Discrete Mathematics

Project

- Recurrence Relations (RR) and Cellular Automata (CA) VII

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Contents

1 Introduction

This project is about Recurrence Relations and Cellular Automata which has been done using mathematica tool. The project contains 4 problems. All the problems have been succesfully completed. Each problem has been described in detail as well as pictures and graps also used for understanding the problems deeply and making them more interesting.

Exercise 1 The relation recurrence (*RR*) of *n*-digit quartenary {0,1,2,3} sequences with an even number of 1's is

$$a_{n+1} = (4^n - a_n) * 1 + a_n * 3$$

where *n* belongs to number of string length.

For understanding the RR we divided the strings in even and odd number of 1's. Whenever the number of 1's is odd at *n* string length, then we need to add 1 in the new *n+1* string length and reduce even number of 1's from total possibilities at *n* length. Similarly, if number of 1's is even at *n* then we need to add {0, 2 or 3} means 3 at *n+1* length and multiply with even number of 1's at *n* position. However, we also found that we can use the following equation to find the even number of 1's possibilties at any *n* length without depending on previous length

$$a_n = \frac{4^n + 2^n}{2}$$

where *n* belongs to number of string length

The *initial condition* is 3 when *n* = 1 becuase at this position we have total four possibilities (0,1,2,3) and we are only allowed to take those possibilities which have an even number of 1's. So, we get 3 as a initial condition by selecting {1,2,3}. There are 633 825 300 114 115 263 698 305 024 000 *strings of length 50.* Table 2.1 shows the sequence of first 10 values and Figure 2.1 describes the graph of it.

Length(n)	**Total Possibilities**	**# of Odd 1's**	**# of Even 1's**
n = 1	$4^1 = 4$	1	3
n = 2	$4^2 = 16$	6	10
n = 3	$4^3 = 64$	28	36
n = 4	$4^4 = 256$	120	136
n = 5	$4^5 = 1024$	496	528
n = 6	$4^6 = 4096$	2016	2080
n = 7	4^7 = 16 384	8128	8256
n = 8	4^8 = 655 36	32 640	32 896
n = 9	4^9 = 262 144	130 816	131 328
n = 10	4^{10} = 104 8576	523 776	524 800

Table 1.1: First 10 values of *n*-digit quartenary sequences.

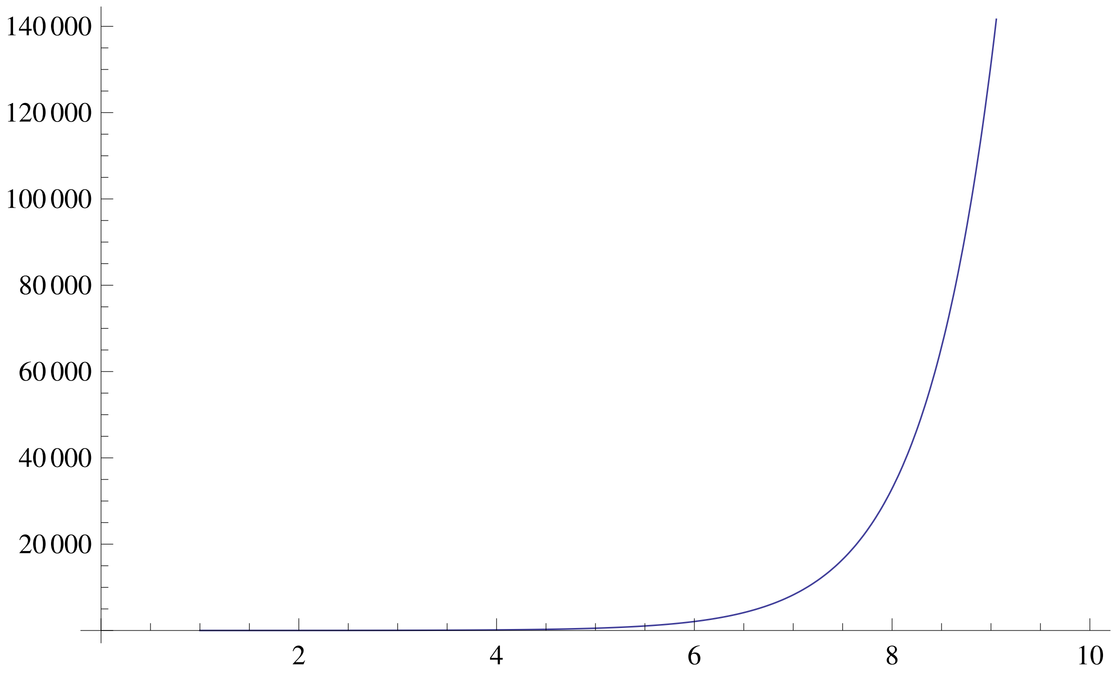


Figure 1.1: This figure illustrates the graph with first 10 values n-digit quartenary sequences.

Exercise 2 For finding *truly chaotic orbit in the logistic map* which seems uniformly spread over the whole interval, we need to first see the bifurcation diagram of logistic map. The bifurcation diagram will help us to find the different variation of orbit at different co-ordinates. In our case, we need to find such value of a where the orbit is uniformly spread over the whole interval. So if we see the Figure 1.2, we can easily see the that blue portion is highly spreaded over the whole graph(0.0 to 1.0) when it goes near to 4.0 which means that the value of a is somewhere there. For finding almost the exact value, we can use the "Get co-ordinates" method in mathematica which will help us to find the appropriate value of a. We found that when a = 3.99 orbit is uniformly spreaded over the whole interval.

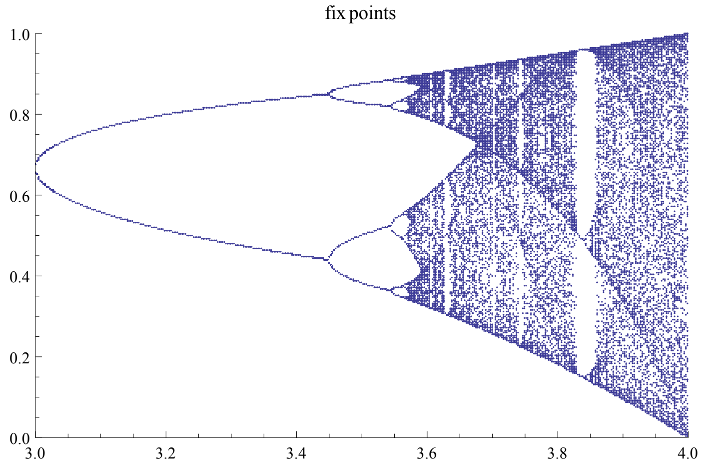


Figure 1.2: Bifurcation Diagram of Logistic Map

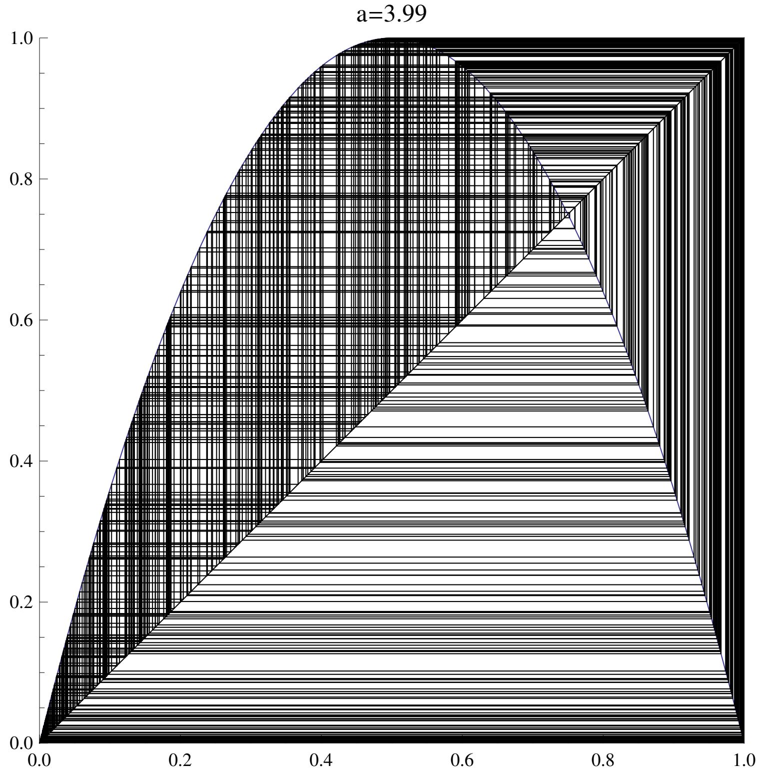
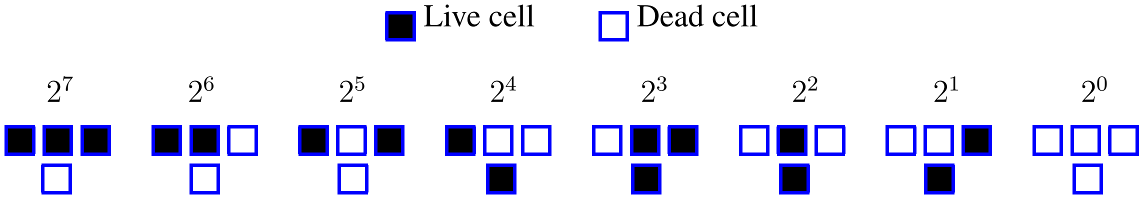


Figure 1.3: Logistic Map with 500 iterations

Exercise 3 As we know that each cell can be in two different states (live or dead) and this states depends on the neigbours cell state which means there are $2^3 = 8$ different states and 2^8=256 different rules. The concept for finding the rule number is "If middle cell and right cell both are white then the middle cell takes the value of the left cell, if they are not both white then the middle cell takes opposite value of left cell."
The following diagram represents the cell states and what happens in next generation.



In our case, "a black cell survives only if it is surrounded by 2 white cells and no birth takes place" which means from above diagram the cell with 2^2 states belongs to our case which results in *rule number 4*. In general for finding the rule number we first find the relavent case and then count all the previous cases into it. However, it is clearly specified in our case that no birth takes place so we should not count the previous case (2^1) in it. If we do so, then the birth takes place which we can easily see in the picture that in case number 2^1 the middle cell is white and the both neighbours cell are not white, so according to defination the middle cell will take the opposite value of left cell. In other words, birth takes place.

By not counting the previous case we get a stable cells means the cell remains in the initial state in all generations. Figure 1.3 is the view of randomly generated seed and executed it under the rule #4.

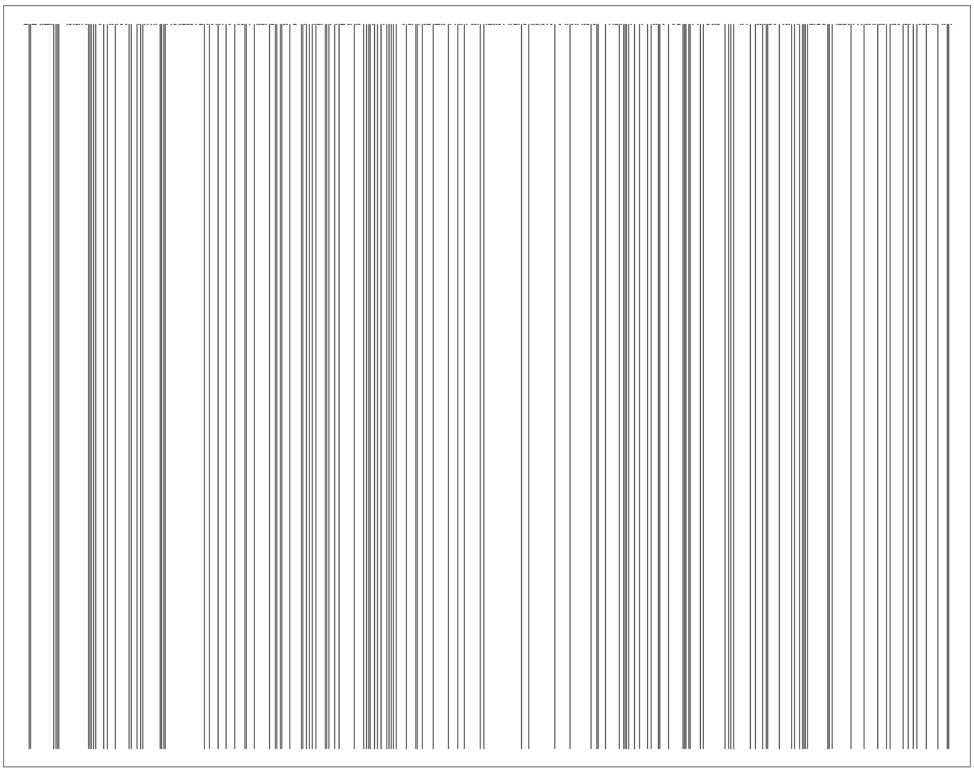


Figure 1.4: This figure illustrates 1D cellular automata rule #4

Exercise 4 We started by setting up the 2D cellular automata with the game of life rule number 224(224, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 1, 1). To plot the gilder we simply looked at a paused gilder and extracted the seed by looking at the coordinates. The most simple start position was a glider formed as an backwards L kicking a ball. We also concluded that the start position of the seed does not matter, as long as they are formed as any of the gliders 4 states. Although, "turning" the start position does affect in which the direction of the glider moves. In all attempts we tried, adding or removing seed from the base glider resulted in death, adding a few seeds can presumable morph it to a oscillator or a still life, but it never happened during our tries. The reason the glider moves as it does, and the

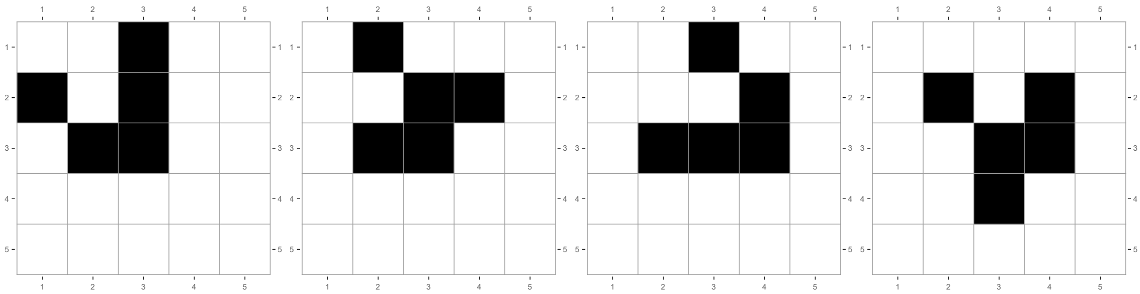


Figure 1.5: The 4 different states of the glider

reason it even exist is because of the 4 main rules of Conway's Game of Life:

1. Any live cell with fewer than two live neighbours dies, as if caused by under-population.
2. Any live cell with two or three live neighbours lives on to the next generation.
3. Any live cell with more than three live neighbours dies, as if by over-population.
4. Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction.

Looking at figures 1.5 we can see that in the initial state that (2,1) and (1,3) will die, they only have 1 neighbour. However, in next picture (2,4) and (1,2) will reproduce because they had exactly 3 neighbours in the first state. And like this it continues forever, or until it hits something else.
Fun fact: searching "Conway's Game of Life" in google will start a game of life with random seed in the background.