Multi-step Quarterly CPI & GDP forecasts by ARIMA & Dynamic Linear Models

Sarp Uslu

May 14, 2018

Abstract: Quarterly CPI & GDP percent changes for the largest economies are fore-casted using ARIMA, ARFIMA and Dynamic Linear Models. Countries that are ideal for equity investing is identified according to Ray Dalio's asset allocation approach.

1 Introduction

Ray Dalio is one of the most successful hedge fund managers in the world. He led Bridgewater Associates, biggest hedge fund in the world with currently $160 billion under management, for a long time until his recent retirement. Ray is usually considered as a global macro investor and he is known for his models about the economy. In my opinion his success is an indicator of the accuracy of his models. This project is motivated by one of Ray's models.

Firstly, Ray argues that diversification is the only free lunch in investing however within a given country diversification is limited. He states that stocks in US are roughly 60% correlated with each other. Based on this fact, he argues that diversification above 6-8 stocks show diminishing diversification effects. Ideally one would like to have many uncorrelated assets in their portfolio as this leads to lower portfolio volatility.

Secondly, Ray emphasizes the importance of debt cycles as the leading factor in the changes in asset prices. These debt cycles are governed by central banks around the world by either printing money or changing interest rates. In a stimulatory economic environment where interest rates are low and money is easily available, asset prices tend to soar. Prolonged stimulation can lead to undesirable high inflation. In that case central banks increase interest rate and decrease the money supply leading to lower asset prices.

Ray simplifies and categorizes any economy into four seasons depending on their position in the debt cycle. Two main indicators of these seasons are inflation and economic growth. He defines four economic seasons based on the relation between inflation and economic growth. In his view, certain asset classes perform better in certain environments. Below is a diagram showing which asset classes tend to rise under given seasons:

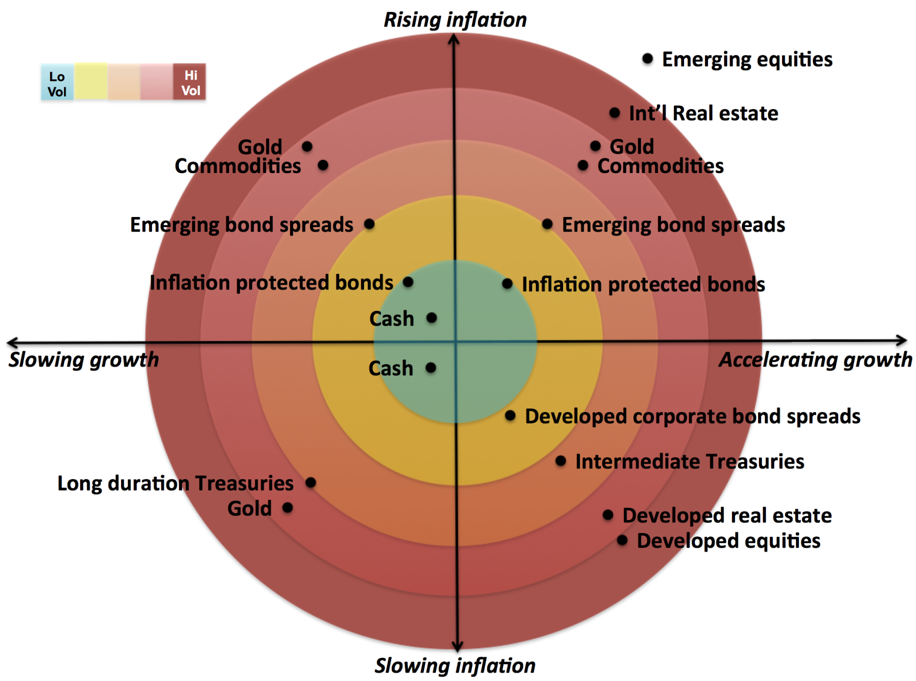

Figure 1: Four economic seasons according to Ray Dalio

In summary, for proper diversification one has to look for global opportunities and these opportunities arise in a given country when there is inflation and economic growth. Therefore, the goal of this project is to use the time series models learned in this course to see how countries compare to one another with respect to expected inflation and GDP growth.

2 Models

2.1 ARIMA & ARFIMA models

I will not describe these models in this report as they were covered extensively in this course. In terms of implementation, "forecast" package in R is used to estimate these models. "auto.arima" and "arfima" functions fit the best models, given the data, according to AIC criterion.

2.2 State Space & Dynamic Linear Models

In state space models, observable process Y_t is a noisy function of an unobservable Markov state process θ_t.

Dynamic Linear Models (DLM) are linear Gaussian state space models. These models can be expressed in the following form:

$$Y_t = F_t\theta_t + v_t \qquad v_t \sim N_m(0, V_t)$$

$$\theta_t = G_t\theta_{t-1} + w_t \qquad w_t \sim N_p(0, W_t)$$

where F_t and G_t are matrices that specify a model. v_t and w_t are Gaussian noise and they are independent of one another and within themselves. V_t and W_t are covariance matrices.

Kalman Filter provides the framework for estimation and prediction of a specified DLM by computing predictive and filtering densities recursively as new data points are observed.

In terms of implementation, "DLM" package in R is used.

2.2.1 Local Level Model

This model is also called random walk plus noise model and it can be defined by the following equations:

$$Y_t = \mu_t + v_t \qquad v_t \sim N(0, V)$$

$$\mu_t = \mu_{t-1} + w_t \qquad w_t \sim N(0, W)$$

This is a DLM with m = p = 1, $\theta_t = \mu_t$ and $F_t = G_t = 1$

2.2.2 Local Trend Model

This model is also called linear growth model. It has the same observation equation as local level model but it has time-varying slope for the behavior of μ_t. This model is described by the following equations:

$$Y_t = \mu_t + v_t \qquad v_t \sim N(0, V)$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + w_{1,t} \qquad w_{1,t} \sim N(0, \sigma_{w_1})$$

$$\beta_t = \beta_{t-1} + w_{2,t} \qquad w_{2,t} \sim N(0, \sigma_{w_2})$$

This is a DLM with:

$$\theta_t = \begin{pmatrix} \mu_t \\ \beta_t \end{pmatrix} \qquad G = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad W = \begin{pmatrix} \sigma_{w_1}^2 & 0 \\ 0 & \sigma_{w_2}^2 \end{pmatrix} \qquad F = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

2.2.3 Basic Structural Model

Suppose, for example, that a series Y_t is the sum of a trend component $Y_{L,t}$ and a seasonal component $Y_{S,t}$:

$$Y_t = Y_{L,t} + Y_{S,t} + v_t.$$

We can construct a DLM for each component, so that

$$\begin{aligned} Y_{L,t} &= F_{L,t}\theta_{L,t} \\ \theta_{L,t} &= G_{L,t}\theta_{L,t-1} + w_{L,t}, \qquad & w_{L,t} \sim \mathcal{N}(0, W_{L,t}) \end{aligned}$$

and

$$\begin{aligned} Y_{S,t} &= F_{S,t}\theta_{S,t} \\ \theta_{S,t} &= G_{S,t}\theta_{S,t-1} + w_{S,t}, \qquad & w_{S,t} \sim \mathcal{N}(0, W_{S,t}) \end{aligned}$$

3 Methodology

3.1 Data Source

Quarterly CPI and GDP time series are obtained from OECD (Organisation for Economic Cooperation and Development) for some of the largest economies in the world. Investigated countries are: Australia, Canada, Germany, France, Great Britain, India, Italy, Japan, Korea, Turkey and United States. For a given country, typical data starts from 1965 and goes up to 2018, covering 213 quarters.

3.2 Training

Initial train set covers about 170 quarters and grows by one in each iteration. As the the train set grows models are re-fitted. Forecast horizon is 4 quarters long. Rolling forecasts are made by shifting the horizon by one at each iteration. This approach is shown below for the US GDP quarterly data:

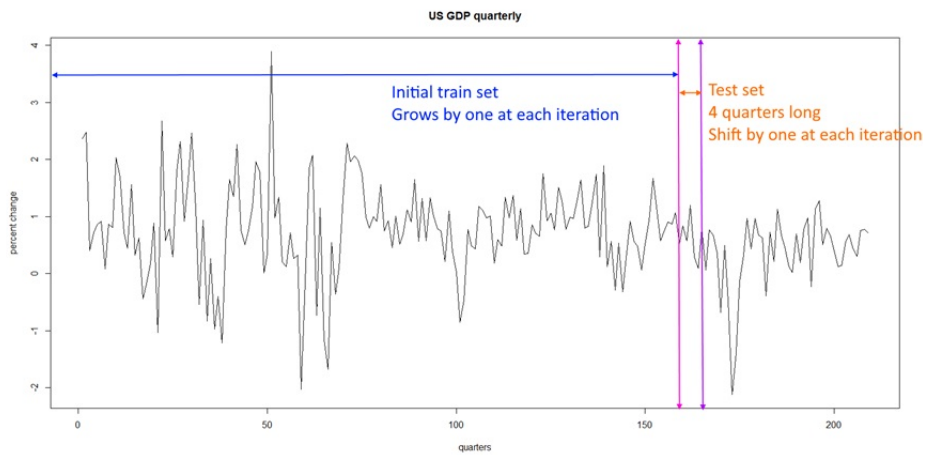

Figure 2: Visual description of the training process

Five models, as described in section above, are used for forecasting: ARIMA, ARFIMA, local level model, local trend model, Basic Structural Model (BSM). Errors of these forecasts are collected and averaged based on steps ahead. Below is the mean absolute error of multi-step errors for US GDP quarterly change forecasts for the last 40 quarters.

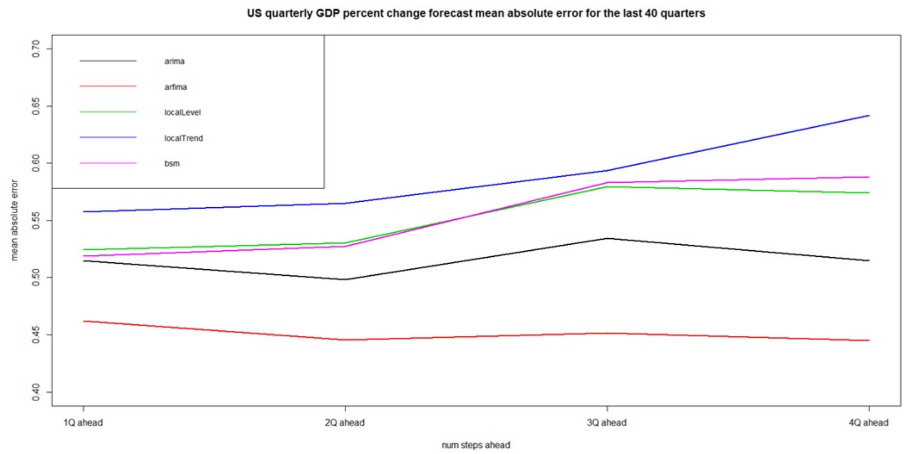

Figure 3: Comparison of mean absolute error of five models for multi-step forecasts

This process is repeated for all the countries for CPI and GDP forecasts. Aggregated results are presented in the next section.

4 Forecasting errors

As seen in Figure 3, forecasting performance of models are not significantly different. Although not shown here this was the case for other countries and CPI forecasts as well. Overall local trend model tends to have the largest errors whereas ARIMA seems to be the overall best model. Since errors are similar, I average the errors one more time. This time with respect to different models. The purpose here is to observe how forecasting errors grow as the forecast horizon increases independent of the model used.

4.1 CPI multi-step forecast errors

As expected, forecast errors grow as the forecasting horizon increases. As seen in Figure 4 below, 4 quarter ahead forecast errors approximately 3 times as large as one quarter ahead errors. Assuming that most central banks try to keep the CPI around 2 percent, the error for 4 quarters ahead is quite large.

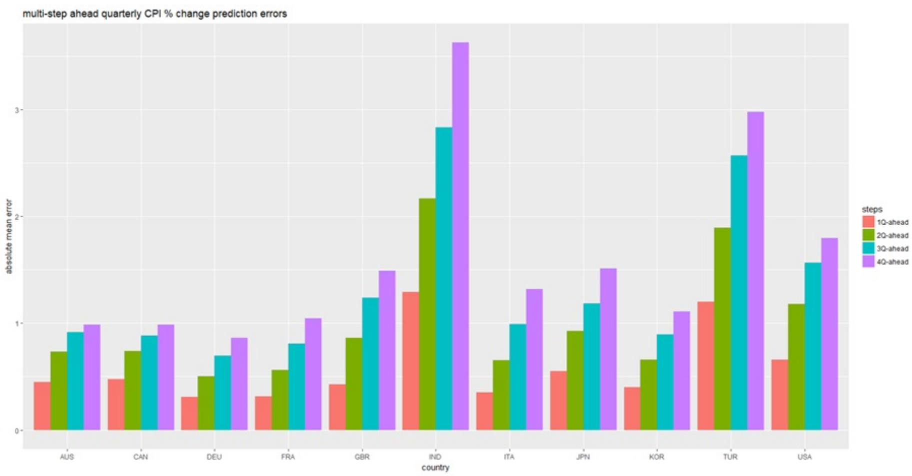

Figure 4: Multi-country and multi-step CPI forecast errors

4.2 GDP multi-step forecast errors

QoQ% GDP forecasts show a different pattern. Errors do not increase as the forecast horizon increases. However, this doesn't mean that GDP is easily predictable. It means that the errors are large overall even for one quarter ahead forecasts.

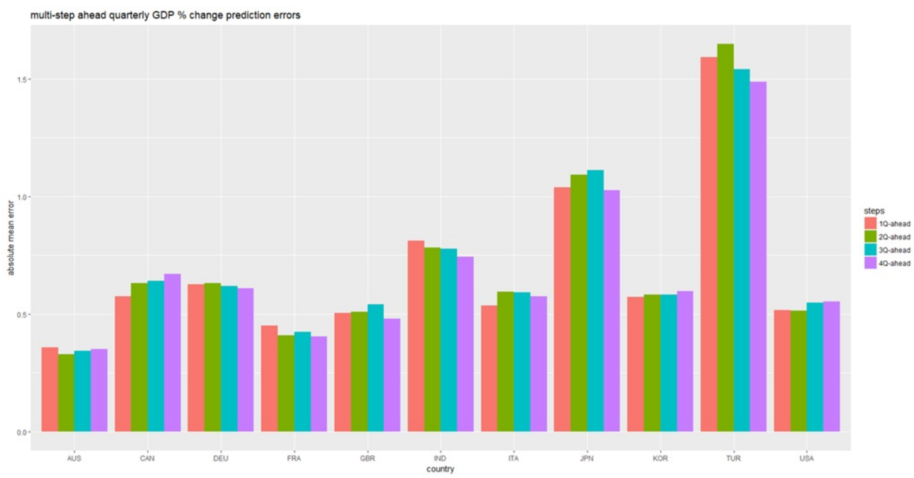

Figure 5: Multi-country and multi-step GDP forecast errors

5 Results & Conclusion

Finally, I forecast QoQ % change for CPI and GDP with the lowest error model for the **next year**. Then, I calculate the percent change over 4 quarters of QoQ % changes. The purpose here is to identify the countries with accelerating economic growth and inflation growth for the next year (as of 5/13/2018). Below is the visual summary of this process:

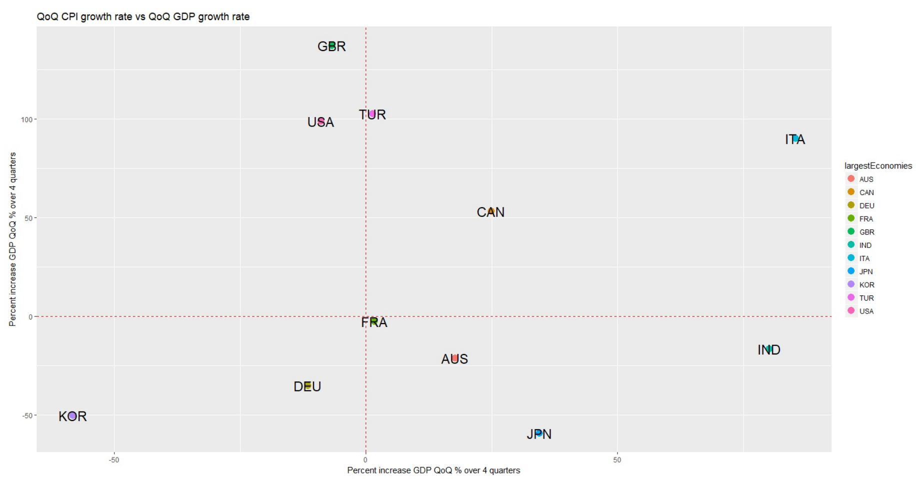

Figure 6: GDP growth rate vs CPI growth rate

According to these results Italy and Canada are expected to experience accelerating economic growth and rising inflation.

Following the discussion in the introduction, which is based on Ray Dalio's approach, these are ideal countries for equity investments for the upcoming year. Some argue that such economic environments are especially good for value investing approach.

References

Harvey, Andrew C, and Simon Peters. "Estimation procedures for structural time series models". *Journal of Forecasting* 9, number 2 (1990): 89–108.

Petris, Giovanni, Sonia Petrone, and Patrizia Campagnoli. "Dynamic linear models". In *Dynamic Linear Models with R*, 31–84. Springer, 2009.

Tsay, Ruey S. *Analysis of financial time series.* Volume 543. John Wiley & Sons, 2005.