[L]}) \\ &= \sigma(w_{m1} \cdot a_1^{[L-1]} + w_{m2} \cdot a_2^{[L-1]} + \dots + w_{mn} \cdot a_n^{[L-1]} + b_m^{[L]}) \\ &= \sigma\left(\sum_{i=1}^n w_{mi} \cdot a_i^{[L-1]} + b_k^{[L-1]}\right) \end{aligned}$$

## Getting Matrix Dimensions

From above operations we know:

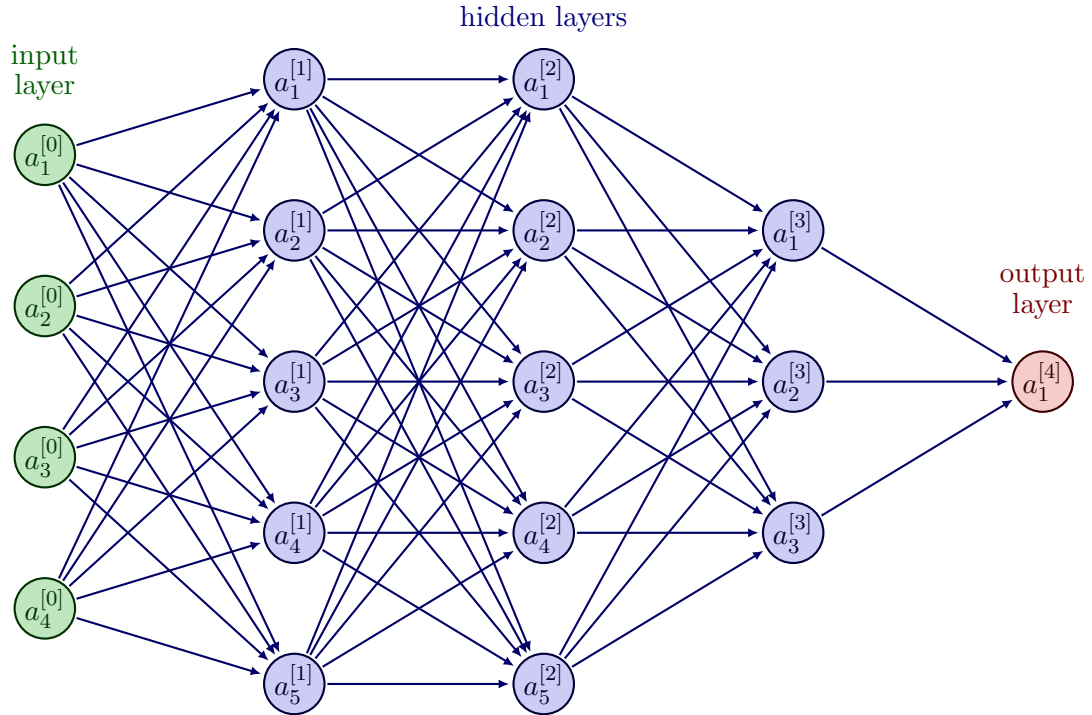
$$\begin{aligned}
 w_{11} \cdot a_1^{[L-1]} + w_{12} \cdot a_2^{[L-1]} + \dots + w_{1n} \cdot a_n^{[L-1]} + b_1^{[L-1]} &= z_1^{[L]} \\
 w_{21} \cdot a_1^{[L-1]} + w_{22} \cdot a_2^{[L-1]} + \dots + w_{2n} \cdot a_n^{[L-1]} + b_1^{[L-1]} &= z_2^{[L]} \\
 w_{31} \cdot a_1^{[L-1]} + w_{32} \cdot a_2^{[L-1]} + \dots + w_{3n} \cdot a_n^{[L-1]} + b_1^{[L-1]} &= z_3^{[L]} \\
 &\vdots \\
 w_{m1} \cdot a_1^{[L-1]} + w_{m2} \cdot a_2^{[L-1]} + \dots + w_{mn} \cdot a_n^{[L-1]} + b_1^{[L-1]} &= z_k^{[L]}
 \end{aligned}$$

The equations look like a system of linear equations from linear algebra. To get the dimensions, we're gonna treat them like it. Let's rewrite the system as matrices.

$$\begin{bmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,n} \\ w_{2,1} & w_{2,2} & \dots & w_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{k,1} & w_{k,2} & \dots & w_{k,n} \end{bmatrix} \begin{bmatrix} a_1^{[L-1]} \\ a_2^{[L-1]} \\ \vdots \\ a_n^{[L-1]} \end{bmatrix} + \begin{bmatrix} b_1^{[L]} \\ b_2^{[L]} \\ \vdots \\ b_k^{[L]} \end{bmatrix} = \begin{bmatrix} z_1^{[L]} \\ z_2^{[L]} \\ \vdots \\ z_k^{[L]} \end{bmatrix}$$

$$W_{k \times n}^{[L]} \times A_{n \times m}^{[L-1]} + b_{k \times 1}^{[L]} = Z_{k \times m}^{[L]}$$

## Backward Propagation

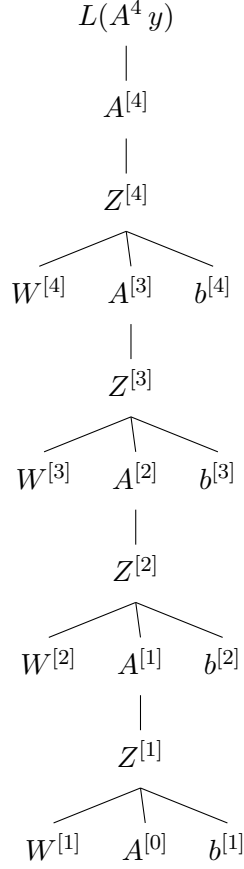


We know that in the first step of backward prop, we are given  $dA^{[L]}$ . Using  $dA^{[L]}$  and the given formulas below, we calculate  $dA^{[L-1]}$ ,  $dW^{[L]}$  and  $db^{[L]}$ .

## Formulas

$$\begin{aligned} dZ^{[L]} &= dA^{[L]} \cdot \sigma(Z^{[L]}) \\ dW^{[L]} &= \frac{1}{m} dZ^{[L]} \cdot A^{[L-1]T} \end{aligned}$$

Remember the tree structure.



Instead of Andrew's notation, we're gonna use the following notation.  $\frac{\partial L}{\partial Z^{[L]}} = dZ^{[L]}$ ,  $\frac{\partial L}{\partial W^{[L]}} = dW^{[L]}$ . Now let's take some derivative

$$\underbrace{\frac{\partial L}{\partial Z^{[4]}}} = \underbrace{\frac{\partial L}{\partial A^{[4]}}} \cdot \underbrace{\frac{\partial A^{[4]}}{\partial Z^{[4]}}}$$

$$\underbrace{\frac{\partial L}{\partial Z^{[3]}}} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}}}_{\frac{\partial A^{[4]}}{\partial A^{[3]}}} \cdot \underbrace{\frac{\partial A^{[3]}}{\partial Z^{[3]}}}$$

$$\underbrace{\frac{\partial L}{\partial Z^{[2]}}} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}} \cdot \frac{\partial A^{[3]}}{\partial Z^{[3]}} \cdot \frac{\partial Z^{[3]}}{\partial A^{[2]}}}_{\frac{\partial A^{[4]}}{\partial A^{[2]}}} \cdot \underbrace{\frac{\partial A^{[2]}}{\partial Z^{[2]}}}$$

$$\underbrace{\frac{\partial L}{\partial W^{[4]}}} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}}}_{\frac{\partial A^{[4]}}{\partial Z^{[4]}}} \cdot \underbrace{\frac{\partial Z^{[4]}}{\partial W^{[4]}}}$$

$$\underbrace{\frac{\partial L}{\partial W^{[3]}}} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}} \cdot \frac{\partial A^{[3]}}{\partial Z^{[3]}}}_{\frac{\partial A^{[4]}}{\partial A^{[3]}}} \cdot \underbrace{\frac{\partial Z^{[3]}}{\partial W^{[3]}}}$$

$$\underbrace{\frac{\partial L}{\partial W^{[2]}}} = \underbrace{\frac{\partial L}{\partial A^{[4]}} \cdot \frac{\partial A^{[4]}}{\partial Z^{[4]}} \cdot \frac{\partial Z^{[4]}}{\partial A^{[3]}} \cdot \frac{\partial A^{[3]}}{\partial Z^{[3]}} \cdot \frac{\partial Z^{[3]}}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}}}_{\frac{\partial A^{[4]}}{\partial A^{[2]}}} \cdot \underbrace{\frac{\partial Z^{[2]}}{\partial W^{[2]}}}$$